Reply to Comment tc-2021-216-RC1

Q1: Authors used Singular Spectral Analysis to examine variations of the Arctic and Antarctic sea-ice extents (SI), and of the atmospheric surface pressure (AP) in both hemispheres (NH and SH). There exists a range of periods (an annual, 1/2, 1/3, 1/4 and 1/5). Some relationships between sea ice and pressure are built. The manuscript was written well. Some improvements need to be made before the manuscript is accepted. The results presented in this study are statistical, the mechanisms behind the results lack. This is the main shortcoming.

A1: We can only agree. So far, we have not yet been able to produce a full model based on a precise mechanism. We do no perform a «statistical analysis». We extract in an objective way (SSA) periodical or quasi-periodical cycles from two a priori distinct data sets. Our «observations only» conclusion is the remarkable similarity (not to say identity) of the two sets of periods, which implies that some explanation must be found. We propose hypotheses but the observations are important enough in our view to be shared, so that others have a chance to uncover the full mechanism.

Q2: For SSA method, what is the difference between SSA and other methods (Circulant SSA, EMD)?

A2: There are roughly three families of tools that can be used to analyze a signal (time series): Fourier analysis, wavelets and what we will call «*ad hoc*» analyses. For the first two, the signal is projected on a basis of imposed orthogonal functions. In the ad hoc analyses, the orthogonal basis is built from the information carried by the signal itself. These often take advantage of the properties of embedding, as exhibited by the descending Toeplitz or Hankel diagonal matrices (*e.g.* Lemmerling and Van Huffel, 2001) combined with the powerful Singular Value Decomposition (*e.g.* Golub and Reinsch, 1971).

These three approaches should not be considered as opposed: given sufficient patience one can obtain the same results in the end. The reason why Fourier invents the method that bears his name is because he has to solve a precise set of physical/mathematical problems: uncoupling the variables in the heat equation, and expressing the solution as a combination of pure oscillations, namely the forcing of daily variations (*e.g.* Fourier, 1822). The Fourier transform can be seen as a kind of cross correlation of the signal with an infinite series of different frequencies. This requires that for its all duration the signal be stationary. This property is clearly not that of seismic waves propagating in a heterogeneous, attenuating and dispersing ground. This is why Jean Morlet modified the infinite sinuses into «Morlet» wavelets (*e.g.* Morlet et al., 1982). The two methods appear for different problems in different physical cases and are pushed to their limits. Geophysicists (again) introduced a new method, Singular Spectrum analysis (SSA), in order to fill gaps in paleoclimatic sedimentary series, when none of its statistics is known (*e.g.* Vautard and Ghil, 1989). As briefly mentioned above, in these ad hoc methods the basis on which the signal is projected is built from the information contained in the signal. Therefore (unlike in Fourier analysis) one cannot predict whether a certain eigenvector will have a given pure frequency, and unlike in wavelets one cannot know ahead of time whether a certain amplitude characteristic will be in such or such wavelet scale.

What is the physical reality embedded in each of these transformations (what is the physical nature of a given component)? The answer is simple: none. The only goal is to distribute the energy contained in a signal over a new reading grid that may better allow to answer the original questions. As another example, cross correlating the horizontal component of the geomagnetic field at a given observatory with an infinite sinus («beginning» at minus infinity and «ending» at plus infinity) has no physical reality. Then, as we do not impose a perfect orthogonal basis, how can we be sure that our eigenvectors (that are orthogonal having been obtained with SVD) are the most «perfect» as possible? This is the question of separability in SSA (treated for example by Golyandina and Zhigljavsky, 2013; chapters 2.3.3 and 2.5.4).

Without going into too much detail, another way to treat this problem is to ask within the frame of SSA how can our Hankel/Toeplitz matrices be the best possible. There are several approaches. We have selected «iterative SSA» that, through iterations of rotations in the space of eigenvectors associated with (f.i.) a «varimax» criterion, optimizes the separability of components (e.g. Hubert, 1985). In conclusion, given its results, SSA is a remarkably powerful method in certain specific cases but it is in no way a «express button» tool.

For this reason (lack of «automatizability»), Bógalo et al (2021) recently proposed the new «circulating» SSA (cSSA). When classical SSA builds its Hankel/Toeplitz matrix from signal values (lines and columns are elements of the signal), cSSA builds a matrix in which lines and columns are auto-covariances (**E**) of the signal:

$$\gamma_m = \int_0^1 f(\omega) exp(i * 2 * \pi * m * \omega) = E\{x_t, x_{t-m}\}$$

These are used to force the automatic extraction of the most oscillating components. The new estimator, the covariance, carries with it the whole problems linked to estimators in general (cf. Claerbout, 1976; Papoulis, 1984).

Again, these methods should not be opposed one to the other : iSSA has been developed for geophysical problems, cSSA for economic questions. If the various « recipes » are followed carefully, extractions of components should end in the same way.

Let us end with Empirical Mode Decomposition (**EMD**). Here again, a geophysicist investigates non stationary signals (Huang et al., 1998). The only point that is common with SSA is that orthogonal bases are directly built from the signal, but are the most oscillating as possible, contrary to SSA, using the properties of the Hilbert transform. These are used to calculate the envelopes and instantaneous phases of any signal. Both EMD and SSA are less sensitive to stationary signals, unlike Fourier. Huang et al., (1998) calculate intrinsic mode functions (IMF, whose properties they define) along the time space signal, leading to the following algorithm : One first determines the maxima ogf the upper envelope, interpolates them with splines and the same is done for the lower envelope. The two curves must contain the whole of the signal. At the first step, the difference between the two envelopes and the original signal is the first mode (or component). Then one iterates. Their may be problems with the regularity of the Hilbert transforms and the spline interpolations (Claerbout 1976) inducing errors in the component amplitudes. Once again, is applied correctly, EMD and SSA should give the same answer.

This short and simplified «tutorial» hopefully answers the referee's central question. The only real differences lie in the scientific questions that are being asked and led to their construction.

Q3: Lines 38-42 Authors should gave some reasons for the dissymmetry of Arctic and Antarctic sea ice extent

A3: This is not a simple question to answer. Several authors (cited in the paper) have attempted to answer in a semi-qualitative way. Reasons for the dissymmetry include ocean/continent boundaries (grounding ice vs sea-ice). The resulting wind and current patterns are also important. The quasi-circular shape of the Antarctic continent, ice extent, free sea-passage contrasts with the lack of an easy sea-passage in the Arctic. Does the referee suggest adding a few lines along this?

Q4: Lines 47-54 The large-scale atmospheric oscillation in the Arctic is not only AO. The factors influencing the Antarctic climate include forcings in the Indian, Atlantic, and Pacific Oceans. Authors only introduced Pacific factors.

A4: This is correct. But in Le Mouël et al. (2019) we showed that most if not all climatic indices reveal the same cycles (periodicities). We can add a few lines and repeat this reference.

Q5: While authors represent the results of the SSA analysis for a given period ( for example annual cycle), please display the spatial pattern of pressure and sea ice at maximum and minimum values, and explain the relationship between air pressure and sea ice pattern

A5: Only the pressure patterns are available in 2D. Sea-ice extent is a 1D series that does not allow the interesting exercise suggested by the reviewer. And so far we acknowledge that we do not have a clear physical mechanism to explain the relationship between air pressure and sea ice pattern. We hope some of the readers of our paper (observations) will be inspired bu them to hypothesize a mechanism…

Q6: Authors exhibited the 1/2 year period. There is a common phenomenon in the Southern Ocean. There are a lot of literatures

A6: Indeed. The ½ year period is for instance also found in polar motion (length of day) (Le Mouël et al, 2019b), in sunspots (Le Mouël et al., 2020; Courtillot et al., 2021), in the magnetic field (Cliver  et al., 2004; Le Mouël et al., 2019c) and of course in the Sun-Earth distance.

Q7: "The semi-annual oscillation (SAO) in the middle and high latitudes is an important and well known component of the Southern Hemisphere climate. An overview of the early literature on the SAO is given by van Loon (1967), and a reexamination of the phenomenon and its causes is presented by Meehl (1991)".  The semi-annual oscillation and Antarctic climate Part 1-4 depict SAO and its effect on the Antarctic climate.

A7: We agree with this remark pointed out by the reviewer. The 6-month component plays an important role in climatology, being present for instance in climate indices as well as pressure variations. Actually, this oscillation has been encountered in many other fields. First of all in geomagnetism: we know since Bartels (1932) that magnetic indices over the entire Earth record this component, whose amplitude varies with latitude. Bartels concluded in favor of an internal origin, strongly linked to the revolution around the Sun. We also find a 6 month periodicity in the length of day (which is a global parameter, e.g. Lambeck, 1980), in sunspots numbers (e.g. Lockwood, 2001) and of course, even if it is modest, in the Earth-Sun distance variation. In summary, the 6 month component is found in many different geophysical and heliophysical fields. We argue that there may exist a general forcing mechanism with a sequence of harmonic components at 1, 1/2, 1/3, 1/4, 1/5 years. This sequence could help us (or others) to uncover its nature.

Q8: For 1/3, 1/4, and 1/5 period, what mechanisms behind these periods are there?

A8: Indeed an important question To our knowledge only the mechanics of turbulent fluids could explain this observation (in a geophysical context). If a rotating sphere or cylinder is forced with a frequency w, this frequency and its harmonics should be encountered in the movement of the fluid. We briefly address this in paragraph 5.4 page 23. We note that whereas there is a unique solution in the case of the cylinder, the problem for a sphere has still to be solved. This suggestion is certainly not the only one and we hope other suggestions will be encouraged by our results.

Q9: Lines 389-390 “The phase lag between sea-ice extent and pressure decreases from -35 to -60 days (~ degrees; Figure 6b).” Pressure precedes sea ice extent 35-60 days for semi-annual period? Why? There is an increasing trend. Why?

A9: Except for the fundamental 1 yr, all harmonics have the same phase lag of 30 days in 42 years. But this is calculated as a phase difference between two signals using a Hilbert transform. All we can say is that the sign of the phase difference indicates that over the analyzed window pressure variations precede sea-ice extent variations. We would need to know the data series from a 0 time origin. Still, the phase lag of 30 days in 42 years is a robust physical estimate that must have a physical origin/explanation. We now must seek which one. Same general conclusion: we have exciting and robust observations that indicate an as yet unknown physical mechanism, most likely contained in Laplace’s celestial mechanics.

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