Controls on Greenland moulin geometry and evolution from the Moulin Shape model

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Abstract. Nearly all meltwater from glaciers and ice sheets is routed englacially through moulins. Therefore, the geometry and evolution of moulins has the potential to influence subglacial water pressure variations, ice motion, and the runoff hydrograph delivered to the ocean. We develop the ***Mou***lin ***Sh***ape (MouSh) model, a time-evolving model of moulin geometry. MouSh models ice deformation around a moulin using both viscous and elastic rheologies and ~~models~~ melting within the moulin through heat dissipation from turbulent water flow, both above and below the water line. We force MouSh with idealized and realistic surface melt inputs. Our results show that, under realistic surface melt inputs, variations in surface melt change the geometry of a moulin by approximately ~~20~~10% daily and ~~by~~ over 100% seasonally. These size variations cause observable differences in moulin water storage capacity~~,~~ and moulin water levels~~, and subglacial channel size~~ compared to a static, cylindrical moulin. Our results suggest that moulins are important storage reservoirs for meltwater, with storage capacity and water levels varying over multiple timescales. ~~Representing~~Implementing realistic moulin geometry within subglacial hydrologic models ~~would~~may therefore improve the representation of subglacial pressures, especially over seasonal periods or in regions where overburden pressures are high.

1 Introduction

Surface-sourced meltwater delivered to the glacier bed drives the evolution of the subglacial hydrologic system and associated subglacial pressures (e.g., Iken and Bindschadler, 1986; Müller and Iken, 1973) The efficiency of the subglacial system, in turn, changes the flow patterns of the overlying ice on daily, seasonal, and multi-annual timescales (e.g., Hoffman et al., 2011; Iken and Bindschadler, 1986; Moon et al., 2014; Tedstone et al., 2015; Williams et al., 2020). Thus, glacial hydrology is a crucial factor in short-term changes to glacier and ice sheet dynamics (Bell, 2008; Flowers, 2018).

On the Greenland Ice Sheet~~,~~ (GIS), surface meltwater can take multiple paths, depending on its origin ~~location~~. In the accumulation zone, meltwater may percolate through snow and firn, remaining liquid (Forster et al., 2014) or refreezing (MacFerrin et al., 2019). In the ablation zone, meltwater runs over bare ice, coalesces into supraglacial streams, and pools into supraglacial lakes (e.g., Smith et al., 2015). These surficial water features – rivers, streams, lakes, aquifers, etc. – direct meltwater into englacial features that can deliver the water to the bed of the ice sheet (Andrews et al., 2014; Das et al., 2008; Miège et al., 2016; Poinar et al., 2017; Smith et al., 2015). Englacial features include moulins, which are near-vertical shafts with large surface catchments (~1–5 km^2 per moulin, Banwell et al., 2016; Colgan and Steffen, 2009; Yang and Smith, 2016), and crevasses, which are linear features with limited local catchments (~0.05 km^2 per crevasse, Poinar et al., 2017). Together, moulins and crevasses constitute a substantial fraction of the englacial hydrologic system in the ablation zone of the GIS.

Water fluxes through the englacial system, and therefore to the subglacial system, are non-uniform in space and time. Quantifying these temporal variations in water fluxes to the glacier bed requires understanding the time evolution of the supraglacial and englacial water systems that deliver it. Ongoing research is making great strides in characterizing the supraglacial water network (Germain and Moorman, 2019; Smith et al., 2017; Yang et al., 2016). For instance, field observations from Greenland indicate that much of the supraglacial water network terminates into crevasses and moulins (McGrath et al., 2011; Smith et al., 2015) and that moulins are important modulators of surface melt inputs to the ice sheet bed (Andrews et al., 2014; Cowton et al., 2013; Mejia et al., 2021; Smith et al., 2021).

Our knowledge of moulin sizes, scales, and time evolution has largely been informed by exploration and mapping of the top tens to hundred meters of a few moulins (Benn et al., 2017; Covington et al., 2020; Gulley et al., 2009; Holmlund, 1988; Moreau, 2009). These sparse field data indicate that moulin shapes deviate greatly from simple cylinders. Furthermore, deployments of tethered sensors into Greenland moulins have encountered irregularities including apparent ledges and plunge pools (Andrews et al., 2014; Covington et al., 2020; Cowton et al., 2013), and seismic (Röösli et al., 2016) and radar (Catania et al., 2008) studies suggest constrictions below the depths of human exploration. These direct near-surface and indirect deep observations suggest that moulin geometry evolves a high degree of complexity at all depths.

State-of-the-art subglacial hydrology models are forced by meltwater inputs that enter the system through crevasses or moulins. These models generally represent the geometry of moulins in a simplified and time-independent manner, for instance as a static vertical cylinder (e.g., Hewitt, 2013; Hoffman et al., 2016; Werder et al., 2013) or cone (Clarke, 1996; Flowers and Clarke, 2002; Werder et al., 2010). The basis for the cylindrical simplification arises from the assumption that depth-dependent variations in moulin size are small relative to the vertical scale of the moulin. The basis for time independence is the assumption that the moulin capacity is, again, small relative to that of the subglacial system. However, neither of these assumptions have been tested. Here, we explore the extent to which ~~time evolution of~~ an evolving moulin geometry ~~affects~~can impact moulin water level, capacity, and water volume, each of which can impact the ~~rate~~evolution of the subglacial ~~meltwater input and subglacial pressure in channelized regions of the bed.~~system.

We present the Moulin Shape (MouSh) model, a new, physically based numeric model that evolves moulin geometry over diurnal and seasonal periods. The MouSh model can be coupled to subglacial hydrology models to more completely characterize the time evolution of the englacial and subglacial hydrologic systems, which are intimately linked.

2 Moulin physical model

We develop the Moulin Shape (MouSh) model, a numeric model of moulin evolution that considers ice deformation and ice melt associated with the dissipation of energy from turbulently flowing meltwater (Fig. 1). We include here a detailed description of the model framework and each module that influences the time-evolving geometry of the modeled moulin (Fig. 2a).

2.1 Moulin geometry coordinate system

We discretize our model in the vertical (z) and radial (r_1 and r_2, or generally r_m) directions, treating the moulin as a stack of egg-shaped (semi-circular, semi-elliptical) holes in the ice that both change in size and move laterally relative to each other. We calculate moulin geometry (elliptical radii r_1 and r_2) and water level ~~(~~h_w~~)~~ with a 5-minute timestep dt. Model calculations are performed in cylindrical coordinates, where $\Pi(z)$ is the perimeter of the semi-circular, semi-elliptical moulin, using Ramanujan's approximation:

$$\Pi \approx \pi r_1 + \frac{1}{2}\pi[3(r_1 + r_2) - \sqrt{(3r_1 + r_2)(r_1 + 3r_2)}\,] \tag{1}$$

Here, r_1 and r_2 are the minor and major radii, respectively, for each node in the vertical direction. The minor radius r_1 is also the radius of the half-circle.

We calculate the cross-sectional area A_m of the semi-circular, semi-elliptical moulin as follows:

$$A_m = \frac{\pi r_1}{2}(r_2 + r_1) \tag{2}$$

The plan-view orientation of the radii and the coordinate system, as detailed on a remotely sensed moulin, are indicated in Fig. 2b-d. The elliptical shape was chosen to reflect the observation that supraglacial meltwater flows into a moulin along a single side above the water line. This asymmetry leads to a nonuniform, noncircular geometry above the water level~~, which can affect the total amount and evolution of water storage at high water levels.~~. This choice is in line with observations of a ~~Greenland~~GIS moulin becoming more elliptical over time (Röösli et al., 2016). For simplicity, MouSh also contains an option to set the moulin cross-sectional geometry to a circle, rather than an egg (see Supplement S2~~).~~.2.2).

Each module is also dependent on the depth varying hydrostatic and cryostatic pressures. We subtract the cryostatic pressure~~,~~ P_i~~,~~ from the hydrostatic pressure~~,~~ P_w~~,~~ to ~~get~~calculate the total depth-dependent ~~effective~~ pressure ~~N~~P at all vertical levels z within the moulin:

$$P_i = \rho_i g(H_i - \cancel{b}z) \tag{3a}$$

$$P_w = \rho_w g(h_w - \cancel{b}z) \tag{3b}$$

$$\cancel{N}P = P_w - P_i \tag{3c}$$

where H_i is the ice thickness; h_w is the height of the water above the bed~~;~~ (moulin water level); z is the vertical coordinate; ρ_i and ρ_w are ice and water density, respectively; and g is gravitational acceleration (Table 1). Note that P is not effective pressure, which is defined as $P = P_i - P_w$ (Cuffey and Paterson, 2010). In ~~this~~our formulation, positive pressures ~~causes~~cause outward expansion of the moulin walls (radial growth), and negative pressures reduces the size of the moulin (radial closure). We use a flat bed at sea level for all model runs presented here~~, so~~; bed elevation is $z = b = 0$.

2.2 Ice deformation modules

We represent the deformation of the ice with the simplest possible combination of elastic and viscous components: a Maxwell rheology, where elastic and viscous deformation occur independently, without interaction (Turcotte and Schubert, 2002). The Maxwell model comprises an elastic element (a spring) and a viscous element (a dashpot) in series and is standard in geophysical modeling. The response timescale in our Maxwell model is equal to $(E \times A \times \tau^2)^{-1}$ where E is Young's modulus, A is the viscous flow law parameter, and τ is stress (Table 1; Turcotte and Schubert, 2002). The Maxwell timescale is thus roughly 10–100 hours for typical Greenland ice. ~~On timescales shorter than the Maxwell timescale, ice deformation is primarily elastic. On longer timescales, viscous deformation dominates.~~

Elastic deformation is described in Sect. 2.2.1. We represent total viscous deformation in two modes: (1) radial opening and closure of the moulin, which changes the size of the moulin (Sect. 2.2.2), and (2) vertical shear of the moulin, which changes the shape but not the size of the moulin (Sect. 2.2.3).

2.2.1 Elastic deformation

Field measurements indicate that, nearly universally during the melt season, the water level in a moulin varies at a sub-hourly timescale (Andrews et al., 2014; Covington et al., 2020; Cowton et al., 2013; Iken, 1972). This variability is shorter than, but comparable to, the Maxwell timescale for ice (10–100 hours~~; see Sect. 2.2~~); therefore, we must assume that elastic deformation plays a role in the response of the ice to variations in moulin water level.

Weertman (1971, 1973, 1996) applied dislocation fracture mechanics principles to vertical glaciological features: water-filled crevasses. These equations have applied to supraglacial lake drainages (Krawczynski et al., 2009) and slow ice hydrofracture (Poinar et al., 2017). However, these problems are Cartesian (linear), not cylindrical, so their solutions are not readily adaptable to a moulin. The stress and deformational patterns around cylindrical boreholes have been well studied in the rock mechanics literature (Amadei, 1983; Goodman, 1989; Priest, 1993). We therefore base our description of the stress field surrounding the moulin on that of a fluid-filled borehole in a porous rock medium, described by Aadnøy (1987) and based on the Kirsch equations, which describe stresses surrounding a circular hole in a rigid plate (Kirsch, 1898). We assume plane strain and approximate our moulin as a stack of such plates with analogous holes (Goodman, 1989). A subtle difference is that our moulin shape is not circular, but egg-shaped: half circular, half elliptical.

At each vertical level z in the moulin, we apply Hooke's Law to the stress field to calculate the strain, in horizontal cross-section, at all points on the moulin wall and in the surrounding ice for both radii r_1 and r_2. We then integrate these strains from an infinite distance (cylindrical coordinate $r = \infty$) to the moulin wall (r_1, $r_2 = r_m$). A full derivation, based on the stress states in a borehole described by Aadnøy (1987), is in Supplement S1. We express the total radial elastic deformation r_e of a moulin segment as:

$$r_e = \frac{r_m}{E}[(1+\nu\cancel{)(N})(\Delta P - \frac{1}{2}(\cancel{\sigma_x}\Delta\sigma_x + \cancel{\sigma_y}\Delta\sigma_y) + \frac{1}{4}\cancel{(\sigma_x}(\Delta\sigma_x - \cancel{\sigma_y}\Delta\sigma_y)(1-3\nu-4\nu^2) + \frac{1}{4}\cancel{\tau_{xy}}\Delta\tau_{xy}(2-3\nu-8\nu^2)] \quad (4)$$

Here, ~~E~~Δ P is ~~Young's modulus for uniaxial deformation~~the change in cryo-hydrostatic pressure (Eq. 3c) over a time interval, ν is Poisson's ratio~~,~~; r_m is used to refer to r_1 or r_2; and $\cancel{\sigma_x}\Delta\sigma_x$, $\cancel{\sigma_y}\Delta\sigma_y$, and $\cancel{\tau_{xy}}\Delta\tau_{xy}$ are ~~the~~changes in background deviatoric and shear stresses that describe the regional setting of the moulin ~~(typically compressive and of order 100 kPa; Poinar and Andrews, 2021).~~. The model is designed to accept user-defined deviatoric and shear stresses; however, we choose ~~values σ_x~~a neutral surface stress state ($\Delta\sigma_x \cancel{=}= \Delta\sigma_y = \Delta\tau_{xy}\ 0\,kPa$~~, $\sigma_y = 50\,kPa$, and $\tau_{xy} = -50\,kPa$. However,~~) for experimental simplicity and because ~~the deviatoric and shear~~these stresses and their changes over time are poorly constrained~~, we also test the~~. This simplification ~~$\sigma_x = \sigma_y = \tau_{xy} = 0$ (Supplement S2), which simplifies the~~reduces elastic deformation r_e:

$$r_e = \frac{r_m}{E}(1+\nu\cancel{)N})\Delta P \quad (5)$$

Unlike viscous deformation and melting, elastic deformation is instantaneous. However, we take advantage of the observation that elastic deformation is driven by changes in the cryostatic and hydrostatic pressures~~.~~ (Supplement S1.5). ~~Therefore, we express Eq. 4 and Eq 5 as an elastic 'deformation rate' for non-zero (Eq 6a) and zero (Eq 6b) stresses:~~

Therefore, we express Eq.

$$\cancel{dr_e = \frac{1}{E}\left(r_m(1+\nu)\frac{dN}{dt} + [(1+\nu)(N - \frac{1}{2}(\sigma_x+\sigma_y)) + \frac{1}{4}(\sigma_x-\sigma_y)(1-3\nu-4\nu^2) + \frac{1}{4}\tau_{xy}(2-3\nu-8\nu^2)]\frac{dr_m}{dt}\right)dt} \quad \cancel{(6)}$$

4 and Eq 5 as an elastic 'deformation rate' for varying (Eq. 6) and constant (Eq. 7) surface stresses:

$$\frac{dr_e}{dt} = \frac{1}{E}\left(r_m(1+\nu)\frac{dP}{dt} + [(1+\nu)(-\frac{1}{2}(\frac{d\sigma_x}{dt}+\frac{d\sigma_y}{dt}) + \frac{1}{4}(\frac{d\sigma_x}{dt}-\frac{d\sigma_y}{dt})(1-3\nu-4\nu^2) + \frac{1}{4}\frac{d\tau_{xy}}{dt}(2-3\nu-8\nu^2)]\right) \quad (6)$$

$$dr_e = \cancel{\frac{(1+\nu)}{E}\left(r_m\frac{dN}{dt} + N\frac{dr_m}{dt}\right)}\frac{r_m}{E}(1+\nu)\left(\frac{dP}{dt}\right)dt \quad (7)$$

Equations 6 and 7 assume that both effective pressure and moulin radius vary smoothly over the time interval in question, which is generally true for small timesteps (5-minutes in our model). ~~We apply Eq. 6 or 7 to both moulin radii, the semi-circular radius r_1 and the semi-elliptical major radius r_2, separately. When water is above the flotation level, elastic deformation opens the moulin at all depths below the water line. When the water level is below flotation, which is the typical case, elastic deformation closes the moulin at all depths. The values of the surface stresses $\sigma_x\sigma\sigma_x$, $\sigma_y\sigma\sigma_y$, and $\tau_{xy}\tau\tau_{xy}$ determine the sign of the deformation above the water line.~~The dominant term in Eq. 6 is the first term, since $\frac{dP}{dt}$ (~1 kPa over a typical hour during the melt season) greatly exceeds the rate of change of the surface stresses (~1 kPa over a year), as explained in the Supplement S1. Equation 7 is commonly used for dilatometer testing in rock mechanics (Goodman, 1989).

2.2.2 Viscous radial opening and closure

Moulins close when they lose their water source at the end of a melt season (Catania and Neumann, 2010). Similarly, boreholes close if they are not filled with drilling fluid with a density ~~similar to~~like ice (Alley, 1992). Our modeled moulin is intermediate to these edge cases because it typically contains water. When the moulin is filled with water to the flotation level, it will stay open at its base and viscously close at and below the water level. ~~When the water level is above flotation, the~~The moulin will viscously open in regions where hydrostatic pressure exceeds the cryostatic pressure. When the water level is below flotation, which is the typical case, viscous deformation shrinks the moulin at all depths.

We calculate strain rate ~~($\dot{\varepsilon}$)~~$\dot{\varepsilon}$ from the total depth-dependent ~~effective~~ pressure ~~N~~P (Eq. 3c) using Glen's Flow Law:

$$\dot{\varepsilon} = \cancel{F_*} F^* A(T_i, P_i) \cdot \cancel{\left(\frac{1}{3} N\right)^n} \left(\frac{1}{3} P\right)^n \tag{8}$$

where ~~F_*~~F^* is the flow law enhancement factor~~, and~~; $A(T_i,P_i)$ is the flow law parameter; and n is Glen's Flow Law exponent. For the flow law parameter, we use the standard relationship from Cuffey and Paterson (2010, Eq. 3.35), which is a function of ice temperature T_i and ice pressure P_i.

We follow borehole studies by Naruse et al (1988) and Paterson (1977) to write strain, ε, in the radial direction as

$$\varepsilon = ln\left(\frac{r_f}{r_0}\right) \tag{9}$$

where a moulin with initial radius r_0 and final radius r_f underwent radial strain of ε.

We use the time derivative of Eq. (9) to calculate the change in moulin radius due to viscous deformation:

$$dr_v = r_m \, exp(\dot{\varepsilon} \, dt) - r_m \tag{10}$$

with strain rate given by Eq. (8). This is the same relationship used by Catania and Neumann (2010).

2.2.3 Shear deformation

We use Glen's Flow Law to calculate the change in shape of the moulin due to regional-scale ice flow. This deforms the entire moulin in bulk, shearing it in the vertical and shifting it laterally downstream, without changing its radii. Basal sliding is not currently included in the model. To represent deformation, we discretize the moulin as a stack of plates with elliptical (or circular) holes with a thickness *dz* and represent deformational ice flow as displacement between these plates.

We calculate the rate of deformational ice flow u_d in the downstream direction from ice temperature ~~T~~T_i and pressure ~~N~~P_i, surface slope α, a constant ~~enhancement factor F_*, and ice thickness~~F^*, and H_i, using Glen's Flow Law (Cuffey and Paterson, 2010):

$$u_d = \cancel{2F_*(\rho_i g\alpha)^n} 2F * (\rho_i g\alpha)^n \cdot \int_b^{H_i} A(T_i, P_i)(H_i - z)^n dz \tag{11}$$

where b is the ice sheet bed. We obtain ice deformation rates of ~20 m yr^{-1}, which is typical of the ablation zone in western Greenland (Ryser et al., 2014).

2.3 Phase change modules

The second mode that changes the geometry of the moulin is ice ablation from or accretion to the moulin walls. During the melt season, the flow of water into and through the moulin generates turbulence, which as it dissipates acts to melt back the moulin walls, expanding the size of the moulin. There is also a small component of melting due to temperature differences between the water and surrounding ice. Outside the melt season, conduction of latent heat into the surrounding ice causes stagnant water to freeze back onto the moulin walls, contracting the size of the moulin.

2.3.1 Refreezing

Refreezing occurs in cold ice when water flow is absent or slow enough that the rate of heat conduction into the surrounding ice drops the water temperature to the freezing point. These conditions occur primarily outside the melt season. When these conditions are met, we apply a radial freezing ~~term~~r_f, which is parameterized economically, following Alley (2005):

$$dr_f = 2\frac{T_i - T_{pmp}}{L_f}\cancel{\sqrt{\frac{k_i C_p}{\pi \rho_i}}}\sqrt{\frac{K_i C_p}{\pi \rho_i}}\left(\sqrt{t_t} - \sqrt{t_t - dt}\right) \qquad (12)$$

Here, $T_i - T_{pmp}$ is the depth-varying difference between the far-field temperature (prescribed as from borehole temperature observations) and the moulin water temperature, which is taken as the pressure melting temperature T_{pmp}. L_f is the latent heat of fusion; K_i is water's thermal conductivity; C_p is the specific heat capacity of ice. The refreezing rates ~~thus~~ evolve exclusively based on the elapsed time since the cessation of turbulent flow~~,~~ t_t.

We calculate the change in moulin water volume from freezing, V_{frz}, by summing the refrozen ice thickness in a ~~timstep~~timestep, dr_f, around the perimeter of the moulin at all depths z, and converting ice volume to water volume:

$$V_{frz} = \frac{\rho_i}{\rho_w}\int_b^{h_w} \Pi(z) r_f(z)\, dz \qquad (13).$$

2.3.2 Moulin wall melting

During the melt season, turbulent energy dissipation from water flowing through the moulin melts back the moulin walls. The dissipation of turbulent energy and the associated melting of the surrounding ice will increase the local moulin radius. We parameterize turbulence in two separate spatial domains: (1) within the water column of the moulin, where r_1 and r_2 are evolved uniformly, and (2) above the water level along the side of the moulin, as supraglacial input falls to the water level, where only r_2 is evolved.

The parameterizations of turbulently driven melting we use in both regimes rely on three simplifications. First, the volume of water moving through each vertical model node is constant within each time step. This ensures that water mass is conserved and that all model elements below the water line are water filled; however, this eliminates the potential long-term storage of meltwater within plunge pools caused by non-uniform incision into the ice. Second, all energy generated from turbulent dissipation is instantaneously applied to melting the surrounding ice. This neglects any heat transport within the water, which is a common approximation in subglacial models (e.g., Hewitt, 2013; Schoof, 2010; Werder et al., 2013). Third, we also make the simplifying assumption that meltwater entering the moulin is at 0°C and at the pressure melting temperature T_{pmp} at all points below the water line, ~~though~~although we do not model the impact of this temperature change on melting because moulin water temperatures are unknown.

Submerged zone: Below the water line, the vertical velocity of the water is dictated by the hydraulic gradient within the system and the local cross-sectional area of the moulin. Under such conditions, head loss – the departure of the hydraulic head from that calculated by Bernoulli's equation – reflects the energy dissipated as heat. We parameterize head loss using the Darcy–Weisbach equation, which relates water velocity u_w to changes in the hydraulic gradient dh_w/dl (head loss per unit length along flow), via the hydraulic radius R_h and a dimensionless friction factor f. Because water velocity is constrained by mass balance within the system, we calculate the head loss dh_w/dl as follows:

$$\frac{dh_w}{dl} = \frac{u_w^2 f}{8 R_h g} \tag{14}$$

The differential element dl represents the path length over which the water experiences head loss: $\cancel{\partial l = \sqrt{\partial x^2 + \partial z^2}} dl = \sqrt{dx^2 + dz^2}$ for horizontal distance dx and vertical drop dz. The friction factor f is a unitless model parameter that controls the rate of head loss within the system. Its value thus directly affects the amount of melting. Most subglacial models fix the Darcy–Weisbach friction factor, with values ranging from 0.01 to 0.5 (e.g., Colgan et al., 2011b; Schoof, 2010; Spring and Hutter, 1981) or use equivalent values of Manning's n (e.g., Hewitt, 2013; Hoffman and Price, 2014). Alternatively, other models parameterize channel roughness using a geometry-dependent friction factor (e.g., Boulton et al., 2007; Clarke, 2003; Flowers, 2008). Thus, MouSh has options for fixed or variable f.

The friction factor within the submerged zone is indicated by f_m and in the open channel zone by f_{oc}. To explore the impact of the chosen friction factor, we complete a sensitivity study (Sect. ~~2.3 and 3.2) where we vary the friction factor in water filled sections, f_m, over an expected range, centered on f_m = 0.1.~~2.5.2 and 3.2). We use a constant $f_m = 0.1$ for all other model runs presented.

Because we approximate the moulin as a half-circular, half-elliptical cylinder with perimeter Π, the hydraulic radius R_h of a water filled node is:

$$R_h = \frac{A_m}{\Pi}$$ (~~16~~15).

To calculate moulin wall melting, we use a simple energy balance equation, following previous work (e.g., Gulley et al., 2014; Jarosch and Gudmundsson, 2012; Nossokoff, 2013):

$$\rho_i C_w (T_{pmp} - T_i) \frac{dA_m}{dt} + \rho_i L_f \frac{dA_m}{dt} = \cancel{Q\left(\rho_w g \frac{dh_w}{dl}\right)} Q_{out}\left(\rho_w g \frac{dh_w}{dl}\right)$$ (~~17~~16)

where C_w is the heat capacity of water. The first term represents the energy needed to warm the surrounding ice to the pressure melting temperature of water T_{pmp}. Equation (~~17~~16) can be rearranged and combined with equation (14) to provide the area of ice melted:

$$dA_t = Q_{out}\left(\rho_w g \frac{u_w^2 f}{4R_h g}\right)\left(\rho_i C_w (T_{pmp} - T_i) + \rho_i L_f\right)^{-1} dt \qquad (\sout{18}17)$$

~~Where~~where Q_{out} is the discharge from the moulin-subglacial system as dictated by the subglacial model component (Sect. 2.4.2); and ~~$T_i - T_{pmp}$~~$T_{pmp} - T_i$ is the temperature difference between the water (prescribed to be at the pressure melting point) and the surrounding ice~~, which we can~~. We vary ~~from site to site around Greenland~~T_i based on observations as described in Table 1~~.~~ and Sect. 2.5.2. Note that Eq. ~~18~~17 determines the area of ice that is removed through melting. For each time step, we reframe Eq. ~~18~~17 into radial melting within an egg-shaped moulin using information about the previous geometry and the assumption that melting occurs uniformly around the perimeter:

$$dr_t = 2dA_t \,/\, \left[\pi\left(5r_1 + 3r_2 - \sqrt{(3r_1 + r_2)(r_1 + 3r_2)}\right)\right] \qquad (\sout{19}18).$$

Equation ~~19~~18 is simplified when considering a circular geometry ($r_1 = r_2$).

Unsubmerged zone: Above the water line, a variety of complex processes drive melting. A first-principles approach would be to quantify melting due to the potential energy loss of falling water, following the work on terrestrial waterfalls (e.g., Scheingross and Lamb, 2017). However, nearly all waterfall-based parameterizations rely on abrasion between waterborne sediment and the substrate as the primary mechanism of erosion. Instead, we implement a simple parameterization for open-channel flow with the understanding that the complexities of thermal erosion are not completely captured. In our model, open-channel melting occurs only on the up-glacier wall of the moulin and follows two ad-hoc rules based on the slope between the vertical nodes: (1) open-channel turbulent melting is applied if the slope of the upstream moulin wall allows water to flow over it; and (2) a small, prescribed amount of melting is applied when the upstream wall slope is vertical or overhung, because while water cannot flow directly along the ice, spray and other processes likely drive some amount of melting. These cases are respectively (1) the open-channel zone and (2) the falling water zone.

In the open-channel zone, we use a similar approach as for melting below the water line. However, the hydraulic radius R_h is adjusted to reflect the observation that water runs down only one wall of the moulin, and a higher friction factor is used to parameterize complex geometries. Due to the presence of a discontinuity between open-channel and water-filled regions (at the water line), we parameterize the hydraulic radius of open channel flow as $R_{h_{open}} = 0.5r_2$. We also use a higher open channel friction factor f_{oc} of 0.8 to parameterize observed extensive scalloping (e.g., Gully et al., 2014; Covington et al., 2020). We apply melting to only the elliptical side of the moulin, defined by r_2 ~~derived~~ using Eq. 18. Note that the hydraulic radius prescribed for open-channel flow is likely larger than the small region over which water is flowing in the natural system (Fig. ~~2a,d~~2). Further, the resulting open channel melt dA_{oc} is applied only to the major radius to calculate the change in open channel radius dr_{oc}.

In the falling water zone, there is very limited interaction between the moulin walls and the water. For simplicity, we assume that a small fraction, f_p, of the potential energy lost as water falls is deposited into the moulin walls, perhaps as the kinetic energy of spray. The change in radius due to this process is as follows:

$$dr_{mf} = f_p \cancel{\frac{(\rho_w/\rho_t)gQ}{L_f \Pi}} \frac{(\rho_w/\rho_i)gQ_{out}}{L_f \Pi} \mathrm{dt}$$

(~~20~~19)

We set f_p to 0.1 for all model runs presented here.

We add the volume of ice melted to the water already in the moulin, similarly to Eq. 12 for V_{frz}. We calculate the change in moulin water volume from melting by summing the melted ice thickness, r_{mf}, around the perimeter of the moulin at all depths z, and converting ice volume to water volume:

$$V_{wallmelt} = \frac{\rho_i}{\rho_w} \cancel{dt \int_b^{H_t} \Pi(z) r_{mf}(z) + A_{oc}(z) + A_t(z)\, dz}$$

$$\cancel{(21} dt \int_b^{H_i} (\Pi(z) dr_{mf}(z) + A_{oc}(z) + A_t(z))\, dz \qquad (20).$$

2.4 Water flux into and out of the moulin (Mass conservation)

Water balance within the moulin and the subglacial channel is dictated by recharge from a supraglacial stream (Q_{in}, ~~Sect. 2.4.1~~described below), discharge through a subglacial channel (Q_{out}, Q_{base}; ~~Sect. 2.4.2~~described below) and any change in volume due to melting or refreezing, such that the volume of water in the system (V) is:

$$\frac{dV}{dt} = Q_{in} - Q_{out} + Q_{base} + \cancel{\frac{(dV_{wallmelt} - dV_{frz})}{dt}} \quad \cancel{(22} \frac{(dV_{wallmelt} - dV_{frz})}{dt} \qquad (21).$$

The ~~integral~~final term varies in space and time, with high ~~melt~~ rates of volume lost to melt above the water line during the melt season (when $Q_{in} > 0$), and moderate ~~melt~~ rates of volume lost to melt at and below the water line during and after the melt season, when there is water flow through the moulin ($Q_{out} > 0$) and refreezing below the water line throughout the winter (when $Q_{in} = Q_{out} = 0$). The MouSh model can also accept an additional prescribed ~~base flow~~baseflow Q_{base} directly to the subglacial module. We design ~~base flow~~baseflow as a loose approximation of additional subglacial water inputs from varied upstream sources, including other moulins on the same subglacial channel, regional basal melt, and the addition and removal of meltwater from subglacial storage. ~~Base flow~~Baseflow is generally required to maintain realistic moulin water levels. In the moulin runs forced by realistic Q_{in}, we represent subglacial flow from ~5 surrounding moulins by prescribing ~~base flow~~baseflow as five times the running 5-day mean of Q_{in}. In other model runs, we do not include ~~base flow~~baseflow. The addition of ~~base flow~~baseflow is designed to represent the widespread seasonal evolution of surface melt; its inclusion maintains a slightly larger subglacial channel than would otherwise occur, which reduces otherwise unrealistically large daily swings in modeled moulin water level~~.~~ (Supplement S2.2.5).

2.4.1 Meltwater runoff from the ice-sheet surface

We force the MouSh model with time-varying water inputs from the supraglacial environment, Q_{in}. We use two different Q_{in} scenarios: a simple diurnal cosine with ~~maximum and minimum discharges ranging between ~1 and~~a mean discharge of 5 m^{3}s^{-1}, in rough agreement with observations near the margins of the GIS (Eq. ~~23~~22, Chandler et al., 2013; McGrath et al., 2011; Smith et al., 2017); and realistic supraglacial discharge over a melt season, determined by using in-situ surface melting data and internally drained catchment size and geometry (Yang and Smith, 2016).

We use the following cosine curve to represent our simplest form of supraglacial discharge into the moulin during sensitivity studies:

$$Q_{in} = cos(\pi(t-19.5)/12) + \cancel{3}\,5 \qquad (\cancel{23}\,22)$$

Here, t is time in hours and Q_{in} is in m^3 s^{-1}. This function has its daily peak at 19:30 hours and a daily minimum at 07:30.

To examine a set of realistic moulins, we select three supraglacial basins from Yang and Smith (2016) and ~~derive~~extract their size, distance from terminus from information provided therein (Basin 1-3; Table 2). We derive surface runoff from MERRA-2 reanalysis (Gelaro et al., 2017; Smith et al., 2017). ~~Surface runoff values for the 2019 melt season were modified using a synthetic unit hydrograph derived for the ablation zone and parameters appropriate for western Greenland (Table 2, Smith et al., 2017). The use of a unit hydrograph parameterizes the time and magnitude adjustments expected from meltwater routing over the ice surface. The parameters for the unit hydrograph were determined during the middle of the melt season and therefore may inaccurately represent routing delays at the beginning and end of the melt season. Further details on supraglacial and internal catchment characteristics are included in Sect. 2.5.2.~~Further details on supraglacial input characteristics are included in Sect. 2.5.3.

2.4.2 Water flow from the subglacial system

We couple the moulin model and a single evolving subglacial channel controlled by melt opening and creep closure (Covington et al., 2020; Schoof, 2010) using a reservoir-constriction model (Covington et al., 2012) that simulate flows between the moulin and subglacial channel. Following Covington et al. (2020), the rate of change of moulin water level h_w is

$$\cancel{\frac{dh_w}{dt}}\, dh_w = \frac{1}{A_m(h_w)}\cancel{\frac{\partial V}{\partial t}}\, dV \qquad (\cancel{24}\,23)$$

With the change in water volume within the system being dV and the volume of the moulin-subglacial system is related to the channel S and the moulin cross-sectional area A_m. The water volume is related to ~~Q_{in},~~ Q_{in}, Q_{base} and Q_{out}, where Q_{out} is the meltwater output from the subglacial channel, defined as follows:

$$Q_{out} = c_3 S^{5/4}\Psi/\sqrt{|\Psi|} \qquad (\cancel{25}\,24)$$

~~Here, S is the subglacial channel cross-sectional area.~~ The hydraulic gradient $\Psi = -\rho_i g \frac{d(h_w-b)}{dL}$ is a linear gradient in h_w to the outlet at a horizontal distance L, where the pressure head is zero. In our calculations, the bed elevation b is zero. Finally, c_3 is a flux parameter:

$$c_3 = \frac{2^{5/4}}{\pi^{1/4}}\sqrt{\cancel{\frac{\pi}{(\pi+2)\rho_w f}}}.$$

$$\cancel{(26}\sqrt{\frac{\pi}{(\pi+2)\rho_w f_{sub}}}. \qquad (25)$$

Equation (~~26~~25) for c_3 follows Covington et al. (2020), who corrected a small error from the original Schoof (2010) formulation.

We use an equation from Schoof (2010) for the time rate of change in subglacial channel cross-section area S, with the first part describing the turbulent melting of the subglacial channel walls, and the second term describing closure due to the pressure of the overlying ice, which is dependent on effective pressure $N = P_i - P_w$:

$$dS = (c_1 Q_{out}\Psi - c_2 N^n S)\, dt \qquad (\cancel{27}26)$$

Here, the constant $c_1 = \frac{1}{\rho_i L_f}$ ~~with ρ_i the ice density~~ and ~~L_f the latent heat fusion of ice,~~ the constant $c_2 = \cancel{2A(T_i,N)n^{-n}}2A_{sub}n^{-n}$ with the Glen's flow law parameters for the subglacial component defined as $\cancel{A = 6\cdot 10^{-24}}A_{sub} = 6\cdot 10^{-24}$ Pa^{-3}s^{-1}.

Replacing Q_{out}, Ψ, and N in Eq. (~~27~~26) yields

$$dS = c({}_1 c_3 S^{5/4}(\frac{\rho_w g h_w}{L})^{3/2} - c_2(P_i - \rho_w g h_w)^n S)\, dt \qquad (\cancel{28}27)$$

Equations (~~24~~23) and (~~28~~27) are numerically solved simultaneously, as in Schoof (2010) and Covington et al. (2021). The parameters used in this module are included in Table 1 and are the same as those used in the englacial component of MouSh, apart from the flow law parameter ~~A~~A_{sub}. In the englacial system, A is calculated from local temperature within the ice column, which can be as cold as -23°C in western Greenland (Iken et al., 1993). This contrasts with the temperature at the ice-bed interface, which must be at the melting point; ~~thus,~~ the subglacial component of MouSh uses a ~~higher~~ fixed ~~A~~A_{sub} value.

In its current configuration, the subglacial module provides a single set of outputs representative of conditions at the moulin. This is primarily because this study focuses on the evolution of a moulin and is not representative of a channel running from a moulin to the terminus in a natural system. A more complex subglacial model would more accurately resolve the spatial changes in subglacial channel geometry and flow.

2.5 Suites of model experiments

To examine the sensitivity of the MouSh model to uncertain parameters, ice and meltwater characteristics, and model choices, and difference from previous moulin parameterizations, we run four suites of experiments. While these experiments do not cover the complete range of possibilities, they were designed to address primary uncertainties in the MouSh model and examine how moulin geometry might vary spatially and temporally.

2.5.1 Quasi-equilibrium and the impact of diurnal supraglacial variability

Under steadily varying conditions such as a repeating diurnal variation, the modeled moulin reaches a quasi-equilibrium state independent of initial conditions with melting opposing viscous and elastic deformation and the only change being driven by shear deformation. ~~We examine the quasi-equilibrium state and the impact of supraglacial variability on this state. Supraglacial runoff Q_{in} is highly variable with seasonal, event, and diurnal variability in surface melting modified by supraglacial drainage basin characteristics as it is routed to a moulin.~~ Moulin water level and shape respond to these patterns of variability. To examine the impact of Q_{in} magnitude (mean) and Q_{in} amplitude (variability), we perform a series of model runs that vary the magnitude of a cosine curve between 1 and 20 ~~m^3s~~$m^3\ s^{-1}$ with a fixed amplitude of 0.5 ~~m^3s~~$m^3\ s^{-1}$ and a series of runs that vary the amplitude of a cosine curve between 0 and 2 ~~m^3s~~$m^3\ s^{-1}$ with a fixed magnitude of 5.0 ~~m^3s~~$m^3\ s^{-1}$. The amplitude ~~as~~is one half the diurnal range. These runs use Basin 1 ice conditions (Table 2; Sect. ~~2.5.4) with no base flow prescribed.~~2.5.3). Further details can be found in Supplement S2.1 and Figures S2-4.

2.5.2 Sensitivity to uncertain parameters

We explored the sensitivity of our results to the values of seven parameters, shown in ~~Fig~~Figs. 3-5, with the prescribed ranges shown in Table 1. We ~~studied~~examined the effect on the water level, the moulin radius at the equilibrium water level, the volume and water storage of the moulin, and the cross-sectional area of the subglacial channel at the end of a ~~ten~~40-day model run. These values reach equilibrium, with daily oscillations superimposed, after ~~3–5~~~15 days. We also tested the dependence of our results on the initial moulin radius, r_0, which we varied across an order of magnitude from 0.65 to 5.0 meters.

We varied the value of a uniform deformation enhancement factor ~~E~~F^* over an order of magnitude (~~E~~F^* = 1 to 9), which affects viscous flow of the ice surrounding the moulin. While the range of enhancement factors tested ~~exceeds that likely to be observed in the field, the variation of an order of magnitude was chosen to match the range of other rheological parameters.~~ here cover a variety of ice conditions, including ice shelves and temperate glaciers, the GIS likely has values between 4 and 6 (e.g., Cuffey and Paterson, 2010). Outside of testing the model sensitivity to the enhancement factor, we assigned $F^* = 5$. We also tested the effect of ice temperature, independent of the enhancement factor. We used five different temperature profiles: cold ice temperatures (mean ~ -15°C, range -23.1°C to the pressure melting point) measured in the center of Jakobshavn Isbræ (Iken et al., 1991); moderate ice temperatures (mean ~ -7 °C, range -13.5°C to the pressure melting point) measured at the GULL site in Pâkitsoq (Lüthi et al., 2015; Ryser et al., 2014); warmer ice temperatures (mean ~ -5°C, range -9.3°C to the pressure melting point) measured at the FOXX site in Pâkitsoq (Lüthi et al., 2015; Ryser et al., 2014); a hypothetical linear profile from -5°C at the surface to 0°C at the bed; and, finally, a fully temperate ice column. These different ice temperature scenarios affected the creep closure rates of ice through the temperature-dependent softness parameter A by approximately a factor of 6 from the coldest profile (Iken et al., 1993) compared to the fully temperate column.

We also examined moulin sensitivity to elastic deformation by varying Young's modulus (E) of the ice column between 1–9 GPa (Vaughan, 1995) and the sensitivity to the values of friction factors for the moulin walls. MouSh has two friction factors: f_m (below the water line) and f_{oc} (above the water line). We varied these friction factors across two orders of magnitude~~.~~ (0.01 to 1). We did not vary the subglacial channel friction factor. Finally, we varied values for basal ice softness A_{sub} over two orders of magnitude (5e-25 to 5e-23) and independently examined moulins over a range of ice thicknesses (670–1570 m) and corresponding distance from the terminus (~20–110 km), which in combination results in variations in hydraulic gradient.

2.5.3 Sensitivity to local conditions

We examined moulins over a range of ice thicknesses and corresponding distances from the terminus (Table 2). Each moulin is associated with a supraglacial basin derived by Yang and Smith (2016). The moulins were selected based on ice thicknesses that broadly represent the range of ice thicknesses within the ablation zone of the western ~~Greenland Ice Sheet~~GIS and supraglacial drainage basin sizes and geometries that were visually similar to nearby drainage basins and approximately representative of the mean supraglacial drainage basin area for the given ice thicknesses (553m, 741m, and 1315m). To derive broadly representative Q_{in} values for each basin, we integrate 3-hourly modeled surface melting from a downscaled version of MERRA-2 (Gelaro et al., 2017) over the surface area of each moulin surface drainage basin. We then use synthetic unit hydrograph parameters derived for a supraglacial basin from western Greenland during the middle of the ~~2015 melt season (Smith et al., 2017) to estimate supraglacial discharge into each moulin.~~ 2019 melt season (Smith et al., 2017) to estimate supraglacial discharge into each moulin. Surface runoff values for the 2019 melt season were modified using a synthetic unit hydrograph derived for the ablation zone and parameters appropriate for western GIS (Smith et al., 2017) with manual dampening of diurnal variability to minimize long periods of no surface melt during the beginning and end of the season. We apply this dampening because the parameters for the unit hydrograph were determined during the middle of the melt season and therefore may inaccurately represent routing delays at the beginning and end of the melt season.

The supraglacial discharge curves for each moulin are only meant to capture the seasonal change in discharge rates and diurnal variability and occasional increases in runoff due to surface melt events during the 2019 melt season. The primary goal of this exercise is to examine season-long and daily differences in model outputs, the variation in each model component (viscous, elastic and phase change), and the relative importance of each component in driving moulin geometry and water level change at different representative locations of the western ~~Greenland Ice Sheet.~~GIS (Figs. 6-9).

2.5.4 Comparison to a cylindrical moulin

Subglacial models generally use a time-invariant vertical cylinder to represent moulins. To investigate and quantify the efficacy of our time-evolving moulin shape model, we drove MouSh and a static cylinder with the same meltwater inputs. We use the time-mean radius at the water level as the radius of the static cylinder; this is 1.6 m and 1.4 m for Basin 1 and ~~1.3 m for~~ Basin 2, respectively. We compared the resulting moulin water level, moulin capacity, subglacial cross-sectional area and meltwater input difference (due to melt generated within the model itself) across these runs. We compared the moulin water level values directly (*cylindrical water level – variable water level*) and ~~compared other metrics~~moulin capacity by percentage difference (*cylindrical – variable*) / (*variable*~~).~~); differences are presented in Figure 10.

2.5.5 Sensitivity to model choices

As part of MouSh development, we made several decisions about how to represent moulin geometry, water inputs, and the associated subglacial system that can directly impact the shape and water level of a modeled moulin. ~~These~~We test the impact of these decisions ~~include~~in a series of experiments, including (1) representing moulin cross-~~Sect.al~~sectional area as a semi-elliptical, semi-circular “egg” instead of as a circle (Sect. 2.1~~ and 2.3.2~~); (2) the inclusion of ~~estimated surface stresses in the representation of~~ elastic deformation (Sect. 2.2.1); (3) the use of a parabolic ice sheet profile to determine the surface slope and distance to terminus for a given ice thickness (Cuffey and Paterson, 2010); (4) the use of prescribed ~~base flow~~baseflow into the subglacial component of the model (Sect. 2.4); and (5) the use of a time-evolving subglacial channel (Sect. 2.4.2).

To explore the impact of our model choices for experiments 1-4, we perform a series of comparisons against a slightly modified seasonal run for Basin 1. This allows us to capture the effect of our choices during periods of increasing and decreasing Q_{in}. We change only the parameter of interest to isolate the effect on moulin water level and moulin capacity, the two variables that most directly affect water flow within the subglacial system. ~~.~~Further description of these runs is included in Supplement S2.2 and resulting differences are highlighted in Figures S5.

The first two choices pertain to the complexity of the model, with our choices being more complex; simplification may be beneficial in some circumstances. ~~Choices 4 and~~In experiment 1, the model is initialized with the same circular geometry as the control run (Supplement S2.2.1) but melting above the water line is uniformly distributed around the moulin perimeter, thus there is only one radius to evolve (Supplement S2.2.2). In experiment 2, we test model sensitivity to the inclusion of elastic deformation (Supplement S2.2.3).

Experiments 3 - 5 reflect the ~~need for a~~simplicity of the current subglacial hydrologic model and would be eliminated if MouSh was configured to function with either specific observational data or with a more comprehensive subglacial model. ~~We also test the impact of the magnitude and diurnal variability of Q_{in} on the timescale for the moulin to reach quasi-equilibrium.~~In experiment 4, we test using a subglacial channel length of one half, and one and one half the length defined in the control run (Supplement S2.2.4). In experiment 4, we prescribe a lower baseflow (Supplement S2.2.5).

~~To explore the impact of our model choices for decisions 1-4~~In experiment 5, we ~~perform a series of experimental comparisons against the seasonal run for Basin 1. This allows us to capture the effect of our choices during periods of increasing and decreasing Q_{in}. We change only the parameter of interest to isolate the effect on moulin water level and moulin capacity, the two variables that most directly affect water flow within the subglacial system.~~

~~To~~ examine the effect of an evolving versus a fixed-~~radius~~geometry subglacial channel (Supplement S2.2.6). The fixed subglacial channel~~,~~ cross-sectional area is set to 1.95 m^2. For these runs we ~~complete a series of runs with the same ice thickness and distance from terminus as Basin 1 but~~ use a simpler Q_{in}, the ~~cosinusoidally~~co-sinusoidally varying function described in Sect. ~~2.4.1. Further description of these runs is included~~2.4.1. Details about this simplification are described in Supplement S2.2.6 with results in Figure S6.

3 Results

3.1 Quasi-equilibrium and dependence on Q_{in}

Under ~~uniform~~a constant supraglacial ~~inputs~~input, the moulin water level, radius, and water capacity reach equilibrium within ~~10~~15 days (red line, Fig. ~~S2~~S2c). However, supraglacial inputs are rarely, if ever, uniform, so under constantly varying conditions, the moulin will reach a 'quasi-equilibrium' state. This is a mean state (geometry, water level, deformation rates) with superimposed variability on the timescale of variations in Q_{in} alone. Therefore, if the forcing is diurnal, the moulin will exhibit diurnal variability from a mean state. The quasi-equilibrium state is also dependent on model characteristics and parameters (Sect. ~~3.2; Supplement S2~~3.2).

The magnitude and amplitude of Q_{in} alter the moulin water level and major radius at the mean water level (a proxy for moulin geometry) in predictable ways (Fig. S2 and Fig. S3). Increasing the diurnal amplitude of Q_{in} increases the diurnal variability and mean moulin water level (Fig. S2b, Fig. S4). This occurs due to the disparate timescales of ice deformation versus melting. The daily increase in Q_{in} raises the water level quickly because the moulin and subglacial channel is slow to expand by melting. Conversely, the nightly fall in Q_{in} is muted by a fast viscous contraction of the moulin and subglacial channel. This behavior drives the daily peak in moulin water level higher above the mean water level than daily minimum water level falls below it (Fig. S2b). The "extra" time spent with higher water levels reduces the visco-elastic closure of the moulin while also increasing turbulent melting, resulting in a larger moulin, as indicated by the moulin radius at the mean water level (Fig. S2c). Higher diurnal amplitudes in Q_{in} magnify this effect.

As the Q_{in} magnitude increases, both the mean water level and its diurnal variability decrease (Fig. S3a-b). This occurs because the moulin becomes larger in response to increasing Q_{in} and subsequent increases in subglacial discharge. As the moulin and subglacial channel widen, they can readily accommodate the fluctuations in Q_{in} with ~~more limited~~lower variations in moulin water level. This accommodation is evident in the moulin radius at the mean water level (Fig. S3c). Higher Q_{in} magnitude drives a linear increase in melt rates within the moulin alongside nonlinear increases in visco-elastic deformation, causing an overall nonlinear increase in mean moulin water level (Fig. S4). However, when moulin water levels exceed flotation, the moulin grows due to both visco-elastic deformation and melting, resulting in a moulin larger than would be expected ~~moulin (red~~based on the equilibrium water level (blue line, Fig. S3c).

3.2 Sensitivity of MouSh to parameter values and deformational processes

A range of ice characteristics affect the time evolution of moulin geometry. These include the initial moulin size, temperature and viscosity of the ice column, viscosity of basal ice, friction factors, and ice thickness. Some of these factors are highly spatially variable (e.g., ice thickness) and others are poorly known (e.g., basal ice viscosity). We quantify the effect of these factors on moulin water level and moulin volume, moulin geometry, and subglacial channel cross-sectional area over both multi-day and diurnal timescales by performing multiple independent sensitivity studies~~ (Sect. 2.3).~~.

We find that moulins reach a quasi-equilibrium~~ within 10 days~~, where the mean moulin water level and the moulin radius at this location ~~are constant. This~~oscillate consistently around a daily mean value, within 15-20 days of model initialization. The quasi-equilibrium value is independent of the initial moulin radius (Fig. 3a–b, Fig 4a–b~~), apart from locations above the water line (Supplement S2.2; Fig. S2.4g) where surface deviatoric and shear stresses impact moulin shape.~~).

~~Three major~~Two primary parameters affect the degree of viscous ~~and elastic~~ deformation in the moulin: the ice flow enhancement factor ~~E,~~F^* and the ice temperature profile ~~T~~$T_i(z$~~), and Young's modulus E.~~). We tested a span of reasonable values representative of ~~Greenland~~glacier and ice sheet ice (Table 1) and found a limited effect on moulin geometry. Equilibrium moulin water level, subglacial channel area, and their diurnal variabilities remain constant (<0.1% change) over the tested range of these parameters (Fig. 3d,f,h ~~&~~and 4d,f,h). Moulin capacity and water storage show ~~moderate~~high sensitivity ($\sim$~~20~~100–150% in equilibrium value and $\sim$~~40~~100–200% in diurnal range) across the range of ~~E~~F^* and ~~T~~T_i scenarios tested; a decrease in moulin capacity and water storage pair with an increase in the diurnal variability for these variables. For instance, varying ~~E~~F^* across an order of magnitude grew the equilibrium major radius by ~~23~~26% and shrank the equilibrium minor radius by ~~44~~72%, with a net effect that moulins had ~~23~~65% less volume and ~~20~~58% less water storage capacity in softer ice (~~E~~F^* = 9) compared to harder ice (~~E~~F^* = 1) (Fig. 3c–d). Similarly, the different ice temperature profiles we tested caused variations of ~~11~~29% in moulin major radius, ~~18~~65% in moulin minor radius, ~~and 24~~63% in moulin capacity and 73% in moulin water storage, with warmer ice hosting smaller moulins (Fig. 3e–f). We also varied Young's modulus E across one order of magnitude, but this affected moulin radius, water volume, and moulin capacity by $\sim$0.01%. This is due to the low magnitude of elastic deformation overall compared to viscous deformation (Fig. 5g).

~~We varied Young's modulus, E, across one order of magnitude. With the highest Young's modulus, moulin major radius increased by 50% compared to the lowest, minor radius decreased by 15%, moulin water volume increased by 38%, and moulin capacity increased by 56% (Fig. 3g–h). The equilibrium water level decreased insignificantly (<0.1%) and the subglacial channel area increased insignificantly (<0.1%) across this range of E. These effects are comparable to those of F_*, which we also varied over one order of magnitude, and T, which changed the englacial flow-law parameter A by approximately a factor of 6.~~

~~In contrast to the above parameters, we~~We find that moulin geometry is strongly sensitive to the choice of basal ice softness and the friction factors used within the moulin (f_m and f_{oc}). Melting due to the dissipation of turbulent energy is partially controlled by the friction factors chosen for the moulin walls. The friction factor above the water line (f_{oc}, “open channel”) does not significantly affect moulin water level (<0.1% change for f_{oc} variations over two orders of magnitude), moulin volume (~~4~~6%), moulin water storage (~~2~~0.1%), or subglacial channel area (<0.1%) over either long or diurnal timescales (Fig. 3m–n and 4m–n). However, like the deformational parameters, the open channel friction factor does affect moulin radii, with the major radius growing by ~~36~~50% as the open channel friction factor increases over two orders of magnitude, and the minor radius decreasing by ~~27~~24%. This dampens the diurnal variability in ~~both~~the major and minor radii by 70% and 24%, respectively (Fig. 4m).

Increasing the friction factor below the water line (f_m) had similar effects to changing f_{oc}. Increasing f_m by two orders of magnitude increased the cross-sectional area of the moulin by 106%, via a ~~15~~10% increase in the major radius and a ~~95~~93% increase in the minor radius. The water volume increased by ~~116~~127% and the storage capacity increased by ~~100~~74% (Fig. 3k–l) while the equilibrium water level and the subglacial channel area changed by <0.1%. Increasing f_m also increased the diurnal variability of the moulin capacity and water storage ~~(Fig. 4k–l)~~by 130% and 126%, respectively, by increasing the diurnal differential melt rate~~.~~ (Fig. 4k–l).

The two parameters which have the largest impact on moulin water level are the basal ice softness ~~A~~A_{sub} and the moulin location on the ice sheet, described jointly by the ice thickness (H_i) and distance from the terminus (L). This sensitivity indicates an interplay among these parameters, the subglacial hydraulic gradient, and moulin water level.

We varied basal ice softness ~~A~~A_{sub} by two orders of magnitude. Softer basal ice increased the size and storage capacity of the moulin: the major radius by ~~21~~23%, the minor radius by ~~25~~23%, the total capacity by ~~88~~41%, and the stored water volume by ~~112~~88% (Fig. 3i–j). These changes also increased the equilibrium water level by ~~57~~34% and the subglacial channel area by ~~24~~14%, unlike tests on englacial parameters (~~E, T~~F^*, T_i, and ~~Y~~E), which did not affect the water level or subglacial channel area. These changes occur because softer basal ice increases the rate of subglacial creep closure, which reduces subglacial channel cross-sectional area, which reduces water throughflow in the moulin and increases water level, which in turn reduces the amount of viscous and elastic radial closure in the moulin. Increasing the basal ice softness to approximately 10^{-23} $Pa^{-3}s^{-1}$ increases the diurnal variability in the sizes of the subglacial channel and moulin (Fig. 4i–j); however, increasing ~~A~~$Asub$ above this value causes moulin water levels to rise high enough that diurnal fluctuations are truncated by the ice thickness resulting in an observed decrease in diurnal range that would not be present in thicker ice (Fig. 4j).

We co-varied ice thickness and distance from terminus using a parabolic approximation for a perfectly plastic ice surface profile (Cuffey and Paterson, 2010~~). Variations~~); this covariance alters the hydraulic gradient of the system. Changes in ice thickness from 670 to 1570 m (80%) increase the equilibrium subglacial ~~conditions~~channel area by ~~20~~24% and increase equilibrium water levels by ~~107~~203% (Fig. 3o–p). Increasing ice thickness and distance from the terminus increases the moulin major and minor radii by ~~4~~7%, increases moulin volume by ~~97~~93%, and increases moulin water storage by ~~114~~235% (Fig. 4p). We also find significant increases in diurnal variability in subglacial channel size (~~28~~29%), water level (~~105~~178%), moulin radii (major radius ~~85~~84% and minor radius ~~22~~24%), moulin volume (130%), and moulin water storage (~~140~~750%) in thicker ice farther from the terminus (Fig. 4o–p).

Overall, we find that MouSh-modeled moulins are primarily sensitive to the friction factors for water flow through the moulin, basal ice softness, and location on the ice sheet (ice thickness and distance from the terminus). The results are less sensitive to englacial material factors that govern elastic and viscous deformation. The observed sensitivity to the ice thickness and distance from terminus signals that moulin geometry can vary spatially. The sensitivity to friction factors and basal ice softness indicates that the values of these poorly constrained parameters should be carefully chosen and kept in mind when interpreting model output.

3.2.1 Contributions to moulin shape

Figure 5 illustrates the role of each process~~ that changes moulin radius under equilibrium conditions:~~, phase change, viscous deformation, and elastic deformation~~. We use standard~~, in determining moulin radius under different hydraulic potential gradients with median model values ~~for all parameters~~ (Table 1). Elastic deformation has little impact on moulin shape or variability (Fig. 5f,g) and ~~we vary~~is persistently an order of magnitude smaller than either viscous deformation or radius evolution due to phase change. Viscous deformation and phase change due to melting peak near the daily maximum water line, with the daily mean of each increasing with increasing ice thickness ~~and distance from terminus. We find that moulin shape~~ (Figure 5f); however, the opposite effect is observed near the bed, where lower mean water levels in moulins in thinner ice increase viscous deformation at the bed; melting also increases in response to the higher hydraulic potential gradient.

At any given depth, viscous deformation and phase change due to melting are similar below the waterline; however, the diurnal variation in these parameters is quite ~~similar across~~ different (Fig. 5g). ~~ice thicknesses, while~~At the mean water level, moulin ~~capacity (Fig. 5a–e) and the diurnal range in moulin radius (Fig. 5g) increase with ice thickness. We also analyze temporal variations in each process (Fig.~~ growth due to melting varies less than 0.04 m day^{-1}, with the shape of the diurnal variability dependent on the parameterization of melting both above and below the water line. In contrast, viscous deformation displays diurnal variations between 0.08 m day^{-1} in the thinnest ice and more than 0.21 m day^{-1} in the thickest ice~~5g). The times of maximum melt and maximum viscous closure are slightly offset, with peak melting occurring during the most rapid decline in viscous deformation (Fig. 5g). This offset aligns with the rising limb of the input hydrograph, when the moulin is small and increases in Q_{in} raise water level and, in turn, elevate englacial melt rates and reduce viscous deformation~~.

~~Melt rates both above and below the water line contribute to moulin growth (Fig. 5f–g). Melt above the water level occurs due to stream or waterfall erosive processes, which in MouSh occur only within a fraction of the total circumference (Fig. 2a,d), which manifests as growth of the major radius. The actual rate of melting, however, is also dictated by the area over which water flow occurs, which under our parameterization is related to the cross-sectional area of the moulin at any given depth (Fig. 5f).~~

~~Elastic deformation, like viscous deformation, closes the moulin except when the water level is above flotation. Elastic deformation rates are generally smaller than viscous rates, except between ~100–300 meters above the bed, where viscous deformation is minimized by cold ice temperatures (Lüthi et al., 2015; Ryser et al., 2014). Diurnally, elastic deformation varies with a similar pattern to viscous deformation, though over less range.~~

3.3 Moulin shape in different environments

We modeled the seasonal growth and collapse of moulins in a range of environments across the ~~Greenland Ice Sheet~~GIS using realistic melt forcings derived for the 2019 melt season (Sect. ~~2.4.1 and Sect.~~ 2.5.3). These model runs varied with respect to ice thickness, moulin distance from the terminus, ~~base flow~~baseflow, and the magnitude, diurnal range, and seasonal evolution of supraglacial inputs (Table 2; Fig. 6a). Overall, we find that moulin setting affects the scale of diurnal and seasonal variability in the size and water capacity of moulins as well as the evolution of subglacial channels (Fig. 6 and 7).

The sizes of all three modeled moulins reach equilibrium with the melt forcing within ~~10~~ ~15 days of the onset of the melt seasons (Fig. 6b–c). As the water flux increases over the next few weeks, each moulin grows in response to increasing supraglacial inputs, both diurnally and with a long-term trend, ~~though~~although this growth is more significant in thicker ice (Fig. 6c and Fig.7). The subglacial channel grows with a similar pattern, but interestingly, the setting and fluxes of Basin 1 and Basin 2 result in very similar subglacial channel cross-sectional areas despite different moulin water levels and capacities (Fig. 6d).

~~Though~~Although the three moulins all evolve in a similar fashion, there are differences in moulin ~~water~~ capacity, ~~equilibrium~~ water level (Fig. 6), overall moulin geometry (Fig. 7), and the magnitude of englacial deformation (Fig. 8). ~~Diurnal variation~~ Basin 3 exhibits the largest seasonal change in moulin capacity ~~is slightly larger~~ in ~~thicker ice,~~part because a lower supraglacial input and subglacial hydraulic gradient results in a smaller subglacial channel and periods where moulin water level is above flotation (Fig. 6). This causes substantial variability of viscous deformation while limiting variations in melt due to ~~higher rates of deformation within the moulin (Fig.~~ changing moulin ~~8a). This occurs because daily~~ water level (Fig. 8a). ~~fluctuations are greater in thicker ice due to the non-linearity~~One of ~~ice creep in conjunction with~~ the ~~linearity~~largest periods of ~~melt-driven~~Basin 3 moulin growth ~~(Sect. 3.1). Furthermore, in thick ice, visco-elastic deformation plays a relatively larger role in moulin evolution (dark purple line in Fig. 8b), for the same reason. The only exception is during periods of low diurnal variability in Q_{in}, which occurred around Day~~occurs starting at day 30~~ of the 2019 melt season (Fig. 6a).~~. During this period, ~~the minimum daily~~ supraglacial inputs ~~are quite high~~experience a step change (Fig. 7a); moulin water levels stayed near flotation ~~for a few~~and were less variable for several days (Fig. 7b), keeping effective pressure near zero and retarding deformation~~, and slightly increasing melt rates~~ (Fig. ~~8b~~8a). In this case, viscous deformation hovers around zero ~~(though causing~~and causes moulin opening~~),~~, resulting in a high ratio of ~~eestatic~~elastic to viscous deformation and a high ratio of phase change to viscous deformation (purple line in Fig. ~~9b). There is an~~8b). Similar behavior also occurs around day 110. Basins 1 and 2 exhibit smaller seasonal variations in moulin capacity because the ratio of melting to deformation stays near one until near the end of the season (Fig. 8b). This occurs because viscous deformation in Basins 1 and 2 is only slightly lower than in Basin 3 and melt rates tend to be higher (Fig 8a) due to increased subglacial discharge associated ~~growth in moulin capacity (Fig. 7c). Ultimately, this is a response to multiple days where melt inputs do not exhibit substantial diurnal variability.~~

~~The ratio of elastic to viscous deformation generally ranges from ~0.4 to ~0.7, depending on ice thickness (Fig. 8b). Elastic deformation rates in the moulin depend on a linear function of ice thickness, while viscous rates are related to ice thickness cubed. Thus, at lower elevations, the elastic contribution is maximized (~0.8 of~~with a higher hydraulic gradient. Further, there are fewer periods where water levels above flotation drive viscous ~~deformation), while at high elevations, significant increases in viscous closure lowers the relative contribution of elastic deformation (~0.4 of viscous deformation).~~

~~This increase in viscous closure in thick ice also minimizes subglacial channel size in thick inland ice (Fig. 6d), despite closure rates being retarded by daily periods of above overburden water pressures.~~opening.

Each moulin has a different daily mean capacity (Fig. 7c). This, in addition to differences in supraglacial inputs, ensures that daily moulin water level variations are substantially different across moulins. Basin 1 exhibits the largest variation in daily moulin water level, followed by Basin 2 (Fig. 9a). Basin 3 shows the lowest daily change; however, this is due at least in part to the fact that water overtops the moulin nearly daily (Fig. 6b and 7m–n). Changing water levels drive changes in moulin and subglacial capacity. Over the melt season, daily change in moulin capacity can be as low as ~~5~~2% during lulls in diurnal melt variability (Basin 3) or as high as ~~31~~12% following a recovery from a low melt day (Basin 1; Fig. 9b). However, in general all moulins display a similar daily change in capacity of ~~~20~~5-10%, with peak values of 12 to 13%.

The subglacial system undergoes diurnal variations in channel size between 1 and ~~21~~20% (Fig. 9c). These changes are similar in magnitude to daily capacity changes within the moulin but exhibit more variability across ice thicknesses~~ and~~. Like changes in moulin capacity, these variations are related to the daily changes in moulin water level (Fig. 9a). This suggests that the time evolution of moulin geometry ~~dampens~~dampen the diurnal pressure fluctuations that drive subglacial channel growth and collapse. Evidence for this can be seen in the temporal pattern of moulin water level and subglacial channel cross-sectional area (Fig. 9a,c). ~~To test this idea, we compared results from static and time-evolving moulins (Sect. 3.4).~~

3.4 Comparison to cylindrical moulins

To examine the role moulin evolution plays in modifying the subglacial hydrologic system, we compared moulin water levels, moulin capacity, and subglacial channel size between model runs with a fully evolving moulin and runs with a static cylindrical moulin. We performed these tests with realistic melt inputs based on the 2019 melt season (Sect. ~~2.4.1,~~ 2.5.3), at moulins with low and moderate ice thicknesses (553 m – Basin 1 and 741 m – Basin 2). We defined the radius of the ~~fixed~~static cylinder as the mean radius at the mean water level: 1.~~4~~6 m and 1.~~3~~4 m for Basin 1 and 2, respectively. This results in fixed moulin cross-sectional areas (~6 ~~m^2 and ~5 m^2) that are~~to 8 m^2) within the range of~~ the~~ spatially invariant moulin cross-sectional areas ~2–10 m^2 often prescribed in subglacial models (e.g., Andrews et al., 2014; Banwell et al., 2013; Bartholomew et al., 2012; Cowton et al., 2016; Meierbachtol et al., 2013; Werder et al., 2013). ~~Inter-comparison of these runs allows us to examine the role moulin geometry has on subglacial pressures (Covington et al., 2020; Trunz, 2021).~~

Comparison of moulin water level~~, moulin~~ and capacity~~, moulin water storage, and subglacial cross-sectional area~~ between ~~fixed~~ static cylindrical and evolving moulins show differences on both the diurnal and seasonal times scales (Fig. ~~10).~~ ~~Moulin water levels (*fixed - variable*) can be substantial (Fig. 10a–b), with short term differences driven by variable melt conditions reaching a maximum of 97 m (Basin 1) and 145 m (Basin 2), but values can also be negative, indicating that the realistic run moulin has higher water levels, up to 46 m for Basin 1 and 25 m for Basin 2.~~10). The ~~long-term daily average differences are -6 m and -35 m for Basin 1 and Basin 2, respectively. These differences are driven by a combination of~~ differences in moulin ~~capacity and subglacial channel size (Fig. 10c–f) and are despite a total increase in the meltwater input~~

~~into an evolving moulin, due to melt generated from turbulent dissipation (less than 2%). These results indicate that diurnal variability is an important component not effectively represented with a cylindrical moulin.~~

~~Generally, the evolving moulin is larger (Fig. 10c–d), stores more water and maintains a larger subglacial channel (Fig.10e,f), which all contribute to the observed difference in water levels. Midway through the melt season, the evolving moulin exhibits capacities only slightly larger than those of the fixed cylinder, but these capacity differences are exacerbated~~water level (both positive and negative) are generally great during ~~higher Q_{in} values (Fig. 10c–d). As meltwater~~lower supraglacial inputs ~~taper~~ at the beginning and end of the melt season~~ (day ~100), the capacity and storage volume in the evolving moulin falls below that of the fixed cylinder, whose volume does not adjust in response to the forcings (Fig. 10c–d). This seasonal evolution is consistent between the two ice thicknesses tested~~, with the relatively limited differences occurring during the highest discharges (Fig. 10a-b). These values are both positive, indicating that the static radius moulin has higher water levels, and negative, indicating that the evolving moulin has higher water levels. Differences in moulin water level can reach nearly 20 m, but are most commonly below 10 m. The seasonal mean water level difference between the static cylindrical and evolving moulin in both basins is less than 1 m.

~~The capacity differences between the variable and fixed moulin contribute directly to dampening the supraglacial input signal and dampening of moulin water levels. This, in turn, drives an increase in subglacial channel size (Fig. 10e–f), both diurnally and over the season. The seasonal difference between the variable and fixed moulin forcing is relatively constant, though punctuated by dips associated with reduced moulin water level differences (Fig. 10a–b).~~

Moulin capacity also displays a clear seasonal pattern; in both basins, the static cylindrical moulin larger than the evolving moulin at the beginning of the melt season with the evolving moulin gradually growing larger as the melt season progresses (Fig. 10c-d). After peak melt (day ~60), the evolving moulin begins to viscously close and gradually becomes smaller than the static cylindrical moulin. The static cylindrical moulin can be more that 100% larger than the variable moulin during the tails of the melt season with the evolving moulin becoming 36% and 42% larger than the static cylindrical moulin during mid-melt season. Overall, the mean capacity difference between the static cylindrical and evolving moulin is less than 5%, with the static cylindrical moulin being slightly larger.

The radius of the cylindrical moulin was chosen to minimize differences with the evolving moulin. This is evident by the limited long-term differences between the two moulins in both Basin 1 and 2. As such, there is limited differences (<1%) between the modeled subglacial channels. We expect difference in moulin water level, moulin capacity, and subglacial geometry to change if the static cylindrical moulin geometry is poorly chosen; if the different or different experimental parameters are used; or the setting changes (e.g., different hydraulic gradients). For example, we use commonly used values of ice softness A for both the moulin and subglacial channel; however, these values are poorly known, and their choice can directly impact the relative importance of moulin shape in dictating moulin water levels and subglacial channel size (Fig. 4).

3.~~4~~.5 Impact of model choices on moulin geometry

Chosen parameterizations have the potential to impact the representation of moulin water level and capacity (Supplement S2). Overall, we find that a circular geometry has limited impact on moulin water level with the circular geometry having water levels that are less than 3 m higher than the egg-shaped geometry, ~~though~~although in nearly all instances the difference is less than 0.~~5m~~5 m (Fig. S5a); however, the impact on capacity is slightly larger (the circular moulin is up to ~~47~~31% smaller) and displays a seasonal trend as the egg-shaped moulin elongates along its elliptical axis (Fig. ~~S5b). Altering the deviatoric and shear stresses used in the calculation of elastic deformation results in minimal changes, primarily above the water line. Moulin water levels are typically within 0.25 m of the control run (Fig. S5c). Prescribing the surface stresses to be zero results in a maximum increase in moulin capacity of less than 10% (Fig. S5d).~~S5b).

Elastic deformation within the moulin is small (Supplement S1 and S2.2.3; Figure 8a). Excluding elastic deformation has a negligible impact on moulin water levels and moulin capacity (< 1%; Figure S5c-d).

In contrast to the previous choices, the distance from the terminus ~~(~~L~~)~~ and the prescribed ~~base flow (~~baseflow Q_{base}~~)~~ can have a substantial impact on moulin water level and capacity (Fig. S5e-h). Distance from the terminus is defined by the position of a given moulin on the ice sheet, and as such is not a choice or parameter per se; however, does directly influence the hydraulic gradient. A shorter L increases the hydraulic gradient and ~~base flow~~reduces both moulin water levels and capacities (Fig. S5e-f). Baseflow is used here to mitigate the use of a simplistic subglacial hydrology model. Reducing the baseflow within the subglacial system increases moulin water levels and reduces moulin capacity (Fig. S5g-h).

Finally, we examine the impact of fixing the subglacial channel cross-sectional area S. Experimental results using a fixed S and a seasonally evolving melt curve resulted in ~~extremely~~unrealistically low ~~and extremely high~~ or zero water levels ~~resulting in~~ during low, early season Q_{in} and complete ~~moulin~~viscous collapse ~~or~~ of the moulin if the subglacial channel size was prescribed to be too large, or persistently high (always above the ice thickness) water levels and runaway moulin growth~~, respectively~~ if the subglacial channel was prescribed to be too small. Therefore, we explore the impact of fixing S using a constant mean Q_{in} with an overlaid diurnal variability (Supplement ~~Sect.~~ S2.2.6). ~~When~~With constant variability, we can easily prescribe the fixed S ~~is smaller than~~ to be the mean value of the time-varying subglacial channel S (1.95 m). In this instance, the fixed S experiment displays a ~~variable S,~~similar mean moulin water ~~levels are higher and exhibit less~~level, but lower diurnal variability ~~while moulin capacity is larger~~than the experiment with a time-varying S (Fig. S6). Further details are included in the Supplement S2.

4 Discussion

4.1 Timescales of moulin formation and evolution

We consider the formation timescales of moulins in the context of the shape evolution of a mature moulin. Using MouSh, we find that in the absence of external forcing, such as time-variable ~~Q_{in},~~Q_{in}, the size of a moulin reaches its equilibrium value in

∼~~1–10~~15 days depending on ice ~~-~~and supraglacial input conditions and initial moulin geometry (Fig. 5g, Fig. S2 and Fig. S3). This relaxation time is comparable to the Maxwell time for ice (10–100 hours), as expected for a linear visco-elastic system. Our relaxation time also compares well to the equilibration timescale defined by Covington et al. (2020) for their modeled moulin – subglacial conduit system, which Trunz (2021) found to be 1–20 days. The most realistically sized moulins in Trunz (2021) had relaxation times closer to 1 day. Their modeled system was governed solely by melt and viscous deformation and lacked elastic deformation; ~~this may explain their modestly longer relaxation time compared to ours.~~however, elastic deformation in MouSh is small explaining why our relaxation times are comparable.

If the process of moulin formation occurs on a timescale shorter than the ~~1–10~~15-day relaxation time, the formation process likely will not influence the overall form of the englacial system at equilibrium. This time range includes hydrofracture during rapid lake drainage (∼2 hours) and slow lake drainage (<∼6 days, e.g., Selmes et al., 2011), and likely also the reactivation of existing moulins in ensuing melt seasons, which, based on the timing difference between surface melt onset and ice acceleration, occurs over multiple days (Andrews et al., 2018; Hoffman et al., 2011). On the other hand, moulin formation by cut-and-closure occurs over years to decades (Gulley et al., 2009), well above the MouSh relaxation time and the Maxwell time for ice~~ and~~, are more likely to create subvertical englacial channels. The interdependence of formation and evolution of these moulins gives us less confidence in applying our model to moulins with cut-and-closure origins. Those moulins primarily occur in temperate near-surface ice within polythermal glaciers (Gulley et al., 2009) and have not been reported on the ~~Greenland Ice Sheet.~~GIS.

4.2 Comparison of modeled and observed moulin geometries

Field observations suggest that moulin geometry evolves a high degree of complexity. Observations include anecdotes of difficulty deploying sensors to the bottom of a moulin, which suggests the presence of kinks, ledges, knickpoints, and other twists (Andrews et al., 2014; Covington et al., 2020; Cowton et al., 2013). Complex geometry revealed during mapping moulins above the water line further suggests that moulins are not simply vertical cylindrical shafts (Covington et al., 2020; Moreau, 2009).

The MouSh model suggests that the energy transfer from turbulent meltwater entering the moulin to the surrounding ice drives highly spatially variable melt rates above the water line. We incorporated the open-channel melt module to allow a large opening to emerge above the water line (Fig. 5a–e and 7). When we run MouSh without the open-channel module~~ (Sect. 2.3.2),~~, the surface expression of the moulin ~~is much smaller than observed in remote sensing images~~becomes dependent on surface stresses and can in some ~~cases, the moulin will~~instances pinch closed~~ at the ice-sheet surface~~. The open channel module also permits the development of an egg-shaped geometry, which is supported by seismic observations and a resonance model of a moulin~~,~~ which ~~suggest~~suggests that ~~the moulin increased~~moulins may increase in ellipticity over time (Röösli et al., 2016).

The value of the open-channel friction factor and the size of the spatial footprint over which melting occurs directly affects the size of the upper, air-filled chamber of the moulin, which differs from when ~~treated~~the moulin is modeled as circular and open-channel melting is applied uniformly around the perimeter (Fig. S5b). MouSh ~~consistently predicts~~can predict ledges at the top and bottom of a consistent diurnal range in water level. Thus, we infer that energetic subaerial water flow drives formation of moulin complexity above the water line, and diurnal fluctuations around a steady multi-day water level drive ledge formation through a differential in melting and ~~visco-elastic~~viscous deformation above and below the water line. Energetic water flow is commonly observed at stream-fed moulins near the peak of the melt season (Pitcher and Smith, 2019) or during and immediately following rapid lake drainage (Chudley et al., 2019). This suggests that complex moulin geometries form during periods of relatively consistent water supply. Conversely, multi-day rises in water level, driven by either the surface water supply ~~(Q_{in})~~ or the basal water supply (baseflow), can erase geometric complexities such as ledges, as seen in MouSh results during a melt event (Fig. 7).

Above the water line, explored moulins in Greenland show highly variable shapes from moulin to moulin (e.g., Covington et al., 2020). Some moulins, for example the FOXX moulin, are nearly cylindrical within the explored depth (~100 m), with radii comparable to what we model (~~~2 meters~~1.5 m). Others~~, like the Phobos moulin,~~ open some tens of meters below the surface to large caverns with radii approaching 10 meters, a similar morphology to karst caves with narrow entrance shafts (Covington et al., 2020). MouSh can produce large openings above the water line if we use a suitably large open channel friction parameter, although we lack a narrow entrance shaft and substantial vertical variability. These differences are due to the inability of model parameterizations to represent complex geometries such as scalloping, plunge pools, and knickpoint migration (Gulley et al., 2014; Mankoff et al., 2017). Indeed, instead of modeling processes above the water line as turbulent open flow, they could, in the future, be modeled using geomorphic parameterizations to model waterfall migration, perhaps resulting in the clearer development of steps and plunge pools. This would require development and inclusion of a supraglacial channel model as well.

Below the water line, MouSh results indicate that a cylinder is a reasonable representation for newly formed moulins in Greenland. However, there are two caveats. First, moulin cross-sectional area, and thus water storage capacity, can vary substantially over the course of a day or season (Fig. ~~9~~6c. and 9b) and features such as englacial crevasses and reservoirs may be present (e.g., McQuillan and Karlstrom, 2021). Second, in instances where moulins are reactivated over multiple melt seasons (Chu, 2014; Smith et al., 2017), there may be substantial deformation, as suggested by cable breakage in boreholes (Ryser et al., 2014; Wright et al., 2016).

Observations show a wide range of moulin volumes above the water line, and moulin volumes predicted by MouSh are sensitive to the ~~value of the open-channel friction factor.~~consideration of turbulent melting and associated parameter choices. Given the flexibility of model results, we should continue to rely on field exploration to measure moulin size and geometry above the water line and make efforts to constrain the parameters that affect sub-seasonal growth and collapse. MouSh results below the water line are less sensitive to uncertain parameter values, so direct observations of underwater geometry would be less relevant for model validation than subaerial observations. Overall, results from the MouSh model demonstrate that moulin geometry evolves substantially over diurnal to seasonal timescales and varies with ice conditions.

4.3 Diurnal water level oscillations and moulin size

Moulin geometry can directly alter the relationship between meltwater inputs and moulin water level changes – the primary driver of subglacial channel evolution (Andrews et al., 2014; Cowton et al., 2013). Field measurements of moulin water levels indicate diurnal oscillations of 3–12% (Covington et al., 2020), ~25% (Andrews et al., 2014), and >20% (Cowton et al., 2013) of overburden pressure with mean water levels of ~70% of overburden. These diurnal fluctuations are larger than those observed in boreholes, which are generally, though not always, thought to sample inefficient components of the subglacial hydrologic system (Andrews et al., 2014; Meierbachtol et al., 2013; Wright et al., 2016).

Our model results agree well with observations of moulin water level: diurnal fluctuations of ~~15–~~approximately 25 to 50% of overburden pressure, with larger absolute oscillations occuring in thicker ice. To explain larger-than-expected daily oscillations (~10%) in thinner ice, Covington et al. (2020) incorporated moulin cross-sectional area as a free parameter into their model. Matching field measurements of water level required a modeled moulin radius of ~5 m (~75 m^2 cross-sectional area) at ice thickness 500 m and a much larger moulin (radius ~20 m and cross-sectional area ~1500 m^2) at ice thickness 700 m (Covington et al., 2020). For comparison, MouSh predicts average radii ~~of~~ ~1.~~3 to 1.~~4 m (~ ~~5~~6 m^2 cross-sectional area) ~~at these~~in similar ice thicknesses using parameters described in Table 2, including substantially larger meltwater inputs compared to Covington et al. (2020). The drastic differences in moulin size despite similar variations in diurnal water level between our study and Covington et al. (2020) cannot easily be attributed to a single factor but may be explained by our limited ability to model processes above the water line, our inclusion of ~~base flow~~baseflow (Fig. S5g–f), substantial differences in meltwater input (e.g., ~~Figs~~Fig. S2 and S3), fluctuations in moulin capacity~~,~~ (Covington et al. (2020) use a fixed moulin geometry), or that their measured water levels were not from the same moulin they mapped englacially. ~~Nevertheless, we observe substantial differences in water level between fixed and variable geometry moulins~~However, our results suggest that ~~are dependent on supraglacial inputs and ice conditions (Fig. 10). Water levels are less variable and generally lower in the~~an evolving ~~moulins compared~~moulin capacity may be important to ~~the fixed cylindrical moulin.~~represent realistic moulin water levels and capacity (Fig. 10). Thus, to match observed moulin water level fluctuations without evolving the moulin geometry, a fixed cross-sectional area substantially larger than the associated subglacial channel may be necessary, as reported in Covington et al. (2020).

4.~~3~~4 Magnitude of viscous moulin deformation

Viscous and elastic deformation drive moulin closure. The role of elastic deformation in the glacial hydrologic system is discussed below~~ (Sect. 4.4);~~; viscous deformation is the primary closure mechanism of moulins, boreholes and subglacial channels (e.g., Catania and Neuman, 2010; Paterson, 1977, Shreve, 1972), with viscous deformation dependent on local effective pressure, ice characteristics, and the geometry of the feature of interest (Flowers, 2015). Viscous deformation within our moulin varies in response to meltwater inputs (Fig. 5g and Fig. 8a) with the highest deformation rates occurring at the water line (Fig. 5f) because at the water line, inward cryostatic pressure is least offset by outward hydrostatic pressure (see Eq. 3).

During our realistic runs, viscous deformation can exceed 0.~~5~~25 m ~~d~~day^{-1} for short periods ~~at~~of the ~~highest elevation~~day at all three moulin locations (Fig. 8a). These deformation rates are substantially larger than measured borehole deformation rates for the primary reasons that boreholes are often at or above flotation due to high subglacial water pressures (e.g., Ryser et al., 2014) or because creep measurements are recorded in much smaller boreholes in colder ice (e.g., Paterson, 1997).

A previous moulin modeling effort focused on understanding moulin closure rates (Catania and Neumann, 2010). Their results indicate that an air-filled moulin will close within a single day at the bed. However, in this instance there is no opposing hydrostatic pressure. While our modeled closure rates are similar to those calculated by Catania and Neuman (2010) near the surface, the moulins modeled here always contain water even at the end of the melt season (Fig. 6b). This continued retention of meltwater is in line with borehole observations that subglacial pressures tend to be highest outside the melt season (Downs et al., 2018) and preclude the presence of completely air-filled moulins in areas where viscous deformation rapidly shuts down the hydrologic system as supraglacial inputs fall.

4.~~4~~5 The role of elastic deformation in ice sheet hydrology

Our model results indicate that the equilibrium moulin geometry is dictated by a balance of visco-elastic deformation and turbulence-driven melting (Fig. 5 and Fig. 8). In both the sensitivity study and realistic model ~~runs~~experiments, visco-elastic deformation generally closes the moulin, while melting of the surrounding ice consistently opens the moulin. The exception is when moulin water levels exceed flotation, in which case all three mechanisms open the moulin. In all model runs, we find that ~~elastic and~~the rates of viscous deformation ~~are of the same order~~exceed elastic deformation by three to four orders of magnitude~~, and that the elastic mode can be between 40% and 80% of the viscous deformation~~ (Fig. 5g and Fig. 8). ~~The importance of elastic~~ Elastic deformation ~~holds even in~~rates are greatest near the water line and at the bottom ~~few hundred meters~~ of the ice column, where stress conditions are similar to those in subglacial models~~ (Fig. 5f). However, the relative importance of viscous and~~ , at a few centimeters per year of closure within a moulin of radius ~1 meter. This moulin size is comparable to that of a typical subglacial channel in our model ($A \sim 2$ m^2, or radius 1.1 m), implying that elastic closure of a subglacial channel would also amount to a few centimeters per year. Elastic closure rates scale linearly with moulin radius; thus, larger moulins or channels would undergo commensurately faster elastic closure. The contribution of elastic deformation ~~in closing the moulin is also dependent on the values of Young's modulus and viscous enhancement factor (Fig. 5 and Fig. 8). Despite extensive study of these parameters, their values are difficult to constrain. Currently, the space of viscous and elastic parameter values could conceivably allow either elastic or~~ relative to viscous deformation ~~to dominate the closure of a moulin. This underscores the importance of including both modes in the MouSh model~~increases with increasing ice thickness (Fig. 5f); at H = 670 meters, viscous deformation is 4000 times larger than elastic deformation at the water line, while at H = 1570 meters, it is 2000 times larger.

Current subglacial hydrology models represent subglacial channel development (opening) by turbulent energy dissipation and destruction (closing) by viscous deformation alone. ~~Some more recent~~However, work involving elastically responding storage elements or elastic flexure of the ice sheet has occurred (Clarke, 1996; Dow et al., 2015), and there have been efforts to use elastic deformation or fluid compressibility to improve numeric stability of channel equations (Clarke, 2003; Spring and Hutter, 1981, 1982). Interestingly, Clarke (2003) chose to use fluid compressibility due to model integration times. Yet, elastic deformation has generally been omitted from current models of ~~subglacial channelization~~glacial hydrology, even when modeling rapid changes in meltwater inputs (< 1 day; e.g., Hewitt, 2013; Hoffman et al., 2016; Werder et al., 2013). ~~This choice is likely because~~ Our investigation of the role of elastic deformation ~~was considered negligible over timescales of subglacial evolution (e.g., days to weeks). However, the importance of elastic deformation~~ in diurnally closing moulins, particularly in ~~thinner~~thicker ice (Fig. 8b and S5c,d), suggests that its exclusion from subglacial channel models ~~could result~~should cause errors of <0.1% and is warranted. On length scales considerably larger than ~1 meter moulins, as well as in ~~the underestimation of channel closure rates when water levels are below flotation.~~problems where elastic flexure is more central to the geometry, elastic deformation remains important. These applications include ice shelves (e.g., Reeh et al, 2003; Walker et al., 2016), large marine-terminating glaciers (Christmann et al., 2021), crevasse opening (Poinar et al., 2017), and rapid supra- and subglacial lake drainage (Dow et al., 2016; Dow et al., 2015; Lai et al., 2021).

~~This leads us to ask why elastic deformation is absent from subglacial models, particularly because its importance relative to viscous deformation is difficult to constrain given the current range of observed Young's modulus (Vaughan, 1995). Hypothetical subglacial channel models that included elastic deformation alongside viscous deformation would show less temporal asymmetry, particularly in thinner ice, where channel closure may be strongly dictated by elastic deformation. Elastic-incorporating models would also likely predict larger diurnal variations in channel size and moulin water level. This in turn would incite stronger local pressure gradients at the bed, increasing connectivity between the channel and the surrounding distributed system.~~

4.~~5~~6 Moulin geometry and the englacial void ratio

Subglacial hydrology models use an englacial void ratio parameter to represent bulk storage and release of meltwater in the englacial system (see Flowers and Clarke (2002) for the best description). Because the englacial void ratio acts as short term, pressure dependent, storage for subglacial models, it can improve the representation of diurnal water pressure fluctuations in subglacial models (Flowers and Clarke, 2002~~) and, if coupled to a dynamical ice model, corresponding diurnal variations in ice flow.~~). This parameter represents bulk behavior and is usually ~~set constant over the model domain, yet it must be~~ tuned by comparing to local observations (e.g., Bartholomaus et al., 2011; Hoffman et al., 2016; Werder et al., 2013). The inclusion of ~~moulins~~time-varying moulin geometry, potentially in addition to time varying representation of englacial fractures (Gajek et al., 2021), that evolve in response to meltwater inputs and subglacial pressures could reduce subglacial model dependence on this ~~highly~~ parameterized englacial storage, particularly in light of observations of time varying englacial features (Church et al., 2020) and meltwater content (e.g., Vankova et al, 2018).

~~Recent work suggests that fluctuations in water level are controlled by the size of the moulin near the water level (Trunz, 2021): moulins with larger cross-sectional areas have lower diurnal variability in water level, if given the same melt input. Furthermore, our results suggest that the amount of water stored in a moulin is highly dependent on local conditions, such as water pressure on daily to seasonal timescales, and ice thickness (Fig. 6c and Fig. 7). Thus, we explore the possibility that detailed model-based information on moulin sizes and shapes could inform the englacial void ratio used in subglacial hydrology models. This would allow time dependence and finer spatial variation, including in the vertical dimension as well as horizontal, than is currently possible with a bulk parameter. Periods of increased supraglacial inputs can require a sizable increase in englacial void ratio for subglacial models to accurately predict moulin water level (Hoffman et al., 2016). During these times, MouSh predicts rapid growth in moulin capacity (Fig. 7 and Fig. 8). This correspondence suggests plausible close ties between moulin size and the englacial void ratio: moulin size modifies englacial storage spatially and temporally.~~

~~MouSh can be used to infer both moulin size and shape, which would effectively change the englacial void ratio in all three spatial dimensions and time. The shape of the moulin imposes new temporal variability on water level and subglacial channel size: moulins with large near-surface chambers that funnel down to become narrower at the water line, for instance, have lower-magnitude and smoother variations in water level compared to cylindrical moulins, whereas moulins with small surface openings that widen toward the water line have larger and peakier water-level variations (Trunz, 2021). Thus, when the shape of a moulin is explicitly resolved, any assumed linear relationship between melt input rates and the range or pattern of oscillations in water level and subglacial channel size breaks down. The relationship also changes with the water level in the moulin; hence it varies in time.~~

MouSh demonstrates that moulin capacity can vary ~~greatly~~ both seasonally and during ~~short periods of~~ large ~~variability~~variations in supraglacial input. Moulin growth rates are largest particularly when water levels are above flotation, ~~maximizing turbulent~~ when both melting and ~~outward visco-elastic~~viscous deformation~~.~~ work to increase moulin capacity. Our results show that moulin capacity changes by ~~~20~~up to 13% daily under realistic conditions (Fig. ~~10~~9b) and ~~~50–100~~87 to 138% over the melt season (Fig. ~~6c and Fig. 8b), with larger changes during periods of large supraglacial input variability and at locations with thicker ice.~~6c). These variations in moulin shape and size may explain difficulties with modeling subglacial behavior during melt events (Cowton et al., 2016), which are sometimes addressed by temporarily increasing englacial storage (Hoffman et al., 2016). Our results with MouSh ~~lead us to recommend~~suggest that modeling moulin shape ~~and size be modeled~~ alongside the evolution of the subglacial system could potentially improve the representation of subglacial pressures, especially during periods of large meltwater variability~~, in order to more accurately predict subglacial water pressures and ice motion.~~; however, additional development is necessary to explore the impact of multiple moulins evolving along with the subglacial system.

Practical limits on model complexity or computational costs may preclude fully time-evolving moulin geometries. While not ideal, an arbitrary static shape ~~is still~~may preferable to a static cylinder (Trunz, 2021). Therefore, we interpret our moulin shape results (Fig. 7) to recommend a representative shape for a static moulin. Below the water line, a cylinder is a reasonable approximation, especially in thinner ice or for newly made moulins, for which full-column ice deformation is minimized. Above the water line, moulin shape is widely variable in time, by location, and across parameter combinations. It is especially sensitive to the friction parameter for open-channel flow (Fig. 3m and Fig. 4m), with low friction values making bottle-shaped moulins that have narrow necks above the water line and larger chambers below the water line, and high friction values making goblet-shaped moulins with open rooms and amphitheaters above the water line atop a narrower geometry below the water line. Exploration of Greenland moulins to date has uncovered multiple goblet-shaped moulins and a few instances of near-cylindrical moulins, but no bottle-shaped moulins (Covington et al., 2020; Moreau, 2009; Trunz, 2021). Overall, our MouSh results support goblet-shaped moulins, although with great variation in the height and width of the upper chamber.

4.~~6~~7 Limitations of the current MouSh englacial – subglacial model

Moulins are a dynamic component of the channelized englacial–subglacial system~~, and~~; therefore, explicitly modeling their evolution can ~~therefore~~ improve the accuracy of ~~englacial–subglacial~~glacial hydrology models~~ (Sect. 3.4).~~. MouSh currently uses a single subglacial channel to represent the entire subglacial system, limiting its accuracy. ~~-~~An optional baseflow term, which parametrizes subglacial water flow from surrounding regions, improves MouSh performance. This ~~base flow~~baseflow, added directly to the subglacial channel, is necessary to produce realistic equilibrium water levels with the realistic supraglacial inputs we prescribed (Fig. 6a). The baseflow value we used does not explicitly represent any specific process because our model runs resolve only a single moulin connected to a single channel, whereas in the real world, multiple moulins feed a network of channels. The idealized baseflow term conceptually connects to multiple potential water sources, including (1) basal melting from geothermal and frictional heating, (2) supraglacial water delivered via nearby moulins that are connected to the same subglacial channel, and (3) water that moves from the channelized system to the surrounding inefficient system at high pressures and then flows back into the subglacial channel at lower water pressures (Hoffman et al., 2016; Mair et al., 2001, 2002; Tedstone et al., 2015).

The addition of baseflow maintains a larger, less variable subglacial channel. This can alternately be achieved by lessening the local hydraulic gradient, thus increasing the mean water pressure along a given reach. This may locally occur where one subglacial channel enters another in an arborescent network (Fountain and Walder, 1998). MouSh currently does not have an interconnected network of channels; however, this is under development (Trunz, 2021).

We use a highly simplified model of the subglacial hydrology system: a single channel that connects the moulin to the ice-sheet margin. Yet, MouSh results clearly indicate that including and evolving a moulin can ~~reduce diurnal and long-term~~alter the hydraulic gradient of the subglacial ~~pressures~~system via time-varying storage in the moulin (Fig. ~~10a). This has implications for~~10), though in our current single moulin configuration, there is limited impact on subglacial channel ~~growth and size (Fig. 10c). Nevertheless~~geometry. Further, MouSh currently lacks a distributed system, which limits its fidelity for assimilating daily meltwater volumes into the subglacial system~~. Realistically~~; realistically, the channelized subglacial system cannot always accommodate the full volume of meltwater produced during summer days, and a portion of this water goes into the distributed system (e.g., Mair et al., 2001, 2002). In our model, however, when the~~ channelized~~ system is overwhelmed, the water level in the moulin rises above what is typically observed, and sometimes even exceeds the height of the ice (Figs. 6b, ~~S2b, S3b~~S4b). The melt-driven opening and creep closure processes in the subglacial model explain this behavior: A lower water input to the moulin (Q_{in}) lowers the water flux into the subglacial system (Q_{out}), which lowers the melt rates that keep subglacial channels open, reducing the size of the subglacial channels and thus further reducing the subglacial water flux. This increases the water level in the moulin. Thus, a reduced rate of surface melt can counterintuitively raise the modeled water level (Fig. 6 around day 30), whereas in reality, much of that water would enter the inefficient subglacial hydrologic system when moulin water levels exceed flotation. If the moulin model were coupled to a two-component subglacial model that represents the inefficient system alongside the channelized system, we would anticipate a much-improved ability to assimilate a wide range of meltwater input rates.

5 Conclusions

Results from the MouSh model show that moulins are not static cylinders. ~~Their shapes oscillate daily by some 30% around an equilibrium value reached within the first week of diurnally oscillating inputs. Daily fluctuations~~Daily fluctuations in moulin capacity change the water volume held in the englacial hydrologic system, which in turn influences the evolution of the subglacial channels that moulins feed. When we represent a moulin as a static cylinder in our englacial–subglacial hydrology model, these daily fluctuations can be ~~substantially over estimated~~overestimated or underestimated, affecting the volume of water stored englacially and the hydraulic gradient of the subglacial system. Modeled moulin size and shape may provide a more realistic representation of moulin water level and the englacial void ratio commonly used in subglacial hydrology models, particularly with future efforts to improve the parameterization of moulin development above the water line. This could be achieved by using an englacial hydrology – channelized subglacial system model, such as the MouSh model we present here, to characterize variability in moulin size and shape, or by coupling moulin models to more complete models of the subglacial system (channelized, distributed, and optionally weakly connected) to make a unified englacial–subglacial hydrology model system. Improving the representation of the englacial–subglacial system to explicitly include moulins would have greatest efficacy during periods of rapidly varying supraglacial input (e.g., during the beginning and end of the melt season and during melt events) and in inland areas with thick ice and high overburden pressures. These are coincident with situations where subglacial models without moulins, or with implicitly static moulins, tend to perform poorly.

Code and Data availability. The Moulin Shape model is publicly available at ~~https://github.com/kpoinar/moulin-physical-model/tree/MouSh-beta-revisions~~https://github.com/kpoinar/moulin-physical-model (we will make a release when revisions are complete). The model results used in the analysis presented here are archived at the University at Buffalo Libraries at http://hdl.handle.net/10477/82587.

Author contributions. L.C.A. and K.P. jointly conceived of and developed the MouSh model. Both L.C.A. and K.P. designed the study, executed the model runs, analyzed the data, produced the figures, and wrote the manuscript. C.T. implemented the subglacial module, participated in discussions, and edited the manuscript.

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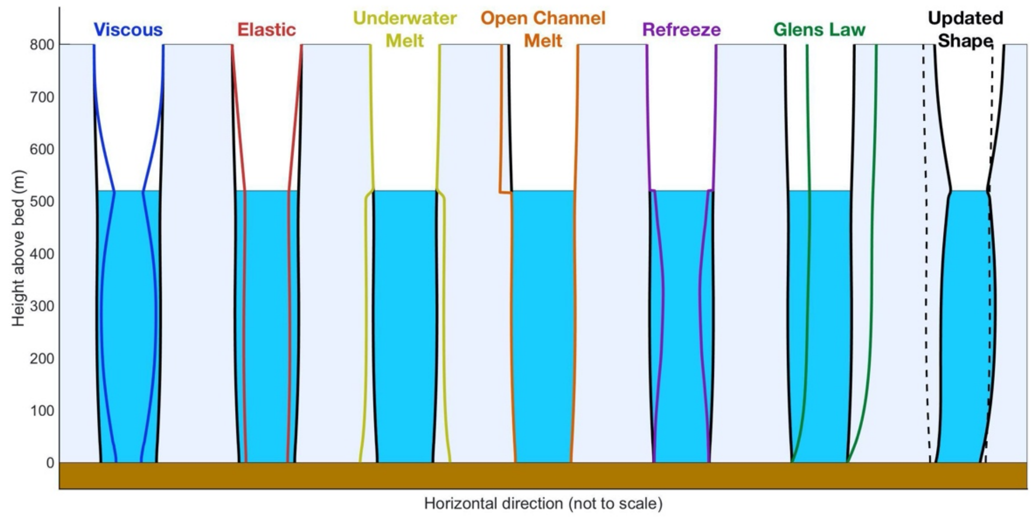
~~Figure~~FIGURE 1



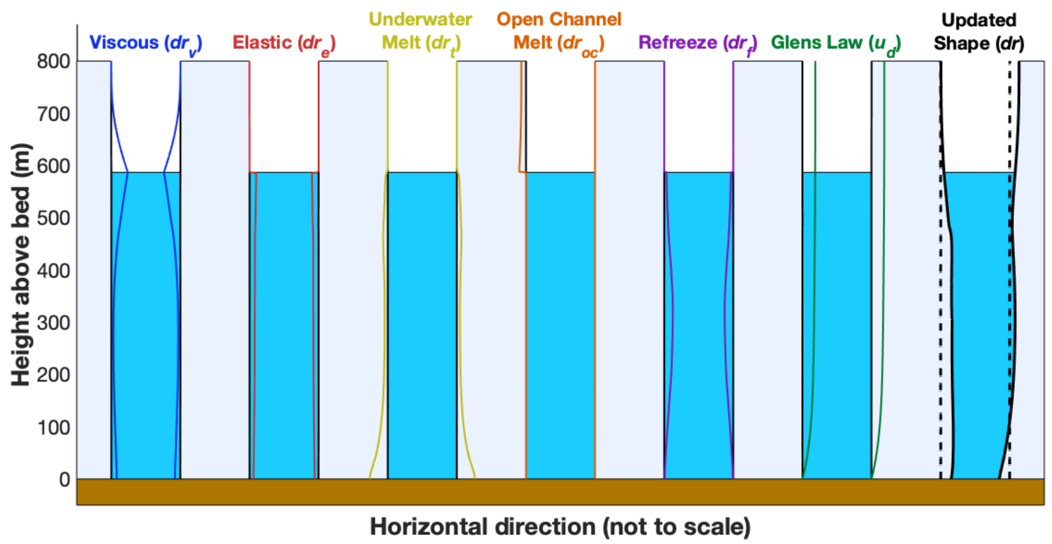
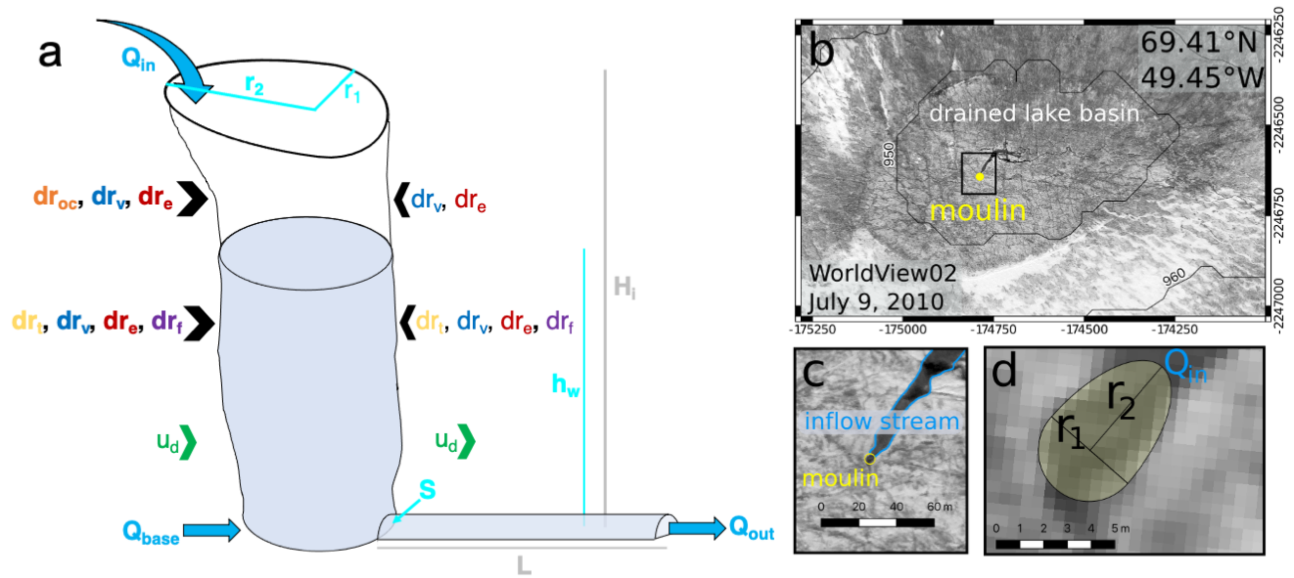


Figure 1. Processes included in the MouSh model. Black lines show a base moulin geometry that each process acts on, and colored lines show the change in moulin geometry (not to scale) due to that process alone. From left to right: changes in moulin geometry due to viscous deformation; elastic deformation; melting by turbulent energy dissipation of flowing water inside the moulin; melting by open-channel water flow along bare ice; refreezing over winter inside the moulin; and deformation due to ice motion prescribed by Glen's flow law. Unlike the other components, elastic deformation is instantaneous, but applied over the model timestep (Sect. 2.2.1; Supplement ~~S2~~S1). The right-most

moulin shows the moulin geometry before (dashed black lines) and after (solid black lines and blue water) ~~a~~several hypothetical model ~~timestep~~timesteps, i.e., the sum of all processes shown in the preceding panels. Changes are not to scale.

~~Figure~~

FIGURE 2



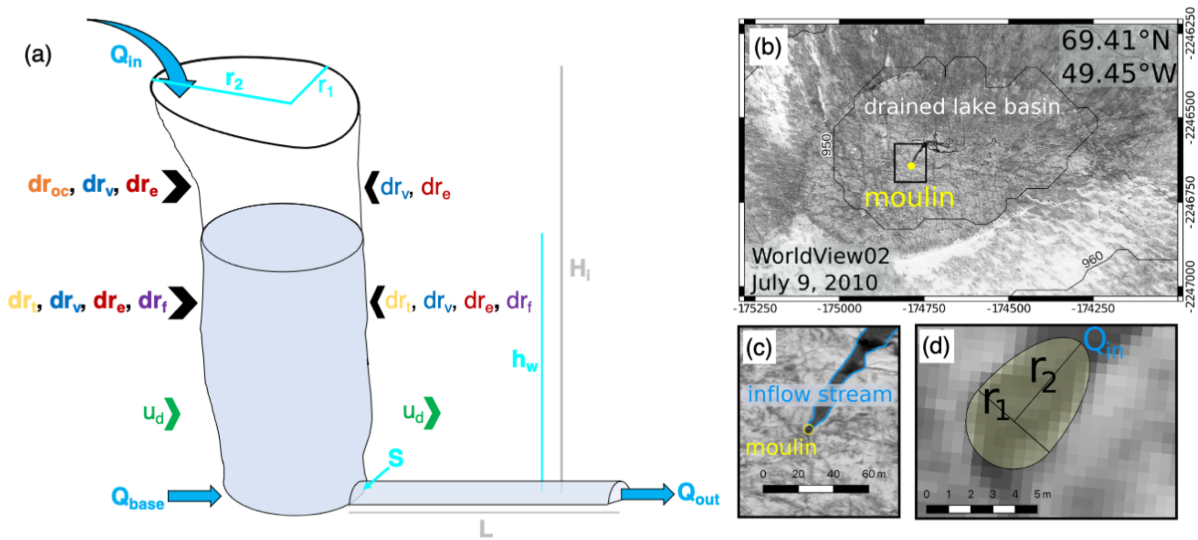


Figure 2. MouSh geometry and surface expression of a moulin and its reflection in the MouSh model. (a) Schematic of MouSh geometry and inputs. Inflow and outflow of the system are indicated by Q_{in}, Q_{out}, and Q_{base}. Time evolving moulin and subglacial parameters include moulin radii (r_1, r_2), moulin water level (h_w), and subglacial cross-sectional area (S). r_1 and r_2 are evolved by dr_{oc}, dr_v, dr_e, dr_f, and dr_t (open channel melting, viscous deformation, elastic deformation, refreezing, and turbulent melting, respectively; colored as in Fig. 1). u_d shears the moulin as prescribed by Glen's Flow Law. Ice thickness and subglacial path length are indicated by H_i and L, respectively. Ice flow is from left to right. ~~Further details are in Sect. 2.~~ Modified from Trunz (2021). (b) WorldView-2 scene from July 2010 of an approximately 1.2 × 0.8 km region surrounding the example moulin (yellow) formed by a drained supraglacial lake. (c) Detail of panel b, with the inflow stream and moulin indicated. (d) Detail of panel c, showing the moulin minor radius r_1, major radius r_2, and water input Q_{in} from the inflow stream, as represented in the MouSh model. Maps generated by authors. WorldView image © 2010 DigitalGlobe, Inc.

~~Figure~~

FIGURE 3

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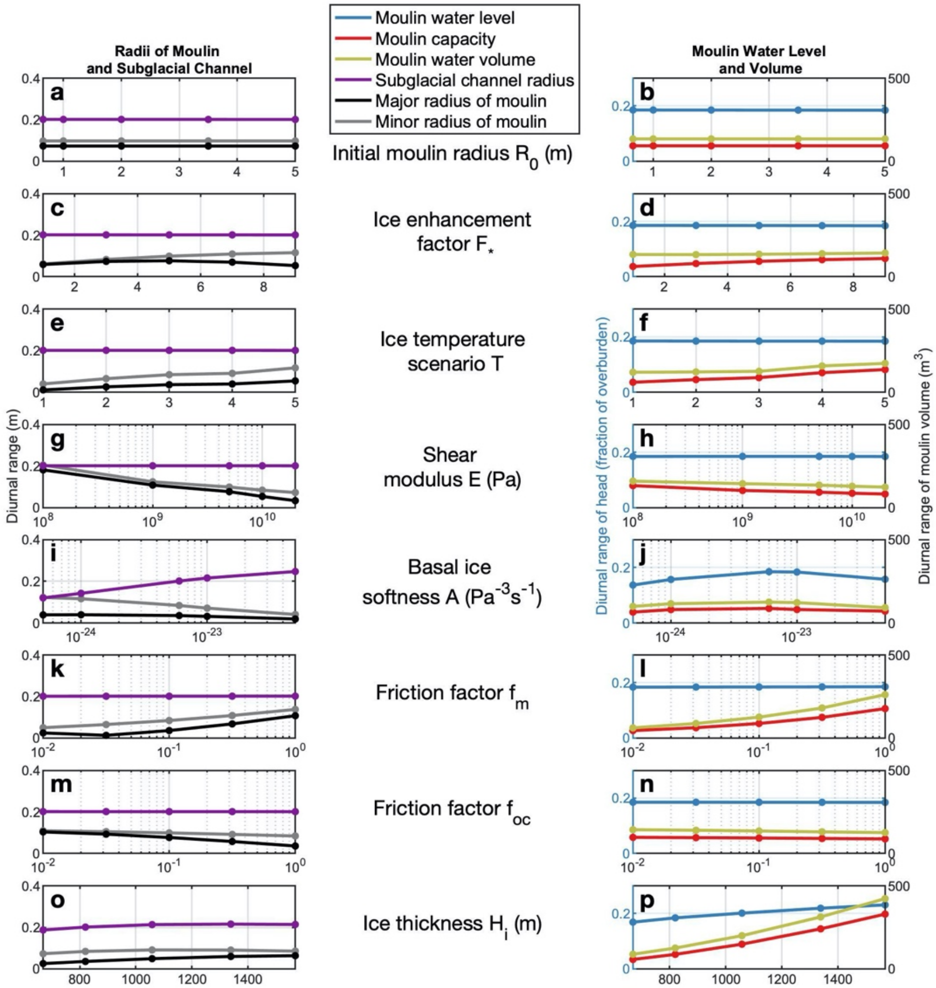


Radii of Moulin and Subglacial Channel
Moulin Water Level and Volume
Moulin water level
Moulin capacity
Moulin water volume
Subglacial channel radius
Major radius of moulin
Minor radius of moulin
Initial moulin radius R_0 (m)
Ice enhancement factor F_*
Ice temperature scenario T
Shear modulus E (Pa)
Basal ice softness A ($Pa^{-3}s^{-1}$)
Friction factor f_m
Friction factor f_{oc}
Ice thickness H_i (m)
Diurnal range (m)
Diurnal range of head (fraction of overburden)
Diurnal range of moulin volume (m^3)

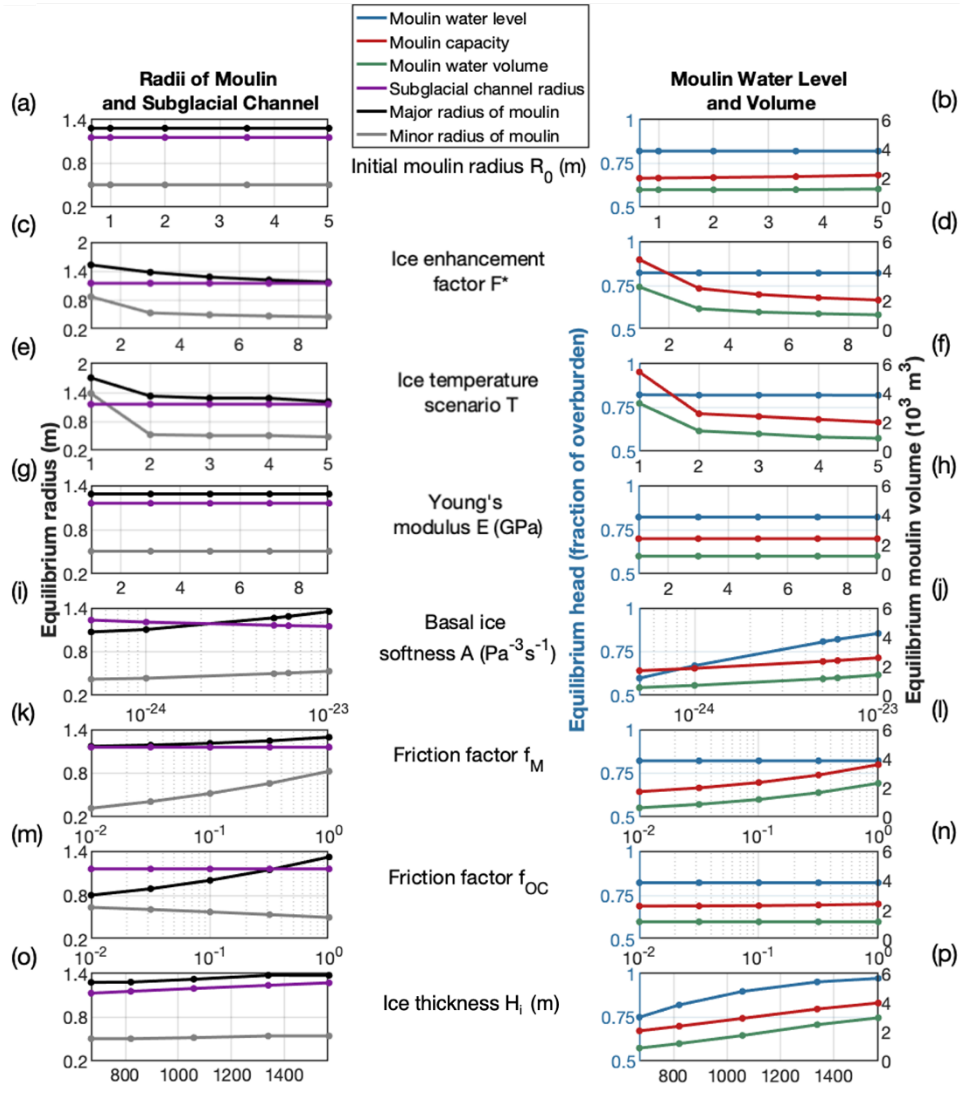
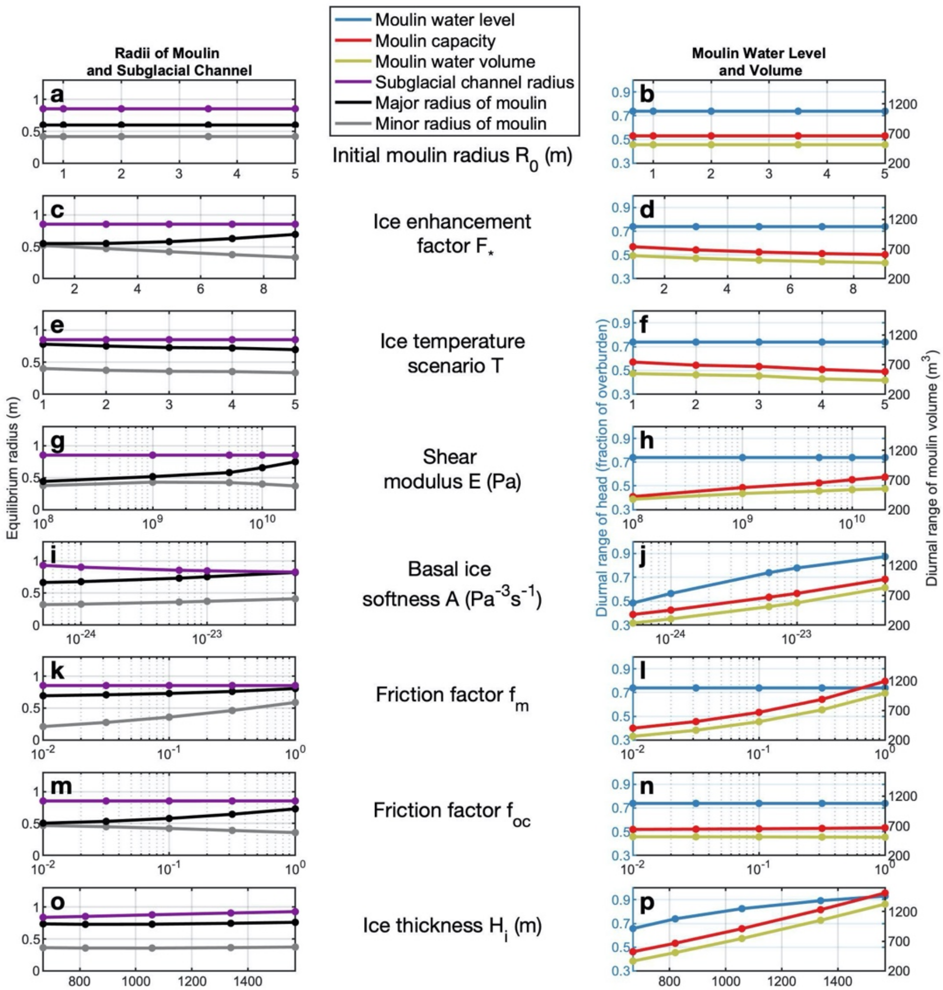


Figure 3. Results of parameter sensitivity studies for ~~10~~40-day MouSh model runs. Shown are the sensitivity of moulin size to initial condition for moulin radius (a–b), enhancement factor for englacial ice (c–d), ice temperature scenario (e–f), Young's modulus (g–h), softness of basal ice (i–j), friction factor for water flow beneath the water line (k–l), friction factor for water flow above the water line (m–n), and ice thickness (o–p). The left column shows the moulin radii (black and grey) at the mean water level and the mean subglacial channel radius (purple) averaged over the final 24-hour period of the ~~ten~~40-day model run. The right column shows the equilibrium water level (blue), moulin ~~volume~~capacity (red), and volume of water in the moulin (~~gold~~green) averaged over the same 24-hour period. Overall, moulin radius is most sensitive to the friction factors, while moulin water level and volume are most sensitive to ice thickness H_i (also an indicator of the hydraulic potential gradient) and basal ice softness A. Note the different y-axis range in panels (c) and (e).

~~Figure~~FIGURE 4



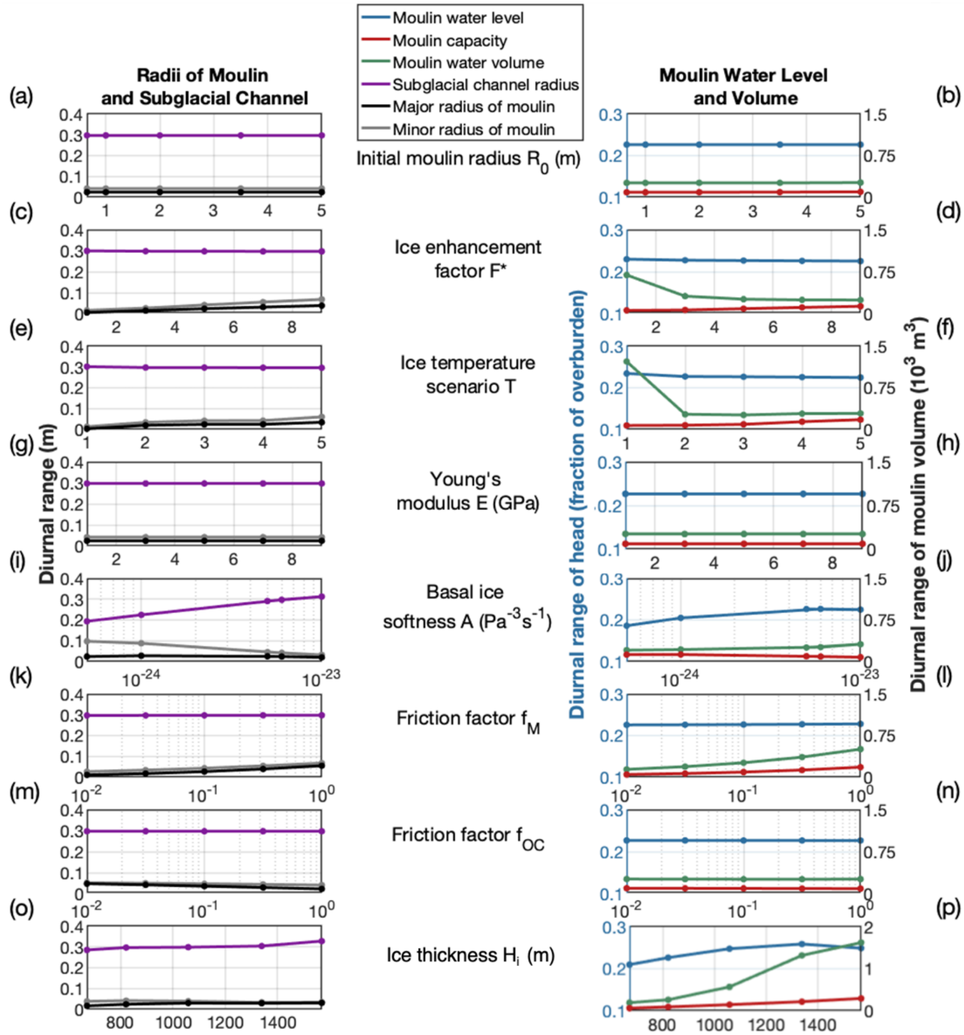
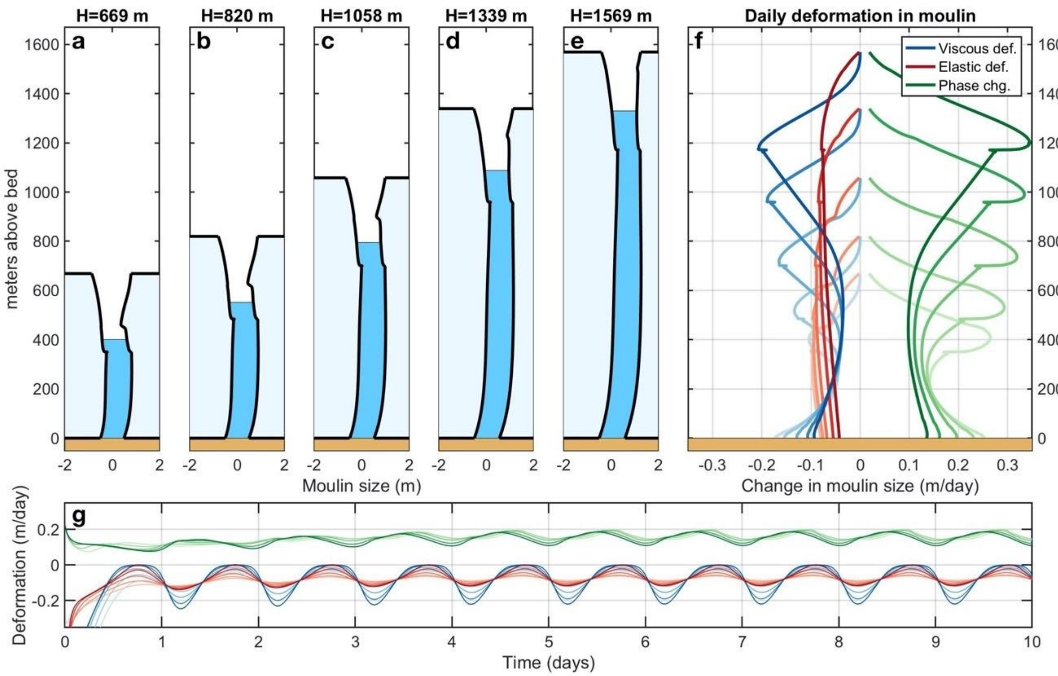


Figure 4. Diurnal variations in moulin sizes in ~~10~~40-day parameter sensitivity runs. Shown are the sensitivity of diurnal variation in moulin size and water storage metrics to initial condition for moulin radius (a–b), enhancement factor for englacial ice (c–d), ice temperature scenario from coldest to warmest ice (e–f), Young's modulus (g–h), softness of basal ice (i–j), friction factor for water flow beneath the water line (k–l), friction factor for water flow above the water line (m–n), and ice thickness (o–p). The left column shows diurnal variations in moulin radii (black and grey) at the equilibrium water level and the subglacial channel radius (purple) in the final 24-hour period of the ~~ten~~40-day model run. The right column shows the diurnal variation in water level (blue), moulin volume (red), and volume of water in the moulin (~~gold~~green) within the same 24-hour period. Note the right y-axis difference in panel (p).

~~Figure~~

FIGURE 5


H=669 m
H=820 m
H=1058 m
H=1339 m
H=1569 m
Daily deformation in moulin
a
b
c
d
e
f
g
meters above bed
Moulin size (m)
Viscous def.
Elastic def.
Phase chg.
Change in moulin size (m/day)
Deformation (m/day)
Time (days)

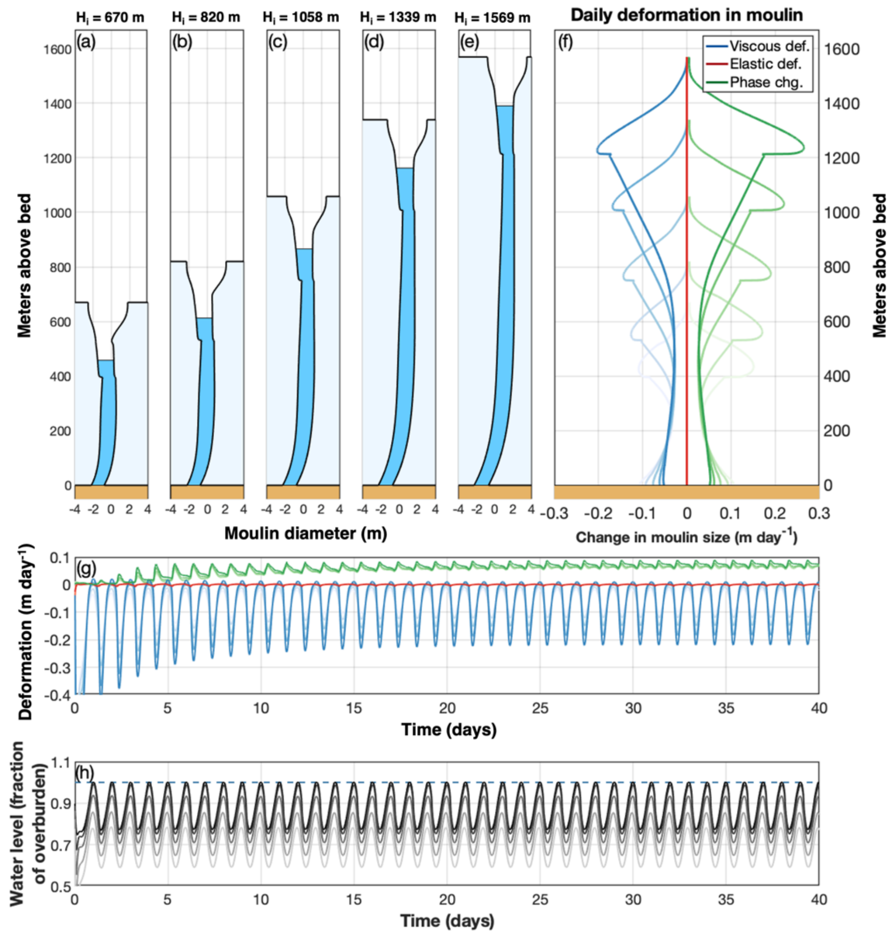
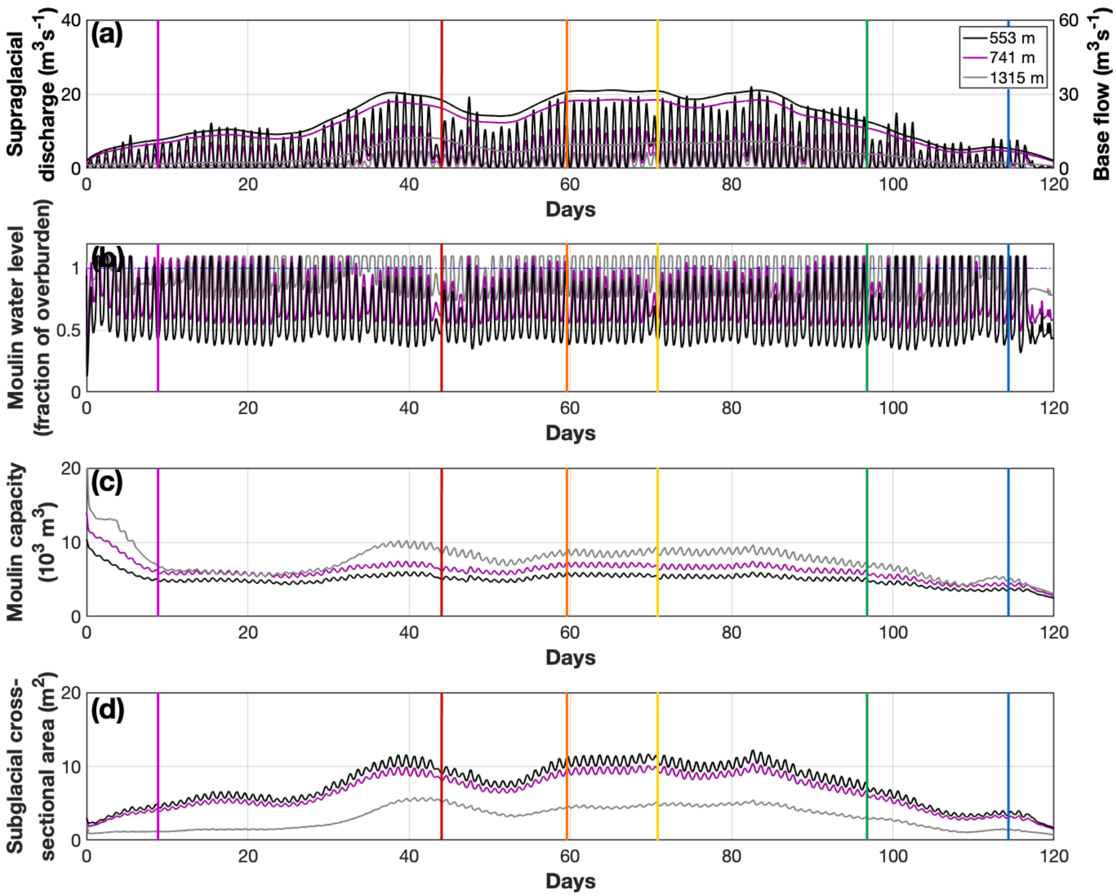


Figure 5. Contributions of viscous deformation, elastic deformation, and phase changes to moulin geometry. (a–e) Equilibrium geometries of five moulins in ice of different ice thicknesses *H* and different distances from the terminus (same as Fig. 6o–p) averaged over the final 24-hour period of a ~~10~~40-day model run. (f) Vertical variation of viscous deformation (blue), elastic deformation (red), and phase change (green) contributions to moulin geometry averaged over the same 24-hour period. Negative values indicate contributions to moulin closure; positive values open the moulin. Darkening shades of each color map to moulins of increasing ice thickness. Closure and opening rates are greatest at the minimum daily water level (which is inferable by the lower notch in the moulin wall). (g) Time series of the components shown in panel f (colors the same) at the mean water level over the entire ~~ten~~40-day model run. The greater diurnal range in water level in moulins in thick ice drives the observed larger diurnal variations in viscous ~~and elastic deformation~~deformation. (h) For reference, moulin water level as fraction of overburden for different ice thicknesses H_i. Lighter greys indicate thinner ice; blue dashed line indicates where fraction of overburden = 1.

~~Figure~~FIGURE 6



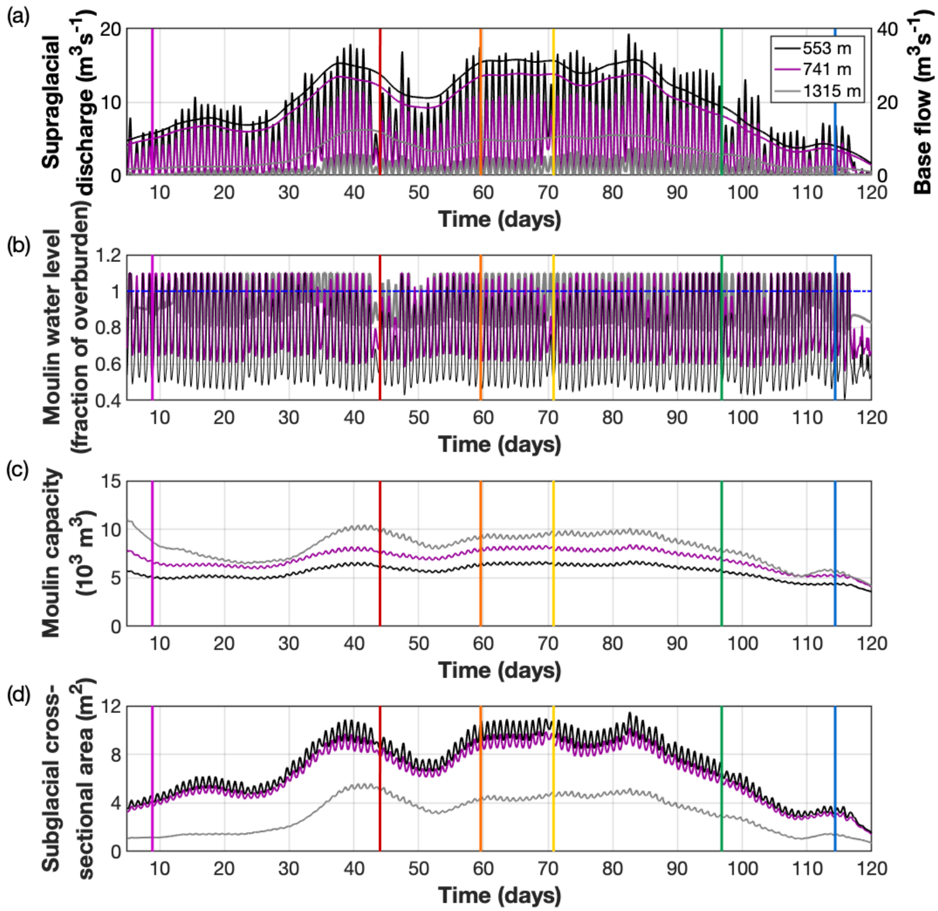
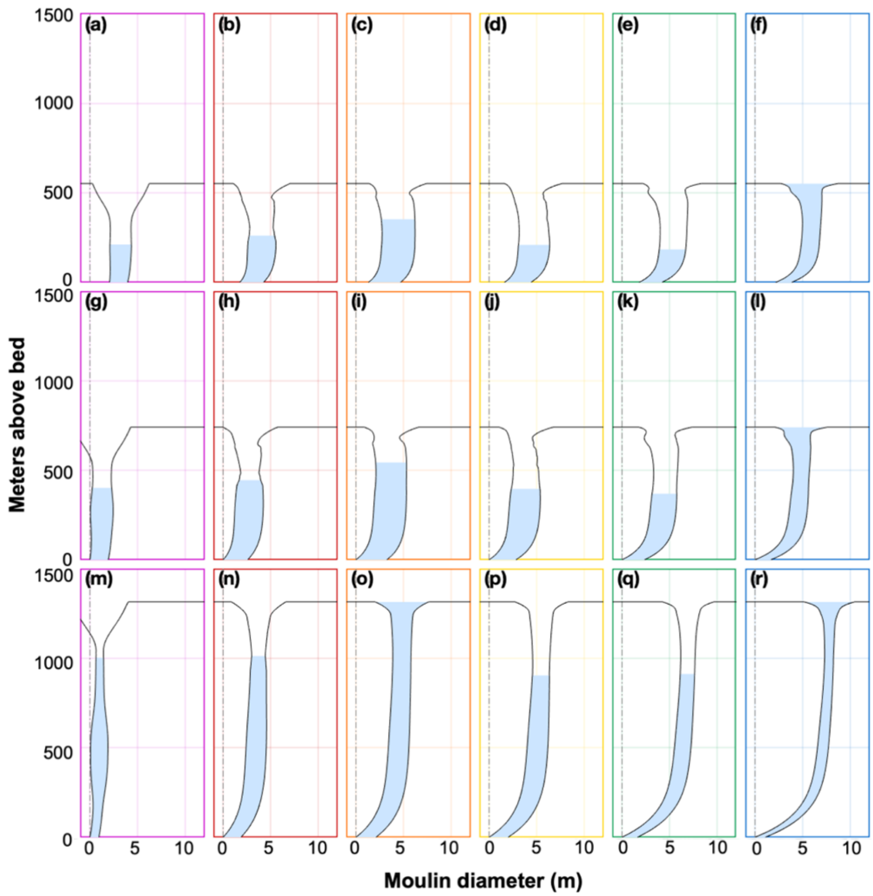


Figure 6. MouSh model runs with realistic supraglacial and ice conditions. The model runs are for a low-elevation ~~basin~~Basin 1 (553 m ice thickness; black lines), mid-elevation ~~basin~~Basin 2 (741 m ice thickness; purple lines), and high-elevation ~~basin~~Basin 3 (1315 m ice thickness; grey lines). (a) Supraglacial discharge into the moulin Q_{in} and prescribed base flow Q_{base}. (b) Moulin water level as a fraction of overburden. Note that the highest elevation moulin exceeds the ice surface most days. (c) Moulin capacity, or the total moulin volume. (d) Subglacial channel cross-sectional area. Colored vertical lines indicate times in Fig. 7. Note x-axes start on day 5.

~~Figure~~FIGURE 7



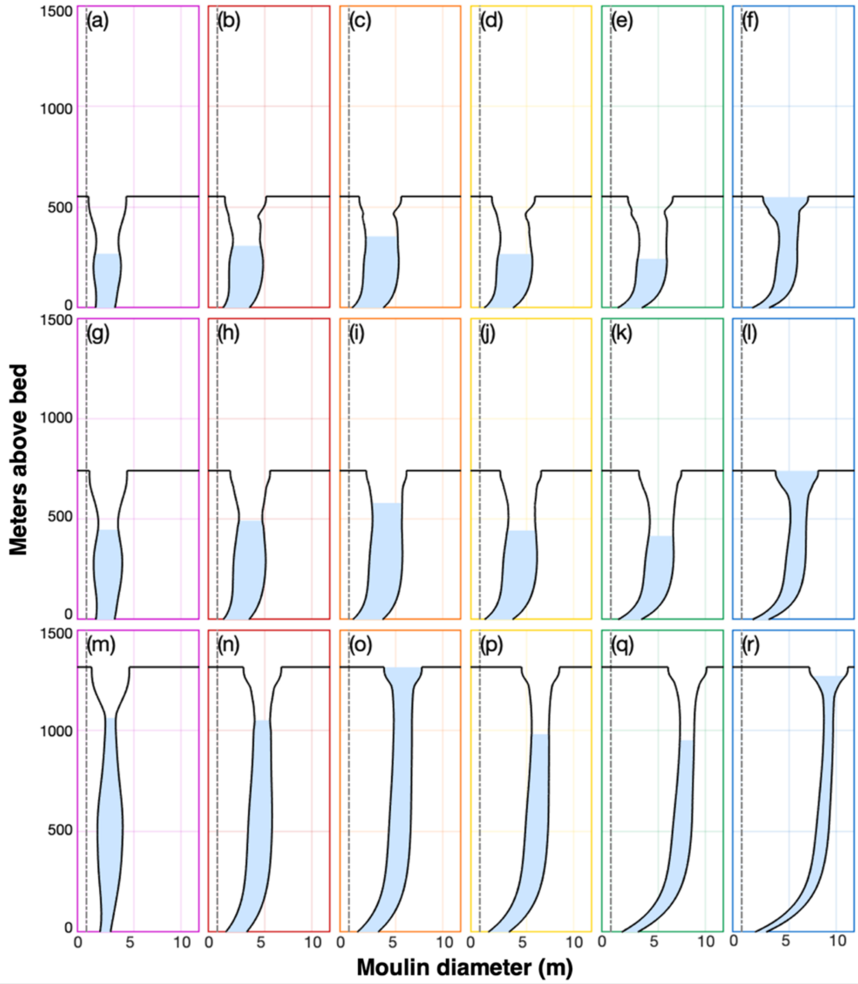
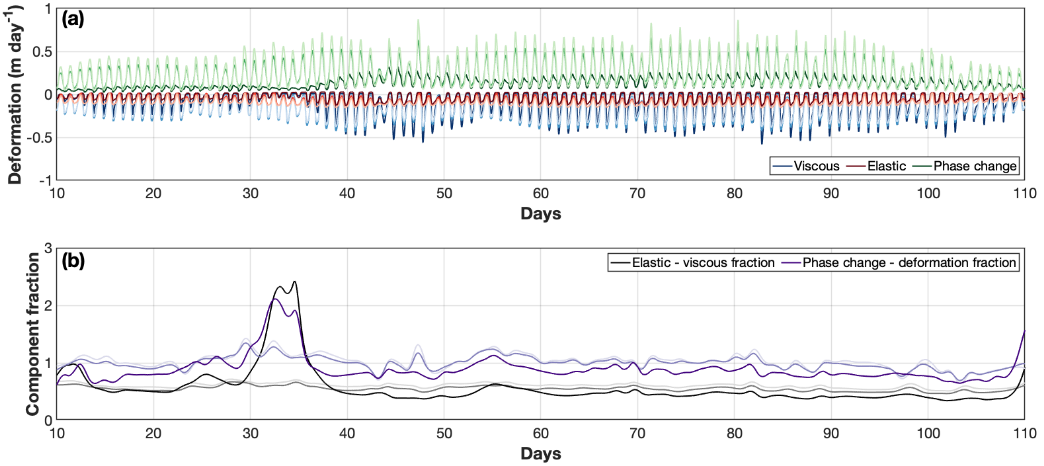


Figure 7. Evolution of moulin geometry over the melt season. Colored boxes correspond to the times indicated with colored vertical lines in Fig. 6. (a–f) Basin 1 with ice thickness of 553 m. (g–l) Basin 2 with ice thickness of 741 m. (m–r) Basin 3 with ice thickness of 1315 m. Axes are not to scale.

~~Figure~~

FIGURE 8



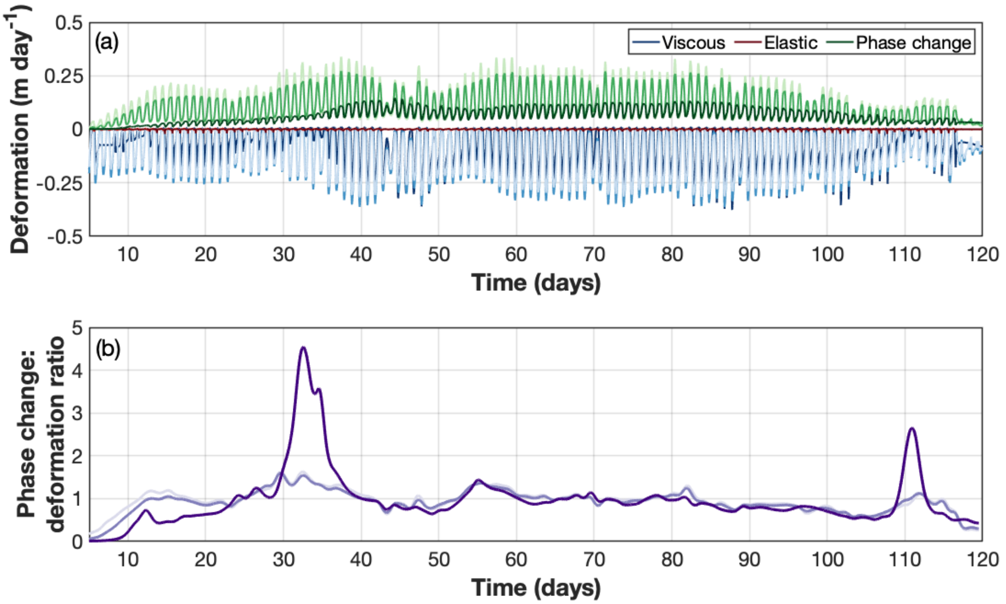
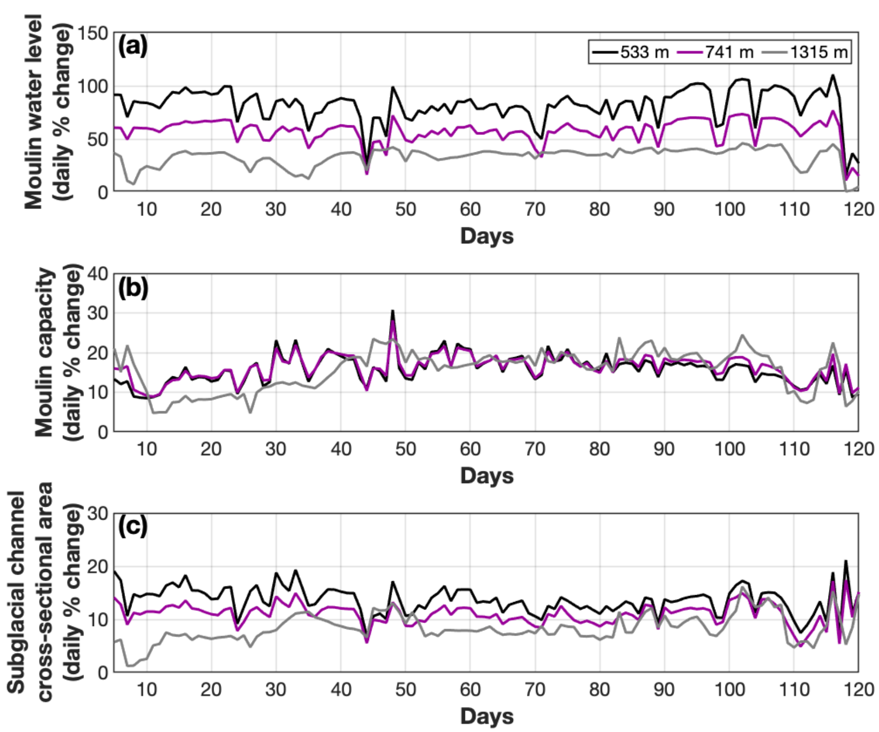


Figure 8. Time series of viscous, elastic and phase change components of moulin evolution and their relative importance in determining moulin geometry. (a) Time varying viscous (blues), elastic (reds), and phase change (melting, greens) components of moulin geometry. ~~(b) The ratio of elastic to viscous deformation (greys) indicates the relative importance of the two deformational processes in moulin evolution. All values are lower than 1, indicating that viscous deformation is always greater. The~~(b) The daily ratio of the total amount of phase change (melting above and below the water line) to total deformation (elastic plus viscous; purples). Values above 1 indicate that melting dominates; values below 1 indicate that deformation dominates. Data is smoothed over 24 h. For both panels, light colors are for Basin 1 (H_i =553 m), medium colors for Basin 2 (H_i =741 m), and dark colors for Basin 3 (H_i=1315 m). Note x-axes start on day 5.

~~Figure~~FIGURE 9



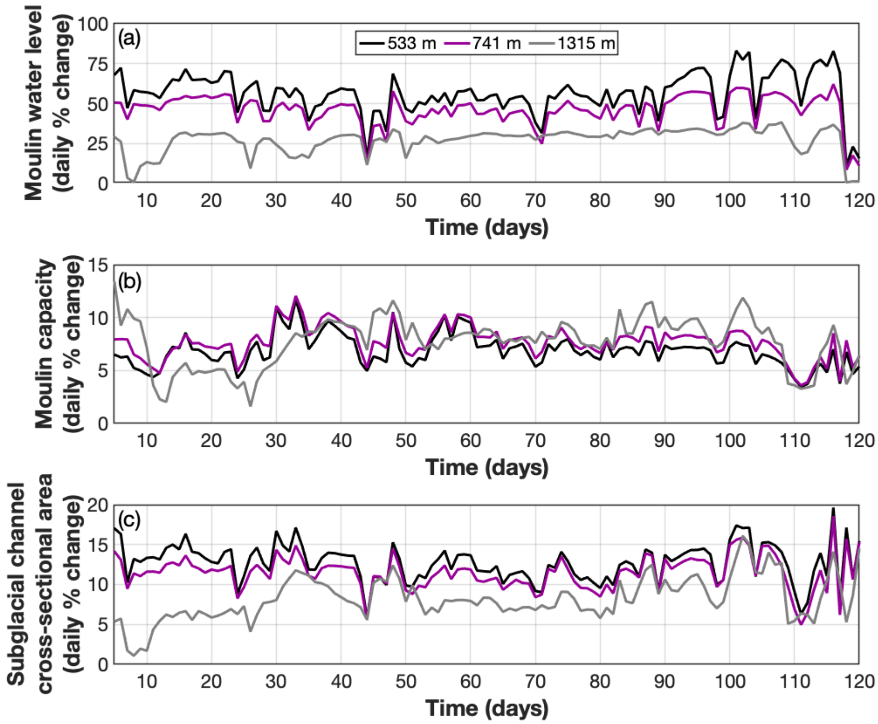
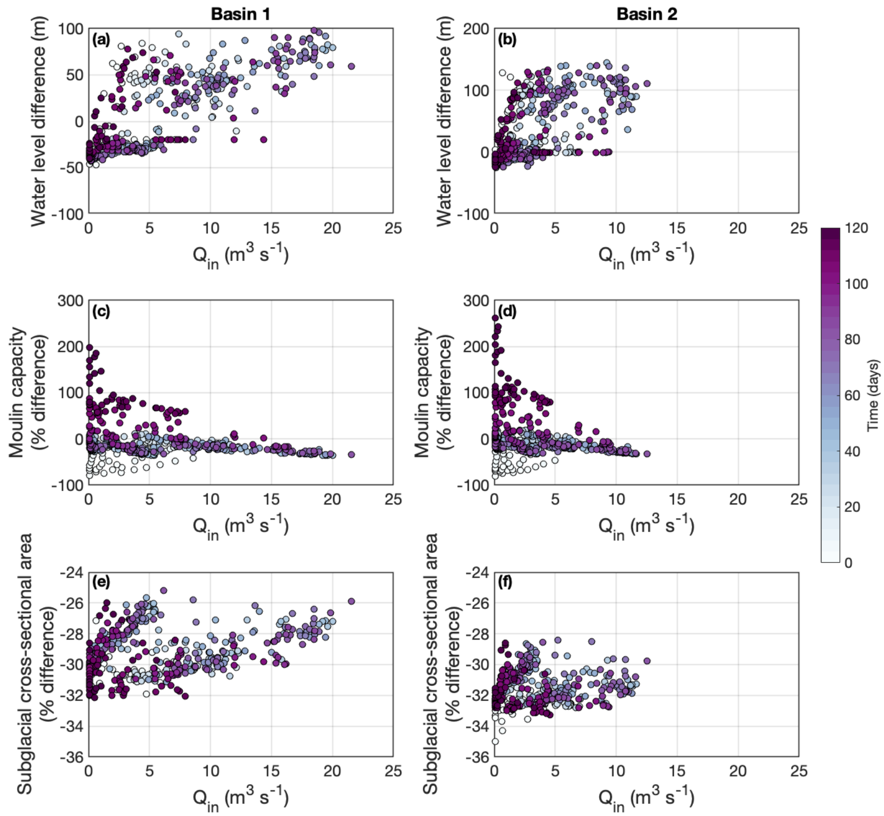


Figure 9. Daily percentage change in moulin variables relative to the daily mean value. (a) Daily percentage change in moulin water level relative to the daily mean water level for Basins 1, 2, and 3 (black, purple, and grey lines, respectively). (b) Daily percentage change in moulin capacity relative to the daily mean moulin capacity. (c) Daily percentage change in the subglacial channel cross-sectional area relative to the daily mean value. For (b–c), colors are as in (a).

~~Figure~~

FIGURE 10


Basin 1
Basin 2
(a)
(b)
(c)
(d)
(e)
(f)
Water level difference (m)
Moulin capacity (% difference)
Subglacial cross-sectional area (% difference)
Q_{in} (m^3 s^{-1})
Time (days)

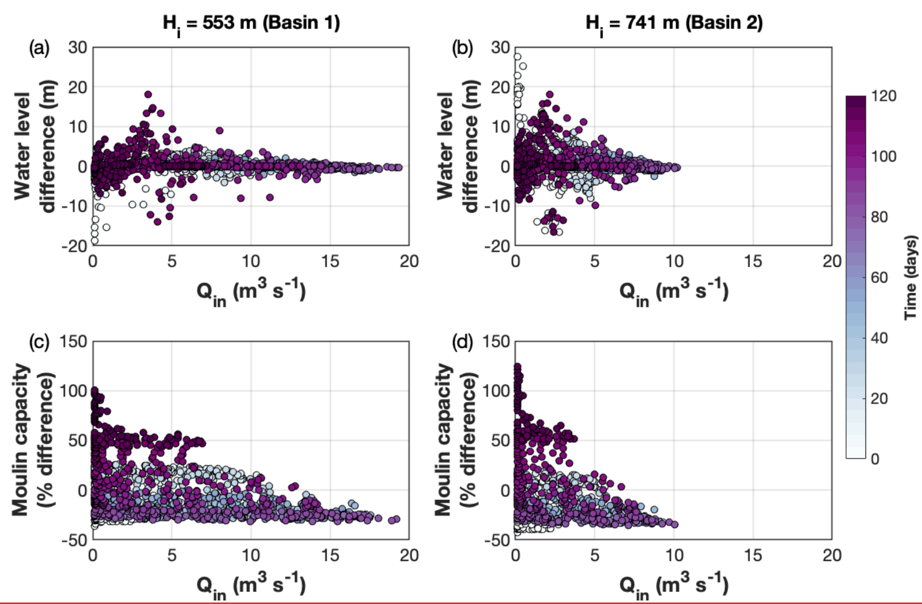

H_i = 553 m (Basin 1)
H_i = 741 m (Basin 2)
(a)
(b)
(c)
(d)
Water level difference (m)
Moulin capacity (% difference)
Q_{in} (m^3 s^{-1})
Time (days)

Figure 10. Difference between variable and fixed moulin geometries for Basin 1 and 2 (ice thicknesses of 553 m and 741 m, respectively). The fixed moulins are cylinders with a fixed radius of 1.6 and 1.4 m ~~(~~for Basin 1~~)~~ and 2 ~~m (Basin 2),~~respectively, which are the time-mean ~~radius~~radii at the ~~equilibrium~~mean water level for the variable moulins. In all instances, the difference is calculated as (*cylindrical* – *variable*) with instances of percentage difference calculated as (*cylindrical* – *variable*) / (*variable*). (a, b) Difference in moulin water level for Basin 1 ~~(black)~~ and Basin 2 ~~(purple).~~respectively, plotted every hour. Negative values indicate periods where the variable moulin water levels are higher than those of the fixed cylindrical moulin.~~-~~ (c, d) Percentage difference in moulin capacity~~.~~, plotted every 2 hours for clarity. When values are negative, the variable moulin is larger than the fixed cylindrical~~ moulin. (e, f) Percentage difference in subglacial channel cross-sectional area. These values are persistently negative, indicating that the subglacial channel is larger with a variable~~ moulin.

Table 1. MouSh model constants and parameter ranges. During realistic runs (Sect. ~~2.4) Median values were~~2.5.3) Values used during realistic model experiments are generally ~~used. In instances where values used differ from~~ the median value~~,~~ of the ~~values used is indicated in parentheses.~~ sensitivity experiment.

~~**Constant**~~		~~**Value**~~		~~**Units**~~
~~ρ_i~~	~~Ice density~~	~~910~~		~~kg m^{-3}~~
~~ρ_w~~	~~Water density~~	~~1000~~		~~kg m^{-3}~~
~~g~~	~~Gravitational acceleration~~	~~9.81~~		~~m s^{-2}~~
~~L_f~~	~~Latent heat of fusion~~	~~335000~~		~~J kg^{-1}~~
~~M_u~~	~~Dynamic viscosity (liquid water)~~	~~0.0017916~~		~~Pa s~~
~~K_w~~	~~Thermal conductivity (liquid water)~~	~~0.555~~		~~J (m K s)$^{-1}$~~
~~C_w~~	~~Heat capacity (liquid water)~~	~~4210~~		~~J (K kg)$^{-1}$~~
~~C_p~~	~~Heat capacity (ice)~~	~~2115~~		~~J (K kg)$^{-1}$~~
~~**Parameter**~~		~~**Median value**~~	~~**Range**~~	~~**Units**~~
~~R_0~~	~~Initial moulin radius~~	~~2.4 (3)~~	~~0.5 to 5~~	~~m~~
~~E~~	~~Ice deformation enhancement factor~~	~~5~~	~~1 to 9~~	~~-~~
~~T(z)~~	~~Ice temperature~~	~~-6 (FOXX profile)~~	~~-23 to 0~~	~~°C~~
~~Y~~	~~Young's modulus~~	~~5 (9)~~	~~1 to 9~~	~~GPa~~
~~A~~	~~Basal ice softness~~	~~6×10^{-24}~~	~~5×10^{-25} to 5×10^{-23}~~	~~Pa^{-3} s^{-1}~~
~~f_M~~	~~Friction factor (under water)~~	~~0.1~~	~~0.01 to 1~~	~~-~~
~~f_{OC}~~	~~Friction factor (subaerial / open channel)~~	~~1 (0.8)~~	~~0.01 to 1~~	~~-~~
~~H~~	~~Ice thickness~~	~~1058 (553, 741, 1315)~~	~~669 to 1569~~	~~m~~

<table>
<tr><th colspan="2">Constant</th><th>Description</th><th colspan="2">Value</th><th>Units</th></tr>
<tr><td colspan="2">ρ_i</td><td>Ice density</td><td colspan="2">910</td><td>kg m^{-3}</td></tr>
<tr><td colspan="2">ρ_w</td><td>Water density</td><td colspan="2">1000</td><td>kg m^{-3}</td></tr>
<tr><td colspan="2">υ</td><td>Poisson’s ratio</td><td colspan="2">0.3</td><td>-</td></tr>
<tr><td colspan="2">C_p</td><td>Heat capacity (ice)</td><td colspan="2">2115</td><td>J (K kg)$^{-1}$</td></tr>
<tr><td colspan="2">C_w</td><td>Heat capacity (liquid water)</td><td colspan="2">4210</td><td>J (K kg)$^{-1}$</td></tr>
<tr><td colspan="2">g</td><td>Gravitational acceleration</td><td colspan="2">9.81</td><td>m s^{-2}</td></tr>
<tr><td colspan="2">K_i</td><td>Thermal conductivity (ice)</td><td colspan="2">2.1</td><td>J (m K s)$^{-1}$</td></tr>
<tr><td colspan="2">K_w</td><td>Thermal conductivity (liquid water)</td><td colspan="2">0.555</td><td>J (m K s)$^{-1}$</td></tr>
<tr><td colspan="2">L_f</td><td>Latent heat of fusion</td><td colspan="2">335000</td><td>J kg^{-1}</td></tr>
<tr><th colspan="2">Parameter</th><th>Description</th><th>Realistic run value</th><th>Range</th><th>Units</th></tr>
<tr><td rowspan="2">A</td><td>A</td><td>Ice softness (englacial)</td><td colspan="2">T_i & F* dependent</td><td rowspan="2">Pa^{-3} s^{-1}</td></tr>
<tr><td>A_{sub}</td><td>Ice softness (subglacial)</td><td>6 x 10^{-24}</td><td>5 x 10^{-25} to 5 x 10^{-23}</td></tr>
<tr><td colspan="2">E</td><td>Young's modulus</td><td>5</td><td>1 to 9</td><td>GPa</td></tr>
<tr><td colspan="2">$F*$</td><td>Ice deformation enhancement factor</td><td>5</td><td>1 to 9</td><td>-</td></tr>
<tr><td rowspan="2">f</td><td>f_{oc}</td><td>Friction factor (under water)</td><td>0.1</td><td>0.01 to 1</td><td rowspan="2">-</td></tr>
<tr><td>f_m</td><td>Friction factor (subaerial / open channel)</td><td>0.8</td><td>0.01 to 1</td></tr>
<tr><td colspan="2">H_i</td><td>Ice thickness*</td><td>553, 741, 1315</td><td>669 to 1569</td><td>m</td></tr>
<tr><td colspan="2">n</td><td>Glen's Flow Law exponent</td><td>3</td><td>-</td><td>-</td></tr>
<tr><td colspan="2">R_0</td><td>Initial moulin radius</td><td>2</td><td>0.5 to 5</td><td>m</td></tr>
<tr><td colspan="2">$T_i(z)$</td><td>Ice temperature</td><td>-6 (FOXX profile)</td><td>-23 to 0 **</td><td>°C</td></tr>
</table>

*H_i defines distance from terminus L and surface slope α based on a perfectly plastic ice surface profile

**including Iken et al. (1993), Lüthi et al. (2015), and Ryser et al. (2014)

Table 2. General ice and moulin input parameters for realistic runs

Parameter	Basin 1	Basin 2	Basin 3
Ice thickness (m)	553	741	1315
Distance from terminus (km)	13.6	24.5	77.1
Catchment size (km^2)	19.8	18.4	55.5
Moulin input, mean diurnal range ($m \cdot s^{-3}$)	11.5	6.7	2.5
Moulin input, maximum value ($m \cdot s^{-3}$)	~~22.1~~19.3	12.8	~~6.~~3.8
Baseflow, mean value ($m \cdot s^{-3}$)	20.2	~~21.2~~17.7	6.2

Elastic deformation around a cylindrical hole in ice

Supplement **S1** for "Controls on Greenland moulin geometry and evolution from the Moulin Shape model", *The Cryosphere*.

Contents

S1.1 Introduction

Here we describe the derivation of the elastic deformation component of the MouSh model. It is based on Aadnøy [1987].

Bernt Aadnøy, a petroleum engineer, derived expressions for the stresses surrounding a borehole (wellbore) through competent rock [Aadnøy, 1987]. He applied the Kirsch [1898] solutions for a circular hole in a plate, stacking many plates to achieve a borehole. He derives an analytic solution for the stress field near a cylindrical borehole through a uniform, solid (non-porous) medium. From the stress solution, we derive the resulting strains using an elastic constitutive relation (Hooke's Law) and integrate the strains to get the total elastic deformation at the borehole wall. We take this borehole through rock as a direct analogue to a moulin through ice.

We treat the moulin as a stack of independent plates, each with a hole in them, of radius a. The radius of the hole in each plate (equivalently, at each elevation z) is independent of the radius in the plate above and below, but generally, $a(z)$ is smoothly varying because the forces at each z are smoothly varying.

S1.2 Aadnøy's setup and stress solutions

Aadnøy [1987] finds the stress field around a borehole by summing the independent stress contributions from three sources: hydrostatic stress ($P = P_w - P_i = \rho_w g(h_w - z) - \rho_i g(H_i - z)$), deviatoric stresses ($\sigma_x$ and σ_y), and shear stress (τ_{xy}). The sign of the pressure P is "positive outward", i.e., net water pressure ($P_w > P_i$) opens the moulin and net ice pressure ($P_i > P_w$) closes the moulin.

S1.2.1 Assumptions

The Aadnøy [1987] solution is based on the Kirsch [1898] equations, which describe the stresses around a hole when the rock is subject to deviatoric stress in one direction, but elaborates from them by adding a second deviatoric stress, a shear stress, and pressure. The Kirsch [1898] and Aadnøy [1987] equations assume that the rock (ice) is a competent linear elastic material. The Kirsch [1898] solution is appropriate for a material stressed below its elastic limit, or roughly one half its compressive strength [Goodman, 1989]. The compressive strength of ice is 3–10 MPa [Fransson, 2009], making the elastic limit 1–5 MPa. This is equivalent to the cryostatic pressure in an empty borehole in ice 100–500 m thick, or the cryo/hydrostatic pressure in a borehole in ice 1–6 km thick that is water-filled to flotation. Because moulin water levels are typically $>\sim 50\%$ the flotation level and ice thicknesses are of order ∼100–1000 m, moulins meet these requirements. We note that toward the beginning or end of the melt season (when water levels are lowest), and in thick ice (>∼1000 m), the ice surrounding the moulin likely approaches or may exceed the elastic limit.

Aadnøy [1987] assumes plane strain in z, i.e., ϵ_z=0 (no vertical deformation anywhere). This is consistent with the assumptions of our overall MouSh model and is the most basic formulation in solid mechanics. The total absence of vertical deformation in the face of finite horizontal deformation can be accommodated by an effective infinite domain in the cross-sectional plane of the moulin (xy). We happen to make this assumption anyway by summing elastic deformation from the point at infinity to the moulin wall (Sect. S1.3).

Alternately, Aadnøy [1987] also presents a plane stress solution. Plane strain is appropriate for thin plates with free surfaces (the top and bottom, z-facing surfaces), which differs from our "stack of plates" domain because our stacked plates have no free surfaces (excepting the topmost and bottommost plates). Aadnøy [1987]'s plane stress solution differs by a factor of $\frac{1+\nu}{1+\nu+\nu^2}$ from the plane strain solution [Goodman, 1989]; for $\nu = 0.3$, this is a change of 7%. The difference is small and plane stress is a less appropriate formulation than plane strain.

S1.2.2 Solution

The Aadnøy [1987] solution is in cylindrical coordinates (r, θ, z). The radius of the hole is a. Figure S1 shows the problem geometry.

The Kirsch [1898] equations for stresses around a hole in an infinite plate made of an elastic material are as follows:

$$\sigma_r = \frac{\Delta\sigma_x + \Delta\sigma_y}{2}\left(1 - \frac{a^2}{r^2}\right) + \frac{\Delta\sigma_x - \Delta\sigma_y}{2}\left(1 + \frac{3a^4}{r^4} - \frac{4a^2}{r^2}\right)\cos 2\theta + \Delta\tau_{xy}\left(1 + \frac{3a^4}{r^4} - \frac{4a^2}{r^2}\right)\sin 2\theta + \frac{a^2}{r^2}\Delta P$$
$$\sigma_\theta = \frac{\Delta\sigma_x + \Delta\sigma_y}{2}\left(1 + \frac{a^2}{r^2}\right) - \frac{\Delta\sigma_x - \Delta\sigma_y}{2}\left(1 + \frac{3a^4}{r^4}\right)\cos 2\theta - \Delta\tau_{xy}\left(1 + \frac{3a^4}{r^4}\right)\sin 2\theta - \frac{a^2}{r^2}\Delta P$$
$$\sigma_z = \Delta\sigma_{zz} - 2\nu\left(\Delta\sigma_x - \Delta\sigma_y\right)\frac{a^2}{r^2}\cos 2\theta - 4\nu\Delta\tau_{xy}\frac{a^2}{r^2}\sin 2\theta \tag{S1}$$

Here, ΔP is the change in pressure around the borehole. In the rock mechanics example, ΔP is equivalent to P, the pressure, because it assumes the borehole was recently drilled. For the moulin case, where the water level fluctuates by the minute, ΔP is the change in pressure over the time interval in question. The same applies to the deviatoric stresses $\Delta\sigma_x$ and $\Delta\sigma_y$ and the shear stress $\Delta\tau_{xy}$: these are changes in the stress field over a time interval.

Applying Hooke's Law to these equations yields the corresponding strain at any point in the domain. Hooke's Law is just a linear combination of the three stresses in Eqn. S1:

$$\epsilon_r = E^{-1}\left(\sigma_r - \nu\left(\sigma_\theta + \sigma_z\right)\right)$$
$$\epsilon_\theta = E^{-1}\left(\sigma_\theta - \nu\left(\sigma_r + \sigma_z\right)\right) \tag{S2}$$
$$\epsilon_z = E^{-1}\left(\sigma_z - \nu\left(\sigma_r + \sigma_\theta\right)\right)$$

where E is Young's modulus (~1 GPa) and ν is Poisson's ratio (~0.3 for ice; unitless).

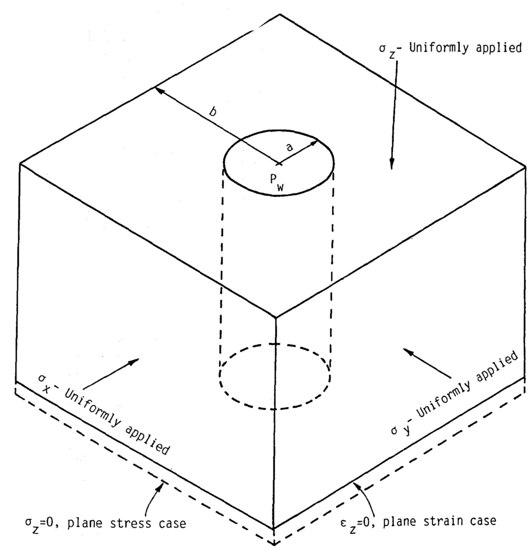


Figure S1: *Problem setup, adapted from Aadnøy [1987], of a borehole in a rock medium. We adapt this to a cylindrical moulin through ice. We use the plane strain case, although the plane stress case is equivalent within* 7%.

S1.3 Integrated elastic deformation

To calculate the radial expansion or contraction of moulin size, we must know the total elastic deformation of the moulin wall. This is the spatial integral of ϵ_r, from the borehole wall ($r = a$) to the end of the domain ($r = \infty$). Deformation will be greatest at the borehole wall ($r = a$) and will fall off to zero as $r \to \infty$.

Integrating Eqn. S2 over $r|_{\infty}^{a}$ entails integrating each stress from Eqn. S1 over the same limits, then summing them together with the appropriate constants involved. So, must simply integrate all the *r-dependent* terms in the Eqn. S1 stresses over $r|_{\infty}^{a}$. We ignore any constant (r-independent) terms in Eqn. S1 because these do not contribute to spatially varying deformation.

Eqn. S1 with the constants removed are as follows:

$$\begin{aligned}
\sigma_r^* &= \quad \left(\Delta P - \frac{\Delta\sigma_x + \Delta\sigma_y}{2}\right)\left(\frac{a^2}{r^2}\right) + \left(\frac{\Delta\sigma_x - \Delta\sigma_y}{2}\cos 2\theta + \Delta\tau_{xy}\sin 2\theta\right)\left(\frac{3a^4}{r^4} - \frac{4a^2}{r^2}\right)\\
\sigma_\theta^* &= -\left(\Delta P - \frac{\Delta\sigma_x + \Delta\sigma_y}{2}\right)\left(\frac{a^2}{r^2}\right) - \left(\frac{\Delta\sigma_x - \Delta\sigma_y}{2}\cos 2\theta + \Delta\tau_{xy}\sin 2\theta\right)\left(\frac{3a^4}{r^4}\right)\\
\sigma_z^* &= \left(-2\nu\left(\Delta\sigma_x - \Delta\sigma_y\right)\cos 2\theta - 4\nu\Delta\tau_{xy}\sin 2\theta\right)\left(\frac{a^2}{r^2}\right)
\end{aligned} \tag{S3}$$

Indefinite integrals of Eqn. S3 are as follows:

$$\begin{aligned}
\int \sigma_r^* dr &= \left(2\Delta P - \left(\Delta\sigma_x + \Delta\sigma_y\right)\right)\left(\frac{a^2}{2r}\right) + \left(\left(\Delta\sigma_x - \Delta\sigma_y\right)\cos 2\theta + 2\Delta\tau_{xy}\sin 2\theta\right)\left(\frac{2a^2}{r} - \frac{3a^4}{2r^3}\right)\\
\int \sigma_\theta^* dr &= -\left(2\Delta P - \left(\Delta\sigma_x + \Delta\sigma_y\right)\right)\left(\frac{a^2}{2r}\right) + \left(\left(\Delta\sigma_x - \Delta\sigma_y\right)\cos 2\theta + 2\Delta\tau_{xy}\sin 2\theta\right)\left(\frac{3a^4}{2r^3}\right)\\
\int \sigma_z^* dr &= 2\nu\left(\frac{a^2}{r}\right)\left(\left(\Delta\sigma_x - \Delta\sigma_y\right)\cos 2\theta + 2\Delta\tau_{xy}\sin 2\theta\right)
\end{aligned} \tag{S4}$$

These all have dimensions of $\mathtt{Pa{\cdot}m}$.

Next, we evaluate definite integrals of Eqn. S4, over $r|_{\infty}^{a}$. Every term in the $r \to \infty$ expressions go to zero. Similarly, all tangential variations (coordinate θ) do not affect moulin size, so we replace all $\cos 2\theta$ or $\sin 2\theta$ with its average absolute value, $\frac{1}{2}$. This gives

$$\begin{aligned}
\int_\infty^r \sigma_r^* dr &= a\left(\quad \Delta P - \frac{1}{2}(\Delta\sigma_x + \Delta\sigma_y) + \frac{1}{4}(\Delta\sigma_x - \Delta\sigma_y) + \frac{1}{2}\tau_{xy}\right)\\
\int_\infty^r \sigma_\theta^* dr &= a\left(-\Delta P + \frac{1}{2}(\Delta\sigma_x + \Delta\sigma_y) + \frac{3}{4}(\Delta\sigma_x - \Delta\sigma_y) + \frac{3}{4}\Delta\tau_{xy}\right)\\
\int_\infty^r \sigma_z^* dr &= \nu a\left(\left(\Delta\sigma_x - \Delta\sigma_y\right) + 2\Delta\tau_{xy}\right)
\end{aligned} \tag{S5}$$

Finally, we take a linear combination of Eqns. S5: a sum with the appropriate coefficients from Hooke's Law (Eqn. S2) to get the strain in the r, θ, and z directions, although we discard strain in the θ or z directions. We thus obtain u_r, the total radial deformation in r, by $u_r = \int_\infty^a \epsilon_r dr$.

$$
\begin{aligned}
u_r &= \int_{\infty}^{a} \epsilon_r dr = E^{-1}\left[\int_{\infty}^{a} \sigma_r^* dr - \nu\left(\int_{\infty}^{a} \sigma_\theta dr + \int_{\infty}^{a} \sigma_z dr\right)\right] \\
&= \frac{a}{E}\left[(1+\nu)\left(\Delta P - \frac{1}{2}(\Delta\sigma_x + \Delta\sigma_y)\right) + \frac{1}{4}(\Delta\sigma_x - \Delta\sigma_y)(1-3\nu-4\nu^2) + \frac{1}{4}\Delta\tau_{xy}(2-3\nu-8\nu^2)\right] \\
&\text{for } \Delta P = \rho_w g(\Delta h_w - z) - \rho_i g(\Delta H_i - z) = \Delta P_w - \Delta P_i
\end{aligned}
\tag{S6}
$$

In the moulin model, we assume that all pressure changes ΔP are due to changes in water level Δh_w and that the ice thickness H_i stays constant. Thus, $\Delta P = \rho_w g(\Delta h_w - z) = \Delta P_w$.

As a check, the integrated displacement u_r (Eq. S6) increases with moulin radius a. This makes sense as a tighter radius of curvature (low a) is more difficult to deform radially (low u_r). Inward deformation (moulin closure) will have negative u_r and outward deformation (moulin expansion) will have positive u_r.

For a typical Greenland Ice Sheet moulin with radius a ~1 meter, the pressure change associated with Δh_w ~1 meter will induce elastic deformation of a few micrometers. This water level change would typically occur over many minutes to a few hours, yielding elastic deformation of up to some 10^{-4} meters per day.

S1.4 Simplest case: Non-varying deviatoric and shear stresses

The deviatoric and shear stresses σ_x, σ_y and τ_{xy} are generally not well know and are variable from place to place in the ablation zone. The spatial variation typically occurs over scales of a few kilometers, and the range may be roughly [–50 kPa +50 kPa].. However, Eq. S6 uses their changes over a time interval, $\Delta\sigma_x$, $\Delta\sigma_y$ and $\Delta\tau_{xy}$. For a moulin advecting through the ablation zone at ~100 m/yr, the stress changes $\Delta\sigma_x$, $\Delta\sigma_y$ and $\Delta\tau_{xy}$ are trivial (~1 kPa) over a melt season. Thus, we make a further simplification that $\Delta\sigma_x = \Delta\sigma_y = \Delta\tau_{xy} = 0$, which yields the most basic expression for radial elastic deformation u_r:

$$u_r = \frac{a}{E}(1+\nu)\Delta P \tag{S7}$$

This is the same equation as is commonly used for dilatometer testing in rock mechanics [Goodman, 1989, page 190].

S1.5 Instantaneous elastic deformation and calculated deformation rates

Elastic displacement is an instantaneous process that occurs in reaction to a change in stress. In the case of a moulin during the melt season, the water level in the moulin changes essentially continuously, which induces continuous changes in pressure (ΔP), which drives continuous elastic deformation (Eqs. S6–S7), although we calculate it only once per timestep. To compare elastic deformation (instantaneous) to viscous deformation (occurring over a time interval), we assume the deformation rate occurs over the entire timestep:

$$\text{elastic deformation rate} = \frac{u_r}{\Delta t} \tag{S8}$$

This is analogous to how we calculate a viscous deformation rate or a rate of refreezing.

More precisely, one could express this in terms of the rate of pressure change, $\frac{\Delta P}{\Delta t}$:

$$\frac{u_r}{\Delta t} = \frac{a}{E}(1+\nu)a\frac{\Delta P}{\Delta t} \tag{S9}$$

This approach assumes that the water pressure varies smoothly over the time interval in question. This is generally true: we run the model at 5-minute timesteps, and the most common discontinuous variations in pressure are likely sourced from rain storms or other sudden melt events (time scales of hours).

References

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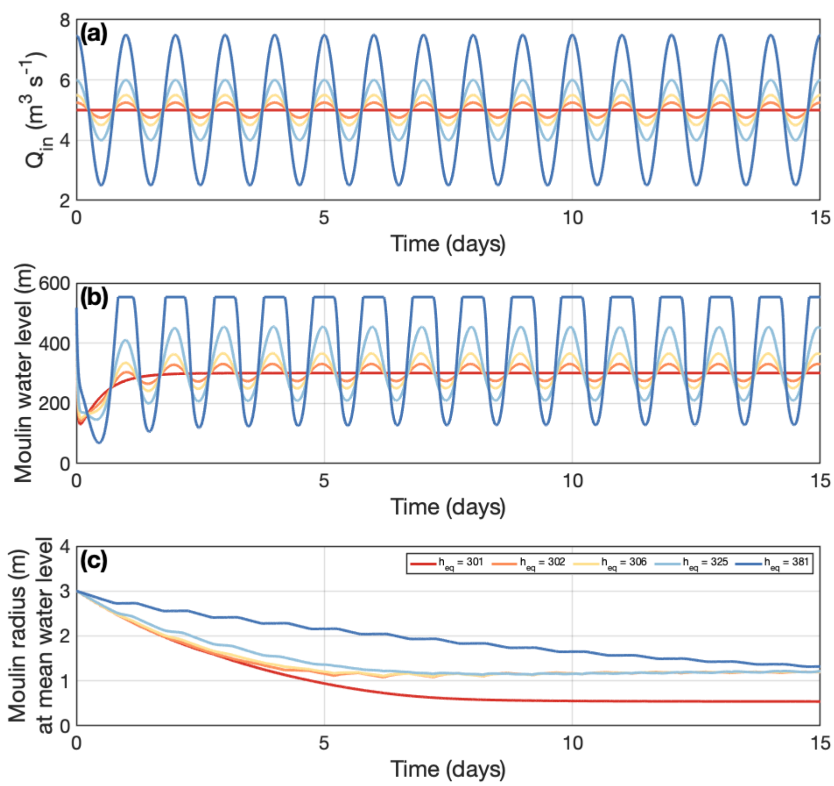
MouSh Sensitivity to model choices

Supplement **S2** for “Controls on Greenland moulin geometry and evolution from the Moulin Shape model”, *The ~~Cryoshpere~~Cryosphere*.

S2.1 The impact of diurnal supraglacial variability

Under steadily varying conditions, the modeled moulin should reach a quasi-equilibrium state independent of initial conditions with melting opposing viscous and elastic deformation below the water line and the only change being driven by shear deformation. We examine the quasi-equilibrium state and the impact of supraglacial variability on this state. Increasing the amplitude of the diurnal Q_{in} signal results in an increase in the mean water level but ~~no~~very little change in the moulin radius at the mean water level apart from an amplitude of zero (Fig. S2). The magnitude of the Q_{in} signal impacts both the mean moulin water level and the radius at that water level (Fig. S3). The ~~increase~~changes in mean moulin water level ~~is~~in response to variations in Q_{in} amplitude and magnitude are non-linear (Fig. S4). Further description is included in Sect. 2.5.1 and 3.1.



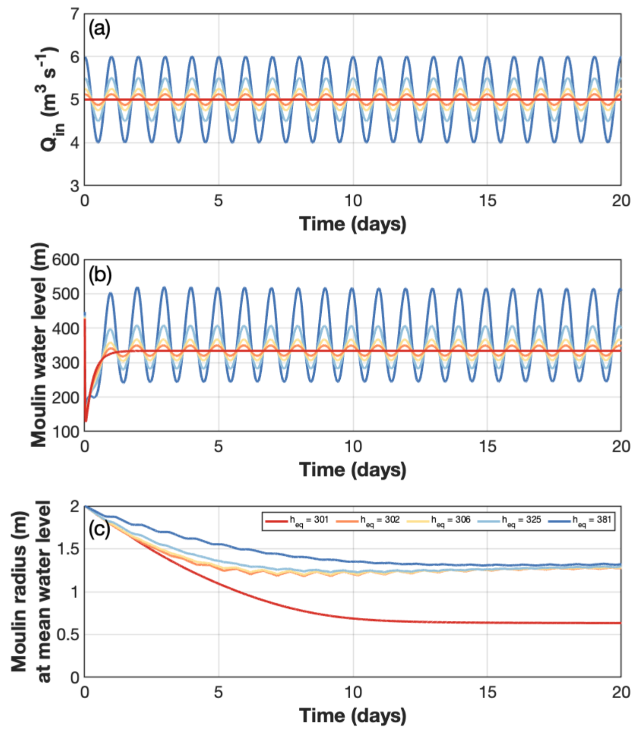
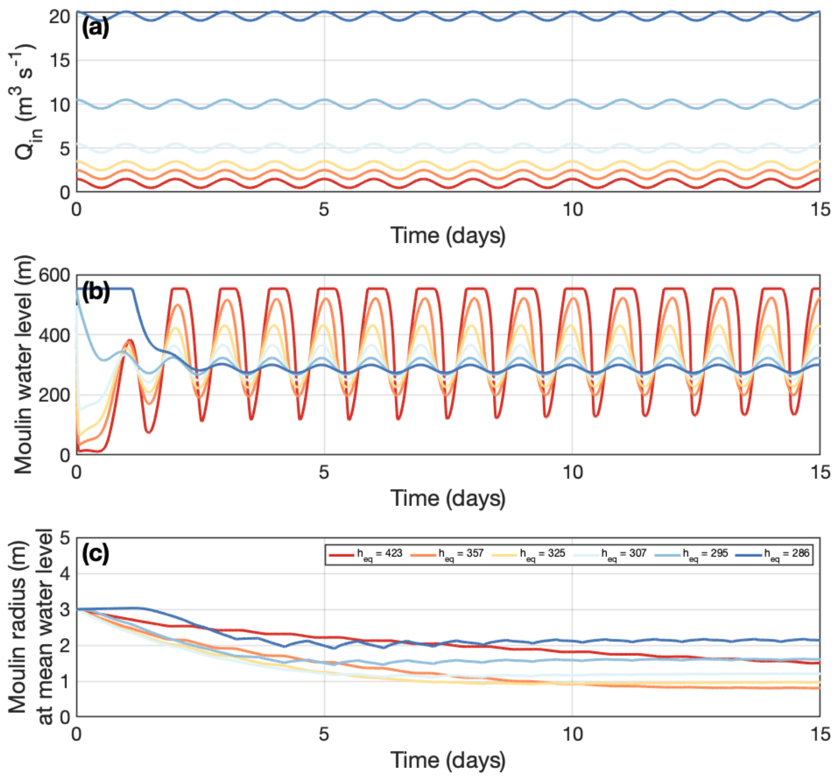


Figure S2. The impact of the Q_{in} amplitude (a) on moulin water level (b) and the major moulin radius at the mean moulin water level over the last 24 h (c) for ~~6~~5 different Q_{in} amplitudes, 0 $m^3\ s^{-1}$ (red), 0.25 $m^3\ s^{-1}$ (orange), 0.5 $m^3\ s^{-1}$ (dark yellow), 1 $m^3\ s^{-1}$ (mid-blue), 2 $m^3\ s^{-1}$ (dark blue). Mean or quasi-equilibrium water levels indicated in (c). All runs have a magnitude of 5 $m^3\ s^{-1}$. Ice thickness is 553 m with flotation at approximately 503 m.



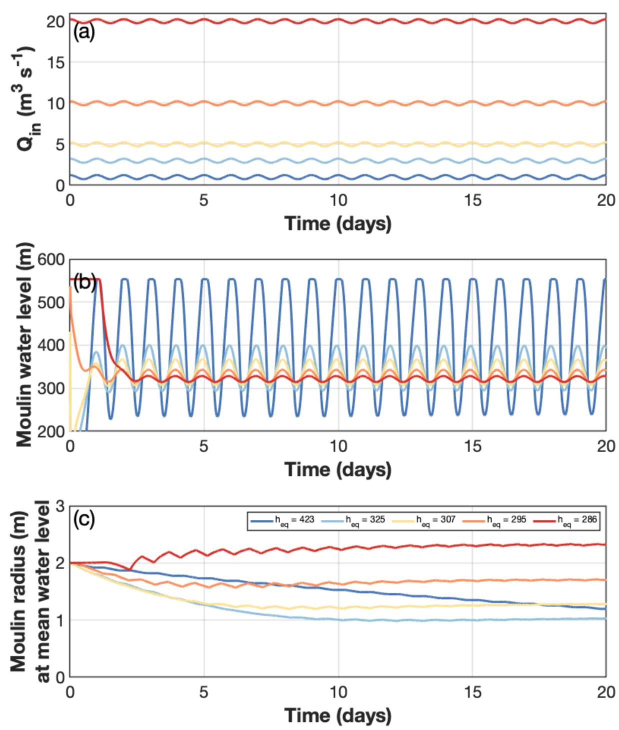
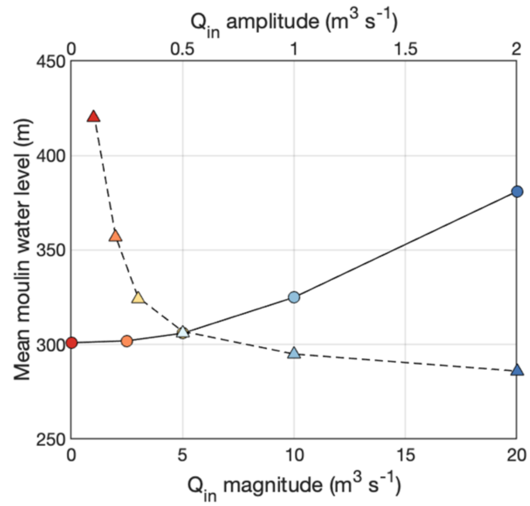


Figure S3. The impact of the Q_{in} magnitude (a) on moulin water level (b) and the major moulin radius at the mean moulin water level over the last 24 h (c) for ~~6~~5 different Q_{in} magnitudes, 1 m^3 s^{-1} (~~red), 2 m3 s-1 (orange),~~ blue) 3 m^3 s^{-1} (light blue), 5 m^3 s^{-1} (dark yellow), ~~5 m3 s-1 (light blue),~~ 10 m^3 s^{-1} (~~mid-blue~~orange), 20 m^3 s^{-1} (~~dark blue~~red). Mean ~~or~~

~~quasi-equilibrium~~ water levels indicated in (c). All runs have an amplitude of 0.5 $m^3 s^{-1}$. Ice thickness is 553 m with flotation at approximately 503 m.



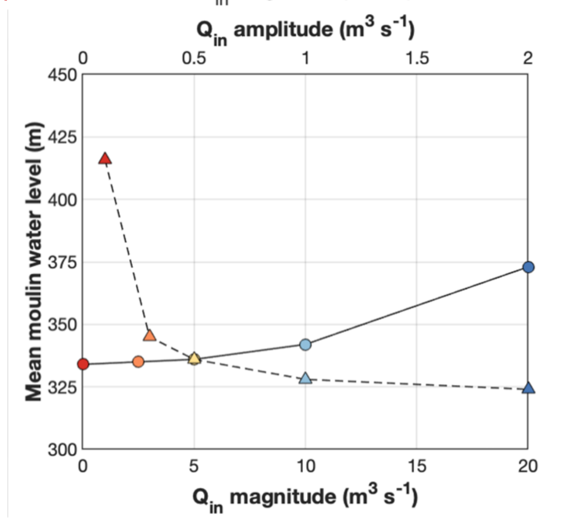


Figure S4. Mean moulin water level as a function of Q_{in} magnitude (~~circles~~triangles) and amplitude (~~triangles~~circles). Colored as indicated in Figures S6 and S7: Q_{in} magnitudes, 1 $m^3 s^{-1}$ (red), ~~2~~3 $m^3 s^{-1}$ (orange) , ~~3~~5 $m^3 s^{-1}$ (dark yellow), ~~5~~10 $m^3 s^{-1}$ (light blue), ~~10 $m^3 s^{-1}$ (mid-blue),~~ 20 $m^3 s^{-1}$ (~~dark~~ blue); Q_{in} amplitudes, 0 $m^3 s^{-1}$ (red), 0.25 $m^3 s^{-1}$ (orange), 0.5 $m^3 s^{-1}$ (dark yellow), 1 $m^3 s^{-1}$ (~~mid~~ light blue), 2 $m^3 s^{-1}$ (dark blue).

S2.2 Sensitivity to model choices

To explore the impact of various parameterizations and model choices, we perform a set of experiments that examine the impact of various parameterizations and model choices on the MouSh modeled moulin water level and moulin capacity, two components of a moulin that can directly impact the englacial and subglacial hydrologic systems. In each model run, all characteristics and forcings are kept the same as in the control run, except the parameterization of interest. Specific details are detailed below, and the results of these exploratory runs are in Figure S5 and Figure S6. All moulin water level differences are presented as *test – control* and percentage differences are *(test – control) / control*.

S2.2.1 Control run

The control run is the Basin 1 experiment detailed in Sect. 2.5.2 and Table 2, with the exception that diurnal variability of Q_{in} is reduced by 30% to prohibit prolonged periods when daily peak water levels overtop the moulin.

S2.2.2 Circular geometry (Experiment 1)

The circular run uses a circular cross-sectional area. In practice, this simply means that open channel melting above the water line is applied uniformly around the moulin perimeter (instead of only to r_2 as in the elliptical formulation) such that the moulin plan view cross-sectional area is circular and only one radius is evolved. Deformation due to elastic, viscous and turbulent melting below the water line is then only calculated for the single radius. This parameterization removes any asymmetry~~ except for that imposed by shear deformation~~. Both runs have the same initial circular plan view cross-sectional area.

The use of a circular geometry has little impact on moulin water level over the course of the melt season (Fig. S5a). Compared to the control run, the circular geometry generally exhibits slight increases in moulin water level (< 0.5 m). These increases become slightly higher at higher Q_{in} values~~; or during one period at the end of the melt season. However, differences never exceed 3 m.~~. The primary difference comes in the moulin capacity~~, which~~: the circular geometry can ~~be up~~ to ~~47~~31% smaller ~~approaching~~than the control moulin as the end of the melt season approaches (Fig. S5b). This difference is concentrated in regions that are not generally water filled except at high water levels, thus has limited impact on moulin water level~~ until the end of the melt season when water levels are highest and the control moulin has evolved most substantially from the circular initial condition.~~. This difference is the result of the control moulin run becoming more elliptical.

~~S2.3 Surface stress impact~~

~~Currently, elastic deformation is based on the Aadnøy (1987), which describes the stresses around a vertical hole. The default deviatoric and stress values are~~ $\cancel{\sigma_x = 0\,kPa,\ \sigma_y = 50\,kPa,\ and\ \tau_{xy} = -50\,kPa}$ ~~, based on a remote sensing study (Poinar and Andrews, 2020). However, these values are generally poorly constrained. We examine higher surface stresses~~ $\cancel{\sigma_x = 0\,kPa,\ \sigma_y = 500\,kPa,\ and\ \tau_{xy} = -500\,kPa}$ ~~(*10x surface stresses*) and the simplest case, deviatoric and shear stresses equal to zero:~~ $\cancel{\sigma_x = \sigma_y = \tau_{xy} = 0\,kPa}$ ~~(*Zero surface stresses*). Our derivation of deviatoric and shear stress equations and out parameterization for time variation can be found in Supplement S1.~~

~~Deviatoric and shear stresses primarily impact the near surface, unsubmerged regions. Therefore, the impact on moulin water level is minimal, except during the beginning and end of the melt season when water levels are high (Fig. S5c). Though overall the difference in moulin water levels between *10x surface stresses* or *Zero surface stresses* and the control run is on the order of +/– 1 m. Zero surface stresses results in a slightly larger moulin (up to 8.5% by the end of the melt season), while 10x surface stresses results in a slightly smaller moulin (up to 29% by the end of the melt season) (Fig. S5d). This comparison suggests that the simplifying case (*Zero surface stresses*) has minimal impact on the model results except during periods when water levels comparatively higher than the mean water level.~~

S2.2.3 Elastic deformation (Experiment 2)

For completeness, we include elastic deformation within the moulin model. Our formulation is dependent on the change in moulin water level and the moulin radius (Supplement 1). Thus, elastic deformation within MouSh is substantially smaller than viscous deformation due to the relatively small moulin radii modeled here. We examine whether the inclusion of elastic deformation impacts moulin water level and capacity by performing a run without elastic deformation.

In its current formulation, the exclusion of elastic deformation has almost no impact on moulin water level and capacity (Fig. S5c-d). This comparison suggests that the simplifying case (*no elastic deformation*) has minimal impact on the model results.

S2.2.4 Distance from terminus (Experiment 3)

In our simple parameterization of the subglacial model, the hydraulic gradient is set by the water level in the moulin and the distance from the terminus. Because the hydraulic gradient exerts an important control over both the subglacial channel and moulin evolution, we examine the impact of different subglacial lengths (L). We compare the control run, L = 13.6 km, to model runs with one half, L = 6.8 km, and one and one half, L =20.4 km while using the same ice thickness (553m). This change directly impacts the hydraulic gradient calculated in Eq. ~~28~~24.

Modifying the distance from terminus and the associated hydraulic gradient can result in substantial changes to both the moulin water level (+/– ∼~~200m~~100m) and moulin capacity (+/– ∼~~40~~30%; Fig. S5e-f). ~~Reducing~~Shortening L reduces both moulin water level and moulin capacity. Lower water levels reduce water velocities and allows viscous and elastic deformation to increase, resulting in a smaller moulin. While increasing L results in higher moulin water levels and a larger moulin. Higher moulin water levels increase turbulent melting linearly and reduce viscous and elastic deformation non-linearly. In addition, with a longer L, the moulin has more instances of water level being above floatation, which permits viscous and elastic deformation to open the moulin. The difference in moulin water levels tends to be exacerbated during higher Q_{in} values (Fig. S5e), resulting in larger differences from the control run during the middle of the melt season and less impact during the onset and cessation of melting.

S2.2.5 Base flow (Experiment 4)

Our simple parameterization of the subglacial system means that the model represents only a single moulin and a long channel. This is an oversimplification of what is generally a complex arborescent network (e.g., Werder et al., 2013) with multiple moulins along a single channel (Andrews et al., 2014). To parameterize this connectivity, we prescribe a base flow term to be 5 times the 5-day lagged moving average moulin input directly into the subglacial channel~~.~~ (Fig. 6a). This definition removes diurnal signals but preserves melt events and the seasonal pattern of melt. Without baseflow, MouSh can produce unrealistically high~~-~~ water levels with realistic meltwater inputs. While an alternative would be to either substantially dampen the diurnal variability or increase moulin inputs, we believe that our current approach best approximates the natural system. Unfortunately, prescribing a larger initial subglacial cross-sectional area does not mitigate the above problem because moulin and subglacial channel size are not dependent on the initial conditions after ~~several days.~~the first few weeks. Here we examine the impact of reducing the base flow to 2 times the 5-day lagged moving average.

The prescribed base flow acts to maintain a larger subglacial channel and permits more rapid growth due to melting; this behavior is non-linear~~ (see Eq. 28).~~. Therefore, reducing the amount of base flow into the subglacial system reduces the ability of the subglacial channel to accommodate the large diurnal swings in Q_{in}. Therefore, a reduction in base flow results in higher moulin water levels for much of the model run (Fig. S5g). Interestingly~~ though~~, during diurnal minimums, the water levels are lower in the low base flow run relative to the control~~.~~ (negative values in Fig S5g). This is likely due to greater moulin growth (increased turbulent melting and reduced or negative viscous and elastic deformation) associated with higher water levels. The moulin capacity difference displays a clear seasonal pattern (Fig. S5h). ~~Early in~~During the tails of the melt season, the lower base flow run exhibits a ~~smaller~~similar capacity ~~due~~ to ~~less total water in~~ the ~~system~~control, but as diurnal variability and maximum daily water levels increase ~~to above floatation~~, the low baseflow moulin begins to grow relative to the control ~~run~~moulin.

S2.2.6 Static subglacial geometry (Experiment 5)

The MouSh model is meant to model moulin geometry. However, to permit water flow through the moulin, we include a simple time-evolving subglacial channel (Sect. 2.4.2). A fixed subglacial channel would, in essence, provide the simplest subglacial component. However, a fixed subglacial channel results in both extremely low and extremely high moulin water levels when Q_{in} values are both high and low, respectively; therefore, during ~~a run~~an experiment with seasonally evolving Q_{in}, the subglacial channel size must be chosen very carefully to produce vaguely realistic moulin water levels and capacities. Therefore, we examine the impact of fixing the subglacial cross-sectional area S using a ~~fixed cosinusoidal run~~co-sinusoidal supraglacial of ~~80~~40 days~~.~~ (as described in Eq. 22). For this comparison, we fix S =~~2~~1.95 m^2 (*Fixed S*), which is ~~approximately~~ equal to the mean value of the subglacial channel cross-sectional area in the control ~~run~~experiment; this minimizes differences between the runs.

The moulin with a fixed subglacial cross-sectional area has similar quasi-equilibrium water levels but less diurnal variability ~~in both moulin water level and water storage than the moulin~~(Fig. 6b). The model run with a ~~variable~~fixed subglacial ~~cross-sectional area (Fig. S6).~~channel also displays a slightly lower radius at the mean water level of the last 10 experiment days. The primary conclusion here is that the moulin geometry and variability is, at least in part, driven by the characteristics of the subglacial hydrologic model used. Such dependency is not uncommon in models of the glacial hydrologic system. For example, the presence of modeled channels is dependent on the prescribed location of supraglacial inputs and prescribed conductivity of the surrounding system. Therefore, the subglacial model used with the MouSh model should be carefully considered.

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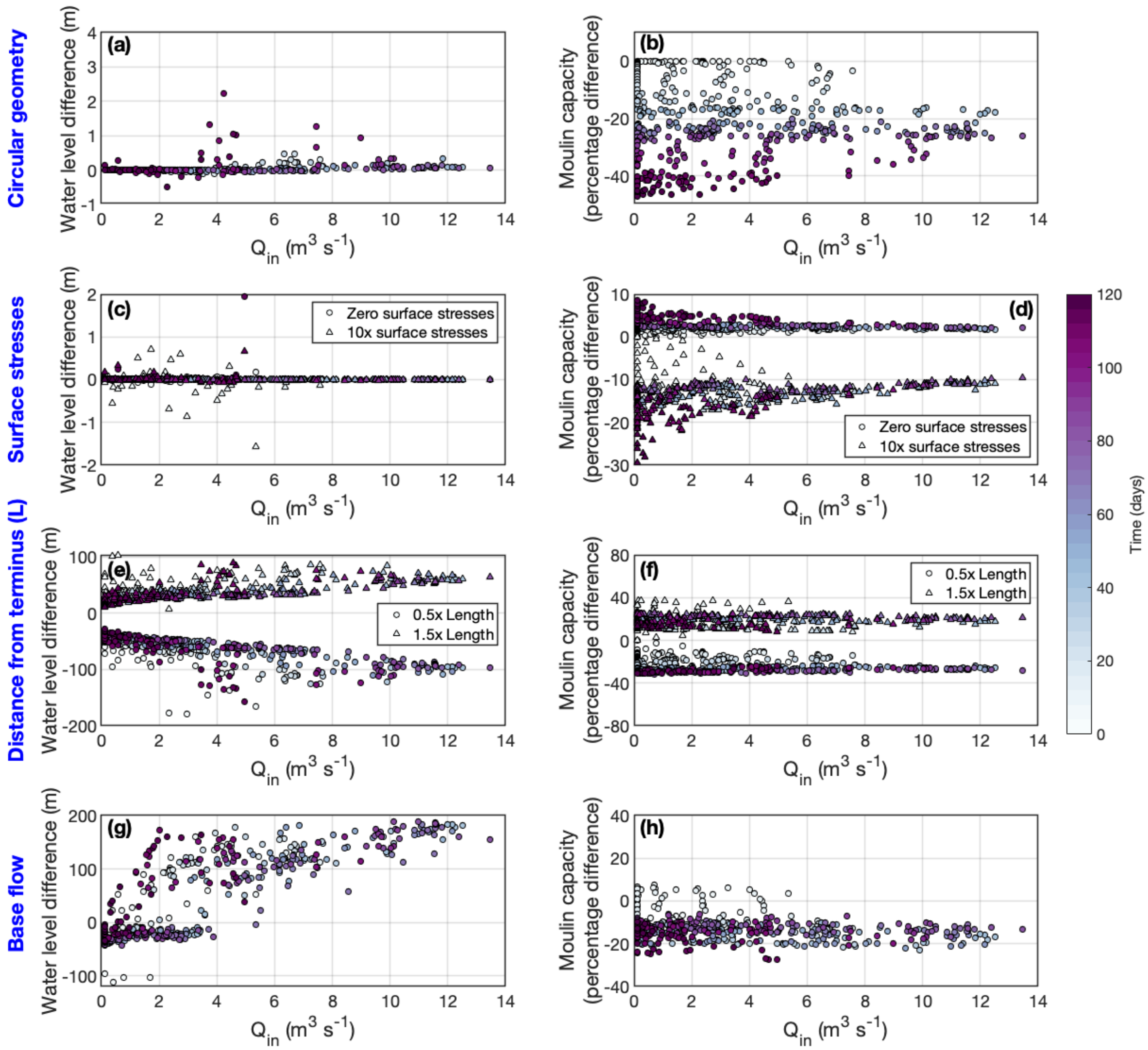


Circular geometry
Surface stresses
Distance from terminus (L)
Base flow
(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
Water level difference (m)
Moulin capacity (percentage difference)
Q_{in} (m^3 s^{-1})
Zero surface stresses
10x surface stresses
0.5x Length
1.5x Length
Time (days)

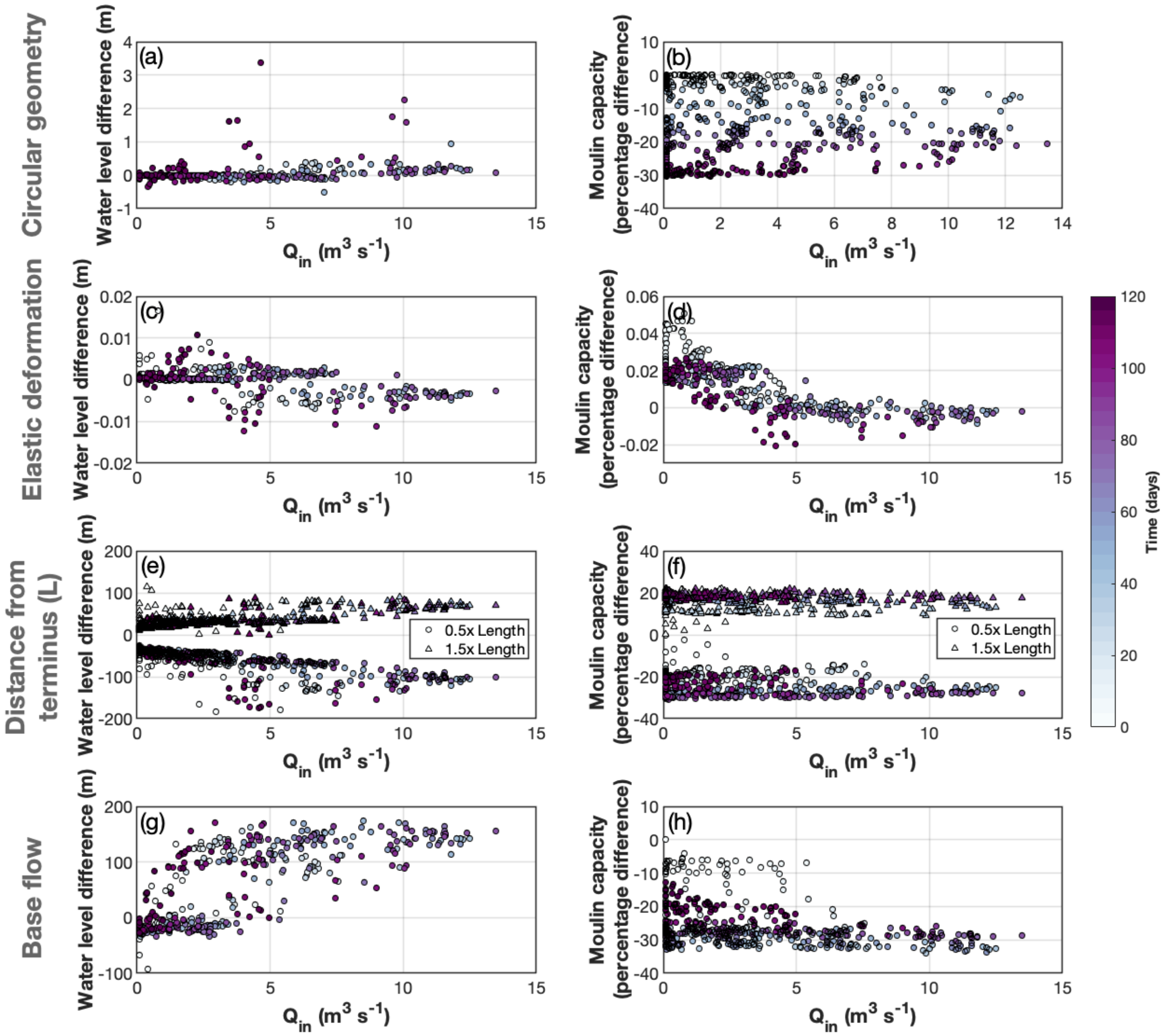
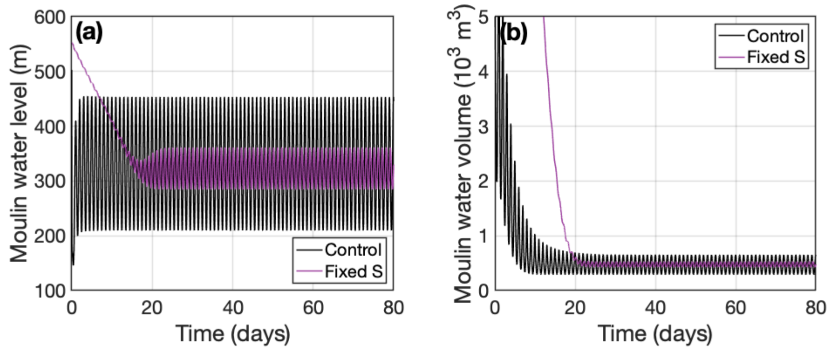


Figure S5. Moulin water level and capacity differences relative to the Basin 1 (H_i = 553 m) control run for a circular geometry (a-b); ~~varying surface stresses~~elastic deformation (c-d); varying distances from the terminus or subglacial path length (L; e-f); and a reduced baseflow (g-h). In all panels on the left-hand side, the differences are experiment minus control. In all panels on the right-hand side, moulin capacity is plotted as a percent difference from the control run such that positive values indicate a capacity larger than the control run and negative values indicate a capacity smaller than the control run.



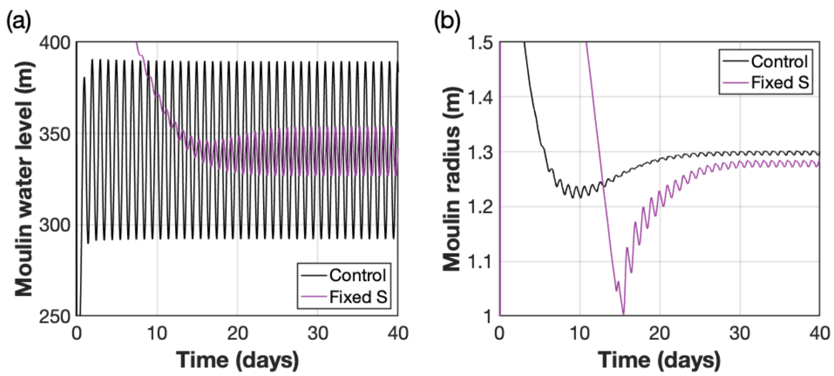


Figure S6. Moulin water level (a) and moulin radius at the mean water ~~volume~~level (b) from ~~experimental runs with a run~~experiments with variable subglacial *S* (*Control*; black) and a fixed subglacial cross-sectional area of ~~2~~1.95 m^2 (*Fixed S*; purple).

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