Author's point-by-point response to the second review of tc-2021-240 by reviewer 1.

Dear Editor Florent Dominé, dear Reviewers

we are very thankful to reviewer 1 for again taking the time to thoroughly review our manuscript. The additional comments were of great value for improving the understandability of the manuscript at places, where it was still difficult to follow. We hope that with the new changes the theory behind our model became clear.

We thank Florent Dominé for his effort and support in editing this manuscript. We further thank both reviewers for their constructive comments, which clearly helped to improve the quality of the manuscript.

Best regards,
Falk and Aslak

Minor remarks

- lines 54, 261: I don't think theer is the need of a capital letter after a colon.

  Author response: fixed.

- line 98: you should start to give the exact definition $\frac{1}{\rho}\frac{\partial\rho}{\partial t} = \mathrm{tr}(\dot{\varepsilon})$

  Author response: Addressed together with the following comment.

- Equation (5): the way Eq. (5) is obtained looks like a magic trick! Why not just saying that you are making the assumption that $\dot{\varepsilon}_h = \dot{\varepsilon}_{xx} + \dot{\varepsilon}_{yy} = 0$ instead of adding it to one term to subtract it right after? This part is a bit confusing and it is difficult to get a correct meaning of all these different terms introduced for $\dot{\varepsilon}_{zz}$.

  Author response: We agree that the derivation of the general expression of the strain softening correction is difficult to follow. We therefore reformulated the section by starting with the general definition of the strain rate tensor and its relation to the densification rate (previous comment) as well as by giving explicitly the assumptions needed for disentangling horizontal divergence and strain softening (this comment). While the following changes look severe, we mainly changed the order of equations, so that the general definition of the relation between strain rate and densification rate is understandable when it is introduced and to then keep a logical structure after doing so.

  Review:
  L80: "A firn densification model in a Lagrangian formulation expresses the densification rate of a firn layer as a function of external forcing parameters and internal parameters, representing its current state. The external parameters are generally

time-variable. In a climate-forced model these are the temperature T and **the accumulation rate $\dot{a}$ or the** overburden load σ**, derived from** $\dot{a}$. As an internal parameter most firn models only consider the current density ρ of the firn layer. **Additionally, for newly** formed snow layers at the surface the initial snow density ρ_0 **is required** as a boundary condition**, which** in most applications **is** assumed to be **a site-specific constant**. The densification rate of a climate-forced model, **here** denoted with the subscript c, is hence given in the form of

$$\left(\frac{\mathrm{D}\rho}{\mathrm{D}t}\right)_{\mathrm{c}} = f(T, \sigma, \rho), \tag{1}$$

with the time t. **As such climate-forced models however neglect the effects of horizontal divergence and strain softening, the given densification rate will differ from its actual value.**

Generally, at a specific point in the firn the state of the strain rates is described by the symmetric strain rate tensor

$$\dot{\boldsymbol{\varepsilon}} = \begin{pmatrix} \dot{\varepsilon}_{xx} & \dot{\varepsilon}_{xy} & \dot{\varepsilon}_{xz} \\ \dot{\varepsilon}_{xy} & \dot{\varepsilon}_{yy} & \dot{\varepsilon}_{yz} \\ \dot{\varepsilon}_{xz} & \dot{\varepsilon}_{yz} & \dot{\varepsilon}_{zz} \end{pmatrix}, \tag{2}$$

which consists of two normal horizontal strain rate components ($\dot{\varepsilon}_{xx}$, $\dot{\varepsilon}_{yy}$), the normal vertical strain rate component ($\dot{\varepsilon}_{zz}$) and three shear components ($\dot{\varepsilon}_{xy}$, $\dot{\varepsilon}_{xz}$, $\dot{\varepsilon}_{yz}$). Following Morris et al. (2017), its trace defines the volumetric strain rate $\dot{\varepsilon}_{\mathrm{vol}} = \mathrm{tr}(\dot{\boldsymbol{\varepsilon}})$ which in firn is related to the densification rate according to

$$\dot{\varepsilon}_{\mathrm{vol}} = -\frac{1}{\rho}\frac{\mathrm{D}\rho}{\mathrm{D}t}. \tag{3}$$

In the case of climate-forced models all components of $\dot{\varepsilon}$ except for the vertical strain rate are assumed to be zero and the strain rate tensor reads as

$$\dot{\boldsymbol{\varepsilon}}_{\mathrm{c}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \dot{\varepsilon}_{zz,\mathrm{c}} \end{pmatrix}. \tag{4}$$

Thereby, not only the volumetric strain rate is reduced to the vertical strain rate $\dot{\varepsilon}_{zz,\mathrm{c}}$, but also the latter will differ from the general case, meaning that $\dot{\varepsilon}_{zz,\mathrm{c}} \neq \dot{\varepsilon}_{zz}$. Eq. 3 for the climate-forced model case then takes the form of

$$\dot{\varepsilon}_{zz,\mathrm{c}} = -\frac{1}{\rho}\left(\frac{\mathrm{D}\rho}{\mathrm{D}t}\right)_{\mathrm{c}}. \tag{5}$$

Here, we aim to derive the actual densification rate $\mathrm{D}\rho/\mathrm{D}t$ as a function of the densification rate given by a climate-forced model $(\mathrm{D}\rho/\mathrm{D}t)_{\mathrm{c}}$. The derivation is complicated by the fact that when horizontal divergence is active alongside strain softening, both effects are entangled.

This is the case when the normal horizontal strain rates do not balance**, i.e.** $\dot{\varepsilon}_{\mathrm{h}} = \dot{\varepsilon}_{xx} + \dot{\varepsilon}_{yy} \neq 0$.

To simplify the situation, we assume that horizontal divergence only has a negligible influence on the pressure, which is justified for the second firn stage, where already a significant overburden pressure has built up **and the pressure component by** horizontal compression can be **neglected. With this assumption both effects** can be taken as being independent of each other **and we can correct for them separately. This we do by first applying the strain softening correction that is derived in the following and then correcting** for horizontal divergence **subsequently, using** the layer thinning scheme by Morris et al. (2017) and Horlings et al. (2021).

For the derivation of the strain softening correction, the above assumption means that $\dot{\varepsilon}_{\mathrm{h}} \ll \dot{\varepsilon}_{zz}$ **and hence the volumetric strain rate in Eq. 3 is approximated by the vertical strain rate, so that** $\dot{\varepsilon}_{\mathrm{vol}} \approx \dot{\varepsilon}_{zz}$**. We obtain**

$$\frac{\mathrm{D}\rho}{\mathrm{D}t} = -\dot{\varepsilon}_{zz}\rho, \tag{6}$$

which describes how a strain softening corrected vertical strain rate $\dot{\varepsilon}_{zz}$ translates back into **a corresponding** corrected densification rate.

By combining Eqs. 5 and 6 it then follows that

$$\frac{\mathrm{D}\rho}{\mathrm{D}t} = \frac{\dot{\varepsilon}_{zz}}{\dot{\varepsilon}_{zz,\mathrm{c}}} \left(\frac{\mathrm{D}\rho}{\mathrm{D}t}\right)_{\mathrm{c}}. \tag{7}$$

Hence, the densification rate output of a climate-forced firn model can be corrected for the effect of strain softening by multiplication with a scale factor that is given by the ratio of the corresponding vertical strain rate components. We aim in the following to determine this scale factor."

- line 118: I am not sure that phase is adapted here! The constitutive relation is given on the form of a tensorial relation between Cauchy stress and strain rate, which can be decomposed in two sets of relation: one tensorial between deviatoric stress and deviatoric strain-rate and one scalar relation between isotropic pressure and rate of change in volume. Phase in a classical meaning is more referring to ice and air for example for snow, which is not the meaning here I guess?

Author response: Here, we follow the terminology of Duva and Crow (1994), who write:

> "A key feature of our modelling is that the reinforced porous material is regarded as a two phase material consisting of a rigid reinforcing phase and a homogeneous, porous creeping phase."

To be consistent with the original literature, we prefer to keep the wording as it is, but add a sentence, that these phases shall not be confused with the ice and air phase that are often used for describing firn.

Review:
L122: "Firn is a compressible material. **Following Duva and Crow (1994),** its deformation is given by two coupled constitutive relations which represent a rigid **reinforcing phase** and a compressible phase**, that are not to be confused with the ice and air phases often used for describing firn. The rigid phase is defined** by the **tensorial** relation between deviatoric stress and deviatoric strain rate, as given by Glen's flow law for the deformation of incompressible ice (Glen, 1955; Nye, 1957). The **scalar** relation of the compressible phase is set between the **isotropic pressure and the volumetric strain rate**."

- line 125: it should be mentioned that the two factors a and b are function of the density. The way it written here using "factor" makes think there are constants.

  Author response: We agree and changed the term "weighting factors" to "density dependent weighting coefficients".

- line 170 k_1 and k_2 are not written the same way as in (10) and (11).

  Author response: fixed.

- line 172: not clear what you mean by the two versions of Eq. (12)?

  Author response: We explicitly write it out now to be clear.

  Review:
  L173: "Equations 8 to 12 are applicable independent of whether the additional strain rate components are considered. They can be formulated both in terms of the strain softening corrected model and the climate-forced model, with the only difference being whether the macroscopic effective strain rate is computed from Eq. 2 or 4. In particular, k_1, k_2 and $\tau_{\mathrm{ice},zz}$ do not differ between the two cases as they are assumed to be independent of the additional strain rate components. **Thus, Eq. 12 can analogously be formulated for the climate-forced model case as**

$$\dot{\varepsilon}_{zz,\mathrm{c}} = \frac{k_2}{2\eta_{\mathrm{ice,c}}}\tau_{\mathrm{ice},zz}, \tag{13}$$

  where the effective ice viscosity in the climate-forced model $\eta_{\mathrm{ice,c}}$ will be higher than the actual effective ice viscosity η_{ice}.

  2.3 The scale factor

  By dividing **Eqs. 12 and 13**, the strain rate ratio in Eq. 7, which is the scale factor, can be expressed in terms of a solid ice viscosity ratio:

$$\frac{\dot{\varepsilon}_{zz}}{\dot{\varepsilon}_{zz,\mathrm{c}}} = \frac{\eta_{\mathrm{ice,c}}}{\eta_{\mathrm{ice}}}. \tag{14}$$

  "

- line 189: the fact that it is difficult to estimate the strain-rate components should be emphasized. Instead of *In summary, all components of* could be *In summary, if all components of* for example.

  Author response: We agree and change the paragraph accordingly. Also we add a sentence saying that even knowing some of the components (e.g. the horizontal strain rates) and correcting for it improves the estimate of the densification rate.

  Review:
  L198: "In summary, the variable r_{v} corresponds exactly to the scale factor that is sought. **If all strain rate components in r_{h} are known, only Eq. 17 is left** to be solved for r_{v} to obtain the scale factor for correcting the densification rate of a climate-forced model for the total effect of strain softening. **But even if merely some of the external strain rate components in Eq. 19, e.g. the horizontal strain rates, are known, this approach can be used to correct for their contribution to strain softening enhancement of the densification rate.**"

- lines 303, 342: I think there is a confusion about temperature, which is taken constant and uniform. If I understand that seasonal variation of temperature can be neglected, changes in temperature with depth should be accounted for, which seems not the case here? In the CFM paper, it is mentioned that T is the temperature of a specific parcel of firn and thus is not uniform (function of depth and eventually of horizontal coordinates also). Is there an existing temperature profile at EGRIP that could confirm that the temperature is uniform in the firn?

  Author response: The CFM model does allow for variable temperature forcing, and so it is simple to apply a variable temperature in the model. However, in practice firn temperatures below 10 m change very little (See e.g. Dahl-Jensen et al. (1998) or Orsi et al. (2017)). This is because vertical velocities are relatively large near the surface and downward advection of cold surface temperatures overwhelms any heating from below. For the same reason, Herron and Langway had so much success with a firn model that uses the 10 m temperature as the only temperature input. Nonetheless, a recent warming trend can be observed in the firn temperatures in Greenland, but with it being only on the order of 1°C its affect on firn densification is minor and can be neglected. This is especially the case regarding the purpose of our work in which we do not want to produce exact estimates of certain firn properties, but to study the general effect of strain softening on firn densification.

  Review:
  L313: "Temperature evolution **is neglected** in our **model** experiments, as **we aim to assess the general impact of strain softening on firn densification and thereby study processes occurring** in the second firn stage, **where temperature is approximately stable. At this depth** seasonal temperature variations are dampened by heat conduction **and only a recent warming trend**

remains, which for North Greenland lies on the order of 1°C **(Orsi et al., 2017) and, hence, has a minor impact on firn densification.**"

L352: "We force the model with a constant temperature of -29.9°C. This is the seasonality-corrected mean of the 10 m temperature recorded between June 2019 and January 2021 at the PROMICE weather station at EGRIP (Fausto and van As, 2019; Fausto et al., 2021). Using a constant temperature input is justified because we are mainly interested in firn processes occurring below a depth of 10 m, where seasonal variability of temperature is smoothed out by heat conduction **and the impact of the general warming trend in Greenland is minor**. Further, we do not expect a significant spatial variability of temperature over this relatively small study region."

- line 355: why not $295\,\mathrm{kg\,m^{-3}}$ as mentioned above?

  Author response: We changed this to accommodate an earlier review. The effect of such a small change is minimal on the deeper density profile, as the densification is very rapid near the surface, so we have decided to keep it as it is, but now start the sentence with a clarification, that we are specifically referring to the ice sheet wide studies.

  Review:
  L370: "**In the ice sheet wide studies,** new surface layers are formed with a density of $315\,\mathrm{kg\,m^{-3}}$, following Fausto et al. (2018)."

- line 387: indicated

  Author response: fixed.

- Figure 5: I don't understand why all the density measurements in Greenland and Antarctica (the red dots in Fig. 5) are not compared with the model results for this last application?

  Author response: This may sound like a small task, but it really require a full (separate) calibration-validation study before this would be useful. We would estimate this would add many pages of text, new data sets, and multiple figures. A lot of work that we further feel would ultimately strongly detract from the focus of the study. So, we leave this for future work - In our opinion - this is simply beyond the scope of the present study. As we have said before, there is no advantage in comparing a wrong model to the data. The dots are there to illustrate that strain softening has some effect at the sites that were used to tune the model. We make that more clear in the figure caption now.

  Review:
  "**Figure 5.** Study on the contribution of strain softening to firn densification in terms of the firn thickness over the dry zone of the Greenland ice sheet (GrIS, a-c) and the Antarctic ice sheet (AIS, d-f). (a & d) Modeled total firn thickness when strain softening is considered without the tuning bias correction. (b & e)

Absolute firn thinning contribution caused by strain softening. (c & f) Relative firn thinning due to strain softening, which also illustrates by how much the densification process is accelerated by strain softening. Red dots indicate the drill locations of the firn cores that were used for tuning the empirical Herron-Langway model **and illustrate that some of these firn cores were considerably affected by strain softening**. Outside the dry zone of GrIS the surface velocity is shown with brighter colors indicating faster flow."

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