Ice thickness and water level estimation for ice-covered lakes with satellite altimetry waveforms and backscattering coefficients:

Supplementary Information

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Supplementary Text 1:

Quantification of the remaining systematic biases in altimetric water levels

Here we provide an analysis of the source and magnitude of possible remaining systematic biases based on Jason-2 and Jason-3. The analysis for Jason-1 and Jason-2 would be similar but with a slightly different length of overlapping periods. Assuming that the real LSH variation during the overlapping period is $H(t)$, LSH observations from Jason-2 during the overlapping period can be represented by the following equation:

$$H_1(t_i) = H(t_i) + \theta_1(t_i) + \alpha \tag{S1}$$

where $H_1(t_i)$ is the LSH observation obtained by Jason-2 at time t_i, subscript i indicates the i-th Jason-2 observation during the overlapping period, θ_1 represents the random error for each Jason-2 LSH observation, and α denotes the systematic bias between Jason-2 observations and the real LSH. Similarly, the LSH observation from Jason-3 during the overlapping period can be represented by:

$$H_2(t_j) = H(t_j) + \theta_2(t_j) + \beta \tag{S2}$$

where H_2, j, θ_2, and β have the same meaning as H_1, i, θ_1, and α but for Jason-3. Removing the systematic bias between Jason-2 and Jason-3 is equivalent to estimating the value of $\alpha - \beta$, wherein we use the difference between the mean LSHs from Jason-2 and Jason-3 during the overlapping period:

$$system\ bias = \frac{1}{N}\sum_{i=1}^{N}[H_1(t_i)] - \frac{1}{N}\sum_{j=1}^{N}[H_2(t_j)] = (\alpha - \beta) + \frac{1}{N}\sum_{i=1,j=1}^{N}[\theta_1(t_i) - \theta_2(t_j)] + \frac{1}{N}\sum_{i=1,j=1}^{N}[H(t_i) - H(t_j)] \quad \text{(S3)}$$

where N is the number of observations during the overlapping period (for simplicity, we assume Jason-2 and Jason-3 to have the same number of observations during this period). Based on Equation (S3), the remaining systematic biases are caused by sensor random errors (termed θ_1 and θ_2) in each observation and the difference in sampling dates between Jason-2 and Jason-3 (termed $H(t_i)$ and $H(t_j)$).

The random errors θ_1 and θ_2 have standard deviations σ_1 and σ_2, which are ~ 0.1 m for both Jason-2 and Jason-3 in large lakes. The bias caused by the difference in sampling time is hard to predict because it is related to the LSH variation itself. However, an extreme example can be used to estimate the upper bounds of this uncertainty source. The difference caused by the sampling time is maximized when every pair of $[H(t_i)$-$H(t_j)]$ is positive or negative. Therefore, we can assume that the LSH constantly decreases during the overlapping period and each pair of Jason-2 and Jason-3 observations have a constant delay smaller than one repeat cycle. In this way, for each pair of i and j, $H(t_i) > H(t_j) \geqslant H(t_{i+1})$. Therefore, we can derive the upper bounds of the possible remaining systematic bias caused by the sampling time as:

$$\left(\frac{1}{N}\sum_{i=1,j=1}^{N}[H(t_i) - H(t_j)]\right) \le \left(\frac{1}{N}\sum_{i=1}^{N}[H(t_i) - H(t_{i+1})]\right) = \frac{1}{N}[H(t_1) - H(t_{N+1})] \le \frac{1}{N}\Delta H_m \quad \text{(S4)}$$

where ΔH_m is the maximum LSH change during the overlapping period. Based on Equation (S3–S4) and the error propagation formula, the maximum possible remaining systematic bias can be written as:

$$\sigma(System\ bias) =$$

$$\sqrt{\sigma\left(\frac{1}{N}\sum_{j=1}^{N}\theta_1(t_i)\right)^2+\sigma\left(\frac{1}{N}\sum_{j=1}^{N}\theta_2(t_j)\right)^2+\sigma\left(\frac{1}{N}\sum_{i=1,j=1}^{N}[H(t_i)-H(t_j)]\right)^2} \le \sqrt{\frac{{\sigma_1}^2}{N}+\frac{{\sigma_2}^2}{N}+\frac{\Delta Hm^2}{N^2}} \quad \text{(S5)}$$

In general, the overlapping period of Jason-2 and Jason-3 is ~9 months so *N* is ~27. For Jason-1 and Jason-2 the overlapping period is ~8 months which gives an *N* of ~24. If the lake is large enough (such as GSL and GBL) such that it is covered by interleaved ground tracks, the overlapping period between Jason-1 and Jason-2 can be extended to ~ 3.5 years corresponding to an *N* of ~120 while that of Jason-2 and Jason-3 can be extended to ~ 13 months corresponding to an *N* of ~40. Here we use an *N* of 25 to represent the most common situations. In addition, the intra-annual maximum LSH changes for large lakes are most likely ~ 1 m. Therefore, the maximum remaining systematic bias can be estimated with equation (S5) as $(0.1^2/25 + 0.1^2/25 + 1^2/25^2)^{1/2} = 0.049$ m. We can conclude that under the worst case, the remaining systematic biases can be around 5 cm, and it is mostly caused by the difference in sampling time between different sensors.