Reply to Referee Comments 2

We first would like to thank the reviewer (Henning Löwe) for the positive and insightful comments. We take all the comments into account, reply in italic text and will update accordingly in the revised manuscript.

Review of "Stochastic analysis of cone penetration tests in snow" by Lin et al

Main comments

The paper analyzes snow micro-penetrometer (SMP) signals in terms of stochastic jump-diffusion dynamics. The analysis is very interesting and adds novel aspects to the interpreation of SMP data. The paper is well written, the topic is suitable for TC and the results warrant publication. I have mainly two questions I would like to ask an answer for in the present work:

- Independence. Naively one is tempted to assume that the signal must be the result of one (and only one) underlying stochastic process, which is the disordered microstructure of snow. The present model rather assumes that the penetration leads to a situation with contributions from different stochastic processes which are assumed to be independent. While I can imagine how such a situation may originate from the physics, it would be helpful if this assumption of independence (between the jump characteristics and the diffusion) could be further assessed. Along these lines we have previously seen that interpreting the signal as a shot noise process with three parameters (λ, δ and f_0) we always end up with correlations between estimates of λ, and δ, which obviously cannot be convincingly separated by such a model. Since δ roughly translates to $1/D^{(1)}$ of the present model, I wonder if these parameters still show correlations. In addition,for large λ and small σ_ξ a jump process with drift could "tend" to diffusion. Therefore potential correlations of the latter parameters with $D^{(2)}$ are relevant too. In the simplest case it would be sufficient to provide mutual scatter-plots of estimated parameters λ, $D^{(1)}$, σ_ξ and $D^{(2)}$ for the profile in Sec 3, but maybe there are even rigorous ways to answer this question.

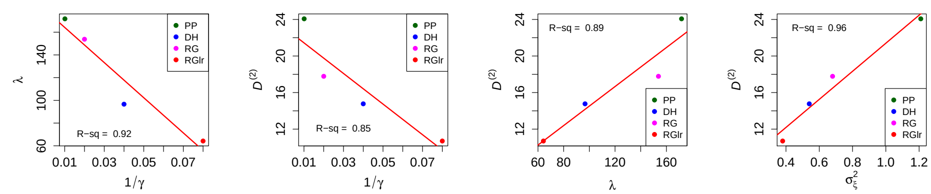
  Thank you for the interesting question. We look at the scatter-plots of the parameters $\frac{1}{\gamma}$ and λ, and $\frac{1}{\gamma}$, $D^{(1)}$, σ_ξ^2 and $D^{(2)}$ for all four snow types according to the results of Table 2. We also do the linear *regression of each scatter-plots and the coefficient of determination or R-squared is ≈ 0.9. In our current analysis, we assume the independence between diffusion and jump-diffusion which could give a good insight to interpret the cone penetration process. However, from these plots, we could see the hints for the possible correlations which could be considered as the improvement for current model and we would leave it as an open topic for the later analysis.*

In the jump-diffusion modeling of stochastic time series it is assumed that three random variables, $W(t)$ Wiener process, ξ jump size and $J(t)$ Poisson jump process, are independent. However in general they can be correlated. For instance for correlated $W(t)$ and $J(t)$ one finds

$$\langle W(t)J(t)\rangle = \rho(t)\sqrt{\lambda}t$$

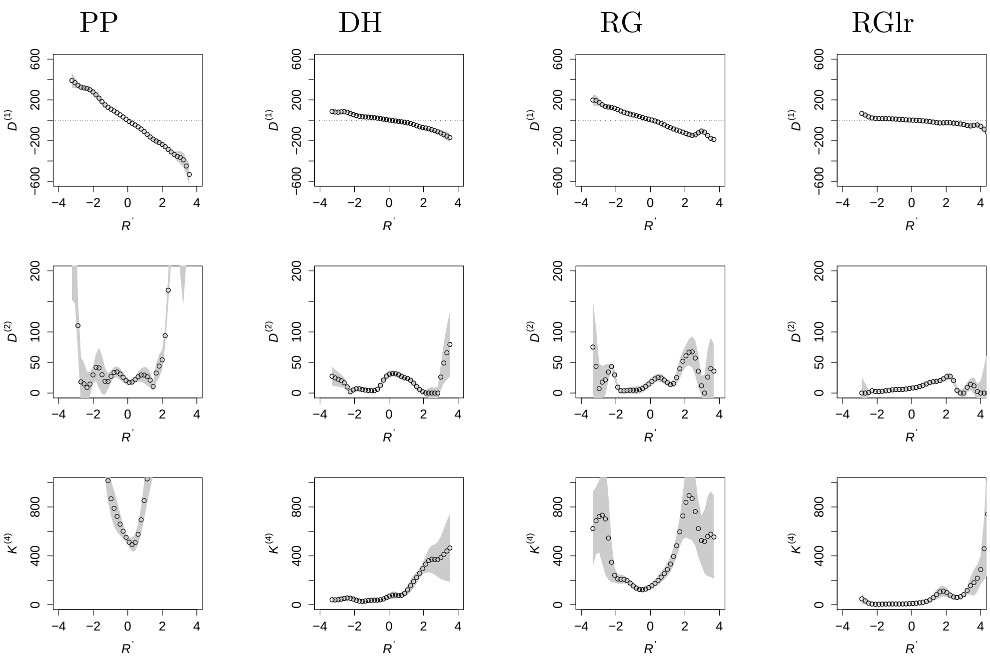
where $\rho(t)$ is the correlation coefficients of $W(t)$ and $J(t)$.

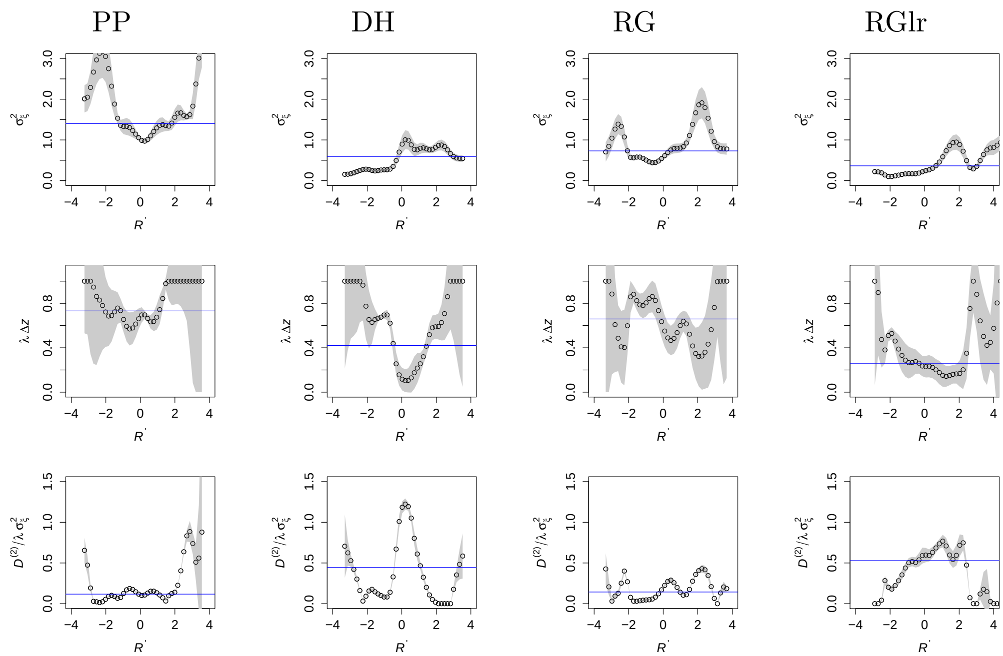
Data-based estimation of $\rho(t)$ is an open topic and we will address this important problem in the near future.



- Kernel width. It might be good to check if a fixed kernel width of 0.6 mm (l168) is a robust choice in view of the statements in the discussion about grain size dependencies: Converting the SSA values given in Tab 1 into a "grain size" (the optical diameter) reveals that diameters range from 0.12 mm for PP to 0.70 mm for RGlr. Now the diffusive contribution is assumed to be a result of steric (grain-grain) interactions in front of the cone, and this process will need a few grain diameters to develop. A fixed kernel of this particular size might thus induce a bias here. A priori grain size information is clearly commonly not available (like for the analysis of hardness profile in Sec 3.2). But it seems relevant to compare, at least for the data from Sec 3.1, how the parameter estimates for the 4 samples compare with those generated from a constant ratio of kernel width and optical diameter. Results could be simply added to existing figures.

Here we calculate the grain size or optical diameter based on the relation $d_{\mathrm{opt}} = \frac{6}{\rho_{\mathrm{ice}} \cdot \mathrm{SSA}}$*. Then, we calculate the average value of SSA for each snow types and find the average grain sizes of {PP, DH, RG, RGlr} to be {0.14, 0.40, 0.29, 0.66} mm. Now we use these grain sizes as the kernel widths for detrending of the snow hardness profiles and the new results for Fig 8 and 9 and Table 2 are as follow:*





Snow type	$L_\mathrm{C} = \frac{1}{\gamma}$ [mm]	$L_\mathrm{J} = \frac{1}{\lambda}$ [mm]	$\overline{D^{(2)}}$	$\overline{\sigma_\xi^2}$	$\overline{\frac{D^{(2)}}{\lambda\sigma_\xi^2}}$
PP	0.008	0.005	28.59	1.40	0.12
DH	0.035	0.010	15.66	0.59	0.44
RG	0.017	0.006	15.94	0.73	0.14
RGlr	0.080	0.016	10.24	0.36	0.53

The new kernel widths do not change the results significantly. Therefore, we decide to keep the constant kernel width of 0.6 mm in order to make it consistent with the field measurement data. The chosen kernel width is also within the range of the smallest and the largest grain size. If we choose the kernel width that is much larger than grain size, there would lead to oversmoothing and the detailed dynamics of cone penetration test could be lost. We also added the remark to the paper, that the results do not change significantly if the kernel width are changed between 0.14 mm and 0.66 mm.

Kind regards, Henning Löwe

Minor comments

(l117): I don't entirely understand why an R dependence of the coefficients is introduced here. Isn't the analysis later only based on constant coefficients, i.e. additive noise? Would everything work also for multiplicative noise?

Our analysis works for multiplicative noise. From our results we can see that the parameters are R dependent (Fig. 8 and 9). When we interpret the results in Table 2, we took the average value as a first order approximation of the parameters in the range of $-2 < R' < 2$ *in order to compare the results of each snow type.*

(l135): Here it might be illustrative to explicitly mention the "triply-stochastic" nature of Eq 11 and that all (ξ,J_t,W_t) are independent.

We now write: "Here, we have triply stochastic processes W_t*,* J_t *and* ξ *which are all independent of each other."*

(Tab 2): Here uncertainties/errors should be included that reflect inter-sample variations of the same snow type.

Uncertainties are now included in Table 2.

(l174): What is the final size of the sub-samples? Is this choice also consistent with grain size $\ll$ sample size in all cases?

The final sizes of the sub-samples vary from (680 - 1500) sample points i.e. (2.72 - 6) mm. The grain sizes varies approximately from (0.14 - 0.66) mm.

(l189): It would be nice to include the correlation lengths estimated from the ACF also in Tab 2 to support this statement. (DH and RGlr appear to be very similar in Fig 8 while the L_c differ by a factor of two)(Fig 9): What is taken as Δz?

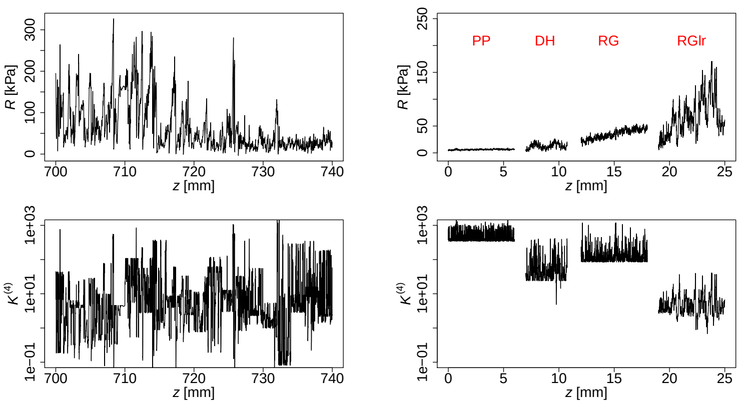
The correlation lengths L_ACF *estimated from the ACF,* (*PP, DH, RG, RGlr*) $= (0.006, 0.025, 0.016, 0.038)$ mm*, are added in the text and Table 2 in the revised manuscript.* Δz *is the resolution of SMP which is 4* μm.

(Fig 10): Top left, this looks like R and not R'?

Corrected.

(Fig 10): Maybe a semilog y scale for $K^{(4)}$ better reveals the differences?

Here is the figure for $K^{(4)}$ *in semilog y scale and we decide to keep it in linear scale since we can observe the differences in* $K^{(4)}$ *between the snow* *types more clearly in linear scale.*



(Fig 10): It would be good to include also $\lambda\,\Delta z$ and $D^{(2)}/\lambda\sigma_\xi^2$ in this figure. The subfigures can be safely reduced a bit in height.

Since the uncertainties of λ are relatively large especially for the extreme values of R', we mainly determine $K^{(4)}$ and σ_ξ^2 which could give more consistent results.

(l234): This is such a statement which might be affected by the choice of the kernel width...

As shown in the main comment, our choice of kernel width does not significantly change the results.