Authors final response to Reviewer 1
TC-2022-97

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September 20, 2022

Note: Reviewers comments are given in blue font whereas the author response reads in black.

General comments:

This manuscript highlights the computational efficiency of using the method of separation of variables to solve the diffusion equation rather than direct numerical methods. While this is well-known, the application to the basal temperature evolution in an ice column had previously only been done for infinitely deep ice; this manuscript considers the case of a shallow ice sheet and performs some example calculations for different initial conditions and boundary conditions. Overall, I was somewhat disappointed that the authors did not go into more depth in analysing and describing their results, in particular exploring the wealth of curious trends shown in figure 4 - most of the paper is instead given over to a routine description of the method of separation of variables. In particular, given the stated threshold of 2km for the solution to approach the infinite depth limit, it would be nice to explore what factors set this threshold. Looking at figure 5 there seems to be a rather narrow band of depth values for which T is finite but larger than the MacAyeal solution. I think figure 4b also shows this rather sudden regime change.

We thank the reviewer for such a comment. The authors have accordingly elaborated in the manuscript to understand the physics behind this threshold.

Briefly, we can understand the process as follows. Since we focus on the time required for the base to thaw, it is essential to consider the temperature gradient between the base and the top. The vertical temperature gradient must be supported by the geothermal heat flux. If the surface is too cold, the heat provided by G may not be sufficient to hold a temperature difference large enough (within the column) so that the base reaches the melting point. For a given choice of G, k and T_{air}, there exists a minimum ice thickness $L_{\min}$ that yields a temperature gradient that allows the base to thaw. For thinner columns, the base will remain below the melting point. This further translates into a sudden jump in the potential periodicity shown in Fig. 4b.

Specific comments:

If Equation (6) were given as $\cot\left(L\sqrt{\lambda}\right) = \beta\lambda$, there would be no need to treat $\beta = 0$ as a special case. We thank the reviewer for such a comment. We will express Eq. 6 as $\cot\left(L\sqrt{\lambda}\right) = \beta\lambda$. Figure 4 - the values of the parameters held fixed are not given. Indeed, we will now include a table with the most relevant parameter values. Figure 4d - interesting that T is non-monotonic with L at $-14°C$. Why is this?

This is a quite complex behaviour since there are several factors that must be considered simultaneously. It is illustrative to look at the vertical profiles shown in Fig. 2. The fact that the temperature appears to be non-monotonic with L at $-14C$ is a consequence of two factors: the necessary energy budget to warm an ice column and the vertical temperature gradient. The former increases with L, whereas the latter decreases with L.

For slight variations of the thickness δL near $L = 1.5$ km (while fixing $T = -14C$), the time required to thaw the base is larger regardless of its sign. In other words, it takes longer to reach the pressure melting point both for a thinner and a thicker column. This local minima is a balance between the total energy necessary to heat a column and the fact that a thinner one implies a larger vertical gradient for a fixed temperature difference between the base and the top. Namely, we could consider the effect of these factors explicitly. First, a thinner column requires a smaller amount of total energy to increase the temperature of the column. However, considering the second factor, a thinner column would yield a larger vertical temperature gradient (ultimately yielding a slowdown in the warming rate as the geothermal heat flux is fixed in the BC). The combination of both effects allows for local minima.

The manuscript will be changed to address and discuss this behaviour explicitly.

Figure 4c - this figure shows the most interesting trends, but is barely discussed in the text. Perhaps using θ_L/L as the primary variable instead would clarify the impact of the temperature gradient on the basal evolution.

This comment agrees with the other two referees. A detailed discussion will be included in the manuscript addressing the most interesting trends. Using θ_L/L will be also considered to clarify the impact of the vertical temperature gradient on the basal evolution.

Line 162 - where T saturates to above 25kyr, are we in fact in a limit where T is infinite?

Yes, for certain boundary conditions the base would never thaw. The text will be updated for clarification.

Convergence towards no dependence on the detailed surface boundary conditions as $L \xrightarrow{\infty}$ could be moved to an appendix for better flow of the manuscript.

We have considered the possibility of moving this part to an appendix, but the comparison with previous work (MacAyeal, 1993a; b) fully relies on the limit $L \xrightarrow{\infty}$. In fact, we recover the well-known 7000-yr-periodicity widely used in the literature. While moving it might lead to a better flow of the manuscript, we feel that an important point would be lacking in the main text. We have made modifications to the text however, to try to nonetheless improve the flow.

Technical corrections:

Figure 4 colorbar caption could be oriented to match the axis label.

We thank the reviewer for such a comment. Figure 4 will be changed.