Review for *The Cryosphere* of

Characterizing the Sea-Ice Floe Size Distribution in the Canada Basin from High-Resolution Optical Satellite Imagery

by Denton and Timmermans

The authors analyzed 78 images from the Canada Basin of the Arctic Ocean. The images have 1-meter spatial resolution and span the period 1999-2014 during the months April to September. They identified individual ice floes in the images and analyzed the floe size distribution (FSD), where size is measured by floe area. They have four main results: (1) The FSDs follow a power-law distribution between areas 5×10^1 m^2 and 5×10^6 m^2 (50 m^2 to 5 km^2) with power-law exponents ranging from −1.65 to −2.03. (2) The FSDs are sensitive to the threshold used to separate ice from water in the images. Other studies may have erroneously found two power-law regimes by not setting the proper threshold between ice and water. (3) A linear relationship is found between power-law exponents and sea-ice concentration (SIC), with more negative exponents corresponding to lower SIC. (4) Locations that experience a seasonal cycle in SIC also have a seasonal variation in power-law exponent, but sites with high year-round SIC do not.

The analysis and conclusions of this paper are generally sound. I recommend publication after the authors consider the following comments and suggestions, which are given in page order.

Comments and suggestions

Lines 9-11. "the structure of the FSD is found to be sensitive to a classification threshold value… and an objective approach to minimize this sensitivity is presented." I searched throughout the paper for the objective approach, but all I could find was this (on lines 163-164): "we iteratively increase the threshold above the minimum until the edges of small floes are appropriately delineated." I have no objection to this method, but I wouldn't call it an objective approach that minimizes anything. It sounds like "visual inspection" or "manual selection" to me. If that's what it is, please say so. If I'm missing the objective approach, please provide more detail.

Lines 182-189. This describes the construction of the FSD by binning the data and fitting a line (in log-log space) to the binned values. That's fine, but it's a shame that the authors did not use Maximum Likelihood Estimation to find the best-fitting exponent, which does not require binning the data (and hence avoids the problem of having an adequate number of samples per bin), and which gives a more accurate estimate of the exponent than least-squares fitting of binned data. No changes necessary, I just wanted to bring up this point.

Lines 199-203. The authors choose to use floe area as the measure of floe size, because once the pixels of a floe have been identified, it's a simple matter to count them and multiply by the area per pixel. However, it's also a simple matter to directly calculate the mean caliper diameter (MCD) from the coordinates of the floe pixels. I'm not talking about an approximation that relates the average MCD to the average area (as in Rothrock and Thorndike, 1984) – I'm saying that the MCD can be easily calculated exactly for every individual floe, as follows.

In a pixel-based coordinate system with (0,0) at the lower left corner of the image, let (x_k,y_k) be the pixel coordinates that comprise a floe, denoted by vectors **x** and **y**. Then:

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for angle = 0, 179 do begin                ; Loop over caliper angles from 0° to 179° in 1° increments
  c = cos(angle * pi / 180.)
  s = sin(angle * pi / 180.)
  r = c*x + s*y
  diameter(angle) = max(r) – min(r)  ; This is the caliper diameter at the given angle
endfor
MCD = total(diameter)/180.                 ; Average over all angles of the calipers to get MCD
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There's nothing wrong with using floe area as the measure of floe size, but in 18 previous studies of the FSD, 17 of them have used a diameter or dimensional length, and only one has used area. It would be easier to compare the results of this study with previous results if the authors had used MCD or another length scale. No changes necessary, I just wanted to bring up this point.

Lines 216-218. "the slope of the FSD is valid only for floe areas larger than 50 m^2 and we limit fitting in log-log plots (to estimate *m*) to floe areas between 50 m^2 and 5 km^2 "
How many floes fall outside these bounds, roughly? Is it 10% of all floes? 50%?
Table A1 gives the "number of floes (whole) retrieved" (according to the caption on page 16). Does the number in the table include only the floes between the given bounds, or all floes? If it's all floes, then another column should be included in the table giving the number of floes with area between the bounds.

Line 240 and Figure 3 caption. Some of the images are segmented (divided into ice and water) with high confidence and others with low confidence. Table A1 should indicate which images are segmented with high confidence and which with low confidence.

Figure 3 panels (b) and (c) include "error bars representing one standard deviation" (lines 252 and 254). However, in some cases the number of samples is 1, so it makes no sense to calculate a standard deviation (which is 0) and call it an error bar. The number of samples for a given month and location never exceeds 7, and in most cases it's 1, 2, or 3. It's of dubious statistical rigor and utility to calculate standard deviations and call them error bars in these cases. Furthermore, the error bars make the plots more difficult to read. In my opinion, all the error bars should be removed from panels (b) and (c), and reference to them deleted from the caption.

Figure 3 panels (b) and (c) also include "mean monthly slopes" and "mean monthly SICs."
In my comment below about Table A1, I note that not all the images provide independent estimates of sea-ice properties because some images were acquired on the same day of the same year at essentially the same location. For example, images #39, #40, and #41 are all from 20 May 2013 at the Northern Canada Basin site (see Table A1). Therefore, in calculating mean monthly slopes for May, the values from #39, #40, and #41 should be averaged first, and then their average should be averaged with the other May values from the same site (i.e., images #2, #13, #37, and #63). Perhaps the authors have actually calculated the means in this way. If not, I'm sure it wouldn't make much difference in the final results, but I just wanted to bring up the point about statistical non-independence of the images.

Lines 290-299. This paragraph is about the lack of a relationship between surface air temperature (SAT) and power-law exponent m of the FSD. The lack of a relationship is not surprising. During the Arctic summer, the SAT over ice is pegged at the melting point of ice because the surface is an ice bath (ice and water). For example, see the plots of SAT in Rigor et al. (2000). From the time of melt onset to the time of freeze-up (roughly June through August in the Beaufort Sea) there is essentially no variability in the SAT over ice, so it can't possibly explain the variability in FSD. The solar energy that goes into the ocean melts ice, potentially changing the FSD, but it does not raise the SAT over ice. In my opinion, it would have made more sense to look for a connection between m (FSD) and some other thermodynamic variable that characterizes ocean heat content or energy balance.
Rigor, I.G., R.L. Colony, and S. Martin (2000). Variations in Surface Air Temperature Observations in the Arctic, 1979–97. *J. Climate*, pp. 896-914, https://doi.org/10.1175/1520-0442(2000)013<0896:VISATO>2.0.CO;2

Lines 303-307. The authors state that a power-law probability density function (pdf) of the form $n(a) \sim a^m$ where a is floe area is equivalent to a power-law pdf of the form x^{2m+1} where x is floe diameter. This is correct, but the reason given is wrong. It has nothing to do with the binning of data or the normalization of bin counts. It follows from a straightforward application of basic probability theory – see the notes at the end of this review. Lines 303-307 should be re-written to simply state that a^m is equivalent to x^{2m+1} as a result of basic probability theory.

Line 315. "image not included in our analysis due to partial cloud-cover"
Somewhere in the paper the authors should state how many images were rejected due to partial or total cloud cover, and how those decisions were made. Was it by visual inspection?

Lines 346-350. "seasonal variation in m is more directly related to changes in SIC" (than SAT). Yes, but as noted above, there's no reason to expect a connection between m and SAT.
"Future studies are needed to investigate the relevant dynamics … and thermodynamics…" Yes, undoubtedly the same forces that drive changes in SIC also drive changes in FSD. This raises the question: why try to relate SIC and FSD in the first place? Neither one drives the other, they're both the result of underlying dynamic and thermodynamic forcing. It seems like SIC could never be more than an imperfect reflection of FSD. Imagine an image with a power-law FSD and a certain value of SIC. Now double the size of the image by adding only ocean pixels. The FSD remains exactly the same, but the SIC is cut in half. Conversely, it's easy to imagine a scenario in which the power-law exponent of the FSD changes but the SIC does not. What is the motivation for relating FSD to SIC? Wouldn't it make more sense to look for connections between FSD and, say, wind stress?

Table A1. This is a good and useful table, but it is incomplete. As noted above, it should include the number of floes with area between 50 m^2 and 5 km^2 (i.e., the bounds used in determining the power-law exponent m) as well as the total number of floes (which is given already, I believe).

Also, the table should indicate which images were segmented with high confidence and which with low confidence.

Also, there should be a column to indicate whether an image is from the Beaufort fiducial site, the Chukchi fiducial site, the Northern Canada Basin fiducial site, or another site. I realize that this can be inferred from the latitude and longitude, but it is extremely tedious to go through the table and extract that information.

Another point that should be noted somewhere in the paper is that some of the images were acquired on the same day of the same year at essentially the same location, and therefore do not provide independent estimates of sea-ice properties (FSD and SIC). In particular:
#3 and #4 are both 27 July 2000 at the N. Basin site
#5 and #6 are both 15 Aug 2000 at the N. Basin site
#14 and #15 are both 23 May 2002 at the Beaufort site
#39 and #40 and #41 are all 20 May 2013 at the N. Basin site
#43 and #44 are both 10 June 2013 at the Chukchi site
#45 and #46 are both 12 June 2013 at the Beaufort site
#57 and #58 are both 28 April 2014 at the Beaufort site

I found it useful to summarize the number of images by location and month in a table. Consider including something like this in the paper:

	Beaufort	**Chukchi**	**N. Basin**	**Other**	**Total**
April	5 (*)	2	1	17	25
May	7 (*)	2	7 (*)	4	20
June	2 (*)	5 (*)	1	7	15
July	1	1	3 (*)	3	8
Aug.	1	0	3 (*)	2	6
Sept.	1	0	1	2	4
Total	17	10	16	35	78

The (*) symbol indicates that some of the images are from the same day of the same year at essentially the same location, i.e., not independent samples.
Beaufort site is shown in Figure 1(a) at the letter e.
Chukchi site is shown in Figure 1(a) at the letter c.
N. Basin site is shown in Figure 1(a) at the letter d.
Other sites are shown in Figure 1(a) at white dots.

<u>Minor typographical notes</u>

Line 6. Change “atmosphere and ocean” to “atmospheric and oceanic” as on line 22.

Line 64. “Aeronautical” should be “Aeronautics”

Line 66. “The images AT FIDUCIAL SITES are panchromatic…”

Last sentence on page 3. Please say that the surface air temperature (SAT) is at the 2-meter level.

Figure 1 legend at upper right. The blue circle is labeled “August 1999” but it should be August 2014 as in the figure caption (line 90).

Lines 101-107 and 125-142. This is a description of the floe-identification algorithm. If it's the same algorithm as in Stern et al (2018b) then that should be explicitly stated; if not, no changes necessary.

Line 199. "The floe size may be taken to be any scalar representative of the floe size" – consider re-writing this.

Line 206. Delete "e.g." and change "black dotted and dashed lines" to "black, red, and blue dashed lines"

Line 210. "Fig. 2c through e" – should this be Fig. 2b–d?

Figure 3 caption
(i) Both "grey" (line 246) and "gray" (line 250) are used. Pick one spelling.
(ii) Line 248 refers to the "black dotted line" of 19 June 2014 in panel (a). To me it looks like a solid black line with black circular symbols. See also the legend in the lower left corner of (a).
(iii) Line 255, change sites' to site's

Line 286. "*m* appears to generally shoal with distance to the ice edge" – does this mean with INCREASING distance to the ice edge or with DECREASING distance to the ice edge?

Line 319. "conclude" should be "concluded"

Lines 347-348. This sentence repeats what was just said two sentences earlier at lines 342-343. It is redundant.

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[See comment above about lines 303-307]

Notes on the size distribution of floe diameters vs. floe areas

Let X be a random variable of floe diameters, and let x be a value drawn from X.
Let A be a random variable of floe areas, and let a be a value drawn from A.

Let $F_X(x)$ be the cumulative distribution function of X.
Let $F_A(a)$ be the cumulative distribution function of A.

Let $P\{\cdot\}$ denote the probability of the expression inside the braces.
Suppose A and X are related by $A = kX^2$ where k is a constant, and suppose $a = kx^2$. Then:

$$F_X(x) = P\{X \leq x\} = P\{X \leq (a/k)^{1/2}\} = P\{kX^2 \leq a\} = P\{A \leq a\} = F_A(a) \qquad (1)$$

The probability density function (pdf) is the derivative of the cumulative distribution function.
Let $f_X(x) = dF_X/dx$ be the pdf of floe diameters.
Let $f_A(a) = dF_A/da$ be the pdf of floe areas.

Take the derivative with respect to x of both sides of equation (1) and apply the chain rule of differentiation to the right-hand side:

$dF_X/dx = (dF_A/da)\,(da/dx)$ or:

$$f_X(x) = f_A(a)\,(2kx) \tag{2}$$

Power-law pdf

Suppose the pdf of floe area follows a power law of the form $f_A(a) = ca^m$ where m is the power-law exponent and c is a normalizing constant.
Substituting this into equation (2) along with $a = kx^2$ gives:

$$f_X(x) = c\,(kx^2)^m\,(2kx) = (2ck^{m+1})\,x^{2m+1} \tag{3}$$

which shows that the pdf of floe diameter follows a power law with exponent $2m+1$ and normalizing constant $c' = 2ck^{m+1}$.

Normalizing constants

Let $a_{\min}$ be the smallest floe area. Then the normalizing constant c is determined from:

$$\int_{a_{min}}^{\infty} ca^m da = 1 \qquad \text{so that} \qquad c = -(m+1)\,(a_{\min})^{-(m+1)} \tag{4}$$

where $m+1 < 0$ in order for the integral to be finite. Let $x_{\min}$ be the smallest floe diameter, and suppose $a_{\min} = k(x_{\min})^2$. Then the normalizing constant for the pdf of floe diameter is:

$$c' = 2ck^{m+1} = -2(m+1)\,(x_{\min})^{-2(m+1)} \tag{5}$$

Cumulative distribution functions

The cumulative distribution function of floe area is:

$$F_A(a) = \int_{a_{min}}^{a} c(a')^m da' = \left(\frac{c}{m+1}\right)(a^{m+1} - a_{min}^{m+1}) = 1 - (a/a_{\min})^{m+1} \tag{6}$$

The cumulative distribution function of floe diameter is:

$$F_X(x) = 1 - (x/x_{\min})^{2(m+1)} \tag{7}$$

Neither $F_A(a)$ nor $F_X(x)$ is a power-law distribution. However, the *complementary* cumulative distributions F'_A and F'_X are power laws:

$$F'_A(a) \equiv 1 - F_A(a) = (a/a_{\min})^{m+1} \qquad \text{and} \qquad F'_X(x) \equiv 1 - F_X(x) = (x/x_{\min})^{2(m+1)} \tag{8}$$

The *complementary* cumulative distributions are used in the sea-ice floe-size literature. The power-law exponent of F'_X is twice the power-law exponent of F'_A. The pdfs are now related to F'_X and F'_A by $f_X(x) = -dF'_X/dx$ and $f_A(a) = -dF'_A/da$ (note the minus signs).

Finite upper limit

Suppose the pdf of floe area is a power law, $f_A(a) \sim a^m$, but the largest floe area is $a_{max} < \infty$. Then the cumulative distributions F_A and F'_A are:

$$F_A(a) = \frac{1-\left(\frac{a}{a_{min}}\right)^{m+1}}{1-\left(\frac{a_{max}}{a_{min}}\right)^{m+1}} \qquad \text{and} \qquad F'_A(a) = \frac{\left(\frac{a}{a_{min}}\right)^{m+1}-\left(\frac{a_{max}}{a_{min}}\right)^{m+1}}{1-\left(\frac{a_{max}}{a_{min}}\right)^{m+1}} \tag{9}$$

neither of which is a power law. If $a_{max} = k(x_{max})^2$ where x_{max} is the largest floe diameter then the expressions for $F_X(x)$ and $F'_X(x)$ are the same as in (9) but with exponents $2(m+1)$ instead of $m+1$ and with x_{min} and x_{max} replacing a_{min} and a_{max}. Again, neither $F_X(x)$ nor $F'_X(x)$ is a power law. Their derivatives (the pdfs) are power laws between the bounds x_{min} and x_{max} (or a_{min} and a_{max}).