Reply to Referee Comments 1

We first would like to thank the reviewer (Adrian McCallum) for the positive and insightful as well as detailed comments. We take all the comments into account, reply in italic text and update accordingly in the revised manuscript.

General comments

Thank you for the opportunity to review this very interesting work that examines how variations in stochastic signals resulting from micro cone penetration in snow can be used to discern snow type/microstructure.
Field data is compared with laboratory data to enable characteristic signals to be identified and variation in two noise types (diffusive and jump) is particularly examined to suggest snow microstructure behaviour and thus snow type/composition. This work relies on the assumption that the SMP penetration process is analogous to Brownian motion.
I found it a very interesting paper and I recommend it for publication. Below I make a few specific comments and numerous technical observations which the authors may wish to consider.

Specific comments

My primary comment is that you may wish to consider altering the title/context/frame of your paper. I say this because McCallum has written many papers on Cone Penetration Testing (CPT) in polar snow, some of which examine microstructure assessment using CPT. You may wish to briefly comment on these works in your introduction, or you may wish to refer to your work as 'micro' cone penetration testing, just to differentiate between the large body of McCallum's large-scale (36.7 mm penetrometer) work and the body of work that you discuss here, primarily pertaining to the SMP, Johnson and Schneebeli's work etc. I am happy with whatever you chose to do; but, if you keep it as cone penetration tests, you probably should mention McCallum's work ...

The title is now changed to:
"Stochastic analysis of micro-cone penetration tests in snow"
and McCallum's work is also mentioned in the introduction. See line 28 in new version.

The rest of my comments are essentially of a technical nature.

Technical observations

Please re-examine your tense throughout the document. You start off in past tense but this alters; please review and amend.

We check the used tense accordingly in the revised manuscript.

Now by line #, for your consideration please:
3 "more and more" etc. please re-phase/tighten this sentence.

We now change it to "By using small penetrometer tips at this high vertical resolution, further details of the penetration process get resolved, leading to much more stochastic signals."

5 delete "employing"

Corrected.

8 replace allows with enables

Corrected.

11 probably less-dense not lighter; keep terminology consistent.

We now use "less-dense"

13 single: how do you discern/confirm this? Perhaps reword.

We now outline that these are our interpretation and findings of our analysis.

14 Perhaps: with micro cone penetration tests.

We specifically mention now as "micro-cone penetration tests"

24 Perhaps: supposedly

Corrected.

26 can be resolved

Corrected.

40 Reference re. important applications?

The text is reworded as "These models are now commonly used to characterize the snowpack stratigraphy from SMP measurements." See line 41 in new version.

41 to some of the most ...

Corrected.

47 delete “the” in the fluid

Corrected.

49 what are micro-events? Please better explain.

We reformulate the sentence and explain it in the revised manuscript. See line 49 in new version.

“Due to the sum of several collisions with the molecules in the fluid as illustrated in Fig. 1, the large red particle undergoes a motion described by a stochastic process.”

53 shot noise? correct?

Our interpretation of Poisson jump noise corresponds to the shot noise. We mention it now in the revised manuscript. See line 56 in new version.

57 Please reword this last sentence; perhaps: Via this advanced analysis, we seek more detailed snow characterisation from micro cone penetration test resistance data.

Done. We used your suggestion.

59 Delete “the” Sect. 3

Corrected.

63 explains the equations?

Corrected.

64/5 note that although the drive is constant the actual penetration rate may not be.

We agree. In the paper, we do not mention the penetration rate.

75 fix “as of the”

Corrected. See line 77 in new version.

77 probably just Friedrich () and Rinn () (instead of semi-colon).

Corrected.

85 do you mean: small depth interval (z)? Also “similar”

Yes and corrected.

91 probably: Such a jump-diffusion dynamic ...
Corrected.

107 Wick's theorem: reference?
Added.

114 where here we use ...
Corrected..

117 do you mean: small depth interval (z)?
Yes and corrected.

122 perhaps state: "; this is the same as Eq. 2 but ..."
Corrected..

128 perhaps "is considered instantaneous".
Corrected.

129 Please spell out OU and SDE in Fig. 2 caption.
Done.

129 Please use drift-jump and jump consistently so as not to cause confusion.
We now change jump to drift-jump in Fig. 2 caption.

134 Rephrase "as above"; this is unclear.
Corrected. "as above" is now deleted.

136 "which is a zero-one ..."
Corrected.

137/138 etc. "process were generated"; please change tense to past throughout.

Corrected and we now change the tense throughout the paper accordingly.

139/140 "Left, a pure ..., middle, ... and right, ..."
Corrected. See line 142 in new version.

141 negligibly

Corrected.

142 “Dots” in Fig. 3 caption

Corrected.

143 process, another parameter that we considered was ...

Corrected. See line 146 in new version.

144 proof evidence? Perhaps: to validate our method, based on the KM coefficient ...; then. comma after “Eq. 8”; “were chosen”

Corrected. See line 147 in new version.

146 “as the previous example”

Corrected.

152 Probably: Firstly, small snow samples whose microstructure was fully characterised ... were used to test ... Secondly, ... we analysed one ... and provided ...

Corrected. See line 155 in new version.

156 Fig. 4 caption; final sentence: Sub-samples shown are ...

Corrected.

158 tested

Corrected.

159 Reference for snow types; the samples were prepared.

The reference Fierz et al. (2009) is added. See line 162 in new version.

160 Temperature of sintering? Microstructure was captured.

Sintering temperature is -10 °C *and added in the revised manuscript. See line 164 in new version.*

161 test was conducted

Corrected.

162 on sample preparation..

Corrected.

163 Main sample properties are summarised in Table 1 and the measured hardness profiles ...

Corrected. See line 166 in new version.

167 focussed on the fluctuations of the hardness profiles. Each profile was first detrended.

Corrected. See line 170 in new version.

173 divided

Corrected.

174 were separated

Corrected.

175 We estimated the KM coefficients of each sample ...; how?

We use Eq. 2 to estimate them and mention it in the revised manuscript. See line 178 in new version.

179 data were determined

Corrected.

180 was determined

Corrected.

181 "... 0, and the higher order KM ..."

Corrected.

182 This indicates the presence ...

Corrected.

186 normalizaton, the fixed ...

Corrected.

187 length scale is given

Corrected.

189 Figure 5 caption: Setup of micro cone penetration test; The samples were placed in the cylindrical sample holder ...; Is "Kistler 9207" the type of force sensor?

Corrected in the figure caption. Yes, "Kistler 9207" is the force sensor used in SMP.

Figure 6 caption. The wording here is unclear: "have smallest trend and fluctuation force"; are you using all these terms consistently? In the next sentence you talk of size not force? Please re-examine ...

We now use fluctuation "force" in the figure caption 6.

Figure 7 caption. ... for better visualisation.

Corrected.

197 Perhaps: Results are summarized in Table 2; we discuss these in Sect. 4.

Corrected. See line 202 in new version.

198 Perhaps: Hardness of Field Data or Application to Field Snow Data?

We change the title of 3.2 to "Application to Field Snow Data".

200 The measurements were also performed with a SMP, but the tip had a different sensitivity of ... what was it?? Spatial sampling was again ...

We reword the text as "the tip had a slightly different shape corresponding to the standard version of the SMP (Johnson & Schneebeli, 1999)." See line 205 in new version.

Figure 8 caption. Please reword last sentence; it is difficult to understand.

We now change it to "Comparing the correlation length scales $L_{\mathrm{C}} = \frac{1}{\gamma}$ *where* $D^{(1)} = -\gamma R'$ *with those of the autocorrelation functions (ACF), we find that both length scales have the same ordering of their values for all snow types."*

203 methods was irrelevant, as we subsequently show ... that in principle, the ... really snow data, and that ...

Corrected.

Figure 9 caption. "$<$ 2; we focus our statistical ..."

Corrected. The last line of the caption is also changed to "The blue horizontal lines show the mean values of the respective parameters in the range of $-2 < R' < 2$*."*

206 therefore, we used ...

Corrected.

207 profile was separated ... and detrending was performed on each window ... 0.6 mm, formalised with ... deviation as in our previous analysis of laboratory data.

Corrected. See line 213 in new version.

Figure 10 caption. parameters were determined ... are also plotted to enable better comparison (right column); they are shifted ... reference to the local characteristic snow types from laboratory measurements, *Corrected.*

212 for better comparison

Corrected.

213 Interpretation of these results will be discussed next.

Corrected.

217 "it is found that sufficient large particle"?? Please reword. "In our interpretation, ..."

Corrected. See line 223 in new version.

218 perhaps: "in the immediate surroundings of the SMP, in addition to the pushing aside ..."

Corrected. See line 224 in new version.

219 Delete However; Perhaps: The jump noise may represent (or be representative of) the bond-breaking events occurring directly at the tip of the SMP ...

Corrected. See line 225 in new version.

221 perhaps: it is clear that snow type morphology, shown in Fig. 4, is essential for effective stochastic analysis as outlined herein.

Corrected. See line 227 in new version.

223 We started ...

Corrected.

230 "R', and can be approximated ..."

Corrected.

249 our earlier discussion,
Corrected.

252 “bigger ice structures”: consider rewording/clarifying this sentence: “thicker grain necks”?
We now use “larger grain size”. See line 258 in new version.

255 “allows”? perhaps: enables differentiation between ...
Corrected. See line 261 in new version.

257 perhaps: “With reference to the local characteristic snow types from the laboratory measurements (), we see dynamics that suggest mixtures of different snow types within this depth segment.
Corrected. See line 264 in new version.

261 “the developed methodology appears ... in the field, but further quantitative evaluation is required.”
Corrected. See line 267 in new version.

264 allows differentiation of
Corrected.

268 the denser structures typical of DH and ...
Corrected.

270 Delete: “we have to remember that”
Corrected.

273 Perhaps: Finally, we would ... of a complex material, snow, by a ...
Corrected. See line 278 in new version.

276 Perhaps: types, complementing existing methods.
Corrected. See line 280 in new version.

Reply to Referee Comments 2

We first would like to thank the reviewer (Henning Löwe) for the positive and insightful comments. We take all the comments into account, reply in italic text and update accordingly in the revised manuscript.

Review of "Stochastic analysis of cone penetration tests in snow" by Lin et al

Main comments

The paper analyzes snow micro-penetrometer (SMP) signals in terms of stochastic jump-diffusion dynamics. The analysis is very interesting and adds novel aspects to the interpreation of SMP data. The paper is well written, the topic is suitable for TC and the results warrant publication. I have mainly two questions I would like to ask an answer for in the present work:

- Independence. Naively one is tempted to assume that the signal must be the result of one (and only one) underlying stochastic process, which is the disordered microstructure of snow. The present model rather assumes that the penetration leads to a situation with contributions from different stochastic processes which are assumed to be independent. While I can imagine how such a situation may originate from the physics, it would be helpful if this assumption of independence (between the jump characteristics and the diffusion) could be further assessed. Along these lines we have previously seen that interpreting the signal as a shot noise process with three parameters (λ, δ and f_0) we always end up with correlations between estimates of λ, and δ, which obviously cannot be convincingly separated by such a model. Since δ roughly translates to $1/D^{(1)}$ of the present model, I wonder if these parameters still show correlations. In addition,for large λ and small σ_ξ a jump process with drift could "tend" to diffusion. Therefore potential correlations of the latter parameters with $D^{(2)}$ are relevant too. In the simplest case it would be sufficient to provide mutual scatter-plots of estimated parameters λ, $D^{(1)}$, σ_ξ and $D^{(2)}$ for the profile in Sec 3, but maybe there are even rigorous ways to answer this question.

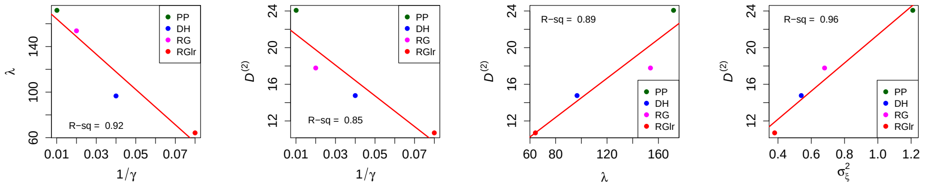
  Thank you for the interesting question. We look at the scatter-plots of the parameters $\frac{1}{\gamma}$ and λ, and $\frac{1}{\gamma}$, $D^{(1)}$, σ_ξ^2 and $D^{(2)}$ for all four snow types according to the results of Table 2. We also do the linear *regression of each scatter-plots and the coefficient of determination or R-squared is* ≈ 0.9. *In our current analysis, we assume the independence between diffusion and jump-diffusion which could give a good insight to interpret the cone penetration process. However, from these plots, we could see the hints for the possible correlations which could be considered as the improvement for current model and we would leave it as an open topic for the later analysis.*

In the jump-diffusion modeling of stochastic time series it is assumed that three random variables, $W(t)$ *Wiener process,* ξ *jump size and* $J(t)$ *Poisson jump process, are independent. However in general they can be correlated. For instance for correlated* $W(t)$ *and* $J(t)$ *one finds*

$$\langle W(t)J(t)\rangle = \rho(t)\sqrt{\lambda}t$$

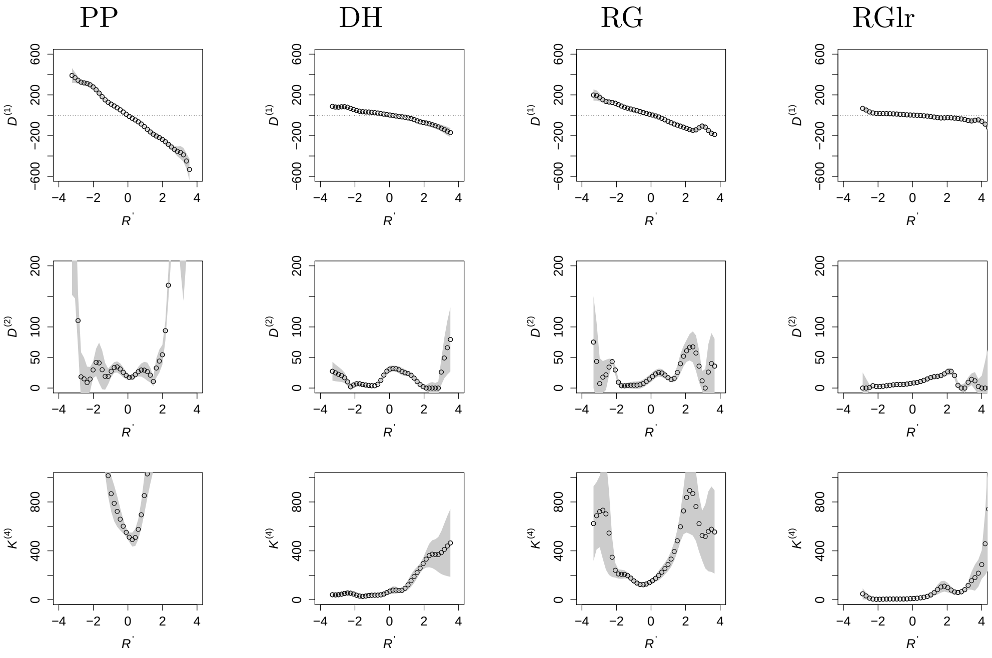
where $\rho(t)$ *is the correlation coefficients of* $W(t)$ *and* $J(t)$.

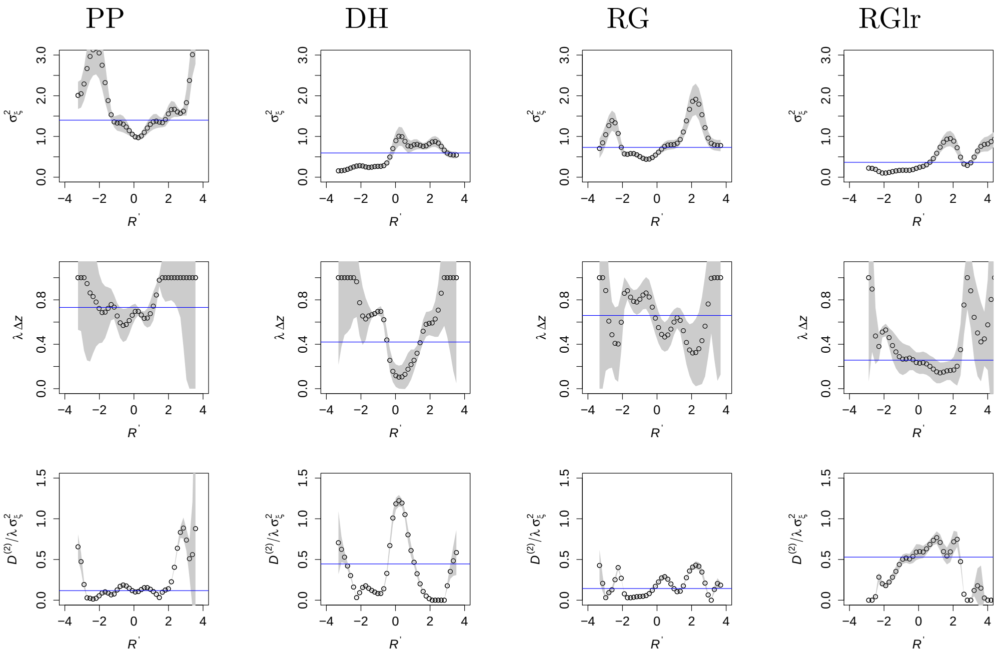
Data-based estimation of $\rho(t)$ *is an open topic and we will address this important problem in the near future.*



- Kernel width. It might be good to check if a fixed kernel width of 0.6 mm (l168) is a robust choice in view of the statements in the discussion about grain size dependencies: Converting the SSA values given in Tab 1 into a “grain size” (the optical diameter) reveals that diameters range from 0.12 mm for PP to 0.70 mm for RGlr. Now the diffusive contribution is assumed to be a result of steric (grain-grain) interactions in front of the cone, and this process will need a few grain diameters to develop. A fixed kernel of this particular size might thus induce a bias here. A priori grain size information is clearly commonly not available (like for the analysis of hardness profile in Sec 3.2). But it seems relevant to compare, at least for the data from Sec 3.1, how the parameter estimates for the 4 samples compare with those generated from a constant ratio of kernel width and optical diameter. Results could be simply added to existing figures.

Here we calculate the grain size or optical diameter based on the relation $d_{\text{opt}} = \frac{6}{\rho_{\text{ice}} \cdot \text{SSA}}$*. Then, we calculate the average value of SSA for each snow types and find the average grain sizes of {PP, DH, RG, RGlr} to be {0.14, 0.40, 0.29, 0.66} mm. Now we use these grain sizes as the kernel widths for detrending of the snow hardness profiles and the new results for Fig 8 and 9 and Table 2 are as follow:*





Snow type	$L_{\mathrm{C}} = \frac{1}{\gamma}$ [mm]	$L_{\mathrm{J}} = \frac{1}{\lambda}$ [mm]	$\overline{D^{(2)}}$	$\overline{\sigma_\xi^2}$	$\overline{\frac{D^{(2)}}{\lambda\sigma_\xi^2}}$
PP	0.008	0.005	28.59	1.40	0.12
DH	0.035	0.010	15.66	0.59	0.44
RG	0.017	0.006	15.94	0.73	0.14
RGlr	0.080	0.016	10.24	0.36	0.53

The new kernel widths do not change the results significantly. Therefore, we decide to keep the constant kernel width of 0.6 mm in order to make it consistent with the field measurement data. The chosen kernel width is also within the range of the smallest and the largest grain size. If we choose the kernel width that is much larger than grain size, there would lead to oversmoothing and the detailed dynamics of cone penetration test could be lost. We also added the remark to the paper, that the results do not change significantly if the kernel width are changed between 0.14 mm and 0.66 mm.

Kind regards, Henning Löwe

Minor comments

(l117): I don't entirely understand why an R dependence of the coefficients is introduced here. Isn't the analysis later only based on constant coefficients, i.e. additive noise? Would everything work also for multiplicative noise?

Our analysis works for multiplicative noise. From our results we can see that the parameters are R dependent (Fig. 8 and 9). When we interpret the results in Table 2, we took the average value as a first order approximation of the parameters in the range of $-2 < R' < 2$ *in order to compare the results of each snow type.*

(l135): Here it might be illustrative to explicitly mention the "triply-stochastic" nature of Eq 11 and that all (ξ,J_t,W_t) are independent.

We now write: "Here, we have triply stochastic processes W_t*,* J_t *and* ξ *which are all independent of each other." See line 138 in new version.*

(Tab 2): Here uncertainties/errors should be included that reflect inter-sample variations of the same snow type.

Uncertainties are now included in Table 2.

(l174): What is the final size of the sub-samples? Is this choice also consistent with grain size $\ll$ sample size in all cases?

The final sizes of the sub-samples vary from (680 - 1500) sample points i.e. (2.72 - 6) mm. The grain sizes varies approximately from (0.14 - 0.66) mm.

(l189): It would be nice to include the correlation lengths estimated from the ACF also in Tab 2 to support this statement. (DH and RGlr appear to be very similar in Fig 8 while the L_{c} differ by a factor of two)(Fig 9): What is taken as Δz?

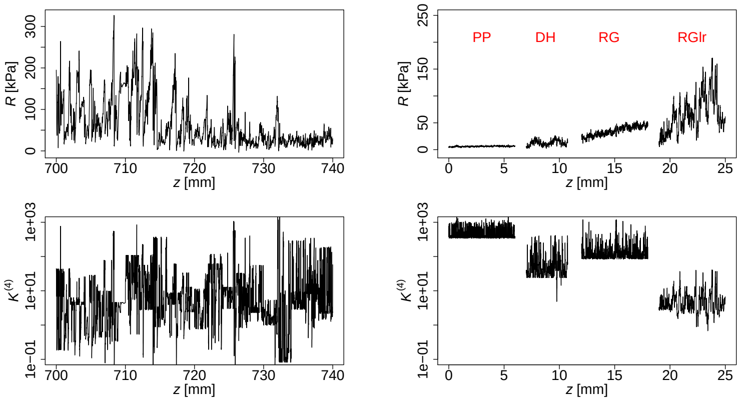
The correlation lengths L_{ACF} *estimated from the ACF,* (*PP, DH, RG, RGlr*) $= (0.006, 0.025, 0.016, 0.038)$ mm*, are added in the text (see line 193 in new version) and Table 2 in the revised manuscript.* Δz *is the resolution of SMP which is 4* μm*.*

(Fig 10): Top left, this looks like R and not R'?

Corrected.

(Fig 10): Maybe a semilog y scale for $K^{(4)}$ better reveals the differences?

Here is the figure for $K^{(4)}$ *in semilog y scale and we decide to keep it* *in linear scale since we can observe the differences in $K^{(4)}$ between the snow types more clearly in linear scale.*



(Fig 10): It would be good to include also $\lambda\,\Delta z$ and $D^{(2)}/\lambda\sigma_\xi^2$ in this figure. The subfigures can be safely reduced a bit in height.

Since the uncertainties of λ are relatively large especially for the extreme values of R', we mainly determine $K^{(4)}$ and σ_ξ^2 which could give more consistent results.

(l234): This is such a statement which might be affected by the choice of the kernel width...

As shown in the main comment, our choice of kernel width does not significantly change the results.