

Dear Lousie, dear reviewers,

Thank you very much for reviewing our manuscript again and for your detailed and constructive feedback. We are very pleased that we were able to improve the readability of the manuscript in particular.

Please find below our responses to your comments and our explanations of further changes to the manuscript.

Best regards,
The authors

Response to Report #1

Reviewer comments are marked in italics.

Minor comments:

- *I believe that the abstract would be more robust if you could quantify some of your main findings.*

We fully agree and have revised the main findings of the abstract.

- *The firm model from Veldhuijsen uses RACMO/ERA5 and the firm model from Medley uses MERRA-2, why different firm models and different reanalysis datasets? does that introduce extra uncertainty?*

We now clarify in the data section: "We include different firm modelling data and altimetry products to test the sensitivity of our results to the choice of data sets and to assess uncertainties."

- *Consider merging sect. 3.5.3 with another section, as it is very short to stand alone*
- We have considered this, but the content of Sect. 3.5.3 does not fit with any of the other sections. We therefore prefer it to stand on its own, in line with its own separate topic.

- *Consider merging sect. 4.4 with another section, as it is very short to stand alone*
- Same as above.

- *R-squard vs R² you use both in the manuscript like in sect 4.2.4 lines 308 and 319. Also you use R-squard in the caption of Fig 9. and R² in the caption of Table 3. Choose one for consistency*

It is true that sometimes we write "R-squard" and sometimes we also use "R²". We would prefer to continue using both, since "R-squard" is the name, while "R²" is the corresponding mathematical symbol. This usage is also consistent with the way we treat other variables, such as the "altimetric residuals" (name) with "r^A" (mathematical symbol).

- *You write rmsm std, and psd with small letters but RCM, EOF, PCA, and SMB in capital letters, is there any reason for the difference?*

This is indeed inconsistent. We have changed the lower case of the abbreviations to upper case.

- *Please go through all the figures again and make sure that they have units, if they are unitless that should be indicated, I have given some examples below.*

All unitless variables presented in the figures are now indicated by (-), as proposed.

All specific comments have been revised as suggested:

- L269. Replace Fig 5 with Figure 5, as it is the beginning of the sentence. As per the submission guidelines.
- L 323. Same as above
- L 395. Same as above
- L540-545. Might be useful with a reference here, maybe this one, <https://www.nature.com/articles/s43017-023-00507-9>
- L527, Is RACMO2.4 not a polar version? referred to as RACMO2.4p1

In the meantime, a paper and a dataset of RACMO2.4p1 have also been submitted.

Therefore we have changed the sentence in L527 and the reference in L529:

„An update of RACMO2.3p2 to RACMO2.4p1 with enhanced physics is now available for 2006 to 2015. This includes (...) (van Dalum et al., 2024).“

Previous reference:

van Dalum, C. and van de Berg, W.: First results of RACMO2.4: A new model version with updated surface and atmospheric processes, in: EGU General Assembly 2023, Vienna, Austria, 24–28 Apr 2023, EGU23-13907, <https://doi.org/10.5194/egusphere-egu23-13907>, 2023.

New reference (preprint):

van Dalum, C., van de Berg, W., Gadde, S., van Tiggelen, M., van der Drift, T., van Meijgaard, E., van Ulft, L., and van den Broeke, M. R.: First results of the polar regional climate model RACMO2.4, EGU sphere, pp. 1–36, <https://doi.org/10.5194/egusphere-2024-895>, 2024.

All comments on the figures have been revised as suggested:

• Figure 4 has no units on the y-axis nor does the colorbar
It is now indicated by (-).

- Figure 7d. what is the unit, if unitless it should be stated or indicated with (-)
- Figure 10b. what is the unit, if unitless it should be stated or indicated with (-)
- Figure 11d. what is the unit, if unitless it should be stated or indicated with (-)
- Figure 12d. what is the unit, if unitless it should be stated or indicated with (-)
- Figure 14. what is the unit, if unitless it should be stated or indicated with (-)
- Figures S6 to S9 have no units on the y-axis

It is now indicated by (-).

- Figure S24. what is the unit, if unitless it should be stated or indicated with (-)
- Figure S26. what is the unit, if unitless it should be stated or indicated with (-)
- Figure S30. what is the unit, if unitless it should be stated or indicated with (-), also can the scaling factor be less than zero, since the colourbar has a minimum extension?

The scaling factor e is estimated by the least squares method and it can be negative. The corresponding regression approach is presented in Eq. A2.

The interesting question might be: Does this scaling make sense for $e < 0$ or what does it mean? For the grid cells where the time series from the firn model must be negatively scaled to match the altimetry time series as best as possible, it can be concluded that altimetry and the firn model cannot be meaningfully combined at the grid cell level because their errors are too large. This is somewhat remedied by the approach finally adopted, which does not scale the firn model time series per grid cell, but rather its dominant PCs.

Response to Report #2

Reviewer comments are marked in italics.

Minor issues:

- *L192: "The stochastic model". I asked about the deterministic/stochastic issues last time and I did not understand your reply. You point me to read this part of the text, but I still cannot see that you describe what is stochastic in Eqn (1). You have a regression onto the PCs and then there is a residual. Where is the stochastic part in that?*

Yes, there is no stochastic part in Eq. 1. Eq. 1 is the deterministic or also called functional model of the least-squares adjustment. The unknown parameters in Eq. 1 are solved by the least-squares method taking into account the variance-covariance information of the altimetric variations. From L192 onwards, the stochastic model or variance-covariance matrix is described verbally rather than by equations. The different weighting for different time periods means that our stochastic model is only a diagonal matrix, so only variances and no covariances are considered.

We agree that the term "stochastic model" may be unnecessary and confusing at this stage, so we have changed the sentence in L192 to read:

"We use different weights for the observations from different time periods."

- *L247-254: This paragraph is still hard to follow. I know you have already worked with it but give it another round, please.*

We have revised the paragraph again and explain why the individual data sets/matrices are merged into a single data set/matrix for PCA. We hope that the reasoning and procedure is now a little clearer.

- *L262: "longer wavelength". Would it not be better with "lower frequency" to distinguish spatial and temporal characteristics?*

We have changed this.

- *278+279: In 278 you have 0.831 and in 279 you have 82%. Should this not be the same number?*

Indeed, this is a typo. We changed it to 83%.

- *Table 2: The line "A1a-A2a", shouldn't this be "different ... based on Ma" and "Mb" for "A1b-A2b"?*

This is correct. In the revision, the names and numbers were adjusted and rearranged, but we forgot to rearrange the corresponding descriptions. We have corrected this. Thanks a lot for pointing this out.

- *L418: "This lessens the actual firm compaction variability" This makes it sound like the real world variability is lessened, but that is not what you mean, right? Perhaps something like "This lessens the modelled firm compaction variability compared to the actual variability..."?*

We agree that the wording is misleading. We have reworded the sentence as suggested.

- *L575-577: Yes, but also to (correctly) modeled variability which just is not covered by the included PCs, right?*

That is correct. Although it is a minor part, it should be mentioned. We have added the following sentence: "A small part of these residual signals may be due to the limitation of our regression which neglects up to 10 % of the potentially correctly modelled variations. However, we attribute the larger part to (...)".