The parameters used in this module are included in Table 1 and are the same as those used in the englacial component of MouSh, apart from the flow law parameter A_{sub}. In the englacial system, A is calculated from local temperature within the ice column, which can be as cold as −23 °C in western Greenland (Iken et al., 1993). This contrasts with the temperature at the ice–bed interface, which must be at the melting point; the subglacial component of MouSh uses a fixed A_{sub} value.

In its current configuration, the subglacial module provides a single set of outputs representative of conditions at the moulin. This is primarily because this study focuses on the evolution of a moulin and is not representative of a channel running from a moulin to the terminus in a natural system. A more complex subglacial model would more accurately resolve the spatial changes in subglacial channel geometry and flow.

2.5 Suites of model experiments

To examine the sensitivity of the MouSh model to uncertain parameters, ice and meltwater characteristics, and model choices and difference from previous moulin parameterizations, we run four suites of experiments. While these experiments do not cover the complete range of possibilities, they were designed to address primary uncertainties in the MouSh model and examine how moulin geometry might vary spatially and temporally.

2.5.1 Quasi-equilibrium and the impact of diurnal supraglacial variability

Under steadily varying conditions such as a repeating diurnal variation, the modeled moulin reaches a quasi-equilibrium state independent of initial conditions with melting opposing viscous and elastic deformation and the only change being driven by shear deformation. Moulin water level and shape respond to these patterns of variability. To examine the impact of Q_{in} magnitude (mean) and Q_{in} amplitude (variability), we perform a series of model runs that vary the magnitude of a cosine curve between 1 and 20 $m^3\,s^{-1}$ with a fixed amplitude of 0.5 $m^3\,s^{-1}$ and a series of runs that vary the amplitude of a cosine curve between 0 and 2 $m^3\,s^{-1}$ with a fixed magnitude of 5.0 $m^3\,s^{-1}$. The amplitude is one-half the diurnal range. These runs use Basin 1 ice conditions (Table 2; Sect. 2.5.3). Further details can be found in Supplement Sect. S2.1 and Figs. S2–S4.

2.5.2 Sensitivity to uncertain parameters

We explored the sensitivity of our results to the values of seven parameters, shown in Figs. 3–5, with the prescribed ranges shown in Table 1. We examined the effect on the water level, the moulin radius at the equilibrium water level, the volume and water storage of the moulin, and the cross-sectional area of the subglacial channel at the end of a 40 d model run. These values reach equilibrium, with daily oscillations superimposed, after ∼ 15 d. We also tested the dependence of our results on the initial moulin radius, r_0, which we varied across an order of magnitude from 0.65 to 5.0 m.

We varied the value of a uniform deformation enhancement factor F^* over an order of magnitude ($F^* = 1$ to 9), which affects viscous flow of the ice surrounding the moulin. While the range of enhancement factors tested here cover a variety of ice conditions, including ice shelves and temperate glaciers, the GrIS likely has values between 4 and 6 (e.g., Cuffey and Paterson, 2010). Outside of testing the model sensitivity to the enhancement factor, we assigned $F^* = 5$. We also tested the effect of ice temperature, independent of the enhancement factor. We used five different temperature profiles: cold ice temperatures (mean ∼ −15 °C, range −23.1 °C to the pressure melting point) measured in the center of Jakobshavn Isbræ (Iken et al., 1991 TS4); moderate ice temperatures (mean ∼ −7 °C, range −13.5 °C to the pressure melting point) measured at the GULL site in Pâkitsoq (Lüthi et al., 2015; Ryser et al., 2014); warmer ice temperatures (mean ∼ −5 °C, range −9.3 °C to the pressure melting point) measured at the FOXX site in Pâkitsoq (Lüthi et al., 2015; Ryser et al., 2014); a hypothetical linear profile from −5 °C at the surface to 0 °C at the bed; and, finally, a fully temperate ice column. These different ice temperature scenarios affected the creep closure rates of ice through the temperature-dependent softness parameter A by approximately a factor of 6 from the coldest profile (Iken et al., 1993) compared to the fully temperate column.

We also examined moulin sensitivity to elastic deformation by varying Young's modulus (E) of the ice column between 1–9 GPa (Vaughan, 1995) and the sensitivity to the values of friction factors for the moulin walls. MouSh has two friction factors: f_m (below the water line) and f_{oc} (above the water line). We varied these friction factors across 2 orders of magnitude (0.01 to 1). We did not vary the subglacial channel friction factor. Finally, we varied values for basal ice softness A_{sub} over 2 orders of magnitude (5×10^{-25} to 5×10^{-23} TS5) and independently examined moulins over a range of ice thicknesses (670–1570 m) and corresponding distance from the terminus (∼ 20–110 km), which in combination results in variations in hydraulic gradient.

2.5.3 Sensitivity to local conditions

We examined moulins over a range of ice thicknesses and corresponding distances from the terminus (Table 2). Each moulin is associated with a supraglacial basin derived by Yang and Smith (2016). ~~The moulins were selected based on ice thicknesses that broadly represent the range of ice thicknesses within the ablation zone of the western GrIS and supraglacial drainage basin sizes and geometries that were visually similar to nearby drainage basins and approximately representative of the mean supraglacial drainage basin area for the given ice thicknesses (553, 741, and 1315 m).~~ To de-

At any given depth, viscous deformation and phase change due to melting are similar below the waterline; however, the diurnal variation in these parameters is quite different (Fig. 5g). At the mean water level, moulin growth due to melting varies less than $0.04\,\mathrm{m\,d^{-1}}$, with the shape of the diurnal variability dependent on the parameterization of melting both above and below the water line. In contrast, viscous deformation displays diurnal variations between $0.08\,\mathrm{m\,d^{-1}}$ in the thinnest ice and more than $0.21\,\mathrm{m\,d^{-1}}$ in the thickest ice.

3.3 Moulin shape in different environments

We modeled the seasonal growth and collapse of moulins in a range of environments across the GrIS using realistic melt forcings derived for the 2019 melt season (Sect. 2.5.3). These model runs varied with respect to ice thickness, moulin distance from the terminus, baseflow, and magnitude, diurnal range, and seasonal evolution of supraglacial inputs (Table 2; Fig. 6a). Overall, we find that moulin setting affects the scale of diurnal and seasonal variability in the size and water capacity of moulins as well as the evolution of subglacial channels (Figs. 6 and 7).

The sizes of all three modeled moulins reach equilibrium with the melt forcing within ∼ 15 d of the onset of the melt seasons (Fig. 6b and c). As the water flux increases over the next few weeks, each moulin grows in response to increasing supraglacial inputs, both diurnally and with a long-term trend, although this growth is more significant in thicker ice (Figs. 6c and 7). The subglacial channel grows with a similar pattern, but interestingly, the setting and fluxes of Basin 1 and Basin 2 result in very similar subglacial channel cross-sectional areas despite different moulin water levels and capacities (Fig. 6d).

Although the three moulins all evolve in a similar fashion, there are differences in moulin capacity, water level (Fig. 6), overall moulin geometry (Fig. 7), and the magnitude of englacial deformation (Fig. 8). Basin 3 exhibits the largest seasonal change in moulin capacity in part because a lower supraglacial input and subglacial hydraulic gradient result in a smaller subglacial channel and periods where moulin water level is above flotation (Fig. 6). This causes substantial variability of viscous deformation while limiting variations in melt due to changing moulin water level (Fig. 8a). One of the largest periods of Basin 3 moulin growth occurs starting at day 30. During this period, supraglacial inputs experience a step change (Fig. 7a); moulin water levels stayed near flotation and were less variable for several days (Fig. 7b), keeping effective pressure near zero and retarding deformation (Fig. 8a). In this case, viscous deformation hovers around zero and causes moulin opening, resulting in a high ratio of elastic to viscous deformation and a high ratio of phase change to viscous deformation (purple line in Fig. 8b). Similar behavior also occurs around day 110. Basins 1 and 2 exhibit smaller seasonal variations in moulin capacity because the ratio of melting to deformation stays near 1 until near the end of the season (Fig. 8b). This occurs because viscous deformation in Basins 1 and 2 is only slightly lower than in Basin 3, and melt rates tend to be higher (Fig. 8a) due to increased subglacial discharge associated with a higher hydraulic gradient. Further, there are fewer periods where water levels above flotation drive viscous opening.

Each moulin has a different daily mean capacity (Fig. 7c). This, in addition to differences in supraglacial inputs, ensures that daily moulin water level variations are substantially different across moulins. Basin 1 exhibits the largest variation in daily moulin water level, followed by Basin 2 (Fig. 9a). Basin 3 shows the lowest daily change; however, this is due at least in part to the fact that water overtops the moulin nearly daily (Figs. 6b and 7m and n). Changing water levels drive changes in moulin and subglacial capacity. Over the melt season, daily change in moulin capacity can be as low as 2 % during lulls in diurnal melt variability (Basin 3) or as high as 12 % following a recovery from a low melt day (Basin 1; Fig. 9b). However, in general all moulins display a similar daily change in capacity of ∼ 5 %–10 %, ~~with peak values of 12 % to 13 %~~.

The subglacial system undergoes diurnal variations in channel size between 1 % and 20 % (Fig. 9c). These changes are similar in magnitude to daily capacity changes within the moulin but exhibit more variability across ice thicknesses. Like changes in moulin capacity, these variations are related to the daily changes in moulin water level (Fig. 9a). This suggests that the time evolution of moulin geometry dampens the diurnal pressure fluctuations that drive subglacial channel growth and collapse. Evidence for this can be seen in the temporal pattern of moulin water level and subglacial channel cross-sectional area (Fig. 9a and c).

3.4 Comparison to cylindrical moulins

To examine the role moulin evolution plays in modifying the subglacial hydrologic system, we compared moulin water levels, moulin capacity, and subglacial channel size between model runs with a fully evolving moulin and runs with a static cylindrical moulin. We performed these tests with realistic melt inputs based on the 2019 melt season (Sect. 2.5.3), at moulins with low and moderate ice thicknesses (553 m – Basin 1 and 741 m – Basin 2). We defined the radius of the static cylinder as the mean radius at the mean water level: 1.6 and 1.4 m for Basins 1 and 2, respectively. This results in fixed moulin cross-sectional areas (∼ 6 to 8 $\mathrm{m^2}$) within the range of spatially invariant moulin cross-sectional areas ∼ 2–10 $\mathrm{m^2}$ often prescribed in subglacial models (e.g., Andrews et al., 2014; Banwell et al., 2013; Bartholomew et al., 2012; Cowton et al., 2016; Meierbachtol et al., 2013; Werder et al., 2013).

Comparison of moulin water level and capacity between static cylindrical and evolving moulins shows differences on both the diurnal and seasonal timescales (Fig. 10). The differences in moulin water level (both positive and negative) are generally great during lower supraglacial inputs at the beginning and end of the melt season, with the relatively limited differences occurring during the highest discharges (Fig. 10a and b). These values are both positive, indicating that the static radius moulin has higher water levels, and negative, indicating that the evolving moulin has higher water levels. Differences in moulin water level can reach nearly 20 m but are most commonly below 10 m. The seasonal mean water level difference between the static cylindrical and evolving moulin in both basins is less than 1 m.

Moulin capacity also displays a clear seasonal pattern; in both basins, the static cylindrical moulin is larger than the evolving moulin at the beginning of the melt season, with the evolving moulin gradually growing larger as the melt season progresses (Fig. 10c and d). After peak melt (day ~ 60), the evolving moulin begins to viscously close and gradually becomes smaller than the static cylindrical moulin. The static cylindrical moulin can be more that 100 % larger than the variable moulin during the tails of the melt season, with the evolving moulin becoming 36 % and 42 % larger than the static cylindrical moulin during mid-melt season. Overall, the mean capacity difference between the static cylindrical and evolving moulin is less than 5 %, with the static cylindrical moulin being slightly larger.

The radius of the cylindrical moulin was chosen to minimize differences with the evolving moulin. This is evident by the limited long-term differences between the two moulins in both Basin 1 and Basin 2. As such, there are limited differences (< 1 %) between the modeled subglacial channels. We expect the difference in moulin water level, moulin capacity, and subglacial geometry to change if the static cylindrical moulin geometry is poorly chosen, if the different or different experimental parameters are used, or if the setting changes (e.g., different hydraulic gradients). For example, we use commonly used values of ice softness A for both the moulin and subglacial channel; however, these values are poorly known, and their choice can directly impact the relative importance of moulin shape in dictating moulin water levels and subglacial channel size (Fig. 4).

3.5 Impact of model choices on moulin geometry

Chosen parameterizations have the potential to impact the representation of moulin water level and capacity (Supplement Sect. S2). Overall, we find that a circular geometry has limited impact on moulin water level, with the circular geometry having water levels that are less than 3 m higher than the egg-shaped geometry, although in nearly all instances the difference is less than 0.5 m (Fig. S5a); however, the impact on capacity is slightly larger (the circular moulin is up to 31 % smaller) and displays a seasonal trend as the egg-shaped moulin elongates along its elliptical axis (Fig. S5b).

Elastic deformation within the moulin is small (Supplement Sects. S1 and S2.2.3; Fig. 8a). Excluding elastic deformation has a negligible impact on moulin water levels and moulin capacity (< 1 %; Fig. S5c and d).

In contrast to the previous choices, the distance from the terminus L and the prescribed baseflow Q_{base} can have a substantial impact on moulin water level and capacity (Fig. S5e–h). Distance from the terminus is defined by the position of a given moulin on the ice sheet and as such is not a choice or parameter per se; however, it does directly influence the hydraulic gradient. A shorter L increases the hydraulic gradient and reduces both moulin water levels and capacities (Fig. S5e and f). Baseflow is used here to mitigate the use of a simplistic subglacial hydrology model. Reducing the baseflow within the subglacial system increases moulin water levels and reduces moulin capacity (Fig. S5g and h).

Finally, we examine the impact of fixing the subglacial channel cross-sectional area S. Experimental results using a fixed S and a seasonally evolving melt curve resulted in unrealistically low or zero water levels during low, early season Q_{in} and complete viscous collapse of the moulin if the subglacial channel size was prescribed to be too large, or persistently high (always above the ice thickness) water levels and runaway moulin growth if the subglacial channel was prescribed to be too small. Therefore, we explore the impact of fixing S using a constant mean Q_{in} with an overlaid diurnal variability (Supplement Sect. S2.2.6). With constant variability, we can easily prescribe the fixed S to be the mean value of the time-varying subglacial channel S (1.95 m). In this instance, the fixed S experiment displays a similar mean moulin water level but lower diurnal variability than the experiment with a time-varying S (Fig. S6). Further details are included in the Supplement Sect. S2.

4 Discussion

4.1 Timescales of moulin formation and evolution

We consider the formation timescales of moulins in the context of the shape evolution of a mature moulin. Using MouSh, we find that in the absence of external forcing, such as time-variable Q_{in}, the size of a moulin reaches its equilibrium value in ~ 15 d depending on ice and supraglacial input conditions and initial moulin geometry (Figs. 5g, S2 and S3). This relaxation time is comparable to the Maxwell time for ice (10–100 h), as expected for a linear visco-elastic system. Our relaxation time also compares well to the equilibration timescale defined by Covington et al. (2020) for their modeled moulin–subglacial conduit system, which Trunz (2021) found to be 1–20 d. The most realistically sized moulins in Trunz (2021) had relaxation times closer to 1 d. Their modeled system was governed solely by melt and viscous deformation ~~and lacked elastic deformation~~; however, elastic deformation in MouSh is small, explaining why our relaxation times are comparable.