Brief communication: Estimating the ice thickness of the Müller Ice Cap ~~using an inversion of the shallow ice approximation~~to aid for a drill site selection

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Abstract. The Müller Ice Cap will soon set the scene for a new drilling project. Therefore, ice thickness estimates are necessary for planning, since thickness measurements of the ice cap are sparse. Here, ~~two~~ three models are presented and compared, i) a simple Semi-Empirical Ice Thickness Model (SEITMo) based on an inversion of the shallow ice approximation ~~(SIA inversion)~~ by the use of a single radar line in combination with the glacier outline, surface slope, and elevation, ~~and~~ ii) an iterative inverse method using the Parallel Ice Sheet Model (PISM)~~. The two methods~~ , and iii) a velocity based inversion of the shallow ice approximation. The velocity based inversion underestimates the ice thickness at the ice cap top, making the model less useful to aid for a drill site selection. Whereas PISM and SEITMo mostly agree about a good drill site candidate. However, the new ~~semi-empirical SIA inversion~~ SEITMo is insensitive to mass balance, computationally fast, and provides ~~better fits~~ equally good fits as PISM.

1 Introduction

The Müller Ice Cap (MIC), located on Axel Heiberg Island in Arctic Canada, is facing a part of the Arctic Ocean, where no full depth ice core has been drilled, thus scientists at University of Manitoba and University of Copenhagen intend to do exactly this. Choosing a good drill site location with stratified ice layers, good time resolution, and age-span is crucial in the dating process needed to interpret the different compounds in the ice and to put it into a climatological sense. However, the MIC remains poorly studied with the exception of a few mass balance (Koerner, 1979; Thomson et al., 2011) and surface velocity studies (van Wychen et al., 2014), leaving the ice thickness poorly constrained. Therefore, knowledge about ice thickness and flow is important ~~. Great ice thickness is of key importance~~ as ice of great thickness and minimal horizontal flow is desirable to increase the ~~possibility~~ probability of reaching ice dating back to the Innuitian ice sheet, referring to the ice sheet in between the Laurentide and Greenland ice sheets during the last glaciation. ~~However, the MIC remains poorly studied with the exception of a few mass balance (Koerner, 1979; Thomson et al., 2011) and surface velocity studies (van Wychen et al., 2014), leaving the ice thickness poorly constrained. This~~ This limited knowledge stands in contrast to one of its neighbouring glaciers, White Glacier, marked in Fig. 1. White Glacier has been studied thoroughly since the late 60s (Müller, 1962; Cogley et al., 1996, 2011) with a strong focus on the mass balance. To ensure a good drill site, ground based radar measurements are necessary. However,

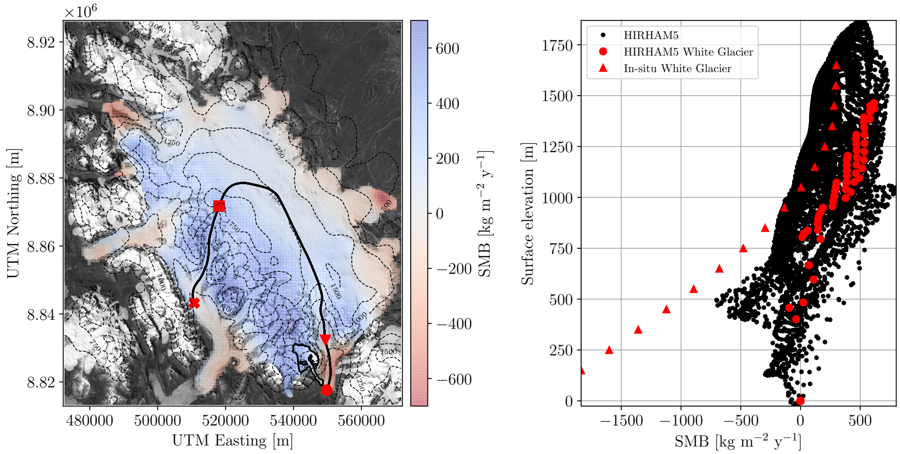

Figure 1. Left: Landsat 8 satellite image of the Müller Ice Cap overlaid with long-term average HIRHAM5 SMB from 1980-2016. Contour lines of surface elevation from ArcticDEM with 250 m increments, and the Operation IceBridge flight line as the solid black line from the red dot to the red cross. The four red symbols have the same location as in Fig. 3, and are present to guide the reader. White Glacier is outlined in black just above the red dot. The coordinates are given in UTM zone 15N. **Right**: Surface mass balance as a function of surface elevation. Red triangles are average in-situ measurements from White Glacier from 1960-91 (Cogley et al., 1996). The black and red dots are long-term HIRHAM5 SMB averages from 1980-2016, with the red dots corresponding to the SMB within the White Glacier polygon.

field work constraints make it impractical and expensive to survey the entire ice cap, and it is therefore necessary to be ~~clever~~ selective when deciding where to conduct ground based radar measurements.

The Ice Thickness Models Intercomparison eXperiment phase 1 and 2~~(~~, ITMIX1 (Farinotti et al., 2017) and ITMIX2 (Farinotti et al., 2021)~~)~~, compare various models of estimating ice thicknesses of glaciers and ice caps from sparse data. In ITMIX1, large differences are found between the models on ice caps in the vicinity of ice divides. Furthermore, it is urged that modellers seek improvements on how to treat these regions to ensure continuity of the subsurface topography around the ice divide. In this study, it is of particular importance that we resolve the area around the ice divide well, as ice divides often qualify as good drill site locations due to limited horizontal ice flow. Furthermore, from ITMIX1 it is not possible to determine what model approaches are to be preferred on ice caps. Thus, to narrow down areas of conducting ground based radar measurements, the ice thickness is modelled using ~~two~~ three different techniques that ~~differs~~ differ greatly in computational demands, input data and model setup. One is a simple ~~inversion of~~ Semi-Empirical Ice Thickness Model (SEITMo) based on the shallow ice approximation (SIA~~inversion~~), only relying on the surface elevation, the ice cap outline and a single Operation IceBridge (OIB) (Paden et al., 2010, updated 2019) flight line with thickness measurement. This is a fast new method that differs from existing SIA approaches due to its semi-empirical nature which makes it less sensitive to mass balance, steady state assumptions and ice flow physics. The second is an iterative inverse method using the Parallel Ice Sheet Model (PISM) as a forward model, which has to be forced with both climate data and an initial guess of ice cap geometry. The third is a velocity based inversion of the SIA as presented in Zorzut et al. (2020), henceforth referred to as Zorzut. This method relies on the surface velocity and slope, alongside an OIB flight line of ice thickness measurements as for SEITMo.

2 Data

Ice thickness measurements taken from the Multichannel Coherent Radar Depth Sounder from ~~Operation IceBridge (OIB)~~ OIB (Paden et al., 2010, updated 2019) are used to validate all models and ~~to perform the SIA inversion~~ in the inversion of the two SIA based methods SEITMo and Zorzut. Only a few flight lines cross parts of the ice cap, ~~why~~ and a single flight line acquired on 30.03.2017 is used as it covers most parts of the ice cap as seen in Fig. 1.

The surface elevation is obtained from the Arctic Digital Elevation Model (ArticDEM, Porter et al. (2018)) and is shown as contours on Fig. 1. To ensure continuity over the entire ice cap, the OIB surface elevations are not used when present, thereby only using the ice thicknesses from OIB.

As climate inputs to initialise PISM, SMB and 2-m temperatures are obtained from the Regional Climate Model HIRHAM5 (HIRHAM5) (Langen et al., 2017; Mottram et al., 2017). Since steady state is assumed in the models presented here, the long-term average of these fields are used. They are calculated from the yearly model results of HIRHAM5 from 1980-2014 and 1980-2016 for the 2-m temperature and the SMB, respectively. The long-term average SMB is shown in Fig. 1, and compared to in-situ measurements from White Glacier (Cogley et al., 1996). Furthermore, a uniform geothermal heat flux of 0.055 $\mathrm{W m^{-2}}$ is used based on Minnick et al. (2018).

The initial guess of ice cap geometry for PISM is obtained from ArcticDEM and the bedrock topography from Farinotti et al. (2019). The Farinotti et al. (2019) bedrock is based on a flow or drainage basin approach, resulting in missing values in-between the basins, and also a misinterpretation of the ice divide. Such misinterpretations are to be expected since the Farinotti et al. (2019) bedrock is ~~the result of a global glacier thickness estimate, and is thereby not tuned to the Müller Ice Cap~~based on a flowline approach and relies on the Randolph Glacier Inventories.

Lastly, to perform the velocity based inversion of the SIA in Zorzut, the 120 m surface velocity mosaic from NASA MEaSUREs ITS_LIVE (Gardner et al., 2018, 2019) is used.

All data are interpolated onto a 100 m grid ~~and~~ for SEITMo and Zorzut, and a 900 m grid for ~~the SIA inversion and PISM, respectively. Thus, gaps~~ PISM. The coarser grid used for PISM is chosen to reduce the computational time. Gaps in the before mentioned bedrock topography are filled in the interpolation process. The initial PISM ice thickness is calculated from the interpolated versions of surface and bedrock elevations.

3 Methods

3.1 ~~Inversion of the shallow ice approximation~~Semi-Empirical Ice Thickness Model (SEITMo)

The SIA is one of the oldest ice flow models, which has proven to show good results on ice sheets and glaciers in areas with no to little basal sliding. Combining this with the simplicity of it and how computationally cheap it is, still makes it a good and often used zero order approximation. Thus, a simple semi-empirical inversion of the SIA to estimate ice thicknesses from sparse data is tested.

From theory the SIA without sliding reads

$$Q = \frac{2A}{n+2}\tau_b^n H^2, \quad (1)$$

where Q is the horizontal ice flux, A the rate factor, n the creep exponent, H the ice thickness and τ_b is the basal shear stress given by

$$\tau_b = \rho g H \alpha. \quad (2)$$

Here ρ is the density of ice and α is the surface slope. Substituting eq. (2) into eq. (1) and introducing the constant c,

$$c = \frac{2A}{n+2}\rho^n g^n, \quad (3)$$

implies that eq. (1) can be reduced to

$$Q = c\alpha^n H^{n+2}. \quad (4)$$

Isolating the ice thickness thus results in

$$H = Q^{\frac{1}{n+2}} \alpha^{\frac{n}{n+2}} c^{\frac{1}{n+2}}. \quad (5)$$

Linearity can be obtained by introducing the three tuning parameters; $k = \log\left(c^{\frac{1}{n+2}}\right)$, $a = \frac{1}{n+2}$, and $b = \frac{n}{n+2}$, resulting in

$$\log H = a \log Q + b \log \alpha + k. \quad (6)$$

Hence, the assumption is that if the ice thickness is known on parts of the ice cap and the surface slope and ice flux are known everywhere on the ice cap, one can perform a least squares regression to obtain the tuning parameters a, b, and k in the areas with known ice thicknesses and apply those in the areas with unknown ice thickness. Thereby, assuming that the areas with known ice thicknesses are representative of the entire ice cap. Note that the empirical regression is insensitive to global multiplicative errors in Q as that will be accounted for by adjustments to k. The approach is semi-empirical because the form of eq. (6) is justified by the SIA theory, but the parameters are tuned empirically. The least squares estimates of a and b are not necessarily consistent with the exponents in eq. (5), which was derived for isothermal ice with no sliding. However, the empirical calibration has freedom to adjust a and b so that it better matches the physical reality.

3.1.1 Surface slope

The surface slope is obtained from ArcticDEM by smoothing the surface elevation using a Gaussian filter with a standard deviation of 250 m, from which the surface slope is calculated. The smoothing is done to prevent surface depressions on the ice cap, thus ensuring that there is a downward slope from all top points of the ice cap all the way to the ice margin. To prevent

$H \to \infty$ in low sloping areas a minimum slope of 0.01 is introduced on the ice cap. This corresponds roughly to 0.6°, and is relatively lower than the 2°-5° minimum slope used in other similar models (Farinotti et al., 2009, 2017). Furthermore, the surface slope is set to be zero outside of the present day ice cap margin.

It should be acknowledged that an optimal surface elevation smoothing alongside an optimal minimum slope can be found by searching a bigger parameter space and using a minimum misfit approach. However, this would increase the computational costs of the model, which is why the smoothing has been chosen with care instead. The chosen smoothing also implies that the minimum slope threshold is only used in a very few areas.

3.1.2 Ice flux

The ice flux of the MIC, and most ice caps and glaciers in general, is not known, which is why the balance flux is calculated. This is done based on the top model approach described in Quinn et al. (1991). The top model approach starts at the very top of the ice cap, from where the ice flows downslope into all eight neighbouring cells depending on the surface slope. From there the analysis moves on to the cell of second highest elevation, until the cell of lowest elevation is reached. From every cell all eight neighbouring cells are assigned a weight, w, such that

$$w(\alpha) = \begin{cases} \alpha^3 & \alpha > 0 \\ 0 & \alpha \leq 0 \end{cases}. \qquad (7)$$

The ice then flows from cell$_{i,j}$ to cell$_{i,j+1}$ by

$$Q_{i,j+1} = Q_{i,j+1} + \frac{w_{i,j+1}}{\sum\limits_{i,j \in nbh} w_{i,j}} Q_{i,j}, \qquad (8)$$

where nbh is the neighbourhood of cell$_{i,j}$. At the very top of an ice cap the ice flux is equal to the SMB, and all areas further down the ice both have a flux input from the SMB at that given point, and from the ice flowing in from higher elevations. In this model we use a uniform SMB of ones in every ice covered cell, as it has been shown that the SMB only has little influence on the modelled ice thickness (Zinck, 2020). This is due to the fact that any multiplicative errors in SMB can be captured in k in eq. (6). Do notice, that this might not hold in areas with negative SMB. According to in-situ measurements from White Glacier and HIRHAM5 SMB (Fig. 1), the areas with negative SMB are restricted to the outlet glaciers and the glacier tongues at the margin, which are anyway outside the area of interest as surface melt at the ice core site should be avoided.

Once again, to prevent $H \to \infty$ in eq. (6), a minimum ice flux of $10^{-3}\ \mathrm{m\ s^{-1}}$ per grid cell area is introduced.

3.2 PISM

To estimate the ice thickness using PISM (stable version 1.1.4, Bueler and Brown (2008)), an iterative inverse method as presented in van Pelt et al. (2013) and Koldtoft et al. (2021) is used, with the hybrid model in PISM acting as a forward model. PISM is forced with climate data as described in Sect. 2 and run for 2000 years in hybrid mode, i.e. a coupled SIA and shallow shelf approximation (SSA) (Bueler and Brown, 2008). This means that PISM is forced with a constant climate thereby assuming steady state. This is chosen due to the limited knowledge about the past climate on MIC. However, it should be noted that the PISM inversion is known to work best when a variable climate forcing is applied.

In the first of a total 10 iterations, PISM is forced with an initial guess of geometry, with surface elevation from ArcticDEM (S_{ref}) and bedrock elevation from Farinotti et al. (2019). The surface elevation is kept constant when initialising every iteration assuming that ArcticDEM represents the ground truth. However, the bedrock topography is adjusted after each iteration (n) by adding bedrock in areas with too low surface elevation (S_n) as compared to S_{ref}, and vice versa removing bedrock in areas with too high surface elevation. Thus, the bedrock topography used in iteration $n+1$ (B_{n+1}), is given by

$$B_{n+1} = B_n - K(S_n - S_{ref}), \tag{9}$$

where B_n is the bedrock used to force PISM in iteration n. K is a relaxation parameter which ensures that the bedrock topography is not overcompensated (van Pelt et al., 2013; Koldtoft et al., 2021). In this study we use a varying relaxation parameter with elevation, to prevent overcompensation in bedrock and ice build-up on the outlet glaciers west of the ice cap (see Fig. 1). K is set to 0 below 500 m and increases linearly to 0.5 at 1000 m above sea level, from where it is kept constant. Thus, under the assumption of steady state, the idea is that the bedrock should come closer to the true bedrock after each iteration. It is important to note that this is not a guarantee.

Since the properties of the bedrock below the ice cap are unknown, a bigger parameter space of till friction angles (ϕ) and enhancement factors (E) are tested using the following possible combinations of parameter values

$$\phi \in \{10, 20, 30, 40\} \tag{10}$$

$$E \in \{1, 3, 6\}. \tag{11}$$

To test convergence and find the most suitable parameters for ϕ and E, the root mean squared error (RMSE) between the following misfit metrics are evaluated:

- Modelled surface elevation and ArcticDEM.
- Modelled ice thickness and OIB ice thickness (both the entire flight line and the part in between the triangle and the square on Fig. 1).
- Modelled surface velocity and median surface velocities obtained through feature tracking of Landsat 8 images (Zinck, 2020).

For a further description and analysis the reader is referred to Zinck (2020).

3.3 Velocity based inversion of the SIA

The third method used to estimate the ice thickness is a velocity based inversion of the SIA as presented in Zorzut et al. (2020), henceforth referred to as Zorzut. The SIA including sliding is formulated as

$$H = \left(\frac{(U_s - U_b)(n+1)}{2A(fg\rho \sin\alpha)^n} \right)^{1/(n+1)}, \tag{12}$$

where U_s is the surface velocity, $U_b = f_a U_s$ is the basal velocity which is assumed to be proportional to the surface velocity. In this model the rate factor, A, is set to be $2.4 \cdot 10^{-25}$ $\mathrm{s^{-1} Pa^{-3}}$, $g = 9.8$ $\mathrm{m\ s^{-2}}$, $f = 0.8$ and $n = 3$. Eq. (12) can thus be formulated as a minimisation problem by keeping f_a as a free tuning parameter. To perform the minimisation surface velocities as described in Sect. 2, surface slopes calculated from the smoothed ArcticDEM as described in Sect. 3.1.1, and thickness measurements of the same part of the OIB flight line as for the SEITMo are used. In the minimisation process all values of f_a in between 0.001 and 0.999 in increments of 0.001 are tested. The f_a resulting in the lowest root mean squared error (RMSE) compared to the OIB ice thicknesses in between the red triangle and square (see Fig. 1), is applied to the entire ice cap.

4 Results

After solving the least squares regression of the SIA inversion SEITMo using the OIB ice thicknesses, the parameters a, b, and k are applied in combination with the surface slope and ice flux over the entire domain to move from one to two dimensions. The modelled ice thickness is also transformed into bedrock elevation by using ArcticDEM. The OIB cross section with corresponding SIA inversion SEITMo bedrock is shown in Fig. 2, where the surface elevation is from ArcticDEM (blue-white intersection), the ice thickness in white is from OIB, resulting in the bedrock given by subtracting OIB from ArcticDEM in brown. The SIA inversion SEITMo is only tuned to the non-shaded part of the ice cap, from the red triangle to the red square in Fig. 1 and 2, since it is the main ice cap which is of interest in this study and because the uniform SMB used in the SIA flux calculation is not valid at low elevations. Fig. 3 (left) shows the modelled bedrock topography (bottom) and ice thickness (top) of the entire ice cap. The RMSEs of the ice thickness compared to OIB are 145 and 126 by including and excluding the outlet glaciers, respectively. Further, the mean absolute deviation (MAD) of the entire OIB flight line is 109 .

In the PISM method, 12 different combinations of till friction angles and enhancement factors are tested. Based on the before mentioned convergence metrics described in Sect. 3.2 a till friction angle of 10° and an enhancement factor of 6 is the parameter combination with the best results. The ice thickness after the tenth iteration is used as the main PISM result, from which the bedrock topography is calculated using ArcticDEM. The PISM bedrock of the OIB cross section is shown in Fig. 2, from where it can be seen that PISM overestimates the ice thickness at the outlet glaciers, though it is capturing the ice cap well. The right side This overestimation is due to the fact that the bedrock is not adjusted on below 500 m due to the relaxation parameter K. Hence, the model is not expected to do well in this area, but only on the main ice cap. The middle panel of Fig. 3 shows the ice thickness (top) and corresponding bedrock topography (bottom) of the entire domain. The PISM ice cap covers a greater area than the present day Müller Ice Cap, since no margin criteria has been used when running PISM. The RMSEs

After performing the minimisation problem of Zorzut f_a is found to be 0.884 which is applied to the entire ice cap. Like for SEITMo and PISM the modelled ice thicknesses are turned into bedrock elevations by subtracting them from ArcticDEM. The Zorzut bedrock topography of the OIB flight line can be seen in Fig. 2. Likewise, the ice thickness and bedrock topography of the entire domain can be found in the right panel of Fig. 3. In the zoom-in of Fig. 2 it can be seen that Zorzut underestimates

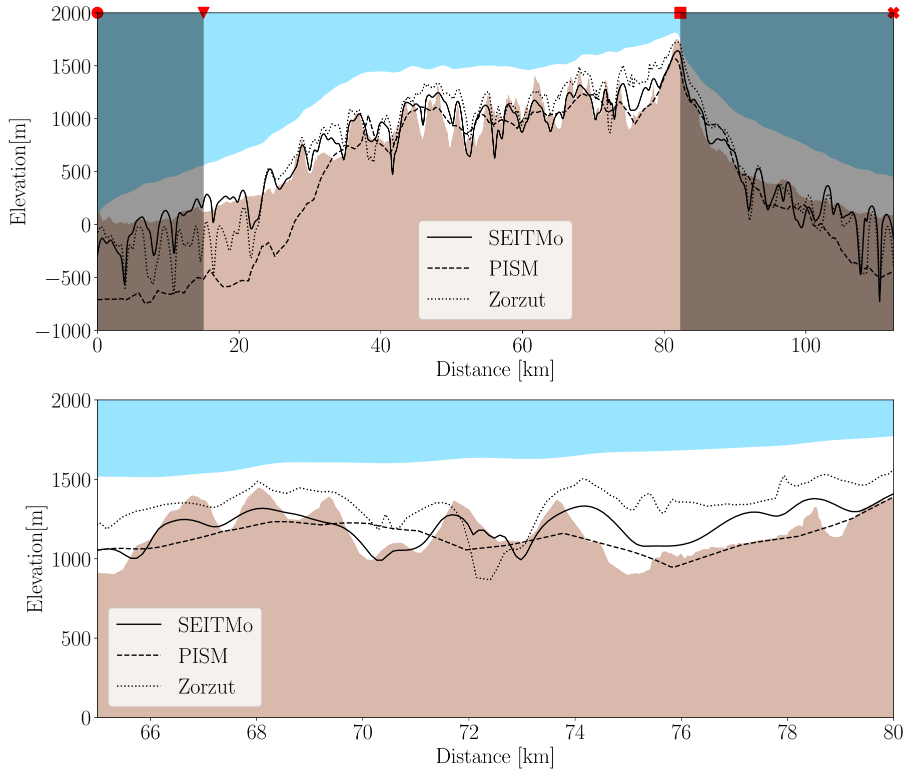

Figure 2. OIB cross section with the ~~SIA inversion and~~ SEITMo, PISM, and Zorzut bedrocks, and red distance marks in the top as marked in Fig. 1. The outline of the ice cap is based on surface elevation from ArcticDEM and OIB ice thicknesses.

the ice thickness in the area between 74-80 km. This is because of the relatively low ice velocities in the vicinity of the ice divide.

The RMSEs of the three models compared to the OIB thicknesses ~~including and excluding the outlet glaciers are 399~~ between 40-80 km in Fig. 2 are 136 m~~and 329~~ , 134 m, and 204 m for SEITMo, PISM, and Zorzut, respectively. Likewise the mean absolute deviations (MADs) of the same part of the flight line are 112 m, ~~respectively. Including the outlet glaciers the MAD compared to OIB is 302~~ 108 m, and 165 m for SEITMo, PISM, and Zorzut, respectively. Thus, PISM and SEITMo perform equally good in the area of interest whereas Zorzut significantly worse which is most likely due to its constraints to the surface velocity.

A drill site candidate (DSC), which has to be further investigated with ground penetrating radar, is proposed based on an ensemble of modelled ice thicknesses, surface melt, and surface velocities (Zinck, 2020). The DSC location is shown in Fig. 3 and the corresponding ~~SIA inversion~~ SEITMo and PISM thicknesses are 579 m and 535 m, respectively. The ~~SIA inversion~~ SEITMo thickness has been resampled onto the 900 m PISM grid before extracting the ice thickness to allow for a more fair comparison between the methods. It should be noted that the Zorzut thickness has not been included in the DSC selection. It is

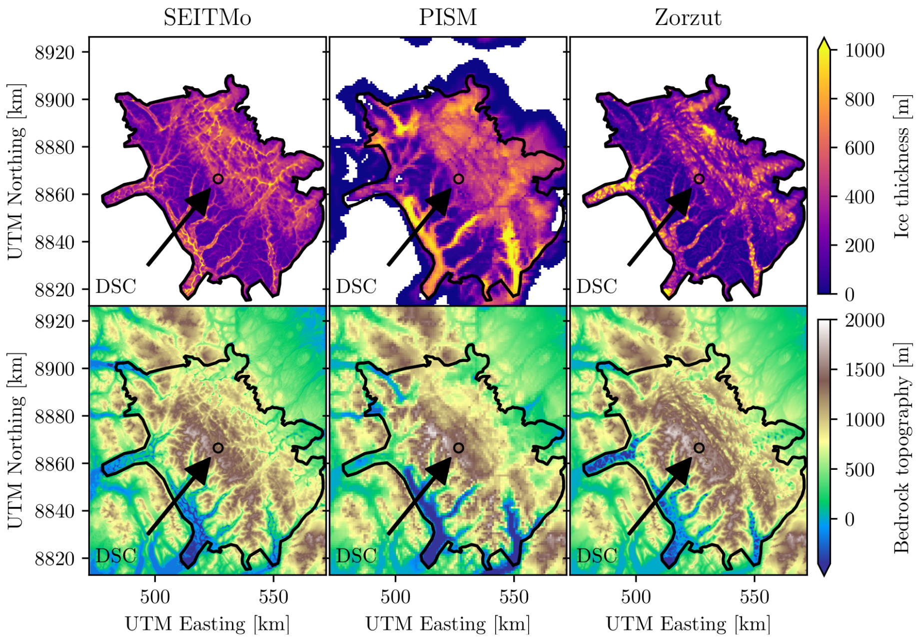

Figure 3. Left: ~~SIA inversion~~ SEITMo thickness (top) and bedrock topography (bottom). ~~**Right:**~~ **Middle:** PISM ice thickness (top) and bedrock topography (bottom). **Right:** Zorzut ice thickness (top) and bedrock topography (bottom). The black polygon in all figures marks the present day ice cap margin as used in ~~the SIA inversion~~SEITMo and Zorzut. The proposed drill site candidate (DSC) is marked in all figures. Coordinates are given in UTM zone 15N.

however noted that the 560 m thickness at the DSC estimated using the Zorzut approach falls between the PISM and SEITMo estimates.

5 Discussion

The ~~SIA inversion~~ SEITMo offers a fast estimate of ice thicknesses from sparse data. However, a stream like pattern from the ice flux is highly visible in the ice thickness showed in the top left of Fig. 3. Furthermore, sudden peaks in ice thickness is visible both in Fig. 2 and 3, due to either low surface slope or ice flux as a result of the logarithm in eq. (6). One solution to these issues is to apply a smoothing to both the surface slope and the ice flux, as applying a smoothing to only one of them will increase the issue of the other. Such fine scale features are not trustworthy in simple models like the ~~SIA inversion~~SEITMo, which is why the ~~SIA inversion~~ SEITMo result is interpolated onto a coarser grid before it is used in the DSC assessment, thereby reducing the impact of these features.

The key assumption in the SIA is that the ice deforms by simple bed parallel shear. Such an approximation is reasonable on ice sheets where the horizontal extent is much greater than the vertical. Whether this holds on MIC is highly debatable. However, while SIA is the justification for the functional form of eq. (6), the model is eventually empirically tuned to reproduce thickness measurements. If sliding has a significant impact on ice flow the regression will attempt to capture this effect by adjusting the free parameters (a, b, and k). For this reason we caution against a naive interpretation of the parameters in terms of e.g. a SIA flow law exponent (eq. (5)).

~~A piece of information which is not used in the SIA inversion is the ice velocity. Ice velocities have been found to be useful in ice thickness inversions, since they give a strong constrain on the horizontal ice fluxes. However, our new SIA approach is completely insensitive to multiplicative errors in SMB and Q, why ice velocities are expected to provide limited additional information.~~

Compared to the ~~SIA inversion, the PISM method~~ SEITMo, PISM is both computationally heavy and relies on much more input data. ~~The two most noticeable features of the PISM result is the overestimation of ice thickness on the outlet glaciers (see Fig. 2) and the greater ice cap extent as seen~~ Focusing on the main ice cap in Fig. ~~3. The latter is a result of the calving criteria chosen when running PISM, or rather the choice of not having a calving criteria at the present day margin. Implementing a calving criteria at the present day ice cap margin results in unrealistic high ice velocities in the margin area (Zinck, 2020). This increases the computational time due to the dynamic time step in PISM, while the ice thickness on the main ice cap remains more or less the same as when no margin criteria is applied (Zinck, 2020).~~

~~The overestimation of ice thickness on the outlet glaciers is a result of the HIRHAM5 SMB, which is highly positive over the mountains on the southwestern side of the ice cap (~~2, PISM performs equally well as SEITMo. The outlet glaciers are of course not captured well due to the chosen relaxation parameter which has no effect below 500 m. If the desire was to capture the entire domain well, one could keep the relaxation parameter as a constant and instead apply a tuning to the SMB. From the right panel of Fig. 1 ~~). It~~ there is an indication that HIRHAM5 in general overestimates the SMB. Further, it is a known issue that HIRHAM5 deposits too much precipitation in up-sloping rough terrain (Schmidt et al., 2017), which might be the case over Axel Heiberg Island. ~~Comparing to in-situ SMB measurements from White Glacier (Fig. 1), there might be an indication that HIRHAM5 in general overestimates the SMB on the Müller Ice Cap with too small mass balance gradients at low elevations.~~ It should be noted that Axel Heiberg Island is on the very edge of the HIRHAM5 domain (Mottram et al., 2017), which might have an influence on the SMB. ~~This~~ However, the computational cost of the iterative PISM inversion severely limits the size of the model parameter space it is practical to search. Introducing additional parameters — e.g. to scale and offset SMB — would increase the dimensions of the parameter space hypercube. This was not feasible in the time allotted for this project. This is considered to be a major drawback of the iterative PISM approach and underscores the advantage of the ~~SIA inverions~~ SEITMo's in-sensitivity to SMB.

Zorzut is equally fast as SEITMo but it performs the worst out of all three models when compared to the OIB thicknesses. Of special interest is the area between 74-80 km in Fig. 2 from where it can be seen that Zorzut highly underestimates the ice thickness. This is because of the low velocities in the vicinity of the ice divide and the nature of the SIA which is not valid at ice divides. The remotely sensed velocity estimates will tend to have a poor signal to noise ratio when ice velocities are small. Therefore, it is expected and of no surprise that the Zorzut approach will have worse performance near the ice divide, as is indeed observed (see Fig. 2). This also makes the model considerably less ideal in the search for a possible drill site candidate where minimal horizontal flow is desired. Furthermore, the uncertainty on surface velocity products often exceeds the actual magnitude of the velocity in these slow flowing areas.

The overall thickness and topography patterns are similar between the two models. The SIA inversion has a significantly lower all three models. SEITMo and PISM have comparable RMSEs and MADs, whereas Zorzut has a relatively higher both RMSE and MAD, also when the outlet glaciers are excluded. However, the SIA inversion is only trained and validated against the . Even though SEITMo and Zorzut are only tuned to the part of the OIB flight line , and not elsewhere on the ice cap. Nonetheless, it still performs much better than PISM on the outlet glaciers, which were not included in the least squares regressionin between the red triangle and square (see Fig. 1 and 2) they still seem to perform well on the rest of the flight line. This suggests that the SIA inversion thickness is both models are also valid outside of the OIB flight line. The good fits of the SEITMo in combination with its low computational costs and limited data requirements makes it an excellent candidate for fast ice thickness estimates. This is especially the case in areas with limited knowledge about past and present SMB.

The drill site candidate suggested here should be taken with caution, as further in-situ measurements of ice thickness, surface melt, and surface elevation are highly recommended. Nonetheless, the DSC is based on an ensemble of thickness estimates, making the site a strong candidate.

6 Conclusions

Two Three methods of estimating the ice thickness distribution of the Müller Ice Cap were presented. Firstly, a simple inversion of the shallow ice approximation (SIA Inversion) , and secondly, Semi-Empirical Ice Thickness Model (SEITMo) based on the SIA. Secondly, an iterative inverse method using PISM as a forward model. And thirdly, a velocity based inversion of the SIA (Zorzut). The general ice thickness pattern of the ice cap is similar to a large degree in between the two all three models. However, PISM is not able to catch the outlet glaciers, for which the ice thickness is highly overestimated. Furthermore, the SIA inversion shows the best results in comparison Zorzut highly underestimates the ice thickness on the top of the ice cap in the vicinity of the ice divide, which is unfortunate as this is also the key area of interest in terms of a possible drill site. Further, SEITMo and PISM show equally good results when compared to thickness measurements from Operation IceBridgeboth including and excluding the outlet glaciers. It is demonstrated that the methods also differ greatly in computational demands and needs of input data, making the SIA inversionSEITMo, which is light on both, far more favourable. Finally, one of the main advantages of the SIA inversionSEITMo, besides the computational speed, is how little data the model relies on. This stands in huge contrast to the amount of data that PISM relies on. It also entails that the SIA inversion SEITMo method shows potential to be applied for global glacier thickness estimates, especially since it performs good on ice caps, which flowline methods often used in such estimates struggle with. This implies that one would have to do regional calibrations of the three tuning parameters for glaciers and ice caps with no ice thickness data available.

Data availability. Modelled PISM and SEITMo ice thicknesses can be downloaded from https://doi.org/10.5281/zenodo.4290039 and the surface velocities used to evaluate PISM can be downloaded from https://doi.org/10.5281/zenodo.4290041

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References

Bueler, E. and Brown, J.: Shallow shelf approximation as a “sliding law” in a thermomechanically coupled ice sheet model, Journal of Geophysical Research, https://doi.org/10.1029/2008JF001179, 2008.

Cogley, J., Adams, W. P., Ecclestone, M. A., Jung-Rothenhäusler, F., and Ommanney, C. S. L.: Mass balance of White Glacier, Axel Heiberg Island, N.W.T., Canada, 1960–91, Journal of Glaciology, 42, 548–563, https://doi.org/10.3189/S0022143000003531, 1996.

Cogley, J. G., Adams, W. P., and Ecclestone, M. A.: Half a Century of Measurements of Glaciers on Axel Heiberg Island, Nunavut, Canada, ARCTIC, 64, 371–375, https://doi.org/10.14430/arctic4127, 2011.

Farinotti, D., Huss, M., Bauder, A., Funk, M., and Truffer, M.: A method to estimate ice volume and ice thickness distribution of alpine glaciers, Journal of Glaciology, 55, 422–430, https://doi.org/10.3189/002214309788816759, 2009.

Farinotti, D., Brinkerhoff, D., Clarke, G., Fürst, J., Frey, H., Gantayat, P., Gillet-Chaulet, F., Girard, C., Huss, M., Leclercq, P., Linsbauer, A., Machguth, H., Martin, C., Maussion, F., Morlighem, M., Mosbeux, C., Pandit, A., Portmann, A., Rabatel, A., and Andreassen, L. M.: How accurate are estimates of glacier ice thickness? Results from ITMIX, the Ice Thickness Models Intercomparison eXperiment, The Cryosphere, 11, 949–970, https://doi.org/10.5194/tc-11-949-2017, 2017.

Farinotti, D., Huss, M., Fürst, J., Landmann, J., Machguth, H., Maussion, F., and Pandit, A.: A consensus estimate for the ice thickness distribution of all glaciers on Earth, Nature Geoscience, 12, https://doi.org/10.1038/s41561-019-0300-3, 2019.

Farinotti, D., Brinkerhoff, D. J., Fürst, J. J., Gantayat, P., Gillet-Chaulet, F., Huss, M., Leclercq, P. W., Maurer, H., Morlighem, M., Pandit, A., Rabatel, A., Ramsankaran, R., Reerink, T. J., Robo, E., Rouges, E., Tamre, E., van Pelt, W. J. J., Werder, M. A., Azam, M. F., Li, H., and Andreassen, L. M.: Results from the Ice Thickness Models Intercomparison eXperiment Phase 2 (ITMIX2), Frontiers in Earth Science, 8, 484, https://doi.org/10.3389/feart.2020.571923, 2021.

Gardner, A. S., Moholdt, G., Scambos, T., Fahnstock, M., Ligtenberg, S., van den Broeke, M., and Nilsson, J.: Increased West Antarctic and unchanged East Antarctic ice discharge over the last 7 years, The Cryosphere, 12, 521–547, https://doi.org/10.5194/tc-12-521-2018, 2018.

Gardner, A. S., Fahnestock, M. A., and Scambos, T. A.: ITS_LIVE Regional Glacier and Ice Sheet Surface Velocities [Data aqcuired on 1 November 2021], https://doi.org/10.5067/6II6VW8LLWJ7, 2019.

Koerner, R. M.: Accumulation, Ablation, and Oxygen Isotope Variations on the Queen Elizabeth Islands Ice Caps, Canada, Journal of Glaciology, 22, 25–41, https://doi.org/10.3189/S0022143000014039, 1979.

Koldtoft, I., Grinsted, A., Vinther, B. M., and Hvidberg, C. S.: Ice thickness and volume of the Renland Ice Cap, East Greenland, Journal of Glaciology, p. 1–13, https://doi.org/10.1017/jog.2021.11, 2021.

Langen, P. L., Fausto, R. S., Vandecrux, B., Mottram, R. H., and Box, J. E.: Liquid Water Flow and Retention on the Greenland Ice Sheet in the Regional Climate Model HIRHAM5: Local and Large-Scale Impacts, Frontiers in Earth Science, 4, 110, https://doi.org/10.3389/feart.2016.00110, 2017.

Minnick, M., Shewfelt, D., Hickson, C., Majorowicz, J., and Rowe, T.: Nunavut geothermal feasibility study, Topical report RSI-2828, Quilliq Energy Corporation, 2018.

Mottram, R., Boberg, F., Langen, P., Yang, S., Rodehacke, C., Christensen, J., and Madsen, M.: Surface mass balance of the Greenland ice sheet in the regional climate model HIRHAM5: Present state and future prospects, Low Temperature Science. Series A. Physical Science, 75, 105–115, https://doi.org/10.14943/lowtemsci.75.105, 2017.

Müller, F.: Zonation in the Accumulation Area of the Glaciers of Axel Heiberg Island, N.W.T., Canada, Journal of Glaciology, 4, 302–311, https://doi.org/10.3189/S0022143000027623, 1962.

Paden, J., Li, J., Leuschen, C., Rodriguez-Morales, F., and Hale, R.: IceBridge MCoRDS L2 Ice Thickness, Version 1, IRMCR2 20170330 02, https://doi.org/https://doi.org/10.5067/GDQ0CUCVTE2Q, 2010, updated 2019.

Porter, C., Morin, P., Howat, I., Noh, M.-J., Bates, B., Peterman, K., Keesey, S., Schlenk, M., Gardiner, J., Tomko, K., Willis, M., Kelleher, C., Cloutier, M., Husby, E., Foga, S., Nakamura, H., Platson, M., Wethington, Michael, J., Williamson, C., Bauer, G., Enos, J., Arnold, G., Kramer, W., Becker, P., Doshi, A., D'Souza, C., Cummens, P., Laurier, F., and Bojesen, M.: ArcticDEM, https://doi.org/10.7910/DVN/OHHUKH, 2018.

Quinn, P., Beven, K., Chevallier, P., and Planchon, O.: The prediction of hillslope flow paths for distributed hydrological modelling using digital terrain models, Hydrological Processes, 5, 59–79, https://doi.org/https://doi.org/10.1002/hyp.3360050106, 1991.

Schmidt, L. S., Adalgeirsdóttir, G., Gudmundsson, S., Langen, P. L., Pálsson, F., Mottram, R., Gascoin, S., and Björnsson, H.: The importance of accurate glacier albedo for estimates of surface mass balance on Vatnajökull: evaluating the surface energy budget in a regional climate model with automatic weather station observations, The Cryosphere, 11, 1665–1684, https://doi.org/10.5194/tc-11-1665-2017, 2017.

Thomson, L., Osinski, G., and Ommanney, C.: Glacier change on Axel Heiberg Island, Nunavut, Canada, Journal of Glaciology, 57, 1079–1086, https://doi.org/10.3189/002214311798843287, 2011.

van Pelt, W., Oerlemans, J., Reijmer, C., Pettersson, R., Pohjola, V., Isaksson, E., and Divine, D.: An iterative inverse method to estimate basal topography and initialize ice flow models, The Cryosphere, 7, https://doi.org/10.5194/tc-7-987-2013, 2013.

van Wychen, W., Burgess, D., Gray, L., Copland, L., Sharp, M., Dowdeswell, J., and Benham, T.: Glacier velocities and dynamic ice discharge from the Queen Elizabeth Islands, Nunavut, Canada, Geophysical Research Letters, 41, 484–490, https://doi.org/10.1002/2013GL058558, 2014.

Zinck, A.-S. P.: Surface velocity and ice thickness of the Müller ice cap, Axel Heiberg Island, M.S. thesis, Niels Bohr Institute, University of Copenhagen, Denmark, https://doi.org/10.31237/osf.io/6qth3, 2020.

Zorzut, V., Ruiz, L., Rivera, A., Pitte, P., Villalba, R., and Medrzycka, D.: Slope estimation influences on ice thickness inversion models: a case study for Monte Tronador glaciers, North Patagonian Andes, Journal of Glaciology, 66, 996–1005, https://doi.org/10.1017/jog.2020.64, 2020.