Author response for Reviewer 1

I would like to thank the reviewer for commenting on the manuscript constructively with valuable ideas and insights. I appreciate the time and effort you made that will surely improve the quality of the paper. Below are my detailed responses to the Reviewer Comments. The original text is in **black** and my response is in **green**.

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1 General comments

This manuscript presents further development on the precedingly applied perturbation theory. Rather than using a perturbation in elevation, the author is introducing a basal friction term which will be the source of the perturbation. The development of the model leads to the denition of two key parameters, J_0 represent the strength of the initial response to basal lubrication and P_e gives insight on the longer term mode of mass transport. The author used multi-sourced data from 1996-1998 from glaciers in Greenland and Austfonna ice cap to compute P_0 and J_e and compare those to the observe velocity changes in the following twenty years. The application of this 1D model allows to highlight glaciers for which a combination of thickness and initial velocity will render said glacier vulnerable to a lubrication of its bed. The study indicates that for some ocean terminating glaciers in the Greenland ice sheet, multiple feedbacks could make glaciers more sensitive to changing basal condition.

The introduction of basal friction in the perturbation theory model allows to introduce a more realistic way to perturb the model and observe its evolution. This improvement allows to draw conclusions on the stability of glaciers to a lubrication event with respect to its P_0 and J_e which ultimately are based on its geometry and velocity before the perturbation. The result unfortunately show that there is quite a large area of the J_0 against P_e/ℓ graph in which the behaviour of the glaciers is uncertain and their classification as vulnerable or not to basal lubrication is not clear. The author propose two hypothesis that would explain this clustering on the graph with quite dierent effect in term of acceleration. Finally, the author presents some characteristics which would render a glacier vulnerable to lubrication and the potential feedback due to said lubrication.

The description of the model, and development of its equations is clear and well described but some of the figures could be clarified for better readability. As a colourblind reader I have a hard time with the colour choice of Figures 5 and 7. This is not vital to the understanding of the paper as the difference between pale and other colours is still readable but it will help to control the colourscale and/or add a colourbar to help with readability. On panel (B) of these figures, it might also be better to use a smaller marker than the dot to allow a better readability of the lines themselves.

Thank you for the suggestion. I was aware of the colorblind readability for most of the plots but obviously did not tune them to the best. I have redesigned the colormap used in Figures 5 and 7. Now the colormap is based on Roma with a few tweaks about the transparency and the red zone so it can nicely show an asymmetrical pattern of the speed change. I have also added a colormap scale in the lower panel and changed the marker size as suggested. I hope this improves the figure, but please let me know if there is still something that could be done for readability. I'd appreciate it very much.

There are a few more points that might need clarification in the paper as those were not completely clear to me.

• In the description of the data treatment, it is stated that the velocities are interpolated on NoData points and afterwards that a valid vertex should not contain any NoData Values, I expect that at this point the interpolated velocities are not considered as NoData anymore, is that right?

The NoData vertices that cannot be interpolated will be kept, so there is a possibility that the interpolated velocity still contains NoData at the end of the flowline. I have added the following sentence in the data treatment for improved clarity: "We do not extrapolate the glacier speed; therefore, the NoData vertices at both ends of the flowline are still preserved after this step."

• It is clear particularly on the Austfonna example that the flowlines are displaced due to the changes in velocity, I wonder how these changes affect the model result and why the flowlines were computed from the 2018 dataset and not from the ITMIX one.

The ITMIX data set contains only flow speed but not flow velocity (i.e. only scalar values available), making it not ideal for constructing glacier flowline. Nevertheless, one of the ITS_LIVE-derived Basin-3 flowlines (ID #36, the northernmost flowline at Basin-3 in Figure 2B) matches the ITMIX flow pattern. Thus, the model results should reflect the average of the changing flow patterns.

• Only marine terminating glacier are investigated here, is there any limitation of the model that prevents to study land terminating glaciers, or was it a choice of the author?

Yes, it is a choice of the author as land-terminating glaciers should follow the same physical framework (speedups triggered by basal lubrication and diffusion thinning) as marine-terminating glaciers do. The main reason for this choice is that GrIS's marine-terminating glaciers have shown significant mass loss through dynamic thinning, while land-terminating glaciers mainly drain the mass by a negative surface mass balance (see the first paragraph of the main text and the references therein). As a result, marine-terminating glaciers seem to have a larger impact on their mass budget under basal lubrication than land-terminating glaciers do, so they become ideal targets for testing the physical framework presented in this study.

• Ultimately, most of the study investigates J_0 as close to the front as possible, it would be interested on the reasons behind that choice in the paper.

There are two reasons for this choice:

1. The crevasse/mouline formation is more likely to occur at the terminus region than at the divide region (see the updated text in the manuscript for references), which means the terminus region is more likely to be lubricated by meltwater routing.

2. J_0 is proportional to the initial flow speed (Eq. 17). The ice flow away from the front is usually small, and glacier basins tend to have a similar small value of J_0 if using observations from the upper stream, making the metric less useful to determine vulnerability.

I have added a paragraph in Section 4.2 about the justifications above.

• On figure 6, I noted that the spread of the Gaussian kernels is different for both classes of glacier, more spread on J_0 and more defined peak on P_e/ℓ for glaciers with high acceleration and the other way around for the more stable glaciers. Can that be explained by the model? If yes it would probably be worth discussing that pattern.

I have included the frontal retreat data and updated the analysis associated with Figure 6 (see my response to Reviewer 2 for details). Now it looks that the observation described in the comment is linked to (1) various intensity of terminus retreat for each glacier (i.e. some glaciers accelerated due to retreat, not basal lubrication); and (2) small sample size causing a wide spread as seen from the Gaussian kernel (see the attached figure in the response to Reviewer 2). I have rewritten a great portion of Section 4.2 to reflect these changes and thoughts.

• Line 285, it is stated that GrIS land terminating glaciers are insensitive to meltwater forcing. However there are some modelling study that seem to disagree with that statement [e.g. Gagliardini and Werder, 2018]

I agree with this observation and have reviewed the corresponding paragraph. Since this paragraph mainly talks about marine-terminating glaciers, I have removed the latter half of the sentence about land-terminating glaciers for a more focused discussion. The Gagliardini and Werder 2018 paper has been cited elsewhere in the original manuscript.

2 Specific comments

Bellow is a list of more specific comments throughout the manuscript given with line numbers:

• Line 1: “flow” seems misplaced here, isn't “discharge of ice” sufficient?

Yes, “discharge of ice” is sufficient. Changed.

• Line 11: I do not completely understand the usage of “forms” here.

This sentence has now changed to “A combined factor of ice thickness, surface slope, and initial flow speed physically determines how much and how fast glaciers respond to lubricated beds in terms of speed, elevation, and terminus change.

• Line 16: I am not sure of the meaning of “where a good portion” is it for most of Greenland glaciers, or most of the marine terminating glaciers?

Thank you for pointing out this ambiguity. I removed “where a good portion of…” and rewrote this sentence as: “At the Greenland Ice Sheet (GrIS), dynamic discharge of marine-terminating glaciers accounts for 66% of the region’s total mass loss (Mouginot et al., 2019).”

• Line 23: Should “subsurface ocean water” be specified here?

Changed as suggested for improved clarity.

• Line 28: Sentence on this line is unclear and could be rephrased.
To improve clarity, this part of the text has been modified to: “Outside of the GrIS, the primary drivers of the dynamic ice loss remain largely uncertain, although significant melt-induced lubrication and speedup events have been identified around the Arctic.”

• Table 1: K_1 here is a function of x and t but it is stated in the text that it is a constant, perhaps the notation in the table should reflect that.
Changed the notation in the table from $K_1(x, t)$ to K_1 with a note that K_1 is assumed constant.

• Table 1: The primes (') are not defined here perhaps it would be worth defining them as x derivative here.
Added a sentence in the Table 1 caption for clarity.

• Line 86: I suspect that *fracdhdt* stands for *fracdH₁dt*
Yes, that’s correct. Changed to dH_1/dt.

• Figure 3: In this figure and the following, the panel lettering is missing in the figure.
Added the missing panel letters in Figures 3, 4, 5, and 7.

• Line 207: I would prefer the “J_0 against P_e/ℓ” notation than the one used here and further down (line 233, 239).
Changed the notation to “J_0 versus P_e/ℓ plot”.

• Line 210: It seems that a zero is missing in the value of P_e/ℓ.
Thank you for pointing this typo out. Changed to $P_e/\ell > 0.0001$.

• Figure 5: In this figure and following the units should be changed to be consistent with the text.
Changed all velocity-like units (e.g., glacier speed change and J_0) from m/yr to m yr^{-1} and Pe/ℓ units from 1/m to m^{-1}, including those label in Figures 1,2, 5-7.

• Line 292: I am not sure of the meaning of status here.
Changed “resulting in status” to “making the glacier.”

• Line 306: Isn't “and” missing, “and create an other feedback..”
Corrected.

References

Olivier Gagliardini and M. A. Werder. Influence of increasing surface melt over decadal timescales on land-terminating greenland-type outlet glaciers. J. Glaciol., 64(247):111, aug 2018. doi: 10.1017/jog.2018.59.