Dear Referee,

We would like to thank you so much for reviewing our manuscript and giving valuable comments and suggestions. Please find our responses to your comments in blue text and revised manuscript in red text below.

Best regards,

Yanan Wang and co-authors

Response to Anonymous Referee #1

This paper compares the sea-ice "floe perimeter density," as calculated from three models, to satellite observations in the Chukchi Sea (CS) and Fram Strait (FS).

The length, or density (length per unit area), of floe perimeter is a factor in the lateral melting of ice floes in summer, and is therefore a potential diagnostic for models. For a given field of ice floes, the floe perimeter density is a scalar.

The sea-ice floe size distribution (FSD) is the number of floes as a function of floe size. The FSD may be normalized (e.g. by the total number of floes) or not.

The analysis in this paper is all about perimeter density, denoted P_i (P sub i) by the authors, and PD by this reviewer. However, the authors treat PD and FSD as if they are interchangeable or equivalent. They are not. Completely different FSDs can give rise to the same PD, and identical FSDs can give rise to different PDs. There is not a one-to-one correspondence between PD and FSD. The authors point out that a larger PD implies more smaller floes, and this is true, but the PD says nothing about the FSD. In light of this fundamental confusion between PD and FSD, I must recommend that this paper be rejected. Specific comments follow.

PD and FSD

==========

The title implies that the paper is about the FSD, but it is really about the PD.

In the Abstract, lines 11-16 are about the FSD, and lines 17-25 are about the PD, without making any connection between the two.

Suppose n(r) is the number of floes of size r. Consider the case of circular floes. The perimeter of each floe is 2*pi*r and the area is pi*r^2. Therefore the perimeter density is:

PD = INTEGRAL(2*pi*r * n(r)dr) / INTEGRAL(pi*r^2 * n(r)dr)

Now suppose the mean value of n(r) is MU, and the variance is SIGMA^2. Then the above equation yields:

PD = 2*MU / (SIGMA^2 + MU^2)

Now consider two cases:

(1) n(r) has a uniform distribution on [0,L], i.e. n(r) = 1/L.

Then MU = L/2 and SIGMA^2 = (L^2)/12, so PD = 3/L.

(2) n(r) has an exponential distribution with parameter LAMBDA,

i.e. n(r) = (1/LAMBDA) * exp(-r/LAMBDA).

Then MU = LAMBDA and SIGMA^2 = LAMBDA^2, so PD = 1/LAMBDA.

By choosing L = 3*LAMBDA, the uniform FSD has the same perimeter density as the exponential FSD. Same PD, different FSDs.

Now consider a set of circular floes with FSD n(r). Construct a set of elliptical floes with semi-major axis "a" and semi-minor axis "b" such that pi*a*b = pi*r^2. Each elliptical floe has the same area as its corresponding floe in the circular set. Therefore the FSD of the elliptical set is also n(r), by construction. But the perimeters of the elliptical floes are longer than the perimeters of the circular floes, so the PD for the elliptical set is larger than the PD for the circular set. Same FSD, different PDs.

Same number FSD, different area FSD.

Lines 359-361. "positive biases of P_i are closely linked to overactive wave fracture in the models. This suggests accurate parameterisation of wave-induced sea ice breakup is essential for simulating the summer FSD correctly."

The implication here (and throughout the paper) is that the PD tells us about the FSD. But a connection between PD and FSD has not been demonstrated, and the simple theoretical examples in the previous comment show that a connection need not exist.

Figure 6 caption. "(a) Change of FSD arising from lateral melt" and "(b) ... wave induced FSD change"

According to the scale bar in the figure, the panels show the change in perimeter density, not the change in FSD. But here (and throughout the paper) the authors seem to equate PD and FSD.

In summary, the authors have not said how the PD is related to the FSD, and therefore why it can be used to assess the FSD produced by the models. According to my calculations, the PD and FSD are not necessarily related, so any statements or conclusions derived from the analysis of the PD do not necessarily apply to the FSD. Since there is no easy way to rectify the confusion between PD and FSD in this paper, it should be rejected.

Thanks for your comments. Regarding your comment 'the PD and FSD are not necessarily related' and 'there is no easy way to rectify the confusion between PD and FSD in this paper', we disagree with these points and have different opinions.

1) In previous studies, the FSD is not only presented as number density. Perimeter density distribution and floe area fraction are also used. Rothrock and Thorndike (1984) first defined the FSD as both number density n(r) and area fraction f(r). Yes, perimeter density distribution is not defined as FSD by Rothrock and Thorndike (1984), but it is a useful proxy for FSD and widely used in previous FSD studies. For example, previous observational studies (Perovich, 2002; Perovich and Jones, 2014 and Arntsen et al., 2015) applied the total floe perimeter per unit ocean area as a proxy for floe size distribution. As discussed by Perovich (2002), the use of the perimeter of the sea ice floe reduces the impacts of partially captured floes at the edge of the image for FSD retrieval. Besides, perimeter density per unit ice area is also used in FSD models as a metric to show the evolution of the FSD, e.g., Roach et al. (2019) and Bateson et al. (2022). As explained by Roach et al. (2019), "P_{ice} is weighted more heavily by smaller sizes, so Pi is more relevant for thermodynamic melting and freezing of floes", which is important in evaluating the FSD model.
2) The same PD corresponding to different number FSDs or the same FSD corresponding to different PDs does not mean there is no connection between PD and FSD. In previous studies, Rothrock and Thorndike (1984) first defined the FSD as both number density n(r) and area fraction f(r). These two definitions are related by $f(r) = \gamma r^2 n(r)$. Now, if we consider a set of circular floes and a set of elliptical floes with semi-major axis "a" and semi-minor axis "b" with the same number FSD n(r) in a given region. But assume that pi*a*b ≠ pi*r^2. In this situation, although these two groups of floes have the same n(r), their areal FSD f(r) is different. Similarly, if we consider 5 circular floes with the same radius of 10 m and 100 elliptical floes with semi-major axis "a=1 m" and semi-minor axis "b = 5 m" in a given region. Then we can get the same areal FSD, different number FSDs. Even the traditional FSD concepts n(r) and f(r) still shows the same situation that there is not always a unique 1:1 relationship between them. Similarly, it is not abundant evidence to prove that Pi is not a metric related to FSD by showing the same PD corresponding to different number FSDs or the same FSD corresponding to different PDs.
3) Although your example and ours given above means that there is not always a unique 1:1 relationship between the number density distribution and perimeter density distribution or between the number density distribution and area fraction distribution. It is worth noting that most examples are related to the variable shape of floe, i.e., γ is not the same for the two group of circle and elliptical floes given in these examples. However, it is common in FSD studies to make assumptions about floe shape, e.g., FSD models assume a fixed shape parameter γ. This should not be a cause for concern.
4) Additionally, the reason for the same PD corresponding to different number FSDs or the same FSD corresponding to different PDs can also be partially explained by equations (4)-(11) in Roach et al. (2019). They define the notation $\langle r^i \rangle_N$ as the ith moment of the number FSD.

   The fractional sea ice concentration in a grid cell, c, is

$$c = \int_{r_0}^{r_{max}} \gamma r^2 n(r) dr = \gamma \langle r^2 \rangle_N.$$

   then the perimeter density per unite ocean area is

$$P_{ocean} = \int_{r_0}^{r_{max}} 2\gamma r n(r)dr = 2\gamma\langle r^1\rangle_N,$$

The perimeter density per unite sea ice area is

$$P_{ice} = \frac{P_{ocean}}{c} = \frac{\int_{r_0}^{r_{max}} 2\gamma r n(r)dr}{\int_{r_0}^{r_{max}} \gamma r^2 n(r)dr} = 2\frac{\langle r^1\rangle_N}{\langle r^2\rangle_N}$$

Different metrics have different emphases. The area fraction of floe $F(r) = \int_{r_0}^{r_{max}} f(r)dr = \int_{r_0}^{r_{max}} \gamma r^2 n(r)dr = \gamma\langle r^2\rangle_N$, is a positive moment of the number FSD. For P_{ice}, Roach et al. (2019) suggest that "P_{ice} is a negative moment of the FSD, weighted more heavily by smaller sizes, so Pi is more relevant for thermodynamic melting and freezing of floes", which is important in evaluating a FSD model.

Whilst this paper may benefit from some tightening of nomenclature and clarification, useful information about the FSD can still be obtained from the perimeter density. For example, in Figure 2, we explicitly present the perimeter density distribution for floe size categories (perimeter of floes in floe size category i per unit bin width per unit sea ice area) rather than just perimeter density Pi, which therefore provides additional information about whether it is smaller floes or larger floes that contribute to differences in Pi calculated across the distribution overall.

5) The reason why we choose perimeter per unit sea ice area rather than per unit ocean area as in previous observational studies (Perovich, 2002; Perovich and Jones, 2014; Arntsen et al., 2015) can be explained by an example. If we consider a circular floe with a radius r = 5 km in a 10x10 km region. Sea ice concentration $SIC_1 = \frac{\pi}{4}$, $P_{ocean1} = \frac{\pi}{10} km \cdot km^{-2}$ and $P_{ice1} = 0.4\ km \cdot km^{-2}$. We also consider 50 circular floes with radius r = 0.1 km in a 10 x10 km region. $SIC_2 = \frac{\pi}{200} < SIC_1$, $P_{ocean2} = P_{ocean1} = \frac{\pi}{10} km \cdot km^{-2}$ and $P_{ice2} = 20\ km \cdot km^{-2} > P_{ice1}$. Obviously, perimeter per unit sea ice area is more related to the distribution of small floes. The floe perimeter per unit ice area is a more suitable and important metric for the evaluation of FSD models as the development of the FSD models are particularly important in the marginal ice zone, where small-floe-related processes are active, such as the thermodynamic freezing and melting of sea ice floes. Besides, Denton and Timmermans (2022) also suggested the SIC should be considered in studying the temporal evolution of FSD, e.g., for use in sea-ice models. The use of perimeter density per unit sea ice area for FSD models evaluation corresponds to this result.

6) To avoid the confusion caused by the definition, we revised the title to "Summer sea ice floe perimeter density in the Arctic: High-resolution optical satellite imagery and model evaluation".

Comparison of Models and Observations

=====================================

The lack of agreement between the models and the observations, and between the models themselves, is truly remarkable. The histograms of PD are all completely different (Figure 2, panels (a) through (d)). The plots of PD vs. sea-ice concentration (SIC) (Figure 3) show that FSDv2-WAVE has values of PD that are an order of magnitude larger than the observations, with vastly greater variability, and a slope (vs. SIC) with the wrong sign. CPOM-FSD is hardly any better. WIPoFSD has a slope similar to the observations but with PD values five times larger. The model/obs differences are noted by the authors at lines 196-206 and 219-231.

In Section 5, the causes of the differences between observations and models are attributed to three factors (lines 322-324). The first factor, image resolution, cannot explain such large differences (line 327). The second factor, underestimation of SIC in the models, "can partially explain" (line 338) such large differences. The third factor, overactive wave fragmentation in the models, was investigated by dividing each region (CS and FS) into north and south portions, and comparing observations vs. models in these sub-regions. In the CS region, all the observations are in the south sub-region. In the FS region, all the observations are in the north sub-region.

In Figure 7(a) for the CS region, the agreement between observations and FSDv2-WAVE(south) is still terrible, and the agreement between observations and CPOM(south) doesn't appear to be any better than in Figure 3(a). In Figure 7(b) for the FS region, the agreement between observations and FSDv2-WAVE(north) is still terrible, and the agreement between observations and CPOM(north) is not particularly good. In my opinion, the analysis by sub-region has not resolved or shed light on the large differences between observations and models.

Lines 352-353. To investigate "unrealistically high perimeter densities in our study regions" the authors "examined the P_i in the northern regions where wave-induced breakup is negligible. In these regions, most modelled P_i match our observations better."

This seems to be saying that the authors have compared model results from the northern sub-regions with the observations. But for the CS region, all the observations are in the south sub-region, so it makes no sense to compare models in the north with observations in the south. For the FS region, Figure 7(b) does not show that "most modelled P_i match our observations better." I don't see any kind of match between models and observations, nor much improvement over Figure 3(b).

Lines 359-360. "positive biases of P_i are closely linked to overactive wave fracture in the models."

I don't believe that the authors have demonstrated this.

Looking at the big picture, I can only think of two possible explanations for the enormous differences between the models and the observations, and between the models themselves: either the models are complete junk, or the PD is meaningless as a diagnostic of model performance. Do the authors have any thoughts on this?

Thanks for your comments.

1) In response to the following comments, *"the analysis by sub-region has not resolved or shed light on the large differences between observations and models"*,

In the northern Chukchi Sea and the Fram Strait, the change in Pi arising from all FSD evolution processes is almost zero in the two prognostic models during our research period. For checking the Pi in these regions, we recognize the Pi value in the northern region is the value at the end of early spring or the beginning of our research period. This comparison could help us determine whether the differences between observations and models arising from FSD evolution processes in summer (lateral melt and wave fracture) or before that.

Thanks for pointing out it. We will give more explanation about the reason for comparing Pi in northern and southern regions in the Chukchi Sea and the Fram Strait.

2) In response to the following comments, *"Looking at the big picture, I can only think of two possible explanations for the enormous differences between the models and the observations, and between the models themselves: either the models are complete junk"*,

The development of the FSD models is essential to improve confidence in sea ice models. Both prognostic FSD models and the power-law FSD model are in the development phase, although models show differences relative to the observations in small floes, it could partially give the ideas of the FSD evolution at this stage. For example, in Fig. 2h, CPOM-FSD (yellow lines) shows good performance in simulating Pi for floes with radius at hundreds of meter scales. As discussed by Bateson et al. (2022), the inclusion of the novel brittle fracture scheme does improve the model performance in hundreds of meter scales compared with observations. There are still many unknowns of FSD evolution for both observations and models. The evaluation of recent FSD models can provide useful insights for model developers, while until now there has been no detailed and comprehensive study comparing modelling FSD with high-resolution observation at metres to thousands of metres scales on a seasonal scale in a long-term time series. This is the first time evaluating the FSD model by using high-resolution satellite imagery resolving small floes in different regions and long-term periods covering more than 10 years. FSD models are still in development and a work in progress. This study is very valuable in contributing to the future development of FSD models by pointing out the differences between models and observations and the possible reason causing these differences, which could help modelers find a way to improve the FSD simulations in the sea ice models and thus the confidence of climate projection in the future.

3) In response to the following comments, *"the PD is meaningless as a diagnostic of model performance... "*,

At the beginning of this research, we tried to use areal FSD as a comparison metric, showing similar differences between models and observations (models show overestimation for small floes and underestimation for larger floes in general). This difference is not caused by the metric used. For the reasons for the use of perimeter per unit sea ice area, please see our response as mentioned previously.

Other Comments

==============

About the MEDEA imagery used in this study (lines 59 and 88-90), please see Denton and Timmermans (2022), and say briefly how their data set and analysis relates to the present work.

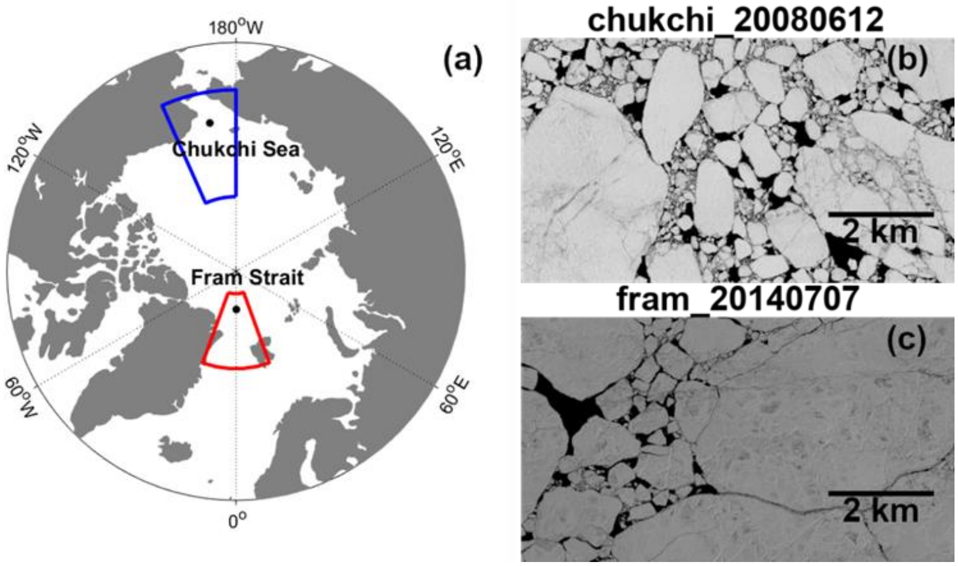
It is an interesting study. The SIC-FSD relationship could give evidence about the processes governing FSD evolution and give us more confidence in the use of perimeter density per unit sea ice area rather than per unit ocean area. Their test of FSD sensitivity to this choice reveals that the FSD can appear divided into two power-law regimes if this choice does not adequately identify small floes. This result corresponds to the difference between Pi derived from 1-m and 0.5-m resolution images (Fig. 4 in our manuscript). Better identifying small floes could expand the power-law range. As in the comments from Referee #2 Fabien Montiel and my response to him, there are also two possibilities leading to these two regimes. One is the limitation of the power-law hypothesis (see details in Montiel and Mokus, 2022). The other one is that small floes are more vulnerable to lateral melt relative to large floes, which causes the deviation of the small-floe distribution prior to the distribution tail (see details in section 4(d) in Hwang and Wang, 2022).

Lines 79-81. "the observations from the Chukchi Sea region captures a more dynamic and fragmented ice condition (e.g., Fig. 1b), while the observations from the Fram Strait capture a less dynamic environment (e.g., Fig. 1c)."

Looking at Figure 1, I don't see that the Chukchi Sea image indicates more dynamic and fragmented ice than the Fram Strait image. The two images look similar to me. How does a single image convey dynamics?

Fig. 1b shows more fragmented sea ice condition and includes more small floes between mid-size floes relative to Fig. 1c.

Thanks for your comments, we have replaced this figure with their partially enlarged image cuts shown below, which can show more details of the difference between them.



Section 3.1.2. What measure of floe size is used? All I can gather is that floe size is characterized by a radius. Is it half the mean caliper diameter? Is it the radius that a circular floe of the same area would have?

In this study, we define floe size as floe effective radius, the radius of a circle that has the same area as a floe. We have added this information about the measure of floe size to the revised manuscript.

Also, at lines 103-105, "we first applied combined filters: median, bilateral and Gaussian filter" and "The smoothing term, KGC algorithm parameter, was set as 0.0001" -- either provide more detail so that the reader can understand what this means, or leave it out and just refer to Hwang et al (2017).

Thanks, we will just cite Hwang et al (2017) in the revised manuscript.

Section 3.1.3. What is the spatial resolution of the sea-ice data? Also, the analysis period is 2000-2014 but AMSR-E is only available 2002-2011 and AMSR-2 is only available since 2012. What sea-ice products (with what resolution) were used during what time periods?

Thanks for your comments. The resolution for AMSR-E and AMSR2 applied in this study is 6.25 km. The resolution for NSIDC sea ice concentration is 25 km.

Yes, in our study period, there isn't SIC derived from AMSR-E in April- August 2000-2001, April-May in 2002 and April-June in 2012. That is one of the reasons that we also applied the SIC from NSIDC, covering April to August during 2000-2014.

Thanks for pointing out this, we have added the explanation of the resolution of the SIC products and the period for each product that we used for analysis.

Lines 121-123. Does 1-degree grid mean 1-degree in latitude and 1-degree in longitude? What does "gx1v6" mean (line 122)? Also, the models are run "for 37 years from 1 January 1980, followed by a 10-year period spin-up" so the spin-up period is 2017-2026 i.e. partially in the future. Is that correct?

Yes, 1-degree grid means 1 degree in latitude and 1 degree in longitude, which is around 30-63 km per gird. gx1v6 is one of the displaced pole grids of the global model, which is approximately 1-degree resolution horizontally (320x384 for horizontal grid with the horizontal resolution of nominal 1 degree). Thanks for pointing out this inaccurate description. The spin-up period is the first 10 years, which is 1980-1989.

We have added a more detailed explanation of the grid used in models and spin-up period as follows.

FSDv2-WAVE model has the displaced 1° gx1v6 grid (320x384 for horizontal grid) over a global domain, which is approximately 1-degree resolution horizontally. The other two models are initiated with the ice-free Arctic and run with the tripolar 1° (129 × 104) grids for 37 years from 1 January 1980, including a 10-year period spin-up during 1980–1989 in a pan-Arctic domain excluding Hudson Bay and the Canadian Arctic Archipelago.

Lines 155-157. "the model also simulates FSD evolution through the floe size parameter r_var ..."

Please define r_var. I see that it varies between r_min and r_max, and I see that it evolves according to four FSD processes, but no definition of r_var is given. What is it?

The r_{var} is defined in the equations 17-19 in Bateson et al. (2020). We will quote that the definition of r_var is given in Bateson et al. (2020) in the revised manuscript.

In addition to the exponent, the model also simulates FSD evolution through the floe size parameter r_{var}, varying between minimum floe radius r_{min} and maximum floe radius r_{max}. r_{var} evolves according to four FSD processes: lateral melt, wave-induced fracture, floe growth in winter and ice advection (Bateson et al., 2020, 2022). The detailed definition of r_{var} is given by Bateson et al. (2020), i.e., a variable FSD tracer reflecting the evolution of FSD in response to the physical processes.

Section 3.3. This section (FSD definition, lines 158-171) is confusing and unnecessarily complicated.

-- "The FSD is usually defined as the floe areal FSD..."

It's confusing to use FSD in the definition of FSD!

-- "By integrating f(r) over floe radius between r and r+dr, f(r)dr (dimensionless) is obtained"

This makes no sense.

Thanks. We revised the definition of FSD as follows.

The FSD is usually defined as the fractional-area distribution f(r)dr and number-density distribution n(r)dr (Rothrock and Thorndike, 1984; Toyota et al., 2006; Perovich and Jones, 2014; Horvat and Tziperman, 2015; Zhang et al., 2015; Hwang et al., 2017b; Bateson et al., 2020). Areal FSD f(r)dr corresponds to the area of floes per unit ocean surface area with radii between r and r+dr.

-- It's really not necessary to introduce the Heaviside function and equation (1) in order to define the FSD, especially since they're not used in the rest of the paper.

-- The cumulative floe number distribution, defined at lines 169-171, is also not used in the rest of the paper.

Thanks for your comment. We will delete the definition of F(r) in equation (1) and only keep the definition of n(r) and f(r).

Line 207. "normalized" perimeter density is not defined in the text. The caption for Figure 2 says "The normalized perimeter density distributions were obtained by dividing the width of every floe size category into P_i at each region." The original P_i has units of 1/km and the "normalized P_i" has units of 1/km^2. In what way is this a normalized quantity? Usually I think of normalization as producing a dimensionless result such as a percentage. What is the point of "normalizing" by dividing by the width of the floe size category to produce another dimensional quantity?

Thanks. Yes, the "normalized" perimeter density distribution is not normalized to 1. P_i is defined as the perimeter of floes per unit sea ice area (units: km km^{-2}). We will define the "normalized" perimeter density p_{ice} (units: km^{-2}) as the perimeter of floes in floe size category i per unit bin width per unit area in the revised manuscript. So, the integration of p(r) will be perimeter density P_{ice}, $P_{ice} = \int_{r_i}^{r_{i+1}} p_{ice}(r)dr$. In Figure 2, what we want to show is the perimeter density at the midpoint of each bin rather than Pi, the integral of the perimeter density over each category. That's why we produce p_{ice}.

Lines 270-271. "we constructed two data sets: monthly changes of P_i arising from lateral melt and FSD changes arising from wave breakup." How were these two data sets created?

These two variables are from model outputs. For the FSDv2-WAVE model, please see the first two terms on the right-hand side in equation (14) given by Roach et al. (2018), representing the growth and melt of existing floes in thickness and lateral size. For CPOM-FSD, the fraction of sea ice area lost due to lateral melting is shown in equation (8) in Bateson et al. (2022). The change in the FSTD f(r, h), per unit time due to fracture by ocean surface waves in both FSDv2-WAVE and CPOM-FSD is given in equation (20) by Roach et al (2018).

Thanks for pointing out this, we will add more information about these two variables in the revised manuscript.

Line 274. "CPOM-FSD produces negative changes in P_i from wave fracture (Figs. 6f and 6h)." But Fig. 6h shows changes arising from lateral melt, not wave fracture. Furthermore, Figs. 6e and 6f show that the change is positive, not negative. Finally, note that the panels in the bottom row of Fig. 6 are labelled (g) (e) (h) (f) from left to right. This might be the source of some confusion.

Line 280. "CPOM-FSD shows a stronger reduction in P_i arising from lateral melt (Figs. 6e and 6g)." But Fig. 6e shows changes arising from wave fracture, not lateral melt. See also the previous comment.

Thanks for pointing out this. Have revised Line 274 as below and the label of the bottom panels in Fig. 6.

The results show that FSDv2-WAVE produces larger positive changes in Pi from wave fracture (Figs. 6b and 6d) in summer relative to CPOM-FSD (Figs. 6f and 6h).

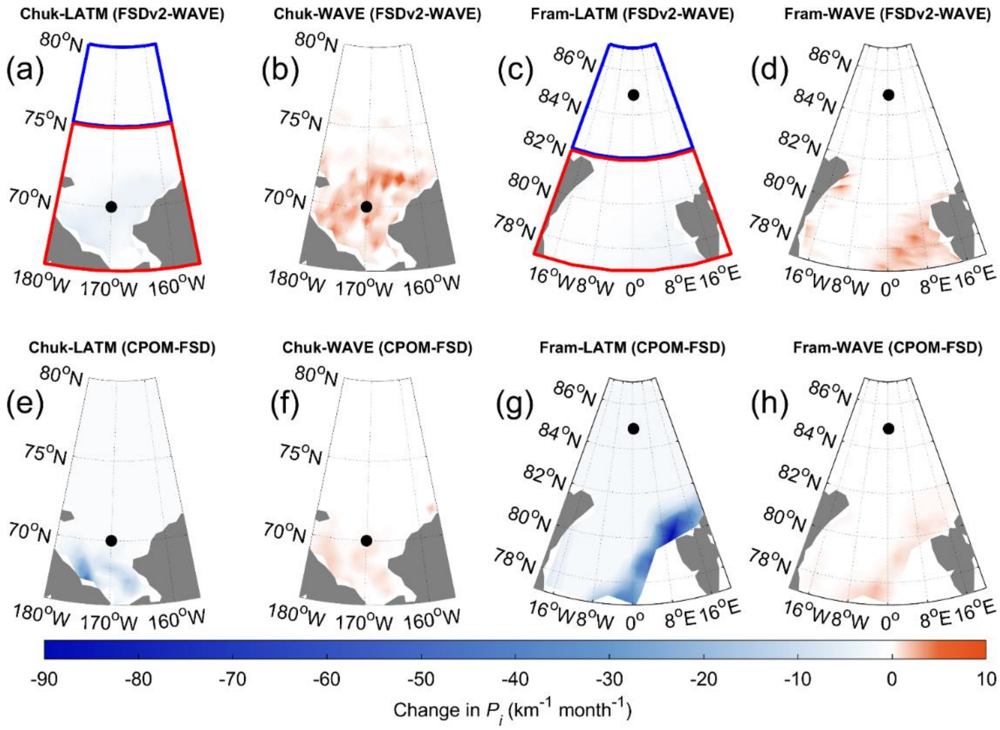


Figure 6: Monthly changes of $\boldsymbol{P_i}$ simulated by the two prognostic models over the period May to July during 2000–2014. (a) Change of FSD arising from lateral melt for FSDv2-WAVE in the Chukchi Sea. (b) is same as (a) but for wave induced FSD change. (c) and (d) are same as (a)and (b) but in the Fram Strait. (e)–(h) is same as (a)–(d) but for CPOM-FSD. The blue and red box in (a) and (c) show the northern and southern region of the two study regions. Black dots indicate the location of observations in the study regions.

Lines 302-303. "The close match between CPOM-FSD and the observations for the northern Fram Strait region..."

I'm looking at Figure 7(b) for the Fram Strait region. The observations (black circles) were acquired in the northern part of the region. The CPOM(north) results are indicated by open yellow circles. I don't see a close match between the black circles and the open yellow circles.

Thanks. The Pi in the northern Fram Strait simulated by CPOM-FSD is of the same order of magnitude of observations. Will use a more rigorous description: "The much closer match between CPOM-FSD and the observations for the northern Fram Strait region suggest no significant wave fracture and lateral melt has occurred in the observation site".

Lines 314-316. "The observation results show clear regional differences between the two study regions, i.e., much larger perimeter density P_i (smaller floes) in the Chukchi Sea region than in the Fram Strait region."

I'm looking at Figs. 3(a) (Chukchi Sea) and 3(b) (Fram Strait). The observations are indicated by black circles. When I look back and forth at the black circles in (a) and (b), I just don't see the "clear" regional differences.

In lines 196-197, observations show a substantial difference in P_i between the two regions (t-test, $t(47) = 6.43$, $p < 0.001$) (Fig. 2a). It shows a significantly higher P_i of 20.77 ± 6.54 km km^{-2} in the Chukchi Sea site than the P_i of 12.16 ± 3.79 km km^{-2} in the Fram Strait site (Fig. 2a). Regional differences shown in observations is at tens km^{-1} scale. For the black circles in 7(a) and 7(b), due to the overestimation of Pi in models, the y-axis limits for Fig. 7 range from 0 to 350 km km^{-2}. At this range, the regional difference in observations is not clearly shown in this plot. But the result from the t-test could statistically support this view.

Lines 320-322. "The observations and WIPoFSD model both show a positive correlation between SIC and P_i ... while the two prognostic models show the opposite (negative) correction."

(Note that the word "correction" should be "correlation").

Thanks. We revised this.

In Figure 3, I see the opposite of what's stated here: observations and WIPoFSD both show NEGATIVE correlations between SIC and P_i; FSDv2-WAVE and CPOM-FSD both show POSITIVE correlations between SIC and P_i.

Thanks. We have corrected.

Supplementary Materials

======================

Equation (1) and following.

-- It's bad notation to use "i" as a subscript on the left-hand side and "i" as an index of summation on the right-hand side. See also my comments below about equation (3) of the main text.

Thanks. We changed P_i to P_{ice} now.

-- The parameter GAMMA is not defined. "Here GAMMA is a floe shape parameter, for example..." (line 22) -- this is not a definition. I gather from equation (1) that the floe perimeter is 2*GAMMA*r, which perhaps defines GAMMA (if so, that should be stated explicitly). In that case, for circular floes, GAMMA = pi, as noted on line 22, but for square floes, GAMMA is not equal to 1, as erroneously stated on line 22. Consider a square floe of side s, perimeter 4*s, area s^2. If r is the radius of a circular floe of the same area, then s^2 = pi*r^2. The perimeter of the square floe is 4*s = 4*sqrt(pi)*r. If this is equal to 2*GAMMA*r then GAMMA = 2*sqrt(pi) = 3.54.

-- Rothrock and Thorndike (1984, hereafter RT84) calculated a "shape parameter" similar to GAMMA, finding that AREA / MCD^2 = 0.66 +/- 0.05, where AREA is the area of a floe and MCD is its mean caliper diameter. In the present context, if AREA = GAMMA*(r^2) and MCD = 2*r then the RT84 shape parameter is GAMMA/4 which implies GAMMA = 2.64 +/- 0.20, which is not too different from the values given on lines 23 and 24.

Thanks for your helpful comments. We will give a clear definition of gamma and have revised all equations in both the supplement and main text.

As r applied here is effective floe radius $r_{eff} = \sqrt{a/\pi}$, the radius of a circle that has the same area a as a floe. We assume $p = 2\gamma r_{eff}$. Here the gamma should be defined as $\gamma = \frac{p}{2r_{eff}} = \frac{p}{2\sqrt{a/\pi}} = \frac{p\sqrt{\pi}}{2\sqrt{a}}$, where p and a are the perimeter and area of a floe respectively. The gamma value we give in L23-L24 (the mean floe shape parameter γ is 2.27 in the Chukchi Sea region and 2.23 in the Fram Strait region) is $\frac{4a}{d^2}$, which corresponda to 4*0.66=2.64 in Rothrock and Thorndike (1984). Thanks for pointing out this, which make us realize that there was a mistake in the value of gamma for the calculation of Pi. Now we have corrected both the equations and the Pi value shown in the results sections.

For $\gamma = \frac{p}{2r_{eff}} = \frac{p}{2\sqrt{a/\pi}} = \frac{p\sqrt{\pi}}{2\sqrt{a}}$, the related value given by Rothrock and Thorndike is $\frac{a}{p^2} = 0.06$, which means $\gamma = \frac{p\sqrt{\pi}}{2\sqrt{a}} \approx 3.62$ in Rothrock and Thorndike (1984). This ratio is for floes with diameters over about 1 km. For our data, if we ignore the floes with caliper diameters smaller than 1 km, $\gamma = 4.57$ in the Chukchi Sea and $\gamma = 4.55$. For $r_{eff} > 0.56$ m (the effective radius of a 1-pixel floe) and $r_{eff} > 945.81$m (the upper limit of floe size simulated in models), $\gamma = 2.17$ in the Chukchi Sea and $\gamma = 1.96$. This ratio between floe perimeter and floe effective radius is closely related to the lower limit that we choose. As in this study, the comparison between models and observations is in the range of 0.07<r<945.81m, we prefer to use the shape parameter calculated within this range. The corrected gamma value is given in the revised manuscript as below.

Here γ is a floe shape parameter, $\gamma = \frac{p}{2r_{eff}}$. From the analysis of MEDEA-derived FSD results, the mean floe shape parameter γ is 2.17 in the Chukchi Sea region and 1.96 in the Fram Strait region, respectively.

The relationship between the areal FSD f(r) and CFND N(r) is revised to $\int_{r_0}^{\infty} f(r)dr = \int_{r_0}^{\infty} \pi r^2 dN(r)$.

P_{ice} for prognostic model P_{prog} is revise to $P_{prog} = 2\sum_{i=1}^{12} \frac{\gamma f_i (r_{i_{max}} - r_{i_{min}})}{\pi r_i c_{ice}}$.

P_{ice} for WIPoFSD is revised to $P_{wipofsd} = \frac{\int_{r_{min}}^{r_{var}} 2\gamma r n(r) dr}{c_{ice}} = \frac{2\gamma(3-\alpha)(r_{var}{}^{2-\alpha} - r_{min}{}^{2-\alpha})}{\pi(2-\alpha)(r_{var}{}^{3-\alpha} - r_{min}{}^{3-\alpha})}$.

P_{ice} for observations is revised to $P_{obs} = \sum_{i=1}^{12} \frac{2\gamma A_{floe_i}}{\pi r_i A_{ice}}$.

-- The meaning of the terms in equation (1) should be explained more clearly. For example: (r_i_max - r_i_min) is the bin width of the i-th bin; n_i is the number of floes in bin i per unit bin width per unit area; therefore their product is the number of floes in bin i per unit area. Therefore the quantity inside the summation is the perimeter of the floes in bin i, per unit area, and the numerator is the total floe perimeter per unit area. After dividing by the sea-ice concentration c_ice, one obtains the total floe perimeter per unit area of sea-ice -- the floe PD.

Thanks for your suggestions. We revised the explanations as follows.

In FSD models, P_{ice} is calculated from number FSD n_i distributed into floe size categories i as follows:

$$P_{ice} = \frac{\sum_{i=1}^{12} (2\gamma r_i n_i (r_{i_{max}} - r_{i_{min}}))}{c_{ice}}, \quad (1)$$

where r_i, $r_{i_{max}}$ and $r_{i_{min}}$ are the midpoint, upper and lower limit for each floe size category i. $r_{i_{max}} - r_{i_{min}}$ is the bin width of floe size category i. n_i (m^{-3}) is the number of floes in floe size category i per unit bin width per unit area. So $n_i(r_{i_{max}} - r_{i_{min}})$ is the number of floes per unit area for each floe size category i. The upper term in the right hand of Eqs. (1) is the perimeter of floes per unit area. By dividing total floe perimeter per unit ocean area by the sea-ice concentration, c_{ice}, we obtain the total floe perimeter per unit sea ice area.

-- Lines 23-24, "From the analysis of MEDEA-derived FSD results..." This sentence belongs in the section on "Calculation of P_i from the observations" at line 47, not in the section on the calculation of P_i from models.

Prognostic models didn't output perimeter density, we also need a gamma for calculating perimeter density in equation (4), which is revised as $P_{prog} = 2\sum_{i=1}^{12} \frac{\gamma f_i (r_{i_{max}} - r_{i_{min}})}{\pi r_i c_{ice}}$. Here we use the same value as in observations for the same floe size range.

Equation (4) (line 33) is the same as equation (3) of the main text (line 181). Also, lines 42-46, including equation (8), are an exact repeat of lines 184-188 of the main text, including equation (4) there. Why repeat the same material in the Supplement?

Thanks for your comments. We deleted the same equation as in the main text.

Minor Comments

==============

Line 173. The units of perimeter density are given as 1/meter but in much of the rest of the paper the units are 1/kilometer. Figures 3 and 7 use 1/km but Figure S1 uses 1/m.

Thanks. We corrected it.

Equation (3) -- notation.

-- On the right-hand side, the index "i" is used in the summation, and on the left-hand side, the index "i" is used as a subscript on P. This is bad notation.

-- Same comment for equation (5) and most of the equations in the Supplementary Materials.

-- The bad notation is easily fixed by dropping the subscript "i" on the perimeter density -- just use "P" (line 173 and following). There's no reason for a subscript.

Thanks. We revised them to P_{prog}, $P_{wipofsd}$, P_{obs}.

-- Also, doubly-subscripted variables r_i_max and r_i_min are confusing and unnecessary. The quantity (r_i_max - r_i_min) is just the bin width of the i-th bin or category. It could be denoted w_i or something else with a single subscript i.

Thanks. We replaced the bin width $r_{i_{max}} - r_{i_{min}}$ as w_i.

Equation (4). GAMMA is not defined. Also, I believe ALPHA = 2.56 in this case, which is perhaps worth repeating here.

Thanks. We defined gamma and repeat the value of alpha in the main text.

Figure 1 caption. The date for panel (b) should probably be 12 June, not 6 June.

Thanks. Revised.

Figure 2. In panels (e) through (k), what is the meaning of "Effective" floe radius?

Thanks. We will define effective floe radius in section 3.3 as follows.

In this study, we define the floe size as effective radius $r = \sqrt{a/\pi}$, the radius of a circle that has the same area, a, as a floe.

Table 2. The "a" and "b" superscripts are missing from the table.

Thanks. Corrected.

Table 3. The caption says "FSDv2-WAVE and WIPoFSD" but the table itself lists FSDv2-WAVE and CPOM-FSD.

Thanks. Corrected.

Citation in the response to Referee #1:

Arntsen, A. E., Song, A. J., Perovich, D. K., and Richter-Menge, J. A.: Observations of the summer breakup of an Arctic sea ice cover, Geophys. Res. Lett., 42, 8057–8063, https://doi.org/10.1002/2015GL065224, 2015.

Bateson, A. W., Feltham, D. L., Schröder, D., Hosekova, L., Ridley, J. K., and Aksenov, Y.: Impact of sea ice floe size distribution on seasonal fragmentation and melt of Arctic sea ice, Cryosph., 14, 403–428, https://doi.org/10.5194/tc-14-403-2020, 2020.

Bateson, A. W., Feltham, D. L., Schröder, D., Wang, Y., Hwang, B., Ridley, J. K., and Aksenov, Y.: Sea ice floe size: its impact on pan-Arctic and local ice mass and required model complexity, 16, 2565–2593, https://doi.org/10.5194/tc-16-2565-2022, 2022.

Perovich, D. K.: Aerial observations of the evolution of ice surface conditions during summer, J. Geophys. Res., 107, 8048, https://doi.org/10.1029/2000JC000449, 2002.

Perovich, D. K. and Jones, K. F.: The seasonal evolution of sea ice floe size distribution, J. Geophys. Res. Ocean., 119, 8767–8777, https://doi.org/10.1002/2014JC010136, 2014.

Roach, L. A., Horvat, C., Dean, S. M., and Bitz, C. M.: An Emergent Sea Ice Floe Size Distribution in a Global Coupled Ocean-Sea Ice Model, J. Geophys. Res. Ocean., 123, 4322–4337, https://doi.org/10.1029/2017JC013692, 2018.

Roach, L. A., Bitz, C. M., Horvat, C., and Dean, S. M.: Advances in Modeling Interactions Between Sea Ice and Ocean Surface Waves, J. Adv. Model. Earth Syst., 11, 4167–4181, https://doi.org/10.1029/2019MS001836, 2019.