Reviewer Comment Response

Dear Dr Karlsson, Elise Kazmierczak

We thank the reviewers for their constructive comments and the improvements that they will bring to the manuscript.

In what follows, the reviewer comments are in black, our responses to the reviewer comments are in blue, and suggested edits to the manuscript are italicised. The line references are to the revised manuscript unless otherwise specified.

In addition to the changes suggested by the reviewers, we have also condensed the appendices for improved readability, by merging all appendices that relate to sensitivity analyses into subsections of Appendix A. Appendix B gives more details on the inversion procedure, and Appendix C gives more details on the new, proposed empirical parameterisation for the effective pressure.

Best regards,
Koi McArthur and co-authors.

1 Author Comments

General comments

In this study, the authors have investigated the coupled interactions between the subglacial hydrological system and the ice sheet through the basal friction coefficient – an important and challenging tuning parameter used in friction laws – and its dependence on the form of the effective pressure. They also proposed a new empirical formulation of the effective pressure. Moreover, they highlighted the importance of subglacial processes on ice-sheet dynamics and conclude the need for geophysical observations of these processes would improve models.

This paper matches the quality criteria required by The Cryosphere. I have only a few comments and recommendations to improve the quality and scientific rigour of the manuscript as well as its understanding for readers that are less familiar with the subject and the tools used.

Thank you for your comments on the paper, which we have addressed below.
My first general comment concerns the choice of the Budd sliding law. Knowing that the value of $m$ has a major influence on ice dynamics, why hasn’t the Budd sliding law with the exponent $m = 3$, which is the most commonly used value instead of $m=1$? Wouldn’t it make it easier to compare to the Schoof sliding law?

Note: we have chosen to use the same convention as Brondex et al. (2019) (their equation 6), such that $m = 1/n$ where $n$ is the exponent in Glen’s flow law, so that the $m$ value can be more easily compared to that of the Schoof friction law. Hence, here $m = 1/3$ corresponds to the value of 3 which the reviewer suggests.

Thanks for this comment. We chose the value $m = 1$ because it has been used in many previous modeling studies (e.g. Åkesson et al., 2021; Åkesson et al., 2022; Yu et al., 2018; Choi et al., 2021; Baldacchino et al., 2022) that use ISSM as well. Using $m = 1/3$ with the Schoof law would lead to the same issue as the $m = 1$ case, of not having the same units of the basal friction coefficient. However, as the reviewer points out, the $m = 1/3$ case will lead to a similar dependence on the basal sliding speed as the Schoof friction law.

To address this, we ran an additional simulation using the Budd friction law with $m = 1/3$. We found strong negative correlations between the Budd basal friction coefficient and the effective pressure (-0.466 for $N_G$ and -0.592 for $N_O$) compared to the correlations when $m = 1$ is used (-0.304 for $N_G$ and -0.0260 for $N_O$). When $m = 1/3$ is used, the Budd basal friction coefficient counteracts the effects of the effective pressure more than in the $m = 1$ case, with higher values in areas of low effective pressure and lower values in areas of high effective pressure. This is particularly noticeable in the region of low/negative effective pressure centered around 2200 km northing and -400 km easting, where a large area of high basal friction coefficient develops for $m = 1/3$. The solution to this would be to raise the cap on effective pressure above 1% of ice overburden pressure, but as we have already shown in Appendix A3 this would mean replacing a significant portion of the GlaDS effective pressure data with an effective pressure that is linearly proportional to the overburden pressure. We have prepared a description of this analysis for a new Appendix in the updated manuscript. The text from this appendix is as follows:

Here, we compare the impact of using a nonlinear exponent $m = 1/3$ in the Budd friction law (Eq. 6; as per Brondex et al., 2017,0; Kazmierczak et al., 2022) to our results with the linear exponent $m = 1$ (as per Åkesson et al., 2021; Åkesson et al., 2022; Yu et al., 2018; Choi et al., 2021; Baldacchino et al., 2022). We follow the same setup outlined in the main
text and in Appendix B, capping the effective pressure at 1% of ice overburden pressure. The L-curve analysis suggests a Tikhonov regularization value of 0.1 is optimal.

The \( m = 1/3 \) case had much smaller variance in the normalized basal friction coefficient compared to the \( m = 1 \) case (Fig. A3). That is, using \( N_G \), the variance of the normalized basal friction coefficient was 0.232 for \( m = 1/3 \) and 0.385 for \( m = 1 \); using \( N_O \), the variance was 0.532 for \( m = 1/3 \) and 0.628 for \( m = 1 \).

Despite the smaller normalized variance in the basal friction coefficients for the \( m = 1/3 \) case, there was a stronger negative correlation between the basal friction coefficient and the effective pressure in areas where ice surface speeds are greater than 10 m a\(^{-1}\) (Fig. A4). Using \( N_G \), the correlation was \(-0.402\) for \( m = 1/3 \) and \(-0.304\) for \( m = 1 \); using \( N_O \), the correlation was \(-0.528\) for \( m = 1/3 \) and \(-0.0260\) for \( m = 1 \) (Fig. A4). This strong negative correlation for the \( m = 1/3 \) case indicates that the basal friction coefficient counteracts the effects of the effective pressure, with high values in regions of low effective pressure and low values in regions of high effective pressure. This suggests that the dependency of the basal shear stress on the effective pressure is too strong when \( m = 1/3 \). This behaviour is particularly clear in the large area of high basal friction coefficient centered around 2200 km easting and -400 km northing in Fig. A3c, which corresponds to an area of low effective pressure (Fig. 2b). Here, the use of the effective pressure cap limits runaway values of the basal friction coefficient; increasing the cap to reduce this area of high basal friction coefficient would mean using less of the GlaDS data. For the \( N_O \) run, the large increase in effective pressure upstream resulted in a strong decrease in the basal friction coefficient upstream (Fig. A3d).

My second general comment concerns the calculation time required for a more complex hydrological model. At what time and space scale does a more complex model (such as GlaDS) make a significant and crucial difference in glacial dynamics and does it compensate for the additional time of calculations used?

The space and time scale depends on the types of questions that are being answered. For short time scales (weeks to months), hydrology is likely to be fairly static. For very long time scales (multiple centuries) the hydrology will change but it will take a lot more computational resources to run a fully coupled ice sheet-subglacial hydrology model on these timescales, so if alterations in the the basal boundary conditions are not important to the ice dynamics modeling, then coupled variable hydrology will not be as important.

The area of interest is also an important factor when considering what spatial and temporal
scales are important for ice-hydrology interactions. For example, the dynamics of ice streams in the Siple Coast region are strongly dependent on ice-hydrology interactions, so coupling on shorter timescales (e.g. yearly to centennial timescales) would be important here (see e.g. Bougamont et al., 2015). Ice-hydrology coupling may also be essential for modelling the onset and location of ice streams (Kyrke-Smith et al., 2015). A recent study in Greenland showed that the impact of geothermal heat flow on basal ice temperatures is elevated when ice sheet and subglacial hydrology models are coupled (Smith-Johnsen et al., 2020), although this kind of analysis has not been tested more broadly across Antarctica or Greenland. In Antarctica, ice-hydrology coupled models may be essential in predicting the magnitude of basal melt and where it is injected into ice shelf cavities to accurately predict the impact on ice shelf melt; however, such analysis has not yet been conducted.

The end goal with ice-hydrology coupling is to have either an accurate parameterization of hydrology for ice dynamics modeling in situations that require too much computational time for hydrology modeling, or a coupled setup that can run efficiently. The former is what we’re aiming for in this manuscript and the latter is something we’re working towards. Because the latter hasn’t been achieved yet (i.e. coupling of hydrology and dynamics in Antarctic) it’s not yet possible to accurately define what the appropriate temporal and spatial scales would be, both for the Denman-Scott catchment, and more broadly across Antarctica.

It’s likely that we have not yet answered well enough the question of when we need ice-hydrology coupling and what impact it makes, which is a strong motivator for further studies of its importance.

My third general comment concerns the figures. If a standardisation does not allow the scales to be the same, it must be stipulated in the text for all figures concerned so that there is no misunderstanding. Also, note that this difference in scale does not allow the same analysis and comparison resolution. Also, it is better to have complete captions (variable symbol-units) and the same than the legend (the text written next to the colorbar). It also is more readable if both limits of the scale are written.

To address this, we have altered sections of our analysis to consider normalised deviations from the mean of the basal friction coefficient across the grounded domain so that the same analysis can be performed for the Budd and Schoof friction laws. This is particularly the case when variances are considered, e.g. line 148 of the original manuscript. We have normalized the basal friction coefficients by their respective means in most updated figures. However,
we keep some of the original – non-normalized – basal friction coefficients where we need to make comparisons, e.g. in comparing $N_G$ with $N_O$ in the Budd friction law where there is a substantial change in mean between the two. Limits of the colorbar have been added for all figures, and where they differ between subplots a note has been added to the caption to mention this. Complete captions have been added for all figures where applicable, though we do not add complete captions to the colorbar labels for figures where it will lead to awkward spacing.

My final general comment concerns how the types of effective pressures are expressed in the text. I think it would be better to define in an equation $N_O$ from the beginning and not to repeat it again in the text. Why choose No and not choose the ‘limited version’ by Brondex et al, 2017 from the start? I don’t quite understand how considering the two brings a lot of added value (especially by comparing figures 6c and 6d). If you decide to keep both, then set a symbol for $N_B$ Brondex to avoid repetitions in the text. Finally, when we see the large difference in $N$ values between $N_O$ and $N_G$ in Figure 3, it would be good to explain how a single variable can be considered with such different values.

We agree that it would be best to define $N_O$ and $N_G$ once and then reference them as such throughout the text. We chose to look at $N_O$ as opposed to the version from Brondex et al. (2017) because $N_O$ has been used in many modeling studies and we wanted to determine whether this was a reasonable value to use for the effective pressure. We will keep the Brondex et al. (2017) analysis and refer to the Brondex et al. (2017) effective pressure as $N_B$ throughout the text.

Fig. 6c and 6d look the same because of the chosen colormap scales. The upper bound of the Fig. 6c and 6d colormap is where water pressure is less than or equal to zero which holds true for the same areas of $N_O$ and the Brondex et al. (2017) effective pressure. These scales have be changed in the updated figures so that the difference between $N_O$ and the Brondex et al. (2017) effective pressure can be seen.

$N_O$ and $N_G$ have such different values because $N_O$ is not actually an effective pressure. It is the overburden hydraulic potential, but is often called the effective pressure, which is not good practice and something we wish to highlight with this manuscript. $N_O$ and $N_G$ are not the same quantity and that is why they differ so markedly. To clarify in the updated manuscript, at line 240-242 we modify:

However, the definition for $N_O$ used here will produce high effective pressures in regions of
thick ice grounded above sea level...

to

However, the definition for $N_0$ here is in fact the overburden hydraulic potential, not an effective pressure, and will produce high effective pressures (i.e. low basal water pressures) in regions of thick ice grounded above sea level...

I therefore propose some small changes in the text or the figure calls to improve clarity and understanding. The main thing is the insistence on the terms ‘‘basal’’ and ‘‘subglacial’’ which for me are important to keep throughout the text. Finally, I also propose to elaborate on more technical details with respect to the tools used and the choice of parameters.

We have added the words subglacial and basal throughout the manuscript where applicable.

Specific comments

L37-L117. Please to specify whether these negative effective pressures are stable, at the steady state, seasonally dependent…Please add information on the stability of this case and also whether it varies over time (depending on the seasons or the tides). It could be interesting to add a sentence mentioning that the presence of zones with negative effective pressure may be persistent and does not lead to instability.

The model reaches near steady state; there are some small temporal changes in water depth in deep pockets on the order of mm per day. There is no seasonality, as there is no surface water input to the subglacial hydrology system and tides weren’t considered in the model setup. Over longer timescales there may be changes due to subglacial lake drainage and/or changes in ice geometry, but these were not apparent in our models. To clarify, on lines 90-94 we have changed: We assume a temperate bed throughout, although regions with zero water input (in the interior or southernmost-regions of the domain) are essentially frozen to the bed. The model is run for 10,000 days, providing outputs including channel size and discharge, distributed system discharge, water depth, and effective pressure.

to

We assume a temperate bed throughout, although regions with zero water input (in the interior or southernmost-regions of the domain) are not an active part of the hydrological system.
The model is run for 10,000 days to near steady state (there are changes in the water sheet thickness in deep pockets on the order of mm day$^{-1}$), providing outputs including channel size and discharge, distributed system discharge, water depth, and effective pressure.

To address low and negative effective pressures persisting at steady state and not causing model instability we add on lines 182-183: *These low and negative effective pressures persist at near steady state and do not cause instability in the GlaDS model.*

L64. Stipulate if the model reaches the steady state after 10,000 days.

See response to previous comment.

L78-L346 : If you don’t add information about the ice rigidity calculation, please add a reference.

The order of the rigidity inversion and the basal friction coefficient inversions has been added in Section 2.2.1 as well as a reference to Appendix B has been added to line 114. We will add to section 2.2.1 lines 139-147, the following text:

*We perform the following inversion procedure. First, we invert for the ice rigidity over the floating portion of the domain. Next, we invert for the Budd basal friction coefficient over the grounded portion of the domain, using an ice rigidity on grounded ice specified by the Paterson function from Cuffey and surface temperatures from RACMOv2.3 (van Wessem et al., 2018). After the Budd inversion we invert for the ice rigidity over the entire domain. We next use the basal friction coefficient estimated using the Budd friction law to compute an initial estimate of the basal friction coefficient for the Schoof friction law. We perform inversions for the basal friction coefficients of the Budd and Schoof friction laws with the ice rigidity from the inversion prior, these are the main simulations discussed in the text that follows (it is worth noting that the Budd friction coefficient converges to the result of the initial Budd friction coefficient inversion). We perform a final rigidity inversion over the entire domain. The cost functions to be minimized for each inversion are described in detail in Appendix B.*

Additionally, the cost function to be minimized during the rigidity inversions has been added to appendix B with an explanation of the various component cost functions (absolute velocity misfit, logarithmic velocity misfit, and Tikhonov regularization).
L140. As you explain that low effective pressures were associated with faster flow, give a brief explanation of this case.

The sentence in the original manuscript is not quite correct; we have changed it as follows and leave analysis of the role of hydrology in ice dynamics to the discussion section:

*We find a positive correlation ($r^2 = 0.291$) between $N_G$ and the basal friction coefficient (Fig. 5a) where ice surface speeds are $\geq 10 \text{ m a}^{-1}$, in the region where ice is more dynamic closer to the grounding line.* From line 193-195. The change in the correlation coefficient is from using $C_{\text{max}} = 0.7$ instead of $C_{\text{max}} = 0.8$.


Budd and Jensen, 1987 has been removed; Huybrechts, 1990 added.

If you use the references, Johnson and Fastook, 2002 and Lebrocq et al., 2009 define the hydraulic potential = 0.

We have removed Johnson and Fastook, 2002 and Lebrocq et al., 2009 which we now see did not actually use $N_O$ and we have added Åkesson et al. (2021); Åkesson et al. (2022); Yu et al. (2018) which do use $N_O$.

L180. Accurately calculating the effective pressure is important improve ice-sheet models. However, effective pressure parametrizations used in Brondex et al., 2017 and Kazmierczak et al., 2022 are simplified for computational purposes and numerical stability, especially when the study is either focused on grounding lines or experiments are done on a continental scale. Please, add the concept of model complexity for the subject under study.

We have changed (lines 244-247)

*Previous studies have investigated alternative parameterizations for the effective pressure in the absence of a coupled ice sheet-subglacial hydrology model.*
Previous studies have investigated alternative parameterizations for the effective pressure due to the computational cost or numerical instabilities associated with coupling of an ice-sheet model to a complex subglacial hydrology model. Full coupling between these models is a recent development in the field (Cook et al., 2022).

L180 : Previous studies have investigated alternative parameterizations for the effective pressure in the absence of a coupled ice sheet-subglacial hydrology model. In Kazmierczak et al., 2022., the ice-sheet model is coupled to the simple subglacial water routing from Lebrocq et al., 2009 by the subglacial water depth and by the flux. This sentence should be modified by [...] complex subglacial hydrology model due to the computational time.

This has been changed, see response to above comment.

L196 : Add a reference on the alpine-like hydrology system.

Added reference Iken and Bindschadler (1986).

L220 : Since equation 1 does not include $m$, this sentence is not clear. Then perhaps add the $m$ in equation 1 and mention that $m = 1$.

Equation 1 has been changed to include $m$ and on lines 120-122 we change:

where $\tau_b$ (Pa) is the basal shear stress, $\alpha (s^{1/2} m^{-1/2})$ is the friction coefficient, $N$ (Pa) is the effective pressure and $u_b$ (m a$^{-1}$) is the basal sliding speed

to

where $\tau_b$ (Pa) is the basal shear stress, $\alpha (s^{1/2} m^{-1/2})$ is the friction coefficient, $N$ (Pa) is the effective pressure, $u_b$ (m a$^{-1}$) is the basal sliding speed, and $m = 1$ is a power law exponent taken to be linear as in Åkesson et al. (2021); Åkesson et al. (2022); Baldacchino et al. (2022); Choi et al. (2021); Yu et al. (2018)
L285-286: Specify with which formulation of the effective pressure this conclusion is made.

This is true for both $N_G$ and $N_O$. We have changed on lines 369-370:

That is, using the Schoof friction law, regions of lower effective pressures tend to also have lower simulated basal friction and faster flow – evidence for the controlling role of the hydrological system.

to

That is, using the Schoof friction law, regions of lower effective pressures (both $N_G$ and $N_O$) tend to also have lower basal friction coefficients – evidence for the controlling role of the hydrological system.

L353: Add a reference for the Paterson function.

In Section 2.2.1 we now include a reference to the Paterson function, as well as an additional paragraph, as per our response to the reviewer’s specific comment about lines 78 and 346.

L362 (eq. C2): Why is $C_{\text{max}}$ (0.8) different to the value used in Brondex et al., 2017 (0.5)?

This is a good question, and there is no clear justification for one value of $C_{\text{max}}$ over another, including the value used in (Brondex et al., 2017). Hence, we test the sensitivity of our inversion results to various values of $C_{\text{max}}$ (we will add this sensitivity analysis as a subsection in an appendix of the revised manuscript). We find that a value of $C_{\text{max}} = 0.7$ yields a basal friction coefficient with the smallest variance and we update the manuscript to use this new value. We will include the following text (lines 463-488; new appendix A5):

Iken’s bound, $C_{\text{max}}$, mathematically describes the idea that the bed can support a maximum stress (Iken, 1981; Schoof, 2005; Gagliardini et al., 2007). In the Schoof friction law (Eq. 7), $\tau_b$ cannot exceed $C_{\text{max}} N$, where $C_{\text{max}}$ represents rheological properties of the till (Brondex et al., 2019) and ranges between 0.17 and 0.84 (Cuffey and Paterson, 2010). To determine an appropriate value of $C_{\text{max}}$ we test the effect of using $C_{\text{max}} = 0.5, 0.6, 0.7,$ and $0.8$ on the Schoof basal friction coefficient using $N_G$ and $N_O$.

We arrive at the same qualitative conclusions as in the main text for all four values of $C_{\text{max}}$. In all cases, the variance of the normalized basal friction coefficient is smaller for $N_G$ than...
for $N_O$, and it is smaller than the variance of the normalized Budd basal friction coefficient. Although the results for $C_{max} = 0.5, 0.6$, and $0.7$ are similar, there is a comparatively large decrease in the mean of the basal friction coefficient and increase in the variance of the normalized basal friction coefficient between $C_{max} = 0.7$ and $C_{max} = 0.8$ for both $N_G$ and $N_O$. For lower values of $C_{max}$, a region of higher basal friction coefficient centered around 2200 km easting and -400 km northing (Fig. A5a) develops in the $N_G$ simulations. Solving Eq. (7) for $C$ yields

$$C = \frac{\left(\frac{\tau_b}{|u_b|^m u_b}\right)^{m/2}}{\left(1 - \left(\frac{\tau_b}{C_{max} N}\right)^{1/m}\right)^{m/2}},$$

(A1)

where we see that in the limit where $\tau_b \to C_{max} N$ then $C \to \infty$. The region of high basal friction coefficient for lower $C_{max}$ also has low effective pressure (Fig. 2a), resulting from $\tau_b$ approaching $C_{max} N$ for larger values of $N$ and the basal friction coefficient compensating this. To prevent potentially infinitely increasing values of the basal friction coefficient it is possible to increase the cap on the effective pressure, but again, this would reduce the area over which the GlaDS data is used and the effect of using modeled hydrology will be less impactful.

The Schoof friction law is a regularized Coulomb friction law which tends towards a Weertman sliding regime when $\tau_b << C_{max} N$ and a Coulomb sliding regime when $\tau_b >> C_{max} N$. Hence, the value of $C_{max}$ will have an effect on what physical processes are being represented in the Schoof friction law. Figure F2 shows $\tau_d/(C_{max} N)$ for various values of $C_{max}$, where values close to zero correspond to a Weertman sliding regime and values close to one correspond to a Coulomb sliding regime. The choice of $C_{max}$ appears to have little effect on where each of the Weertman and Coulomb sliding regimes occur, with distinct locations between the two for all values of $C_{max}$. This suggests that the choice of $C_{max}$ will not have a significant impact on which physical processes are being represented throughout the domain and does not justify the use of one value of $C_{max}$ over another.

Figure 1. First time you mention Denman-Scott catchment. It is better to mention it in the text beforehand.

Line 18-19 changed to: In the East Antarctic, Denman Glacier of the Denman-Scott catchment (Fig. 1) has seen some of the fastest grounding line retreat of the last 20 years.
Figure 4. Could you explain how the very different effective pressures between $N_G$ and $N_O$ do not significantly impact the basal friction coefficient in the Schoof basal sliding law but impact significantly the basal friction coefficient in the Budd basal sliding law.

This is because the domain is largely in a Weertman sliding regime in the Schoof friction law where the effective pressure doesn’t play a significant role in the value of the basal friction coefficient. Using the Budd friction law, the effective pressure plays a large role in the basal friction coefficient throughout the entire domain so the differences between $N_G$ and $N_O$ propagate into the basal friction coefficient. To better explain this we have added the following text into the discussion of the Schoof compared to Budd friction coefficients (lines 206-212):

*Unlike the Schoof friction law, the choice of $N_O$ or $N_G$ has a significant impact on the distribution of the basal friction coefficient (Fig. C4) in the Budd friction law. This is because the upstream portions of the catchment fall into a Weertman sliding regime in the Schoof friction law where $\tau_b << C_{\text{max}}N$ (Fig. F2) and $C^2 \approx \tau_b/u_m^m$. Here, the choice of effective pressure will have minimal effect on the Schoof friction coefficient. In the Budd friction law $\alpha^2 = \tau_b/(Nu_b)$, meaning that the effective pressure plays an important role in determining the basal friction coefficient throughout the entire catchment, which propagates the large discrepancy between $N_O$ and $N_G$ to the basal friction coefficients obtained from using these various effective pressures.*

Table 1: I never used GlaDs but as some parameters are different than in Werder et al., 2013, why? Specificity from Antarctica or this specific catchment? How did you obtain these parameters? The ice flow constant is the same for cavities and channel?

The parameter values were obtained through sensitivity testing in the Antarctic tested against geophysical data Dow et al. (2020), and are generally assumed to be applicable for Antarctic glaciers. We have now expanded on the GlaDS methods which justifies the choice of parameters reported in Table 1. We have added the following text (lines 72-76):

*As discussed in Dow (2023), when the system is overconstricted, the pressures are unrealistically high and the model ceases to converge. When the system is underconstricted, the pressures are below overburden for much of the domain. While there is some variation within the range of acceptable pressures, the output we present is the median and therefore is the most appropriate for representing the hydrology pressure in ice sheet dynamics equations. Future*
work with full coupling of hydrology and ice dynamics can explore sensitivity to different distributed system inputs. Channel conductivity is, similarly, a median value applied in GlaDS in Antarctica (Dow et al., 2020).

In GlaDS application to the Antarctic, the ice flow constant is the same for cavities and channels because they are of a similar size.

Technical corrections

Abstract

L10 Budd friction law

Changed

L10-14 Schoof friction law

Changed

Introduction

L19. Mention the dataset of Adusumilli et al., 2020 in the references. Because the data you mention are not in the paper itself.

Changed

L20. Mention the dataset of Morlighem et al., 2020 in the references.
Center. https://doi.org/10.5067/E1QL9HFQ7A8M.

Changed

L20. Denman Glacier

Changed

L21. << containing 1.5 m of sea level equivalent.>> → source? (rewrite Brancato et al., 2020)

We have changed 1.5 m of sea level equivalent to 1.5 m of sea level equivalent (Brancato et al., 2020)

L26-27. Replace << Coulomb laws >> by regularized Coulomb friction laws. Because in a Coulomb friction law s.s., the basal shear stress is independent of the basal sliding velocity.

Changed

L27. of the of the Budd

Changed

L27 & 29. On the basal sliding velocity

Changed

L28. subglacial water pressure
L29. Provide examples of the << other quantities >> (i.e. rugosity of the bed…) L29. Why unexpected?

By unexpected, we mean that the basal shear stress has a different functional dependency on basal sliding velocity and effective pressure than the basal friction law proposes. To clarify this we have changed (on lines 31-34):

However, the dependency of the basal shear stress on other quantities, or unexpected dependency on sliding velocity and effective pressure, is implicitly captured in the friction coefficient.

to

However, if the basal shear stress does not actually have the functional dependence on the basal sliding velocity and effective pressure proposed in the basal friction law, or if it has a functional dependence on other properties of the bed such as roughness, substrate, or temperature, then this will be implicitly captured in the basal friction coefficient.

L30. basal friction coefficient (2)

Changed

L31. basal sliding velocity

Changed

L32. basal friction coefficient

Changed

L38. Remove (i.e. the ice overburden pressure) or modify it by (i.e. when subglacial water pressure exceeds the ice overburden pressure)
Changed to (i.e. when subglacial water pressure exceeds the ice overburden pressure).

L41. Subglacial water pressure

Changed

Methods.

2.1 GlaDS Setup

L58. Source for the surface velocities? MEaSUREs v2?

These are not actually surface velocities. Instead they are basal sliding velocities from the ISMIP6 experiments (see Seroussi et al., 2019 reference on line 83). The word surface has been changed to basal sliding in line 80 and we have expanded on the description of how these velocities were obtained in the methods (lines 82-85).

Basal water and sliding velocity inputs are taken from the JPL-ISSM ISMIP6 Antarctic control run final time step (Seroussi et al., 2019) which was a thermal steady state simulation using the enthalpy formulation implemented in ISSM (Seroussi et al., 2013) and the Blatter-Pattyn (BP) approximation to the full Stokes equations.

L59. Same comments in L20 about the Morlighem et al., 2020 source

Changed

2.2 ISSM Setup

Reference added

L72. Same remark in Figure 1.

Changed: Denman-Scott catchment now appears before Figure 1 on line 19.

L75. basal friction

Changed

L76. basal friction law

Changed

L77. Same comment in L59.

Changed

2.2.1 Solving for basal friction coefficients

L85. Why eq 2 and eq C2 are not the same? If it’s a mistake, the \((\cdot)^m\) is missing in the denominator. Is it possible to isolate the coefficients of friction in order to make them more visible?

Thanks for catching this. Eq 2 is correct, Eq C2 has been changed to match. The basal friction coefficients have been bolded in the equations and we have clarified that the equations are scalar.

L87. Source of the Iken’s bound missing and please refer to Appendix C.
Added reference (Appendix A5; Iken, 1981), the appendices have been rearranged and A5 addresses the impact of Iken’s bound on the inversions.

L90. **Appendix C, Table C1**

Changed to **Appendix B, Table B1**.

L98. For the No equation complete that B takes negative values below sea level and defines the variables.

We have changed the paragraph from lines 148-154:

We compare the difference in the friction coefficients when we use two different prescriptions for the effective pressure: (1) an effective pressure given by assuming water pressure equals the ice overburden pressure plus the gravitational potential energy of the water $N = \rho_i g H + \rho_w g B$, which we refer to as $N_O$; and (2) the effective pressure taken directly from the GlaDS simulations, which we refer to as $N_G$. We cap the effective pressure at 1% of ice overburden pressure for Budd runs and 0.4% of ice overburden pressure for Schoof runs, due to numerical artefacting that arises for values smaller than this. The impact of these choice of caps is discussed in Appendix D.

to:

We compare the difference in the friction coefficients when we use two different prescriptions for the effective pressure. (1) An effective pressure given by assuming water pressure equals the ice overburden pressure plus the gravitational potential energy of the water $N_O = \rho_i g H + \rho_w g B$. Here $\rho_i$ is the density of ice $\rho_w$, is the density of water, $g$ is the absolute value of the gravitational acceleration, $H$ is the ice thickness, and $B$ is the bed elevation which takes negative values below sea level. (2) The effective pressure taken directly from the GlaDS simulations, which we refer to as $N_G$. We cap the effective pressure at 1% of ice overburden pressure for Budd runs and 0.4% of ice overburden pressure for Schoof runs, due to numerical artefacting that arises for values smaller than this. The impact of these choice of caps is discussed in Appendix A3.

Results 3.1 Subglacial hydrology
L105 and L106. Even if it’s clearly written in L103, mention that the data indicated (length of the channel and the flow) come from the GlaDs modelling.

We have changed on lines 161-162: **Major subglacial hydrology channels form in the Denman-Scott catchment, with significant discharge through both the Denman (Fig. 2a (i)) and Scott Glaciers (Fig. 2a (ii)).**

The GlaDS modeling indicates that major subglacial hydrology channels form in the Denman-Scott catchment as seen in Fig. 2a, with significant discharge through both the Denman (Fig. 2a (i)) and Scott Glaciers (Fig. 2a (ii)).

L108. If the two branches of 80 and 52 km of the Denman channel are figure 2a (iii) and (iv), mention the figure in the text.

Changed.

L111 & Fig. 2d(v). I know that the choice of the limits of the legend is there to allow a better reading, but I find it strange to mention 25 m of thickness whereas the legend stops at 10 m.

The upper limit of Fig. 2d has been increased to 25 m.

L112. We cannot really see on the figure that the strongest flux is toward Denman, so do not mention the figure but rather the data.

We have changed (lines 166-171): **There is substantial water amalgamation with a maximum depth of 25 m in a basal depression (Fig. 2d (v)) that feeds the channels of both the Denman and Scott Glaciers, although with the strongest flux towards Denman (Fig. 2a). The bed topography of this basin feature lies at 1900 m below sea level (Fig. 1a), and subglacial water flows upslope by approximately 1200 m to drain downstream. A second ‘lake-like’ feature feeds the northern branch of the Denman channel and reaches a water depth of ∼8 m (Fig. 2d (vi)).**

There is substantial water convergence with a maximum depth of 25 m in a basal depression (Fig. 2d (v)) that feeds the channels of both the Denman and Scott Glaciers, although with
the strongest flux towards the eastern branch of the Denman channel (Fig. 2a (iii)). The bed topography of this basin feature lies at 1900 m below sea level (Fig. 1a), and subglacial water flows upslope by approximately 1200 m to drain downstream. A second ‘lake-like’ feature feeds the western branch of the Denman channel (Fig. 2a (iv)) and reaches a water depth of \( \sim 8 \) m (Fig. 2d 125 (vi)).

L116-117-118-119 subglacial water pressure

Changed

L121. Show these zones in the Fig. 2b.

These are already shown in Fig 2d. They have now been referenced in the text as follows on lines 176-178: Effective pressure in the GlaDS outputs is lowest in the basin feature (Fig. 2d (v)) and the lake-like feature (Fig. 2d (vi)), reaching -0.4 MPa in the former and -0.25 MPa in the latter (Fig. 2b)

3.2 Ice dynamics and inversion

L127. Schoof friction law (éq. 2)

Changed

L132. slow ice flow (maybe mention the Fig. 1b with the surface speed)

This sentence was a bit unclear so we have changed lines 189-192:

The friction coefficients are relatively lower in the Denman and Scott troughs, and alternating high and low “stripes” are evident in the region of slow flow (ice surface speeds < 50 m a\(^{-1}\)) to the west of the Denman Glacier and south of the Shackleton Ice Shelf (we note that this region was excluded from the GlaDS model; see discussion in Appendix A).

to

The basal friction coefficients are relatively lower in the Denman and Scott troughs, and alternating high and low “stripes” are evident in the region west of the Denman Glacier
and south of the Shackleton Ice Shelf (Fig. 1b (i)) where ice surface speeds are <50 m a\(^{-1}\) (Fig 1a). We note that this region was excluded from the GlaDS model (see discussion in Appendix A1).

L133. Locate the Shackleton Ice Shelf in one of the figures (e.g. Figure 1).

The Shackleton Ice Shelf was added into Fig. 1b, as per the response to the above comment.

L135. a space is missing between Fig. and 4c

Space added

L136. **basal** friction coefficient

Changed

L138. faster **ice** flow (ice surface speeds […]

The paragraph on lines 135-141 of the original manuscript was unclear and has been changed to:

*The Schoof basal friction coefficient estimated using \(N_G\) is smoother compared with that using \(N_O\) (Fig. 4a and Fig. 4d; Appendix B). Here, a smoother basal friction coefficient resulted in lower median differences and root mean square error (RMSE) and higher mean differences between the simulated and observed ice surface speeds for the \(N_G\) simulation over the \(N_O\) simulation (Table 2; Fig. 4c,f). We find a positive correlation between \(N_G\) and the basal friction coefficient (correlation coefficient \(r^2 = 0.291\), Fig. 5a) when considering areas where ice surface speeds are \(\geq 10\) m a\(^{-1}\), in the region where ice is more dynamic closer to the grounding line. Similarly, there is a positive correlation between \(N_O\) and the basal friction coefficient (\(r^2 = 0.281\), Fig. 5b).*

L139. slow **ice** flow (ice surface speeds […]


See response to line 138

L141. **basal** friction coefficient and please refer to (Fig. 5b).

See response to line 138

L133-138-140. The limits given to consider an ice flow faster or slower are not the same, why?

On line 133 of the original manuscript, we used ice surface speeds to describe a specific region of the domain but on lines 138 and 140 we were referring to ice surface speeds throughout the whole domain. The words *fast* and *slow* have been taken out of these sections to avoid confusion.

L143. **Glaciers**

We are referring to the troughs so the *s* should be at the end of *troughs* instead of *Glaciers*. We now have: *Denman and Scott Glacier troughs*.

Discussion

4.2 Effective pressure and basal sliding laws

L182. : the till parametrization used in Kazmierczak et al. 2022 is from Bueler and van Pelt, 2015

The reference has been changed

L200. : The relationship between low effective pressures and low basal friction did not hold for the Budd friction law, *(Fig 3 (a), Fig 4 (e))* despite a stronger negative correlation between the friction coefficient and surface speed for the Budd friction law compared to the Schoof **friction** law *(Fig 5(a),(c)).*
L214. when using a Weertman friction law, which not considered the strong dependence of $\tau_b$ on the effective pressure,

We have changed this to when using a Weertman friction law, which does not consider the strong dependence of $\tau_b$ on the effective pressure,

L215. Schoof friction law

L216-218-221. Regularized Coulomb friction laws

L229. This study by Kazmierczak et al. (2022) examined the impact of different representations of the effective pressure – approximated in turn by height above buoyancy (van der Veen, 1987; Huybrechts, 1990, Winkelmann et al., 2011, Martin et al, 2011), reduced by the subglacial water pressure (modified from Bueler and Brown, 2009) or sliding related to water flux (Goeller et al., 2013) from a simple subglacial water routing model (Lebrocq et al., 2009), and the effective pressure in a till (Bueler and Van Pelt, 2015) – on ice mass loss from Antarctica over the 21st Century.

L 254-255. $\gamma$ or $\delta$ ? use the same symbol
The symbol is now consistently \( \gamma \).

L260. : define Brondex et al., 2017 or write something like […] \( N_O \) on all the domain or, following the condition of Brondex et al., 2017, exclusively below sea level […] Fig. 6a, b, c, d

We have changed:

*and the Brondex et al. (2017) effective pressure*

to

*and the Brondex et al. (2017) prescribed effective pressure (\( N_B, \text{Section 4.2} \)).*

Note that the equations describing the Brondex et al. (2017) effective pressure now have \( N_B \) instead of \( N \).

L264. : define the << saturation term >>

We have changed:

*Fig. 6e shows the saturation term as well as the physically equivalent scatter from the GlaDS output data.*

to

*Fig. 6e shows the saturation term \( (g_x(H)) \) as well as the physically equivalent scatter from the GlaDS output data, \((1 - N_G/p_i - r_i)/(1 - r_i)\).*

\( g_x(H) \) is now defined in lines 327-332 which now reads: \( \text{we suggest a form of the water pressure proportional to the ice overburden pressure multiplied by a term } (r_i + (1-r_i)g_x(H)) \) where \( g_x(H) = H^x/(H^x + H^z) \) is a saturation term such that the water pressure reaches a maximum fraction of flotation in areas of high ice thickness. Here, \( \hat{H} \) is defined in Eq. (9) and \( x \) is defined in Eq. (10).

L267. : 77.0\% (use the same number of significant digits)

45.4 also has three significant figures so we keep 77.0.
L279. : basal friction laws

Changed

L280. : basal friction coefficient

Changed

L285. : Schoof friction law

Changed

L289. : geophysical observations

Changed

Appendix A

L304-308-313. : pressures (it is correct but keep pressure or pressures)

We have changed on line 394 effective pressures do not exist to the effective pressure does not exist.

L305. : domains

Changed

L306. : speeds
L307 + Table A1: $500 \geq v < 1000 \text{ m a}^{-1}$. The sign is not correct?

Yes this is not correct, thank you for catching this, the sign has been flipped.

L320-321: (e.g. Bueler and Brown, 2009; Bueler and Van Pelt, 2015, van der wel et al., 2013, Huybrechts 1990, Kazmierczak et al., 2022; Goeller et al., 2013; Le Brocq et al., 2009; van der Veen, 1987; Winkelmann et al., 2011)

Figure A1. (a)-(b) Effective pressures N (MPa) Note that the color bar scale is not the same. It is better if both limits of the scale are written.

Appendix B

L324-332-333-334-335-336: basal friction coefficient

And following the informations given to this link: https://nsidc.org/data/nsidc0484/versions/2 these references has to be included:


Changed

L336. : the prescribed effective pressure $B_1$? It is not clear... explain in the text (like in the Table B1. Caption) that the prescribed effective pressure is $N_O$.

This was in reference to Table B1, however, this table has been removed from the manuscript as the relevant data can be summed seen Fig. B1 and C4 of the original manuscript.

L337. : effective pressure calculation/representation

We have changed effective pressure used to form of the effective pressure.

Table B1. : basal friction - Schoof basal friction coefficient – Budd basal friction coefficient

See response to your comment on line 347.

Figure B1. Schoof basal friction coefficient

Changed

Appendix C

L344. : B is the bed elevation (m) taking negative value below sea level
This has been cut from the appendices and added to the main text in Section 2.2.1 where it is specified that $B$ takes negative values below sea level.

L348. : same comments in L327

Changed

Figure C1. Budd basal friction coefficient – ‘.’ Missing at the end of the sentence.

Changed

Figure C2. Schoof basal friction coefficient– ‘.’ Missing at the end of the sentence.

Changed

Figure C3. On the figure: Schoof (x2) + specify with No and NG– In the caption: Basal friction coefficients - Schoof basal friction coefficient (x2) Budd basal friction coefficient (x2). Add the units and that the colorbar scale is not the same. It is better if both limits of the scale are written.

The caption now reads: Basal Friction coefficients from inversion (note that the units and the colorbar are not the same in each subplot). (a) Schoof basal friction coefficient ($C$, kg$^{1/2}$ m$^{-2/3}$ s$^{-5/6}$) from $N_G$ run; (b) Schoof basal friction coefficient ($C$, kg$^{1/2}$ m$^{-2/3}$ s$^{-5/6}$) from $N_O$ run; (c) Budd basal friction coefficient ($\alpha$, s$^{1/2}$ m$^{-1/2}$) from $N_G$ run; and (d) Budd basal friction coefficient ($\alpha$, s$^{1/2}$ m$^{-1/2}$) from $N_O$ run.

Figure C4. basal friction coefficients – prescribed → No?

Changed

Figure C5. It is better if both limits of the scale are written. – Caption: ‘.’ Missing at the
end of the sentence.

The limits have been added and the period has been added as well.

Table C1. Budd basal friction coefficient – Schoof basal friction coefficient

Changed

L352. : Budd basal friction coefficient

Changed and added to section 2.2.1 of the main manuscript.

L358. : basal friction coefficient

Changed

L361. : same comment L85

Changed

L362. : Schoof basal friction coefficient – Add a reference or an explanation for the Iken’s bound – basal friction coefficient

This is now in the main text, we have added the reference (Iken, 1981).

L363. : basal friction coefficient

Changed, this is now in the main text.
L364. : **basal** friction coefficient

Changed, this is now in the main text.

Appendix D

L374-376. **ice** overburden **pressure**

Changed

Figure D1. It is better if both limits of the scale are written.

We have removed this figure from the manuscript, we believe that the text in this section is sufficient to get the point across.

Add the units and that the colorbar scale is not the same. It is better if the caption and the legend are the same (with symbol and units) → Effective pressure (N) (MPa)/ friction coefficient for the Schoof friction law C [...] / surface speed (us) (m a-1)

See our response to the above comment.

Figure D2. It is better if both limits of the scale are written.

We have removed this figure from the manuscript and summarized the results in the appendix text, as follows on lines 437-440:

*For a cap of 0.4%, approximately 2% of the domain has an effective pressure that is linearly proportional to the ice overburden pressure; that is, 98% of the effective pressure in the ISSM simulation is derived directly from the GlaDS simulated effective pressure. Increasing the cap to 1% — which is used in the Budd runs — decreases the area over which the GlaDS effective pressure is used to 96% and increasing the cap to 4% decreases the area to 48%.*
Add the units and that the colorbar scale is not the same. It is better if the caption and the legend are the same (with symbol and units)

See our response to the above comment.

Appendix E

L380. : define $\rho_i$, $g$ and $H$ in the $p_i$ equation

Thank you for the suggestion. These variables have already been defined earlier in the text (the introduction of $N_O$) so we don’t redefine them here.

L381. : subglacial water pressure

Changed

L394. : to avoid a confusion with the power law exponent, maybe chose another letter than $m$.

We have changed $m$ to $x$ throughout the manuscript.

L395. : subglacial water pressure ($x^2$) – allowed water pressure allowed

Changed

Figures

Figure 1.
- (a) Bed elevation (unit missing) from Bechmachine v2 o Detail: it’s << Bed elevation >> in the caption and << Bed topography >> on the figure. Use the same formulation.
- (Morlighem et al., 2020) same comment as L20.
- (Rignot 2017) same comment as L327. → And modify by << Rignot et al., 2017 >>
- It misses a ‘.’ At the end of the caption.

The new figure caption now reads: Denman-Scott catchment. (a) Bed topography (m) from Bechmachine v2 (Morlighem, 2020); (b) Ice surface speed (m a\(^{-1}\)) from MEaSUREs v2 (Rignot et al., 2011, 2017; Mouginot et al., 2012, 2017), using a logarithmic color scale; (i) The Shackleton Ice Shelf. The black lines in both panels show the ice catchment outline, defined by the drainage divide and calving front, the grounding line is shown in red, and the GlaDS domain in yellow. The x– and y–axes are eastings and northings, defined in polar stereographic coordinates referenced to WGS84. These maps were made using the Antarctic Mapping Tools (Greene et al., 2017) and RAMP Radarsat Antarctic Mapping Project (Greene, 2022) toolboxes for MATLAB.

Figure 2.
- modify overburden pressure by ice overburden pressure.
- On L114, you mention << the northern branch >> of the Denman glacier, but in the caption you mention in Fig2a (iii) a western branch and in (iv) an eastern branch. I’m confused. Please, is it possible to have a clarification/harmonisation between the figure and the text and also to say which one is longer compared to what is written on L108.

It now reads ice overburden pressure. On line 170, northern branch has been changed to western branch.

Figure 3. Place the figure in the subsection 3.2. (a) Effective pressure (MPa) calculated by GlaDS, \( N_G \), [...] (b) prescribed effective pressure (MPa) [...] 

The caption now reads:

Effective pressure inputs. (a) effective pressure (MPa) from GlaDS, \( N_G \), capped at 0.4% of ice overburden pressure for the Schoof friction law shown here; (b) prescribed effective pressure (MPa) equal to the ice overburden pressure plus the gravitational potential energy of water \( N_O = \rho_i g H + \rho_w g B \).

Figure 4. Caption : surface velocity/ Schoof friction law (2)/ Budd friction law (2)
Ice dynamics outputs (note the colormap limits are not all the same). (a), (d), (g), (j) are basal friction coefficients ($C$ for (a) and (d), $\alpha$ for (g) and (j)), (b), (e), (h), (k) are basal friction coefficients normalized to their respective means (normalized $C$ for (b) and (e), normalized $\alpha$ for (h) and (k)). (c), (f), (i), (l) are the differences between the simulated and observed ice surface velocities ($m a^{-1}$, sim-obs $u_s$). (a), (b), and (c) show outputs from the Schoof friction law with $N_G$; (d), (e), and (f) are from the Schoof friction law with $N_O$; (g), (h), and (i) are from the Budd friction law with $N_G$; and (j), (k), and (l) are from the Budd friction law with $N_O$. In each panel, the black lines are the grounding lines.

Figure 5. Fig. 5b Schoof It is better if both limits of the scale are written. Fig. 5d for $r^2$ = write the same number of significant digits. In the caption: The red line is the linear line of best fit, and the slope of this line is reported ($r^2$).

The colorbar limits are now shown, all correlation coefficients are to three significant digits in the plot and in the text. The caption now reads:

Relationship between the effective pressure and basal friction coefficient for: (a) Schoof basal friction coefficient $C$ with $N_G$; (b) Schoof basal friction coefficient $C$ with $N_O$; (c) Budd basal friction coefficient $\alpha$ with $N_G$; and (d) Budd basal friction coefficient $\alpha$ with $N_O$. In each panel, points are colored by the natural log of the ice surface speed. The red line is the linear line of best fit, with the correlation coefficient reported ($r^2$).

Figure 6. a-b-c-d for the colorbar, maybe write $<< N_G/P_i >> - << N_E/P_i >> - << N_O/P_i >> << N_B/P_i >>$? and explain it completely in the caption. Use the same wording in the legend as in the caption. (f) The difference between the proposed empirical parameterization of effective pressure ($N_E$) and the GlaDS effective pressure ($N_G$) as a fraction of ice overburden pressure.

The colorbars have been changed as suggested. The caption now reads:

Effective pressures as a fraction of ice overburden pressure for: (a) GlaDS output effective pressure $N_G/p_i$, (b) empirical parameterization of effective pressure $N_E/p_i$, (c) typically prescribed effective pressure $N_O/p_i$, and (d) Brondex et al. (2017) prescribed effective
The difference between the empirical parameterization of effective pressure and the GlaDS output effective epressure as a fraction of ice overburden pressure \((N_E - N_G)/\rho_i\). (f) The saturation curve \(g_x(H)\) and the physically equivalent scatter for \(N_G\) \(((1 - N_G/\rho_i - r_l)/(1 - r_l))\). Black lines in (c) and (d) represent the \(N/\rho_i = 1\) contour.

As figure f is explained before fiigure e in the text, I would switch them.

They have been switched.

Table 1.

<table>
<thead>
<tr>
<th>Channel Conductivity</th>
</tr>
</thead>
<tbody>
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</tr>
</tbody>
</table>

Ice density kg m\(^{-3}\)

Changed

Table 2.

<table>
<thead>
<tr>
<th>Sheet Width Below Channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Changed</td>
</tr>
</tbody>
</table>

In the caption, use the same formulation than in the fig. 3 caption: [...] to the ice overbruden pressure [...] formula

Changed
References


K. Cuffey. Burlington, MA.


C. Dow, F. McCormack, D. Young, J. Greenbaum, J. Roberts, and D. Blankenship.


