Authors point-to-point response on Referee Comment #1 to tc-2021-325

General comments

#1

I'm a big fan of proper treatment and discussion of uncertainty. I like the idea behind this paper. Unfortunately, I have significant issues with the implementation of the idea. Specifically, the work is incomplete, and difficult to understand. All work could be considered incomplete because any effort can always be done a bit more or better, but by my understanding of uncertainty analysis this work leaves out a lot of sources of uncertainty, and ignores ways that many scientists reduce uncertainty. Or maybe it is there, but suffers from my second issue - it is difficult to understand.

Response 1:

Thanks for the reviewer's comment. We agree and have made efforts to address the issues. Detailed responses are given below.

Most of the mass balance papers rarely mention the uncertainty evaluation in detail, which is a black box for our beginner in mass balance estimation. On the one hand, it is most perplexing that the differences among the mass balance estimates published in authoritative journals are much greater than their respective uncertainties. For example, for the East Antarctic ice sheet (EAIS), in contrast to the mass gain of 58 ± 43 Gt yr^{-1} from 2003–2020 obtained using the gravity method (Wang et al., 2021) and 90 ± 21 Gt yr^{-1} from 2003–2019 obtained using the altimetry method (Smith et al., 2020), mass losses of 82 ± 9 Gt yr^{-1} from 1999–2009 and 51 ± 13 Gt yr^{-1} from 2009–2017 were estimated by Rignot et al. (2019) using the IOM. Rignot et al. (2019) have done a lot work to reduce the uncertainty, but their IOM mass balance of the EAIS is quite different from others. How this could happen? On the other hand, if we are new to do mass balance assessment using IOM method, how can we make others believe our results. This is our original intention of this paper.

We know that the (combined) standard uncertainty is usually used in the IOM uncertainty estimation. We speculate that Rignot et al. (2019) used this method, thus we think it is hard to help us to find the answer. So we tried to get the uncertainty from the random errors generated by Monte Carlo simulation (the Monte Carlo uncertainty) rather than that the (combined) standard uncertainty in the original manuscript. We agree that this point in the manuscript has not been clearly clarified. Though it is really hard to demonstrate all the uncertainty analysis in a paper, we have tried to give more comprehensive uncertainty estimations in the revised version. We have assessed: (1) the (combined) standard uncertainty based on the uncertainty field of the data sets and the given measurement precision, (2) the Monte Carlo uncertainty based on the uncertainty field of the data sets and the given measurement precision, (3) the uncertainty due to system error, (4) the uncertainty presented by the difference between two datasets, between two methods and between two studies.

Moreover, we have restructured the manuscript as suggested by the Referee #2 and have a greater focus on hammering home the key points in the revised manuscript.

The (combined) standard uncertainty is calculated as followings:

The standard uncertainty of D due to the ice thickness ($U\_D_{T_i}$) and ice velocity ($U\_D_{V_i}$) at the pixel scale can be respectively calculated from Eq. (1) and Eq. (2):

$$U\_D_{T_i} = \frac{U_{T_i}}{T_i} \cdot D_i, \tag{1}$$

$$U\_D_{V_i} = \frac{U_{V_i}}{V_i} \cdot D_i, \tag{2}$$

where the U_{T_i} is the thickness uncertainty of flux gate *i*, the T_i is the thickness, the U_{V_i} is the ice velocity uncertainty, the V_i is the velocity, and the D_i is the ice discharge. And for the whole AIS and ice drainage basins, the D uncertainty due to the ice thickness (U_D_T) and ice velocity (U_D_V) can be calculated from Eq. (3) and Eq. (4):

$$U\_D_T = \sum_{i=1}^{n} \frac{U_{T_i}}{T_i} \cdot D_i, \tag{3}$$

$$U\_D_V = \sum_{i=1}^{n} \frac{U_{V_i}}{V_i} \cdot D_i, \tag{4}$$

where the *n* is the number of flux gates. The D is derived from three components unrelated to and independent of each other. Thus, we used synthetic standard uncertainty to evaluate its accuracy. For the calculations, 917 kg m^{-3} is used for ρ_{ice} and the uncertainties $U_{\rho_{ice}}$ is valued at 5 kg m^{-3} (Griggs et al., 2011). The combined standard uncertainty of D (U_D) is as follows:

$$U\_D = \sqrt{\frac{{U_T}^2}{T} + \frac{{U_V}^2}{V} + \frac{{U_{\rho_{ice}}}^2}{\rho_{ice}}}, \tag{5}$$

To calculate the uncertainty of SMB, we use the relative uncertainty percentage (u_r) of the RACMO2.3p2 SMB, which is nearly 4% (Mottram et al., 2021). The standard uncertainty of SMB (U_SMB) is as follows:

$$U\_SMB = u_r \cdot SMB \tag{6}$$

In this way, the uncertainty could be expressed in terms of the magnitude of the value. But this way is unavailable if the uncertainty data sets are not provided, and on the contrast, the second way, comparing results using different dataset combinations, is not limited by the availability of the uncertainty data. The detailed combinations of thickness and velocity products are shown in Table 2 (the manuscript line 170). This comparison may be of certain reference significance to the data set selection. We also compared the Ds of three flux gates (the original grounding-line flux gate, the flux gate for a 1 km advance, and the flux gate for a 1 km retreat).

The Monte Carlo uncertainty evaluates the uncertainty attribution of random errors through generating the random errors based on the precision of the ice velocity and thickness data products we used, which has been described in details in the original manuscript.

Revised Table 3:

Table 3. Data and method uncertainties of the D and SMB (Gt yr^{-1})

Components	**EAIS**	**WAIS**	**APIS**	**Islands**	**AIS**
$D_{reference}$	895.8	750.2	157.7	132.4	1936.0
U_D					
Standard uncertainty					
U_D($V_{InSAR\text{-}based}$)	33.0	13.3	3.2	10.9	60.4
U_D($V_{Phase\text{-}based}$)	8.9	5.5	1.9	3.3	19.5
U_D($T_{BEDMAP2}$)	265.3	145.0	86.4	50.3	547.0
U_D($T_{Bedmachine}$)	93.1	50.6	30.9	33.5	208.2
U_D($V_{Phase\text{-}based}+T_{Bedmachine}$)	95.6	52.4	31.3	34.5	213.8
Monte Carlo uncertainty					
U_D($V_{InSAR\text{-}based\_re}$)	5.3	2.2	0.3	1.8	9.6
U_D($V_{Phase\text{-}based\_re}$)	0.9	0.6	0.2	0.3	2.0
U_D(V_{re} 20 m yr^{-1})	17.4	10.1	4.0	9.7	41.1
U_D($T_{BEDMAP2\_re}$)	27.9	14.5	11.6	7.3	61.4
U_D($T_{Bedmachine\_re}$)	17.1	5.9	5.7	6.8	35.5
U_D(T_{re} 100 m)	3.2	1.7	3.0	4.8	10.5
Uncertainty due to system error					
U_D(V_{se} 20 m yr^{-1})	138.1	80.1	29.0	53.4	302.6
U_D(T_{se} 100 m)	151.2	105.1	70.4	43.1	369.8
Uncertainty presented by difference					
U_D($V_{InSAR\text{-}based}$-$V_{Phase\text{-}based}$)	7.7	2.6	5.1	1.5	17.1
U_D($T_{BEDMAP2}$-$T_{BedMachine}$)	18.4	32.9	25.7	24.6	101.7
U_D($GL_{Advance}$-GL_0)	-97.2	-107.1	-44.4	24.7	-278.1
U_D($GL_{Retreat}$-GL_0)	-8.8	-7.5	-0.8	-62.5	-79.6
U ($D_{ref}-D_{Rignot}$)	-214.6	-24.9	-172.0	55.3	-356.2
U ($D_{ref}-D_{Garder}$)	-56.2	11.2	-62.3		
U ($D_{ref}-D_{Shen}$)					-171.3
U_MAX(ABS($D_{pixel}-D_{scale}$))	3.2	6.5	1.8	7.4	10.4
Maximum uncertainty					
Max(ABS(U_D_V))	*138.1*	*80.1*	*29.0*	*53.4*	*302.6*
Relative(% of D_{ref})	*15.4%*	*10.7%*	*18.4%*	*40.3%*	*15.6%*
Max(ABS(U_D_T))	*265.3*	*145.0*	*86.4*	*50.3*	*547.0*
Relative(% of D_{ref})	*29.6%*	*19.3%*	*54.8%*	*38.0%*	*28.3%*
Max(ABS(U_D_{GL}))	*97.2*	*107.1*	*44.4*	*62.5*	*278.1*
Relative(% of D_{ref})	*10.9%*	*14.3%*	*28.2%*	*47.2%*	*14.4%*
$SMB_{reference}$	1153.5	649.5	253.6	61.4	2118.0
U_{SMB}					
Standard uncertainty					
U_$SMB_{p2\text{-}27}$ (4%)	46.1	26.0	10.1	2.5	84.7
Uncertainty presented by difference					
U ($SMB_{p1\text{-}35}$-$SMB_{p1\text{-}27)}$	-26.1	-51.6	-119.4	-19.6	-216.7
U ($SMB_{p2\text{-}27}$-$SMB_{p2\text{-}5.5)}$			-51.7		
U ($SMB_{p1\text{-}27}$-$SMB_{p2\text{-}27)}$	-78.5	3.1	39.4	15.6	-20.4

U (SMB_{ref}-SMB_{Rignot})	78.5	-3.1	-39.4	-15.6	20.0
U (SMB_{ref}-$SMB_{Gardner}$)	95.5	108.5	19.0		
U (SMB_{ref}-SMB_{Shen})					217.0
Maximum uncertainty					
Max(ABS(U_SMB))	*95.5*	*108.5*	*119.4*	*19.6*	*217.0*
Relative(% of SMB)	*8.3%*	*16.7%*	*47.1%*	*31.9%*	*10.2%*

#2

I think there needs to be a significant effort added to the discussion of random errors vs. systematic bias. Many scientists assume errors are random, and then use this and the mathematical treatment of uncertainty to reduce errors by summing or averaging over many small regions with random error. This may or may not be correct, but should be discussed in detail in this paper. Limited discussion on the distinction between these two types uncertainty, and their treatments in other works, is problematic.

Response 2: We understand the reviewer's point. We used the Monte Carlo method to estimate uncertainties assuming the observation errors are randomly distributed and considered the system error to estimate uncertainties due to the ignorance of the velocity or thickness change. In the revised manuscript, we have discussed it in detail as suggested and added the (combined) standard uncertainties, which have been provided additional data to discuss.

#3

The authors make some questionable decisions on use of data to assess uncertainty. For example, they use BedMachine and BedMap, but do not address the uncertainty field provided by BedMachine.

Response 3: We did not write well for the older version   We have added add the D's uncertainty estimations from the uncertainty field of provided by datasets of BedMap, BedMachine, InSAR-based and Phase-based velocities both using the mathematical treatment and Monte Carlo method as revised Table 3.

#4

The authors continually refer to an 'uncertainty analysis framework'. What is this? Is 'framework' just a different way of saying 'method'? An "introduction" suggests something new or not seen before. I do not see anything particularly novel in this work. Therefore, if it is not novel, it should at least be comprehensive, but it is not that either.

Response 4: Thank you for your comment. The "uncertainty analysis framework" is a scheme on how to make the uncertainty evaluation of IOM open and transparent. There are more than five research groups to estimate mass balance of Antarctic ice sheet using the IOM. They have done a lot work to reduce the uncertainties. But it is hard to use their method or repeat the work without the details. We presented the details in our uncertainty estimation. For example, to demonstrate the D's details, we showed 58,597 flux gates around the grounding line. We agree that the "framework" expression is inappropriate and have modified it in the revised manuscript.

#5

Finally, I feel that the summary of this work could be a simple and clear table showing the

uncertainty from different sources in both absolute (Gt/decade?) and relative (% of total uncertainty) terms. Table 3 lists "data uncertainties" but not "method uncertainties" and not relative amounts.

Response 5: Yes. We have added the relative terms and the "method uncertainties" term as the revised Table 3.

Specific comments

The 15 gt/yr target has a reference, but it is not clear that this target is necessary. Furthermore, the reference for that states,

> *Determine the changes in total ice-sheet mass balance to within 15 Gton/yr over the course of a decade and the changes in surface mass balance and glacier ice discharge with the same accuracy over the entire ice sheets, continuously, for decades to come.*

And it isn't clear to me how to parse that. Is this 15 Gt/year average over a decade? Or 150 Gt/decade? Or 15 Gt/decade? Something else?

Response: The 15 Gt yr^{-1} is a decadal goal. Scientists are required to determine the changes in the total ice sheet MB within 15 Gt yr^{-1} over the course of a decade, i.e., 15 Gt yr^{-1} per decade or 1.5 Gt yr^{-2}, as well as the changes in the SMB and D with the same accuracy over all of the ice sheets (Board et al., 2019). We have deleted the 15 Gt yr^{-1} target to reduce disputes in the revised manuscript.

Authors use terms without defining them. For example, "scale effect" on L22.

Response: Thank you for your suggestion. We have added the definition of the scale effect in the introduction as suggested in the revised manuscript.

What does it mean to have an uncertainty on an uncertainty? For example, L24 reports "Uncertainties of [...] X +- Y Gt/yr".

Response: "The uncertainty on an uncertainty" or "±Y" is the one standard deviation of 100-times uncertainties estimated using a given measurement precision by Monte Carlo method. X is the averaged uncertainty.

There is often reference to "future D". But everything else is using present or historical data. Why is "D" discussed predictively?

Response: Using the Monte Carlo method, we can estimate the uncertainty of future velocity or thickness with any given precision. But considering the controversy you mentioned, we have deleted such description in the revised manuscript.

L73 is a motivation for the entire work, "the dominant factors influencing the uncertainties in the MB estimation of the AIS remain unclear." I disagree, the dominant factors are known. Perhaps not their relative scales?

Response: Yes, we have corrected. We have changed "the dominant factors" into "the relative scale of factors" in the revised manuscript.

L83 "We introduce an uncertainty analysis framework" <-- where?

Response: As is responded in our reply #4, we agree “framework” is not appropriate to express our intention, which has been revised in the revised manuscript.

L96 "SMB (input)" SMB is output too (runoff, sublimation, evaporation) although runoff is small in Antarctica.

Response: Yes, we have revised this sentence in the revised manuscript.

L104 D equation also uses ice density.

Response: We have added “ice density” in the sentence as suggested in the revised manuscript.

L168 Which "original raster" is extended? The velocity raster? thickness? Both?

Response: Both of the velocity and thickness raster are extended. We have clarified this point in this sentence as suggested in the revised manuscript.

L190-195: This sentence seems out-of-place. It's not about your methods or results of your methods, it is instead about anomalous years. I am unable to see the relevance to the surrounding text or the paper.

Response: We agree and we have removed this sentence in the revised version.

L447-449: "The strategy of using the yearly averaged SMB instead of the annual SMB is acceptable for decadal MB estimation to constrain variability when the long-term trend is required. However, if the research is related to short-term, regional snowfall events, it is better to use the annual SMB to determine the annual variability." <-- but the motivation for this whole paper is about a 15 Gt/year decade average. In several places there is mention of "annual variation" (see also L467). It isn't clear to me that annual variation matters based on the target "15 Gt/year over the course of a decade". Can these annual variations be considered random and therefore reduce when averaged over a decade?

Response: Thanks for raising this point. The decade uncertainty of SMB can be calculated as follows:

$$U\_SMB_{decade} = \frac{\sqrt{{U\_SMB_{yearly}}^2 + STD^2}}{9}, \tag{11}$$

where the U_SMB_{yearly} is the average uncertainty of SMB during the decade (the relative uncertainty percentage u_r is 4%), and the STD is the standard deviation of the decadal SMB. So we can find the uncertainty of yearly SMB for 2007-2019 is 85.4 Gt yr^{-1}, and the uncertainty of decadal SMB for 2007-2019 decrease to 11.9 Gt yr^{-1}, which is within the goal of 15 Gt yr^{-1} per decade. We have clarified this point in the revised version.

L454 - again mention of "future D". Why not also mention "future SMB"?

Response: Using the Monte Carlo method, we can estimate the uncertainty of future velocity or thickness with any given precision. But considering the controversy you mentioned, we have deleted such description of "future D" in the revised manuscript.

L464 "Even ice thickness data with a 100-m precision" What is 100 m precision? Is that like "thick_100 = round(thick/100)*100" ?

Response: We appreciate this insightful comment. “100-m precision” is the measurement precision of 100 m, one standard deviation of thickness measurement. Measurement precision is related to random measurement error and is a measure of how close results are to one another. Measurement precision is expressed numerically using measures of imprecision such as one standard deviation calculated from results obtained by carrying out replicate measurements. We have clarified the description in the revised manuscript.

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