A review of "A daily basin-wide sea ice thickness retrieval methodology: Stefan's Law Integrated Conducted Energy (SLICE)" by James Anheuser et al.

General Comments

The authors introduced a new method for sea ice thickness estimation from satellite snow-ice interface temperature (T_{si}) by using idealized sea ice thermodynamic model. The key idea of their methodology is that thermodynamic sea ice growth rate can be calculated from upward conductive heat flux within the sea ice layer which balances with the latent heat of fusion. In their method, the conductive heat flux is a function of the T_{si} under the linear temperature profile assumption. Therefore, sea ice thickness can be calculated from T_{si} with appropriate initial ice thickness. Furthermore, the authors insist that the introduced method is self-correcting.

However, I have some major concerns about the introduced method. 1) There should be clarifications on the physical conditions (regions and seasons) that meet with the four assumptions they made. 2) More explanation is needed to insist that the method is "effectively" self-correcting. 3) The method seems to be a modeling approach rather than satellite retrieval. 4) Detailed procedure for the bias correction of the satellite T_{si} must be provided.

Associated with the major concerns above, I think there should be significant improvements on the data and methodology before the manuscript is published to The Cryosphere. Therefore, my decision is to reconsider after the major revision. I would like to review the manuscript again after the revision.

Major comments

1. Assumptions in the SLICE method

From L202 to L208, the authors listed the four assumptions used in the SLICE retrieval method. I have concerns about the second and the third assumptions. In my opinion, the second assumption is equivalent to the statement that the temperature profile of sea ice is linear. But if you see the buoy measured temperature profiles, you will find this assumption is not always valid. Such linear profile assumption is generally valid during wintertime. Moreover, even during wintertime, sudden change in air temperature due to warm/cold advection or radiative forcing due to cloud cover can rapidly change surface temperature which makes curves in the temperature profile. The good thing is time-averaged temperature profile during wintertime is close to linear (Shi et al., 2020). The authors would consider shortening of retrieval period of the SLICE method.

The third assumption tells that there is no internal heat source associated with shortwave radiation. In other words, this assumption is valid for the regions where the solar zenith angle is maintained less than zero. The authors should check the validity of this assumption regarding the seasonal variability of the solar zenith angle. There can be sunlight in lower latitude regions during fall and spring. Otherwise, please consider the shortwave radiation effects or justify that the shortwave radiation effect is negligible for the lower latitude regions during the fall and spring seasons.

The other point is that the authors mentioned that the retrieval method should be applied in a Lagrangian sense in L224 but they neglected sea ice motion in the actual calculation (L262). What are the reasons for this? There must be justification for the neglect of sea ice motion. Each sea ice parcel should be tracked and matched with the nearest satellite T_{si} because the equation used in this study is a time-dependent equation. Meanwhile, the neglect of sea ice motion is not the same as focusing on thermodynamic growth. Thermodynamical growth, sea ice motion, and dynamical growth (deformation due to convergence and divergence) should be addressed separately. Consideration of sea ice motion without dynamical growth is possible.

2. Effectiveness of self-correcting characteristic

It was interesting to read the statement in L225 regarding the self-correcting characteristic of the SLICE method. Thicker sea ice indeed grows slower than thinner sea ice with a given T_{si} and vice versa according to equation (7). Therefore, the error in sea ice thickness can be relaxed by the modulation of sea ice growth speed.

However, the relaxation speed of error is important as well. If the speed of relaxation is slow, the effectiveness of self-correcting characteristics will be minor and the initial condition will be the major factor that determines the accuracy of sea ice thickness estimation. In L249-250 and Figure 2, the authors tried to show the effect of the self-correcting characteristic. Although it seems that 0.25 m deviations in the initial condition are decreasing with time, it will be better to specify the improvement quantitatively to know how fast the errors are relaxed. In addition, I suggest conducting a sensitivity test and including the result as an appendix.

I found some doubtful points on the self-correcting characteristic of the SLICE method. In my opinion, if the method is self-correcting, the retrieval result should fluctuate around the true state. Why is the SLICE retrieval (red solid line) the center of red shade instead of the buoy (blue solid line) which is the true state? In addition, I think the sentence “The bias grows with time as the SLICE profile moves away from its initialized thickness” makes a contradiction with the self-correcting characteristic of SLICE.

The significance of self-correcting characteristic is important for the algorithm extension to the past because such characteristic makes the retrieval method relatively independent from the accurate initial condition. If the self-correction is significant, SLICE sea ice thickness records initialized with PIOMAS can be constructed, and it will be more accurate than PIOMAS. To examine this, I suggest comparing the accuracy of the sea ice thickness from the PIOMAS and that from the SLICE initialized with the PIOMAS. There are some widely used independent datasets for validation such as Operation IceBridge (OIB), buoy, upward-looking sonar (ULS), and submarine observations.

3. Retrieval or modeling (significance of this study)

In some sense, the SLICE retrieval method seems to be a thermodynamic sea ice model. The reason is that it simulates sea ice thickness evolution with time, and the result of SLICE retrieval is highly dependent on initial conditions rather than observed data. I think that the SLICE method is a simplified version of the thermodynamic sea ice model introduced by Maykut and Unterstiener (1971) or the PIOMAS. It will be nice for the authors to explain why the SLICE method is satellite retrieval.

Nonetheless, the novel point of this study is SLICE method is independent of the atmospheric reanalysis generally used as the forcing to sea ice model. The most relevant study to the SLICE method will be Kang et al. (2021), which simulates the physical state of a snow-ice system by using a thermodynamic equation set forced by atmospheric reanalysis and nudged by satellite T_{si}. This study has significance in terms of constructing an independent sea ice thickness record, while the physics of SLICE is very simplified compared to Kang et al. (2021) or other thermodynamic sea ice models. I recommend including an ice thickness comparison with the results of Kang et al. (2021). Their results are open to the public, and the authors can find the data repository in their paper. It is worth comparing the performance of the SLICE method with other sea ice models with more sophisticated physics and forced by reanalysis data.

4. Bias correction for satellite T_{si}

The authors mentioned that "The resultant snow-ice interface temperatures were found to require a bias correction of 5 K in order to match buoy snow-ice interface temperatures…"). I have read Lee and Sohn (2015) and remember that the snow-ice interface derived from AMSR-E 6.9 GHz brightness temperatures are validated with buoy measured temperature. The validation result showed that the bias was less than 1 K, which is a very different result from the 5 K bias in the manuscript. Lee and Sohn (2015) also neglected atmospheric/snow absorption.

Regarding this situation, first I thought that it is possibly due to the bias within AMSR-E and AMSR2 measurements. However, the authors stated that the AMSR2 data has been intercalibrated with the AMSR-E data so this may not be the issue. Then, may the version of L3 brightness temperature be a problem? Or simply authors failed to reproduce the T_{si} retrieval algorithm.

This is a very critical issue because sea ice thickness is determined by T_{si}, which is the only real observation used for the sea ice thickness retrieval. The mentioned comparison result between buoy data and T_{si} calculated by the authors showing 5 K bias must be presented (as an appendix) to justify the bias correction procedure. It will be worth reproducing figure 6 in Lee and Sohn (2015).

Minor comments

L29-L37: Please provide more details for relevant studies on sea ice thickness retrieval in order to emphasize the novelty or necessity of SLICE. How are the satellite altimetry methods limited in spatial coverage and temporal resolution (I think the resolution of ICESat-2 is better than passive microwave sensors such as AMSR2 6.9 GHz)? What are the limitations of the other methods? How is this study related to the existing studies?

L63: horizontally and vertically polarized…

L215: Please define negative degree-days in the manuscript and provide what happens if the temperature is positive (melting?).

L221: It is hard to know which equation was used for sea ice thickness calculation. Equation (4) is too general. Did you use equation (8) which is an analytic solution for sea ice thickness, or equation (7) for change in sea ice thickness per unit time and accumulate the thickness changes?

L235-237: Why the retrieval method was initialized with such condition (the day when the 14 d rolling average sea ice growth exceeded 1mm d^{-1})? Please provide the reason.

L400: I think uploading the data produced in this study to the public data repository more fits the data policy of TC journal.

References

Kang, E. J., Sohn, B. J., Tonboe, R. T., Dybkjaer, G., Holmlund, K., Kim, J. M., and Liu, C.: Implementation of a 1-D Thermodynamic Model for Simulating the Winter-Time Evolvement of Physical Properties of Snow and Ice Over the Arctic Ocean, J. Adv. Model. Earth Sy., 13, 3, https://doi.org/10.1029/2020ms002448, 2021.

Lee, S.-M and Sohn, B.-J.: Retrieving the refractive index, emissivity, and surface temperature of polar sea ice from 6.9 GHz microwave measurements: A theoretical development, J. Geophys. Res. Atmos., 120(6), 2293-2305, https://doi.org/10.1002/2014JD022481, 2015.

Maykut, G. A. and Untersteiner, N.: Some results from a time-dependent thermodynamic model of sea ice, J. Geophys. Res., 76, 1550–1575, https://doi.org/10.1029/JC076i006p01550, 1971.

Shi, H., Sohn, B.-J., Dybkjær, G., Tonboe, R. T., and Lee, S.-M.: Simultaneous estimation of wintertime sea ice thickness and snow depth from space-borne freeboard measurements, The Cryosphere, 14, 3761–3783, https://doi.org/10.5194/tc-14-3761-2020, 2020.