



Brief communication: Estimating the ice thickness of the Müller Ice Cap using an inversion of the shallow ice approximation

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Abstract. The Müller Ice Cap will soon set the scene for a new drilling project. Therefore, ice thickness estimates are necessary for planning since thickness measurements of the ice cap are sparse. Here, two models are presented and compared, i) a simple inversion of the shallow ice approximation (SIA inversion) by the use of a single radar line in combination with the glacier outline, surface slope, and elevation, and ii) an iterative inverse method using the Parallel Ice Sheet Model (PISM). The two methods mostly agree about a good drill site candidate. However, the new semi-empirical SIA inversion is insensitive to mass balance, computationally fast, and provides better fits.

1 Introduction

The Müller Ice Cap (MIC), located on Axel Heiberg Island in Arctic Canada, is facing a part of the Arctic Ocean, where no full depth ice core has been drilled, thus scientists at University of Manitoba and University of Copenhagen intend to do exactly this. Choosing a good drill site location with stratified ice layers, good time resolution, and age-span is crucial in the dating process needed to interpret the different compounds in the ice and to put it into a climatological sense. Therefore, knowledge about ice thickness and flow is important. Great ice thickness is of key importance to increase the possibility of reaching ice dating back to the Innuitian ice sheet, referring to the ice sheet in between the Laurentide and Greenland ice sheets during the last glaciation. However, the MIC remains poorly studied with the exception of a few mass balance (Koerner, 1979; Thomson et al., 2011) and surface velocity studies (van Wychen et al., 2014), leaving the ice thickness poorly constrained. This stands in contrast to one of its neighbouring glaciers, White Glacier, marked in Fig. 1. White Glacier has been studied thoroughly since the late 60s (Müller, 1962; Cogley et al., 1996, 2011) with a strong focus on the mass balance. However, field work constraints make it impractical and expensive to survey the entire ice cap, and it is therefore necessary to be clever when deciding where to conduct ground based radar measurements.

The Ice Thickness Models Intercomparison eXperiment phase 1 and 2 (ITMIX1 (Farinotti et al., 2017) and ITMIX2 (Farinotti et al., 2021)) compare various models of estimating ice thicknesses of glaciers and ice caps from sparse data. In ITMIX1, large differences are found in between the models on ice caps in the vicinity of ice divides. Furthermore, it is urged that modellers seek improvements on how to treat these regions to ensure continuity of the subsurface topography around the ice divide. In this study, it is of particular importance that we resolve the area around the ice divide well, as ice divides often

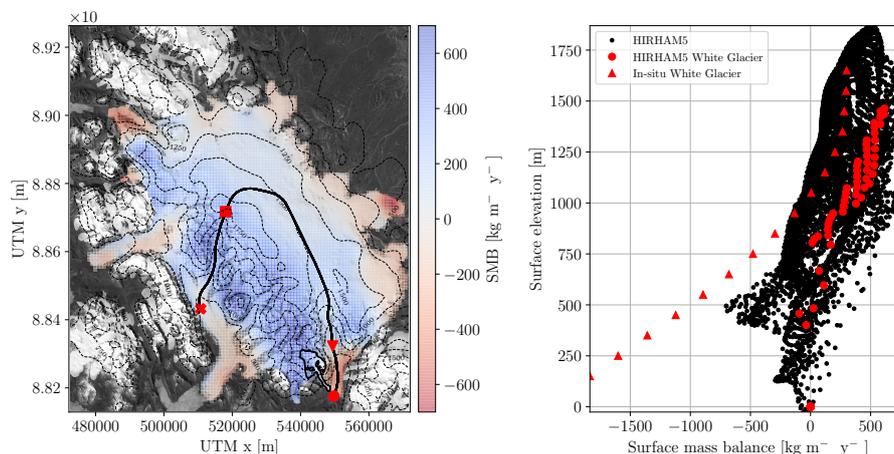


Figure 1. Left: Landsat 8 satellite image of the Müller Ice Cap overlaid with long-term average HIRHAM5 SMB from 1980-2016. Contour lines of surface elevation from ArcticDEM with 250 m increments, and the Operation IceBridge flight line as the solid black line from the red dot to the red cross. The four red symbols have the same location as in Fig. 3, and are present to guide the reader. White Glacier is outlined in black just above the red dot. **Right:** Surface mass balance as a function of surface elevation. Red triangles are average in-situ measurements from White Glacier from 1960-91 (Cogley et al., 1996). The black and red dots are long-term HIRHAM5 SMB averages from 1980-2016, with the red dots corresponding to the SMB within the White Glacier polygon.

25 qualify as good drill site locations due to limited horizontal ice flow. Furthermore, from ITMIX1 it is not possible to determine what model approaches are to be preferred on ice caps. Thus, to narrow down areas of conducting ground based radar measurements, the ice thickness is modelled using two different techniques that differs greatly in computational demands, input data and model setup. One is a simple inversion of the shallow ice approximation (SIA inversion), only relying on the surface elevation, the ice cap outline and a single Operation IceBridge (OIB) flight line with thickness measurement. This is a fast
30 new method that differs from existing SIA approaches due to its semi-empirical nature which makes it less sensitive to mass balance, steady state assumptions and ice flow physics. The second is an iterative inverse method using the Parallel Ice Sheet Model (PISM) as a forward model, which has to be forced with both climate data and an initial guess of ice cap geometry.

2 Data

Ice thickness measurements taken from the Multichannel Coherent Radar Depth Sounder from Operation IceBridge (OIB)
35 (Paden et al., 2010, updated 2019) are used to validate all models and to perform the SIA inversion. Only a few flight lines cross parts of the ice cap, why a single flight line acquired on 30.03.2017 is used as it covers most parts of the ice cap as seen in Fig. 1.



The surface elevation is obtained from the Arctic Digital Elevation Model (ArcticDEM, Porter et al. (2018)) and is shown as contours on Fig. 1. To ensure continuity over the entire ice cap, the OIB surface elevations are not used when present, thereby
40 only using the ice thicknesses from OIB.

As climate inputs to initialise PISM, SMB and 2-m temperatures are obtained from the Regional Climate Model HIRHAM5 (HIRHAM5) (Langen et al., 2017; Mottram et al., 2017). Since steady state is assumed in the models presented here, the long-term average of these fields are used. They are calculated from the yearly model results of HIRHAM5 from 1980-2014 and
45 to in-situ measurements from White Glacier (Cogley et al., 1996). Furthermore, a uniform geothermal heat flux of 0.055 Wm^{-2} is used based on Minnick et al. (2018).

The initial guess of ice cap geometry for PISM is obtained from ArcticDEM and the bedrock topography from Farinotti et al. (2019). The Farinotti et al. (2019) bedrock is based on a flow or drainage basin approach, resulting in missing values in-between the basins, and also a misinterpretation of the ice divide. Such misinterpretations are to be expected since the Farinotti
50 et al. (2019) bedrock is the result of a global glacier thickness estimate, and is thereby not tuned to the Müller Ice Cap.

All data are interpolated onto a 100 m grid and 900 m grid for the SIA inversion and PISM, respectively. Thus, gaps in the before mentioned bedrock topography are filled in the interpolation process. The initial PISM ice thickness is calculated from the interpolated versions of surface and bedrock elevations.

3 Methods

55 3.1 Inversion of the shallow ice approximation

The SIA is one of the oldest ice flow models, which has proven to show good results on ice sheets and glaciers in areas with no to little basal sliding. Combining this with the simplicity of it and how computationally cheap it is, still makes it a good and often used zero order approximation. Thus, a simple inversion of the SIA to estimate ice thicknesses from sparse data is tested.

From theory the SIA without sliding reads

$$60 \quad Q = \frac{2A}{n+2} \tau_b^n H^2, \quad (1)$$

where Q is the horizontal ice flux, A the rate factor, n the creep exponent, H the ice thickness and τ_b is the basal shear stress given by

$$\tau_b = \rho g H \alpha. \quad (2)$$

Here ρ is the density of ice and α is the surface slope. Substituting eq. (2) into eq. (1) and introducing the constant c ,

$$65 \quad c = \frac{2A}{n+2} \rho^n g^n, \quad (3)$$

implies that eq. (1) can be reduced to

$$Q = c \alpha^n H^{n+2}. \quad (4)$$



Isolating the ice thickness thus results in

$$H = Q^{\frac{1}{n+2}} \alpha^{\frac{n}{n+2}} c^{\frac{1}{n+2}}. \quad (5)$$

70 Linearity can be obtained by introducing the three tuning parameters; $k = \log\left(c^{\frac{1}{n+2}}\right)$, $a = \frac{1}{n+2}$, and $b = \frac{n}{n+2}$, resulting in

$$\log H = a \log Q + b \log \alpha + k. \quad (6)$$

Hence, the assumption is that if the ice thickness is known on parts of the ice cap and the surface slope and ice flux are known everywhere on the ice cap, one can perform a least squares regression to obtain the tuning parameters a , b , and k in the areas with known ice thicknesses and apply those in the areas with unknown ice thickness. Thereby, assuming that the areas
 75 with known ice thicknesses are representative of the entire ice cap. Note that the empirical regression is insensitive to global multiplicative errors in Q as that will be accounted for by adjustments to k . The approach is semi-empirical because the form of eq. (6) is justified by the SIA theory, but the parameters are tuned empirically. The least squares estimates of a and b are not necessarily consistent with the exponents in eq. (5), which was derived for isothermal ice with no sliding. However, the empirical calibration has freedom to adjust a and b so that it better matches the physical reality.

80 3.1.1 Surface slope

The surface slope is obtained from ArcticDEM by smoothing the surface elevation using a Gaussian filter with a standard deviation of 250 m, from which the surface slope is calculated. The smoothing is done to prevent surface depressions on the ice cap, thus ensuring that there is a downward slope from all top points of the ice cap all the way to the ice margin. To prevent
 85 $H \rightarrow \infty$ in low sloping areas a minimum slope of 0.01 is introduced on the ice cap. This corresponds roughly to 0.6° , and is relatively lower than the 2° - 5° minimum slope used in other similar models (Farinotti et al., 2009, 2017). Furthermore, the surface slope is set to be zero outside of the present day ice cap margin.

3.1.2 Ice flux

The ice flux of the MIC, and most ice caps and glaciers in general, is not known, why the balance flux is calculated. This is done based on the top model approach described in Quinn et al. (1991). The top model approach starts at the very top of the
 90 ice cap, from where the ice flows downslope into all eight neighbouring cells depending on the surface slope. From there the analysis moves on to the cell of second highest elevation, until the cell of lowest elevation is reached. From every cell all eight neighbouring cells are assigned a weight, w , such that

$$w(\alpha) = \begin{cases} \alpha^3 & \alpha > 0 \\ 0 & \alpha \leq 0 \end{cases}. \quad (7)$$

The ice then flows from cell _{i,j} to cell _{$i,j+1$} by

$$95 \quad Q_{i,j+1} = Q_{i,j+1} + \frac{w_{i,j+1}}{\sum_{i,j \in nbh} w_{i,j}} Q_{i,j}, \quad (8)$$



where nbh is the neighbourhood of cell $_{i,j}$. At the very top of an ice cap the ice flux is equal to the SMB, and all areas further down the ice both have a flux input from the SMB at that given point, and from the ice flowing in from higher elevations. In this model we use a uniform SMB of ones in every ice covered cell, as it has been shown that the SMB only has little influence on the modelled ice thickness (Zinck, 2020). This is due to the fact that any multiplicative errors in SMB can be captured in
100 k in eq. (6). Do notice, that this might not hold in areas with negative SMB. According to in-situ measurements from White Glacier and HIRHAM5 SMB (Fig. 1), the areas with negative SMB are restricted to the outlet glaciers and the glacier tongues at the margin, which are anyway outside the area of interest as surface melt at the ice core site should be avoided.

Once again, to prevent $H \rightarrow \infty$ in eq. (6), a minimum ice flux of 10^{-3} m s^{-1} per grid cell area is introduced.

3.2 PISM

105 To estimate the ice thickness using PISM (stable version 1.1.4, Bueler and Brown (2008)), an iterative inverse method as presented in van Pelt et al. (2013) and Koldtoft et al. (2021) is used, with the hybrid model in PISM acting as a forward model. PISM is forced with climate data as described in Sect. 2 and run for 2000 years in hybrid mode, i.e. a coupled SIA and shallow shelf approximation (SSA) (Bueler and Brown, 2008). In the first of a total 10 iterations, PISM is forced with an initial guess of geometry, with surface elevation from ArcticDEM (S_{ref}) and bedrock elevation from Farinotti et al. (2019). The surface
110 elevation is kept constant when initialising every iteration assuming that ArcticDEM represents the ground truth. However, the bedrock topography is adjusted after each iteration (n) by adding bedrock in areas with too low surface elevation (S_n) as compared to S_{ref} , and vice versa removing bedrock in areas with too high surface elevation. Thus, the bedrock topography used in iteration $n + 1$ (B_{n+1}), is given by

$$B_{n+1} = B_n - K(S_n - S_{ref}), \quad (9)$$

115 where B_n is the bedrock used to force PISM in iteration n . K is a relaxation parameter which ensures that the bedrock topography is not overcompensated (van Pelt et al., 2013; Koldtoft et al., 2021). In this study we use a varying relaxation parameter with elevation, to prevent overcompensation in bedrock and ice build-up on the outlet glaciers west of the ice cap (see Fig. 1). K is set to 0 below 500 m and increases linearly to 0.5 at 1000 m above sea level, from where it is kept constant. Thus, under the assumption of steady state, the idea is that the bedrock should come closer to the true bedrock after each
120 iteration. It is important to note that this is not a guarantee.

Since the properties of the bedrock below the ice cap are unknown, a bigger parameter space of till friction angles (ϕ) and enhancement factors (E) are tested using the following possible combinations

$$\phi \in \{10, 20, 30, 40\} \quad (10)$$

$$E \in \{1, 3, 6\}. \quad (11)$$

125 To test convergence and find the most suitable parameters for ϕ and E , the root mean squared error (RMSE) between the following misfit metrics are evaluated:

- Modelled surface elevation and ArcticDEM.

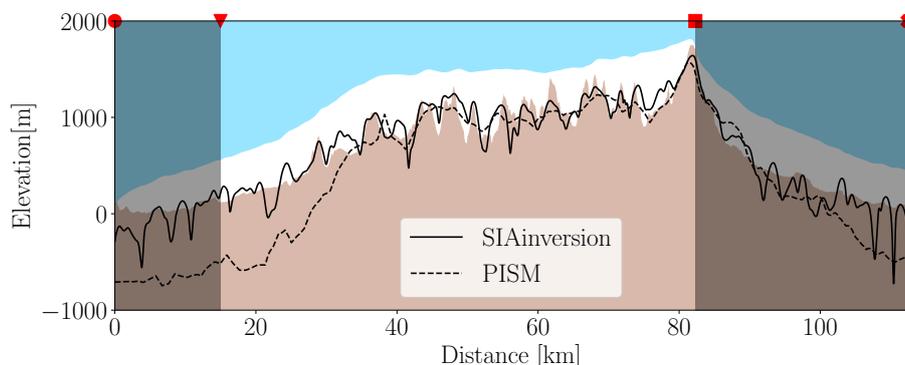


Figure 2. OIB cross section with the SIA inversion and PISM bedrocks, and red distance marks in the top as marked in Fig. 1. The outline of the ice cap is based on surface elevation from ArcticDEM and OIB ice thicknesses.

- Modelled ice thickness and OIB ice thickness (both the entire flight line and the part in between the triangle and the square on Fig. 1).
- 130 – Modelled surface velocity and median surface velocities obtained through feature tracking of Landsat 8 images (Zinck, 2020).

For a further description and analysis the reader is referred to Zinck (2020).

4 Results

After solving the least squares regression of the SIA inversion using the OIB ice thicknesses, the parameters a , b , and k are applied in combination with the surface slope and ice flux over the entire domain to move from one to two dimensions. The modelled ice thickness is also transformed into bedrock elevation by using ArcticDEM. The OIB cross section with corresponding SIA inversion bedrock is shown in Fig. 2, where the surface elevation is from ArcticDEM (blue-white intersection), the ice thickness in white is from OIB, resulting in the bedrock given by subtracting OIB from ArcticDEM in brown. The SIA inversion is only tuned to the non-shaded part of the ice cap, from the red triangle to the red square in Fig. 1 and 2, since it is the main ice cap which is of interest in this study and because the uniform SMB used in the SIA flux calculation is not valid at low elevations. Fig. 3 (left) shows the modelled bedrock topography (bottom) and ice thickness (top) of the entire ice cap. The RMSEs of the ice thickness compared to OIB are 145 m and 126 m by including and excluding the outlet glaciers, respectively. Further, the mean absolute deviation (MAD) of the entire OIB flight line is 109 m.

In the PISM method, 12 different combinations of till friction angles and enhancement factors are tested. Based on the before mentioned convergence metrics described in Sect. 3.2 a till friction angle of 10° and an enhancement factor of 6 is the parameter combination with the best results. The ice thickness after the tenth iteration is used as the main PISM result, from which the bedrock topography is calculated using ArcticDEM. The PISM bedrock of the OIB cross section is shown in Fig.

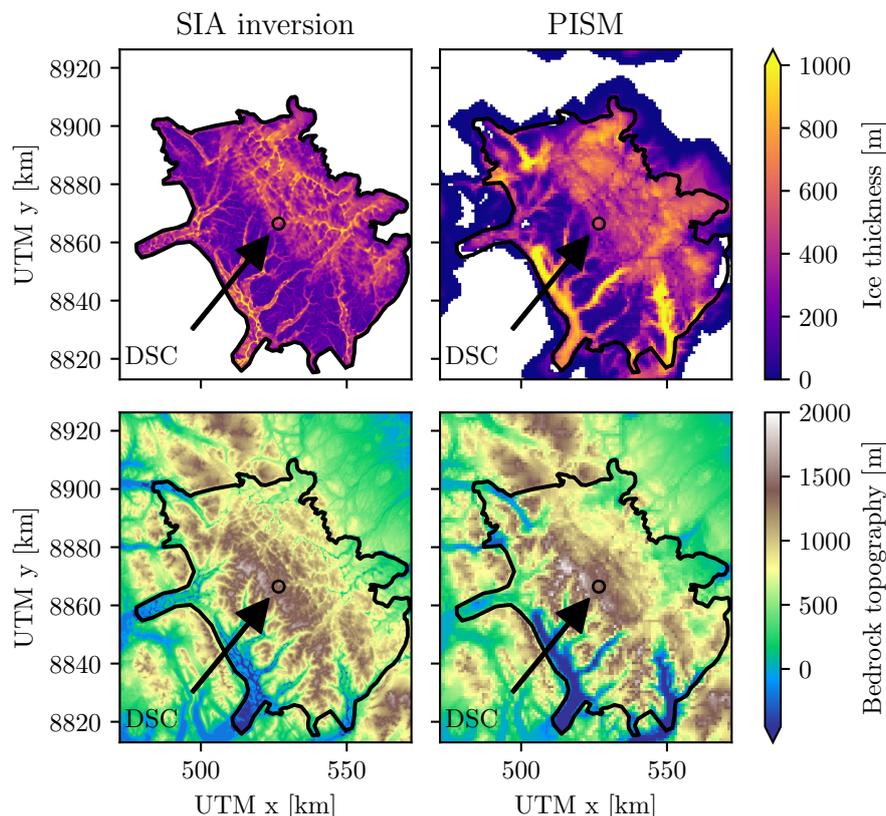


Figure 3. Left: SIA inversion thickness (top) and bedrock topography (bottom). Right: PISM ice thickness (top) and bedrock topography (bottom). The black polygon in all figures marks the present day ice cap margin as used in the SIA inversion. The proposed drill site candidate (DSC) is marked in all figures.

2, from where it can be seen that PISM overestimates the ice thickness at the outlet glaciers, though it is capturing the ice cap well. The right side of Fig. 3 shows the ice thickness (top) and corresponding bedrock topography (bottom) of the entire domain. The PISM ice cap covers a greater area than the present day Müller Ice Cap, since no margin criteria has been used
150 when running PISM. The RMSEs compared to the OIB thicknesses including and excluding the outlet glaciers are 399 m and 329 m, respectively. Including the outlet glaciers the MAD compared to OIB is 302 m.

A drill site candidate (DSC), which has to be further investigated with ground penetrating radar, is proposed based on an ensemble of modelled ice thicknesses, surface melt, and surface velocities (Zinck, 2020). The DSC location is shown in Fig. 3
155 and the corresponding SIA inversion and PISM thicknesses are 579 m and 535 m, respectively. The SIA inversion thickness has been resampled onto the 900 m PISM grid before extracting the ice thickness to allow for a more fair comparison between the methods.



5 Discussion

The SIA inversion offers a fast estimate of ice thicknesses from sparse data. However, a stream like pattern from the ice flux is highly visible in the ice thickness showed in the top left of Fig. 3. Furthermore, sudden peaks in ice thickness is visible both in Fig. 2 and 3, due to either low surface slope or ice flux as a result of the logarithm in eq. (6). One solution to these issues is to apply a smoothing to both the surface slope and the ice flux, as applying a smoothing to only one of them will increase the issue of the other. Such fine scale features are not trustworthy in simple models like the SIA inversion, which is why the SIA inversion result is interpolated onto a coarser grid before it is used in the DSC assessment, thereby reducing the impact of these features.

The key assumption in the SIA is that the ice deforms by simple bed parallel shear. Such an approximation is reasonable on ice sheets where the horizontal extent is much greater than the vertical. Whether this holds on MIC is highly debatable. However, while SIA is the justification for the functional form of eq. (6), the model is eventually empirically tuned to reproduce thickness measurements. If sliding has a significant impact on ice flow the regression will attempt to capture this effect by adjusting the free parameters (a , b , and k). For this reason we caution against a naive interpretation of the parameters in terms of e.g. a SIA flow law exponent (eq. (5)).

A piece of information which is not used in the SIA inversion is the ice velocity. Ice velocities have been found to be useful in ice thickness inversions, since they give a strong constrain on the horizontal ice fluxes. However, our new SIA approach is completely insensitive to multiplicative errors in SMB and Q , why ice velocities are expected to provide limited additional information.

Compared to the SIA inversion, the PISM method is both computationally heavy and relies on much more input data. The two most noticeable features of the PISM result is the overestimation of ice thickness on the outlet glaciers (see Fig. 2) and the greater ice cap extent as seen in Fig. 3. The latter is a result of the calving criteria chosen when running PISM, or rather the choice of not having a calving criteria at the present day margin. Implementing a calving criteria at the present day ice cap margin results in unrealistic high ice velocities in the margin area (Zinck, 2020). This increases the computational time due to the dynamic time step in PISM, while the ice thickness on the main ice cap remains more or less the same as when no margin criteria is applied (Zinck, 2020).

The overestimation of ice thickness on the outlet glaciers is a result of the HIRHAM5 SMB, which is highly positive over the mountains on the southwestern side of the ice cap (Fig. 1). It is a known issue that HIRHAM5 deposits too much precipitation in up-sloping rough terrain (Schmidt et al., 2017), which might be the case over Axel Heiberg Island. Comparing to in-situ SMB measurements from White Glacier (Fig. 1), there might be an indication that HIRHAM5 in general overestimates the SMB on the Müller Ice Cap with too small mass balance gradients at low elevations. It should be noted that Axel Heiberg Island is on the very edge of the HIRHAM5 domain (Mottram et al., 2017), which might have an influence on the SMB. This underscores the advantage of the SIA inverions in-sensitivity to SMB.

The overall thickness and topography patterns are similar between the two models. The SIA inversion has a significantly lower RMSE and MAD, also when the outlet glaciers are excluded. However, the SIA inversion is only trained and validated



against the OIB flight line, and not elsewhere on the ice cap. Nonetheless, it still performs much better than PISM on the outlet glaciers, which were not included in the least squares regression. This suggests that the SIA inversion thickness is also valid outside of the OIB flight line.

195 The drill site candidate suggested here should be taken with caution, as further in-situ measurements of ice thickness, surface melt, and surface elevation are highly recommended. Nonetheless, the DSC is based on an ensemble of thickness estimates, making the site a strong candidate.

6 Conclusions

Two methods of estimating the ice thickness distribution of the Müller Ice Cap were presented. Firstly, a simple inversion of the shallow ice approximation (SIA Inversion), and secondly, an iterative inverse method using PISM as a forward model. The general ice thickness pattern of the ice cap is similar to a large degree in between the two models. However, PISM is not able to catch the outlet glaciers, for which the ice thickness is highly overestimated. Furthermore, the SIA inversion shows the best results in comparison to thickness measurements from Operation IceBridge both including and excluding the outlet glaciers. It is demonstrated that the methods also differ greatly in computational demands and needs of input data, making the SIA inversion, which is light on both, far more favourable. Finally, one of the main advantages of the SIA inversion, besides the computational speed, is how little data the model relies on. This stands in huge contrast to the amount of data that PISM relies on. It also entails that the SIA inversion method shows potential to be applied for global glacier thickness estimates, especially since it performs good on ice caps, which flowline methods often used in such estimates struggle with. This implies that one would have to do regional calibrations of the three tuning parameters for glaciers and ice caps with no ice thickness data available.

Data availability. Modelled ice thicknesses can be downloaded from <https://doi.org/10.5281/zenodo.4290039> and the surface velocities used to evaluate PISM can be downloaded from <https://doi.org/10.5281/zenodo.4290041>

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