Review of Inverting ice surface elevation and velocity for bed topography andslipperiness beneath Thwaites Glacier by Ockenden et al.

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1 Summary

In "Inverting ice surface elevation and velocity for bed topography and slipperiness beneath Thwaites Glacier", Ockenden and co-authors present the application of a transfer function method for inferring basal topography and slipperiness from surface elevations to Thwaites Glacier in Antarctica. The authors claim that the method effectively captures the spatial pattern of variability in ice sheet topography as compared to radar flightlines. However, I am quite skeptical that the method is not being unduly influenced by the mean ice thickness that is being derived from existing thickness products. I am also skeptical of the utility of the product itself, given that it does not closely match observations, the mismatch is systematically biased, and the error estimate that accompanies it does not accurately reflect this mismatch. While I believe that the method is potentially interesting, for this manuscript to be suitable for publication, the authors need to 1) substantially increase the specificity of the description of their method, and 2) take a much more self-critical viewpoint of these results, in particular from the perspective of a reader that might wish to use the resulting product for some kind of downstream analysis.

2 Line-by-line comments

L50 In what sense is linear perturbation theory underused?

Eqs. 1/2 This form of the SSA implies some special coordinate system (flow-aligned, downhill in direction of flow). This needs to be justified to the reader.

L65 Need to specify u and v as constant through depth by assumption.

Eqs. 3-6 Some basic discussion of what assumptions are being made for linear perturbation analysis are necessary here. For example, the assumption that the zero-order terms are spatially constant needs to be stated explicitly, otherwise it's confusing to see how these equations are derived.

L73 Steady state means that the time derivative of the surface and bed elevations are zero. How is it justifiable to make that assumption for Thwaites?

Eqs. 7-10 These need improved typesetting. It's not clear what is the argument of e.g. cos. Also, need to state that the Fourier representation of a variable is denoted by a hat, and that the hatted variables are the Fourier transform of the perturbations.

L92 I think the expression 'variability in the Fourier components of the surface' is a bit opaque. This looks to me to just be the ratio of the fourier coefficients themselves as a function of wavenumber. Is the language about variability a reference to these being the fourier coefficients of the first-order perturbation?

L93 While I recognize that this section is mostly a reproduction of Gudmondsson, it would be helpful to go a bit beyond just presenting these symbols and discuss just a bit what these mean, for readers that aren't already familiar with the antecedent work.

L97 Should $h+\Delta s$ be $h+\Delta h$? They're the same in this case, but the difference should be mentioned for notational consistency.

Section 2.1.3 Why is non-dimensionalization necessary here? it's not at all apparent that the transfer functions that appear in the supplement are simpler, and the inclusion of all this extra notation just makes the paper more confusing, especially given that there's no real motivation for why it's needed in the first place. Furthermore, the section itself is quite unclear. Where does $\bar{C}$ come from? The text here does a lot of hand-waving and should either be substantially expanded or eliminated.

Eqs. 26–28 The arguments (k,l) to all of these terms needs to be retained in order to make clear that this function gives the amplitude of the surface perturbation for a given wavenumber as a function of the same for the bed. Also, I think that the traction transfer function subscripts should be capitalized?

Sec 2.2/Appendix C The inverse problem section is insufficiently described in these sections. Is this inversion being done for every wave number independently, or are they somehow coupled? Why is the 'filtering' method described a more sensible approach than a more common thing like a truncated eigendecomposition? How are the Fourier transforms of the observations performed? Most importantly, how are the error bars that appear in the later figures computed? This shows up qualitatively in the text, but it is not justified nor sufficiently detailed to be reproducible.

Sec 2.3 Real topography does not have a delta function as its Fourier representation. It would be very helpful to see what the model's skill is for recovering topography that varies over multiple wavelengths simultaneously.

L200 I think some care should be given to explaining a bit better why different 'patches' should give different results in overlapping regions. If I understand correctly, this is a direct result of the linearization and the fact that there's different assumed values of $\bar{h}$ and $\bar{u}$ being utilized in each. Is that true? Is there a way to communicate this clearly?

L202 I'm confused by the mention of the SIA here: isn't this all based on the SSA?

Sec 2.4 I'm concerned about the use of the BedMachine prior, particularly in the context of taking the mean of multiple overlapping blocks. It seems to me that if each of these overlapping blocks each has its own a priori mean being calculated from something else, and then these are being themselves aggregated, this is effectively injecting sub-50km scale a priori information. How do I know that what I'm looking at in Figs. 7 and 8 is not just a low-pass filter of BedMachine with some additional wiggles added from the inversion? This 'mean of means' would yield thickness results that 1) are biased too shallow and 2) are less skillful in regions of large bed slope, both of which are evident in the data.

Sec 3.1 The error bounds reported are obviously unreliable. A 1-σ credible interval ought to contain the data in 65% of instances. Simply looking at Figs. 7 and 8 show that this is not the case.

L290 This statement regarding hyperparameters being sensible choices is not shown to be true in the text.