Answers to tc-2022-163 RC2

September 6, 2023

Note:

- The referee comments are shown in black,
- The authors answers are shown in blue,
- *Quoted texts from the revised manuscript are shown in italic and in dark red.*

* The exact pages and line numbers in our responses are subjected to change as the revised manuscript is being prepared.

Review #2

This paper describes the use of SPH for sea ice dynamics. In more detail, it implements VP rheology with elliptical yield curve into a SPH model. This is an interesting exercise and could lead to further work on using SPH on ice dynamics. The paper is worth publishing after some modifications. The comments by this reviewer are, mainly, related to the usefulness and applicability of the method: What is gained by using SPH when compared to FDM or DEM?

On general level:

Paper is very technical and it not easy to follow without a background in SPH. Is there a way to make it easier to read? Considering the readership of TC, effort to do this might increase the number of readers. Even if this reviewer is very familiar with numerical models, cannot go through all the equations of the paper. Authors could consider if such high level of detail needed here or could some parts rely on referencing earlier work? What is new in this description and what is from other sources?

In our opinion, section 2.1 is the only section which is purely theoretical and could be avoided by referencing. It describes rapidly the general concept of SPH and the kernel restrictions and assumptions. However, we feel that omitting it and just referring the theory to previous work for the readers would not give them the tools to understand the reformulation of differential equations of section 2.4 and the importance of the kernel. This is a key component to understand because from it we show that the modifications of the equations for the SPH framework modify the way plastic waves propagate in the medium. Consequently, to address reviewers' comments, we decided to keep section 2.1, but to move it to the Appendix. Sections 2.4 to 2.8 are important for people that want to know how the implementation was done and would like to reproduce or create their own SPH model. If the reviewer insists, more details could be added to specific sections.

The particle size in all simulations is of order of several kilometers. In addition, if the reviewer understands SPH correctly, all quantities in SPH become distributed over even larger area due to smoothing by kernel functions. Discussion on the following five issues in the paper is warranted:

(1) Is your model able to describe discontinuities in the deforming ice field with higher accuracy than typical continuum models (both in the case of opening leads and formation of ridges)?

As stated by the reviewer, the quantities are distributed over a large number of particles according to their smoothing length which also represent the effective resolution. From our dynamic formulation of the smoothing length and the ability of the SPH to move the particles around, we believe that during ridge formation (convergence) the discontinuities will have a high effective resolution because there are a lot of points to capture the ice deformations. On the other hand, in a lead opening (divergence) the edge of the discontinuity can be blurry because of the low number of points, but the shape of the opening is not restricted to a grid which has the advantage to enable smoother edge shape.

This sentence is added at l. 478 in the discussion of the revised manuscript: " … *in space. SPH can fracture and transitions from the continuum to fragments seamlessly since it is not restricted on a grid which also has the advantage of enabling smoother ice edge shape. The ability of the method to move around particles has the interesting property to concentrate them in converging motion increasing the resolution of the model in regions under high stress activity and to scatter them in diverging motion which decreases the resolution in low ice concentration area. This property should result in higher accuracy than typical continuum models. The elastic* …"

(2) Is the resolution of your model higher than typical continuum models?

This depends on the computational power available and the parallelization efficiency of the code. Assuming the same resolution (smoothing lengths equal to the grid cells size) for both techniques the SPH method will adapt its resolution to improve it where the dynamics predict more ice (convergent flow group the particles) and reduce it in areas of low ice concentration (divergent flow scatter the particles). This is different from other continuum approaches where the grid is fixed. Therefore, SPH should improve the overall accuracy because we are usually interested in areas of high ice concentration which have high stress and deformation.

The following sentence is added at l.176 : *"...Note that at its current stage, the model is rudimentary parallelized and a single time step for 40000 particles is of the order of tenths of a second. This implies that the model cannot have a resolution as high as other continuum approaches for the same physical set up. This…"*

(3) is the coarse resolution, or large particle size, due to computational burden?

Yes it is. The SPH method is explicit, which cannot take advantage of the solvers used in FDM like JNFK or Picard-SOR. The SPH explicit formulation forces a really small time step of the order of the hundredths of a second for a smoothing length of 10 km to properly resolve the plastic-wave propagation. This is around 6 orders of magnitude smaller than the time step used in FDM. The efficiency of SPH comes from its great potential in parallelization which, we believe, could compete with FDM on supercomputers for large simulations.

The following sentence is added in the discussion section 2.4 in the revised at l.167: " *The stability criterion imposes a strict limitation on the time step (∼ 10^−4 to 10^−2 seconds for particles of radius between 1 and 10 kilometres) that cannot be avoided by pseudo-time step with a solver because, in the SPH framework, particles are irregularly placed and move around at each time step. This makes the parallelization of particle interactions algorithm mandatory for any practical applications, but the explicit time step avoids possible convergence issues with the use of a numerical solver."*

(4) Does it even make sense to decrease the particle size when VP rheology is used?

At the moment, the dynamic formulation of sea ice used in the model is fairly simple and an increase in resolution (or decrease in particle size) is not really useful. The model uses the VP rheology as a test case because it is well known and makes the comparison with previous work easier, but further development using SPH should step away from it.

The following sentence is at l.72 in the revised manuscript: "*In this work, we use the standard VP sea-ice model with an elliptical yield curve and normal flow rule (Hibler, 1979) as a proof-of-concept. Further development of the SPH model should consider a broader range of rheologies . We…*"

(5) Does an individual particle in your simulation have physical meaning (do they, for example, describe ice floes – you do mention that particle collisions occur and affect your solution so the particles appear to have a physical meaning)?

We believe that yes they do. They can be seen as an unresolved collection of floes scattered within the smoothing length that can compact, ridge over one another, break, etc. However, since particles are points in space they cannot get in contact with one another even when their concentration is 1. Therefore, we suggest the addition of a short length contact force to simulate the collision of particles, but this is beyond the scope of our study.

The following sentence is added in the revised manuscript at l.148: "*Since the particle density ρ is independent of the concentration, the particle concentration Ap is a quantity that measures the compactness of the floes at the particle location but does not relate to the amount of ice carried by a particle. Overall, a particle can be seen as an unresolved collection of floes scattered within the support domain A that can compact, ridge, break and drift apart. Consequently, the concentration can be interpreted as the probability of ice floes carried by a particle to come in "contact" — because particles are points in space, they never touch each other and are repulsed according to the ice strength — with ice floes of another particle."*

Overall, do the authors consider their technique to be closer to continuum model or particle-based model? In Section 3.2 you show that your model follows a continuum solution. While this is what you appear to be aiming for, the example raises a question for the need of the approach presented. What is the advantage of using SPH in this case (or in other examples)? The authors should include a paragraph on this in the discussion; please emphasize what is gained by using SPH.

We consider the technique closer to the continuum model since it is based on the same sea ice dynamic equations. In its current state, the model reproduces very similar behavior as the conventional FDM formulation and doesn't bring much advantages. However, we believe that SPH enables the possibility to describe sea ice as a continuum at large scale with what is already known from the current continuum models (when there are a lot of particles) and explore some new avenues at small scales, where the continuity approximation is questionable. Indeed, the SPH discrete representation of the continuum with particles enables pairwise interactions like contact force and the conservation or transport of individual properties like angular momentum. Those further investigations inspired from DEM models, which cannot be achieved in classical continuum descriptions, are beyond the scope of the current study.

The second last paragraph has been modified to emphasize the current stage and the advantages of the method in the revised manuscript (l.471): "*In its current state, the model reproduces very similar behaviour as other FDM continuum models and does not constitute a large improvement. Nevertheless, we believe that SPH enables the possibility to describe sea ice as a continuum at large scale using what is already known from continuum models and explore some new avenues at small scales, where the continuity approximation is questionable. Indeed, SPH has interesting properties … SPH can fracture and transitions from the continuum to fragments seamlessly since it is not restricted on a grid which also has the advantage of enabling smoother ice edge shapes. The ability of the method to move around particles has the interesting property to concentrate them in converging motion increasing the resolution of the model in regions under high stress activity and to scatter them in diverging motion which decreases the resolution in low ice concentration area. This property should result in higher accuracy than typical continuum models…*"

It would be beneficial for the reader if you would include information on time step lengths and simulation times into your paper so that the reader can estimate how efficient the suggested approach is.

Agreed, at l.168 we added the information on time step length " … *time step (~ 10^-4 to 10^-2 seconds for particles of radius between 1 and 10 kilometers)* …" .

We also added a simulation time example at l.176, which now reads : "... *OpenMP. Note that in its current state, the model is rudimentary parallelized and a single time step for 40000 particles is of the order of tenths of a second. This could be greatly improved by taking advantage of CPU clusters (Yang et al., 2020) and GPUs (Xia and Liang, 2016).* ".

More detailed comments:

L37-38: If ice is thought to behave like a granular material, then is there a reason to believe the emergent properties of sea ice would not depend on floe size? Please comment on this in the text.

In the continuum formulation, the medium is considered as a whole and the ice strength at a given point only depends on the concentration and the thickness. Consequently, even if we can resolve floes of various sizes on a really fine grid, the medium property doesn't change.

We reformulated the following sentence to add some details at l. 43 : "*In practice, the emergent properties of a granular medium still depend on the assumed floe size and the nature of collisions in contrast with the continuous numerical methods which can only account for floe size in the formulation of the constitutive laws.* "

EQ17 & 18: Maybe it is mentioned somewhere in the paper, but is it common to define ice thickness and concentration as independent parameters? Maybe this is a misunderstanding by the reviewer, but at a given point in your simulation domain these two parameters cannot be totally independent, but, for example, A=0 should imply h=0. Could the authors comment on this shortly in the paper?

The ice thickness (h) and concentration (A) are independent prognostic variables in a simple two category Hibler-type model (ice or open water) and in multi-ice thickness category as well. In practice, during melt conditions, h reaches zero first, but is capped at a very small value to avoid a singularity in the A evolution equation that depends on 1/h (see for instance Hibler 1979). When h is capped to a small value, A is typically much smaller than 15%, which is considered outside of the "ice edge" in a continuum model. A continuous solution where A asymptotes to 0 and 1 (without capping for A=0 or A=1) was introduced by Gray, J. & Morland, L.W. (1994) for a more mathematically correct treatment of the mass equation, but this does not have an impact on the ice simulation.

This was clarified at line 107 of the revised manuscript with the following: "*Note that the thickness and concentration are independent prognostic variables in an Hibler-type model which can create a singularity when thickness reaches zero. To avoid this behaviour and for a more mathematically correct treatment of the mass equation, Gray and Morland (1994) introduced a continuous solution where the concentration asymptotes to 0 and 1. However, Hibler's formulation does not have an impact in our test case simulations since there is no melting and the particle thickness or concentration stay far from 0.*"

L375-379: You mention particle size does not affect jamming in your simulations. This is not what one would expect for a granular media. Does this suggest that your approach is not capable to fully represent granular behavior of an ice field (if such exists)? Do the particles have a physical meaning in your simulations? Please elaborate.

From our understanding, this means that the SPH approach is much closer to the other continuous method than to the discrete ones. If the constitutive laws don't take into account particle size and there are no contact forces added between particles then SPH doesn`t incorporate the granularity by default. It only makes it easier to incorporate than for continuous methods. The physical meaning has been answered previously in the comment to answer point (5) above.

We reformulated the following sentence at l.376 to add some details: " … *no ice arch formation for floe sizes ranging from approximately one quarter to one sixteenth of the strait width. In the present model, the constitutive laws prescribe the repulsion of the particles with one another according to the ice strength, which is a function dependent on the ice concentration and mean* *thickness, not on the particle size. We conclude that to enforce granularity within the SPH framework, the constitutive laws need to be adapted to account for contact force and particle size which could then reproduce similar behaviour as observed in DEM. However … "*

L381-384 (also FIG 8): (1) Are the "tree-like" peak stress values in your simulations transient (you use word oscillating stresses) or have your reached somewhat of a steady state in your simulations? If latter, is figure 8 just showing stress waves bouncing around the ice field in your simulation domain, which does not seem physically correct? Please clarify. If this reviewer is correct, the actual arches in your simulations appear to be limited into one close by the outlet. Please comment if you would to see more arches within the deforming ice in full-scale or if you would use DEM?

The "tree-like" peak stresses appear during transient and during the "steady state". However, note that the particles never stop moving even in a steady state because the material becomes viscous. Viscous deformation can also lead to oscillation and large stress and they are physically correct in the sense that in the SPH we approximate sea ice as a viscous-plastic material. However, it is known that SPH can have spurious behavior when the stress is solved at the same location as the particle which can be avoided if necessary (Chalk, C. Stress-Particle Smoothed Particle Hydrodynamics: An application to the failure and post-failure behavior of slopes, 2020).

This has been clarified in section 3.3, l., of the revised manuscript: " *From our experiments, the "tree-like" peak stresses appear during transient and at the steady state. However, the particles never stop moving even in a steady state because the material becomes viscous. Viscous deformation can lead to oscillation and large stress in the material so we hypothesized …*".

And in at l.394 : "*However, a deeper investigation is required to ensure that what is observed is physical. It is known that SPH can have spurious behaviour in some cases when the stress is solved at the same location as the particle centre (as done here) . This can be avoided using stress particles (see Chalk et al., 2020, for details).*"

Indeed, in our simulations the arch only forms close to the outlet. We know that the number of arches will increase within the deforming ice with higher resolution and they would also change location with more complex domain geometry and by changing the boundary condition to be no-slip (they are free-slip currently).

We reformulated l.407 to the following:"... *form an arch. Note that in our simulations the only one arch forms and is close to the outlet. We know that the number of arches would increase if the model is run at higher resolution and they would also change location with more complex domain geometry or by changing the boundary condition to no-slip. Overall, this…* "

L433: You again mention stress networks. Your approach adapts features from continuum models, which cannot present stress networks. Please elaborate clearly in the manuscript if you think your approach can present them reliably or not—and if yes, why does it do so if the underlying rheological model cannot present them.

Even though the approach is based on the continuum models which cannot represent stress networks, we believe that they can be observed with the SPH method because particles interact in a pairwise fashion according to their relative distance and they can move around according to stress. This can create less dense ice areas within the medium which can lead to stress networks. However, we don't know if those are reliable since we only compared them qualitatively. More tests should be done in future work.

We reformulated to the following at line 391: " *Despite the fact that the model solve the same continuum equations as other FDM models, we believe that stress networks can be observed with the SPH method because particles interact in a pairwise fashion according to their relative distance and they can move under the action of wind/ocean forcing and internal sea ice stresses. This can create less dense ice areas within the medium which can lead to stress peaks and lows.*"

REFERENCES:

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