Supplement to: Inverting ice surface elevation and velocity for bed topography beneath Thwaites Glacier

Helen Ockenden¹, Robert Bingham¹, Andrew Curtis¹, Daniel Goldberg¹, and Antonios Giannopoulos¹ ¹School of Geosciences, University of Edinburgh, Drummond St, Edinburgh EH8 9XP **Correspondence:** Helen Ockenden (h.ockenden@sms.ed.ac.uk)

S1 Non-dimensionalisation

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The form of these transfer functions can be simplified by considering them in a non-dimensional form. For this purpose the same scalings as used in Gudmundsson (2003) and Gudmundsson (2008) are employed.

All spatial scales are in units of mean ice thickness (\bar{h}), and stress components in units of driving stress (τ_d).

The non-dimensional form of the equations is obtained using the substitutions $\bar{c} \mapsto \bar{C}$, $\eta \mapsto 1/2$, $\bar{h} \mapsto 1$, $\bar{u} \mapsto \bar{C}$, $k \mapsto k$, $l \mapsto l$, $\gamma \mapsto (m\bar{C})^{-1}$, and $\tau_d = \rho g \bar{h} \sin \alpha \mapsto 1$.

Using these substitution leads to the non-dimensionalised transfer functions, shown in Figure S1:

$$T_{SB}(k,l) = \frac{k\left(1 + m(2j^{2}\bar{C} + 1)\right)}{k + m\left(k + 2kj^{2}\bar{C} + ij^{2}\cot\alpha\right)}$$
(13 non-dimensionalised)

$$T_{UB}(k,l) = \frac{-i\cot\alpha\left(l^{2}m - k^{2}\left(1 + 0.5j^{2}m\bar{C}\right)\right)}{\left(k + m\left(k + 2kj^{2}\bar{C} + ij^{2}\cot\alpha\right)\right)\left((m\bar{C})^{-1} + 0.5j^{2}\right)}$$
(14 non-dimensionalised)
10 $T_{VB}(k,l) = \frac{ikl\left(1 + 0.5j^{2}mC + m\right)\cot\alpha}{\left(k + m\left(k + 2kj^{2}\bar{C} + ij^{2}\cot\alpha\right)\right)\left((m\bar{C})^{-1} + 0.5j^{2}\right)}$ (15 non-dimensionalised)
 $T_{SC}(k,l) = \frac{k}{k + m\left(k + 2kj^{2}\bar{C} + ij^{2}\cot\alpha\right)}$ (21 non-dimensionalised)
 $T_{UC}(k,l) = \frac{\bar{C}\left(3kl^{2}m\bar{C} + 2k + kj^{2}m\bar{C} + 2il^{2}\cot\alpha\right)}{\left(k + m\left(k + 2kj^{2}\bar{C} + ij^{2}\cot\alpha\right)\right)\left(2 + j^{2}m\bar{C}\right)}$ (22 non-dimensionalised)
 $T_{VC}(k,l) = \frac{-klm\bar{C}\left(2i\cot\alpha + 3k\bar{C}\right)}{\left(k + m\left(k + 2kj^{2}\bar{C} + ij^{2}\cot\alpha\right)\right)\left(2 + j^{2}m\bar{C}\right)}$ (23 non-dimensionalised)



Non-dimensionalised shallow-ice-stream transfer functions

Figure S1. Panels (a), (b) and (c) show the non-dimensional shallow-ice-stream transfer functions T_{SB} , T_{UB} and T_{VB} between nondimensional bed topography B and surface topography S; along flow velocity U; and across flow velocity V, respectively. Panels (d), (e) and (f) show the non-dimensional shallow-ice-stream transfer functions T_{SC} , T_{UC} and T_{VC} between non-dimensional bed slipperiness Cand surface topography S; along flow velocity U; and across flow velocity V, respectively. All transfer functions are calculated for mean slipperiness $\overline{C} = 100$, sliding law constant m = 1, and mean surface slope $\alpha = 0.002$. θ is the angle of basal variability to the direction of flow, such that $\theta = 0$ for landforms aligned with the ice flow, and $\theta = 90$ for landforms perpendicular to the ice flow. λ is the wavelength of variability and is non-dimensionalised in terms of \overline{h} , the mean ice thickness, such that $\log_{10}(\lambda/\overline{h}) = 1$ means that $\lambda = 10\overline{h}$. $T_{sb} = 0.1$ means that variability in the surface topography will have one tenth of the amplitude of variability in the bed topography at that angle and wavelength. Similar figures can be seen in Gudmundsson (2008) with $\overline{C} = 99$.

S2 Additional airborne radar flight lines

15 There are 9 airborne radar flight lines from the 2019/2020 field season which cross the region of Thwaites Glacier where the inversion procedure was carried out. 2 of these are presented in the main body of the text, and the remaining 7 are presented here.



Figure S2. a) to g) Comparative plots of inverted bed topography with mean non-dimensional slipperiness $\overline{C} = 100$ with along flow radarsounded bed topography. Bed topography is given as unfiltered bed picks from the 2019/20 airborne surveys; a version of he same filtered to 2 km wavelengths to be more representative of the detail we might expect to image in our inversion; and the bed profile extracted from BedMachine Antarctica (Morlighem et al., 2020). The envelope around the inverted bed topography shows plus or minus one standard deviation. The correlation coefficients (r) and slopes given are the results of a linear regression between the inverted bed or the BedMachine Antarctica bed and the filtered radar bed. h) to n) Profile locations within the inverted grid.

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