We appreciate the constructive comments from two reviewers and the community. Our manuscript will be much improved by their input. We have made changes to our manuscript. In the following responses, we use "**bold**" text for comments, "non-bold" text for our responses, and "*italic*" for changed text in the manuscript.

## Referee #1

### General comments

**The paper by Li et al., 2021 presents an innovative method that aims at correcting glacier velocity overestimation, that are due to accelerations, when using long timespan. The paper is well presented with a clear structure, well written and the Figures are relatively clear.**

**A large share of the paper is dedicated to the description of the method, which is simple in principle, but that could be ambiguous to understand clearly. Consequently, I have few comments that I hope, will help to make the paper more understandable.**

**Among those comments, the definition of the different Premises needs a bit of clarification, and particularly on their area of validity (see below)……**

Response:
In *Premises I and II* now we clarified the correspondence between the short span (months to year) and "OE-free" (< σ) trajectory segments, as well as that between the longer span (5 to over 10 years) and difference of E- and L-velocities (< *k* σ) at the end of the entire trajectory.

(Line 152 in the marked-up manuscript) ***"Premise I:*** *Within a baseline time span (e.g., 1 year or shorter) each segment (from $P_{i-1}$ to $P_i$ in Figure 1b) is relatively short and the E- and L-velocity difference is smaller than $\sigma$. Furthermore, over the map span of n years (e.g., 5-10 years or longer) the accumulated E- and L-distances along the entire trajectory do not deviate significantly from each other, so that the maximum velocity difference in the Lagrangian and Eulerian frameworks (end points of blue and red lines in Fig. 2) is limited within a threshold ($V_{0-n}^{L} - V_{0-n}^{E} \leq k\ \sigma$), where k is a constant and $\sigma$ is the velocity mapping uncertainty."*

(Line 169) ***"Premise II:*** *Within the time span of n years (e.g., over 5-10 years), the velocity field described by $V_{0-1}^{E}$ and $V_{0-n}^{E}$ does not change significantly, so that the line between $V_{0-n}^{L(n)}$ and $V_{0-1}^{L(n)}$ (black line in Fig. 2) and that between $V_{0-n}^{L}$ and $V_{0-1}^{L}$ (blue line in Fig. 2) are approximately parallel to each other. Accordingly, the difference between their simple averaged accelerations is within a threshold ($\left|\bar{a}\left(V^{L(n)}\right) - \bar{a}\left(V^{L}\right)\right| \leq k' \frac{\sigma}{\Delta t_n}$), where $k'$ is a constant and $\sigma$ is the velocity mapping uncertainty and*

$\overline{a}(V^{L(n)}) = \frac{V_{0-n}^{L(n)} - V_{0-1}^{L(n)}}{\Delta t_n}$, $\overline{a}(V^L) = \frac{V_{0-n}^L - V_{0-1}^L}{\Delta t_n}$. *(7)”*

Their validity in different types of glaciers is introduced in the Discussion section. Based on the premises we rephrased sentences of OE corrections to formalize a Theorem:

(Line 190) “***Overestimation Correction Theorem****: Assume that the necessary condition in Premise II is met, spatial acceleration - induced overestimations in long time span velocities* $V_{0-n}^E$ *can be corrected or reduced using the following Correction term, regardless of temporal acceleration:*

$V_{0-1}^E = V_{0-n}^E + Correction + \varepsilon$ , *(9)*

*Correction =* $V_{0-n}^E - V_{0-n}^{L(n)}$. *(10)*

*If Premise I holds (sufficient conditions are met), Correction* $\approx -OE_{0-n}$ *; otherwise,* $|Correction|<|OE_{0-n}|$*, preserving the velocity increases induced by temporal accelerations in the residual term ε (Fig. A2).*”

**…… The discussion also needs to be supplemented with an overview of the method applicability to different glacier types, and specifically fast flowing glaciers (e.g Jakobshavn Isbrae), or with a more complex geometry (e.g Zachariae Isstrøm ice shelf, Getz Ice Shelf or George VI ice shelf). Another interesting point of discussion is the impact of the glacier seasonal variability. Are the corrections significant with respect to the natural variability of glacier flow? While, seasonal signals are not really pronounced in Antarctica, variability can be much greater in Greenland (cf. Joughine et al., 2020).**

**For example, using a 1-year velocity reference for a glacier like Jakobshavn Isbrae, might not be ideal, as the glacier is flowing at more than 15 km/yr (which increases the chances of acceleration along a flowline). Similarly does the premises still holds, for glaciers that are changing directions and not flowing in a straight line (for example the ice shelf of Zachariae Isttrøm before 2000)?**

Response:
(Line 417) The Discussion section is restructured. At the beginning we added an overview statement: “*In this section we discuss the applicability of the proposed method in terms of overestimation-free time span, influence of complex glacier geometry, overestimation in fast flowing glaciers, and comparison with the “Midpoint” method.*”

Then we added three new subsections and restructured one section to make a strong Discussion according to the comments. The four subsection titles are:

(Line 419) “***4.1 Threshold of the overestimation-free time span for trajectory segments***”

(Line 430) “***4.2 Glaciers with complex geometry***”

(Line 443) “***OE correction in fast-flowing glaciers***”

(Line 484) “***Comparison with the “Midpoint” method***”

Please see manuscript for details.

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In the following we present supporting materials for the above Discussion that are not given in the manuscript.

### 4.1 Threshold of the overestimation-free time span for trajectory segments

The choice of a short time, i.e., baseline or reference, span $\Delta t_{Ref}$ (e.g., a few months to a year) for the "overestimation-free" segments along a trajectory in *Premises I* makes sure that the difference between E-and L-velocities within the span is negligible, or less than $\sigma$ (velocity mapping uncertainty, $\sigma$=20 m a$^{-1}$ in this study). It is also the time span of the initial "OE-free" E-velocity map that is used for ice mass tracking and L-velocity computation in premise validation. Determination of this threshold has an implication on validation of *Premise I*, as well as the integration period of the trajectory segments from $P_i$ to $P_{i+1}$ (Fig. 3b). Estimation of $\Delta t_{Ref}$ can be performed in a systematic way. An area of the highest acceleration in a glacier should be selected. Within the area a multi-span E-velocity series $V_{0-i}^{E}$ ($i$=1, 2, ... $n$) can be used to establish a linear relationship between the E-velocity $V^E$ and time span $\Delta t$, $V^E = K\ \Delta t + b$, by a linear regression (red line in Fig. 2). With the known parameters of $b$ and $K$, $\Delta t_{Ref}$ can be calculated as $\Delta t_{Ref} = \frac{\sigma - b}{K}$. For example, *Area 5* in TG in *Experiment 1* has the highest acceleration (Table 1, Fig. 5c). After a regression using the E-velocities in *Area 5*, $\Delta t_{Ref}$ is calculated as 3.2 years ($R^2$=0.96, Fig. R1-1a). Thus, the selected $\Delta t_{Ref}$ of one year for TG in *Experiment 1* is justified.

Alternatively, if the E-velocities are not available, multi-span L-velocities along a profile on the main trunk of a glacier may be established from an available short span E-velocity map (e.g., 1-year map in PIG, Fig. R1-2a). Along the profile the highest acceleration (location "A" in Fig. R1-2a) is localized where a multi-span L-velocity series can be computed (Fig. R1-2b). Using this L-velocity series and the above regression method, the "OE-free" time span $\Delta t_{Ref}$ can also be estimated. For example, given $\sigma$=20 m a$^{-1}$ and a series of computed multi-span L-velocities from 1- to 10-years at location "A" in PIG (Fig. R1-2) and "B" in Jakobshavn Isbrae (JI, Fig. R1-6), we estimated $\Delta t_{Ref}$ as ~3.0 months ($R^2$=0.99, Fig. R1-1b) and ~1.4 months ($R^2$=0.86, Fig. R1-1c) for PIG and JI, respectively.

Based on the above analysis results of $\Delta t_{Ref}$ in TG (3.2 years), PIG (3.0 months) and JI (1.4 months), it appears that the threshold of an "OE-free" time span is strongly related to the ice flow dynamics of the glaciers. Given a known $\sigma$, shorter spans of $\Delta t_{Ref}$ should be selected for trajectory segments in validation of *Premise I* and L-velocity integration in faster flow glaciers. We suggest that an analysis of multi-span L-velocities and a regression for $\Delta t_{Ref}$ be performed before extensive historical velocity mapping would be carried out.

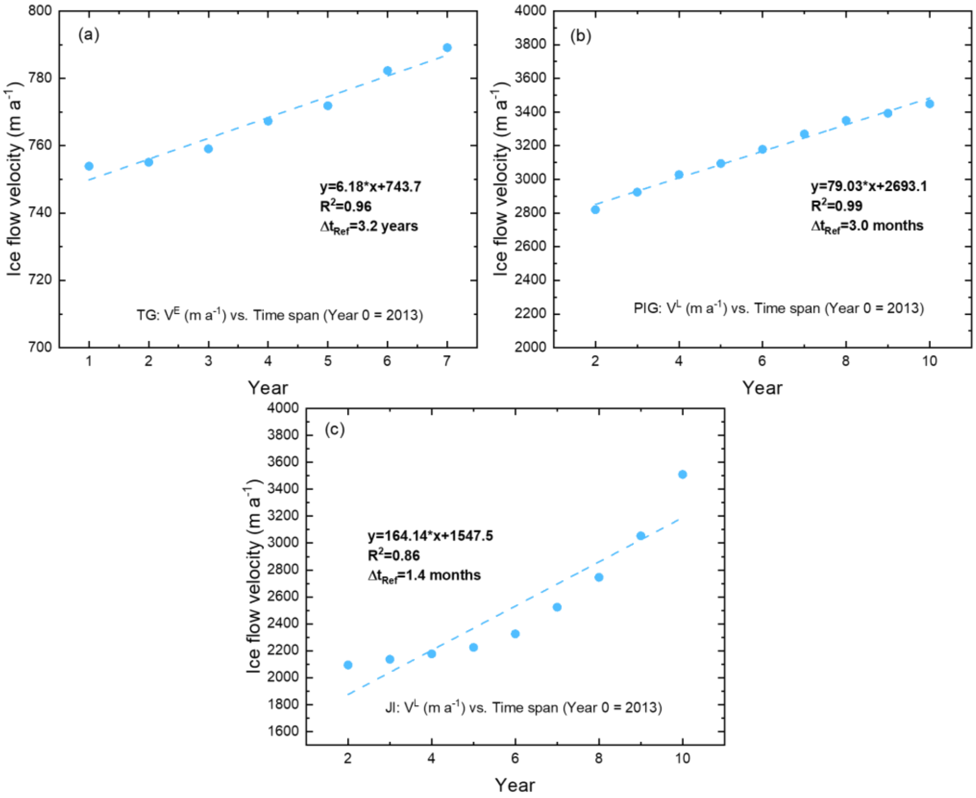

Figure R1-1. Linear regression of multi-span velocities vs. time span is performed to estimate the "OE-free" time span. (a) Totten Glacier (TG): 7-years of E-velocities in *Area 5* (Fig. 4), $\Delta t_{Ref}$=3.2 years; (b) Pine Island Glacier (PIG): 10-years of L-velocities at location "A" near grounding line (Fig. R1-2), $\Delta t_{Ref}$=3.0 months; and (c) Jakobshavn Isbrae (JI): 10-years of L-velocities at location "B" along the main trunk profile (Fig. R1-6), $\Delta t_{Ref}$=1.4 months.

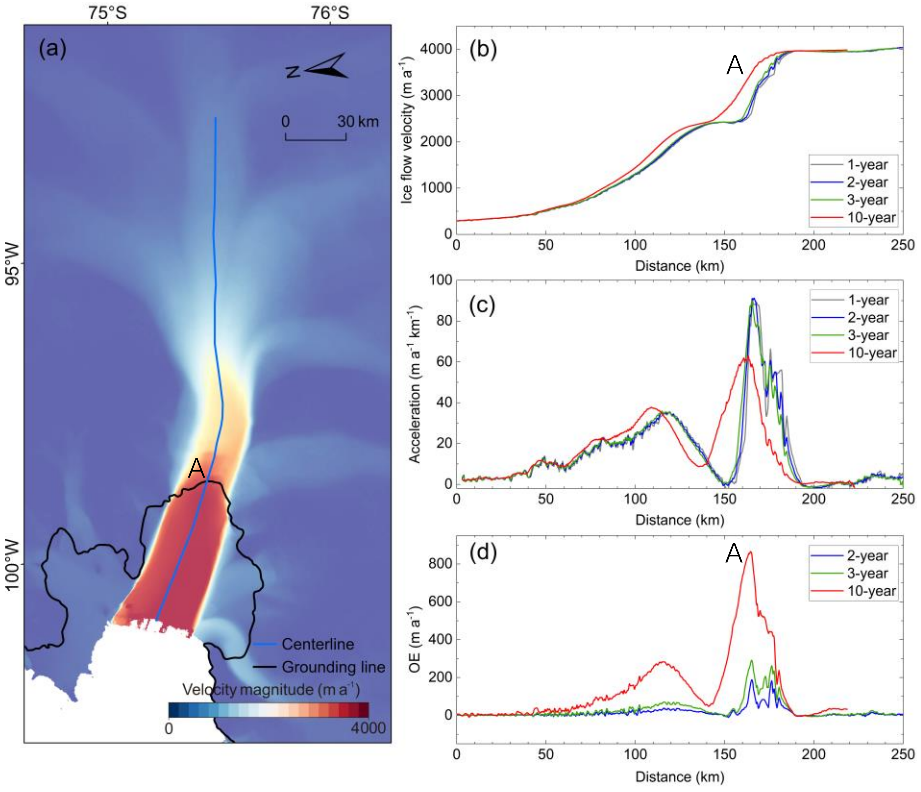

Figure R1-2. (a) Velocity, grounding line, and profile position along the main trunk of PIG; (b) E-velocity of 1-year span along the profile from ITS_LIVE, and L-velocities of 2-, 3-, and 10-year spans calculated from the 1-year span E-velocity map; (c) L-acceleration of the corresponding time spans along the profile, and (d) Estimated OEs caused by the L-velocities of different time spans. Please note that the L-velocities are only used for simulation estimation of time span $\Delta t_{Ref}$. In validation of *Premises I* and *II*, we used actual measurements of multi-year span E-velocities.

## 4.2 Glaciers with complex geometry

Furthermore, within a longer time span (e.g., over 5 - 10 years) in *Premise I* the difference between the E- and L-velocities accumulated over all segments along the entire trajectory, i.e. the end-point deviation between the red and blue lines in Fig. 2, is measured with a more tolerable threshold of $k$ times of $\sigma$ ($k\ \sigma$). Although the OEs of the trajectory segments are controlled by $\Delta t_{Ref}$, ice mass moving along a curved flow line over this long span may result in an additional discrepancy along the entire trajectory.

As suggested, we examined the complex geometry issue in 5 Antarctic glaciers, including George VI, Abbott, Dotson, Crosson and Getz (Fig. R1-3a) where the velocity ranges from ~100 m a$^{-1}$ to 1,000 m a$^{-1}$. We used one-year velocity maps of 2013 of 5 glaciers from ITS_LIVE to derive 20-year span L-velocities along trajectories in the significantly curved sections of the glaciers (Fig. R1-4). The computed straight and curved lengths vs. time span in the 5 glaciers are illustrated in Fig. R1-3b. The statistics of the corresponding E- and L-velocities are given in Table R1-1.

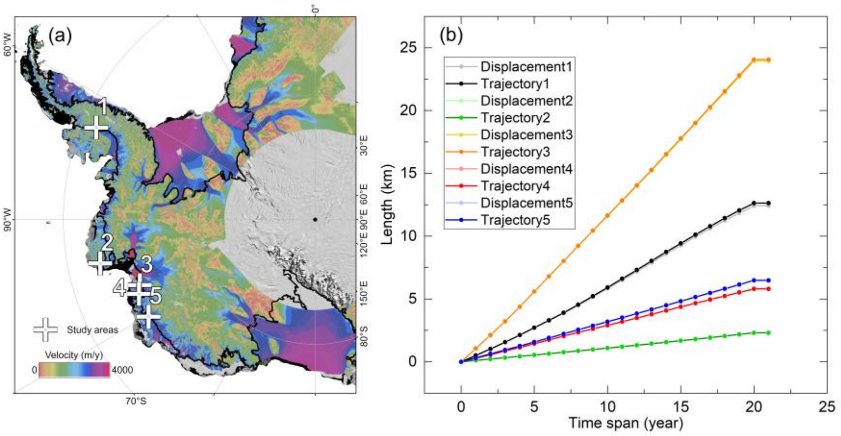

Figure R1-3. (a) Locations of 5 glaciers with complex geometry, including George VI, Abbott, Dotson, Crosson, and Getz. (b) Computed straight (displacement) and curved (trajectory) lengths vs. time span in 5 glaciers.

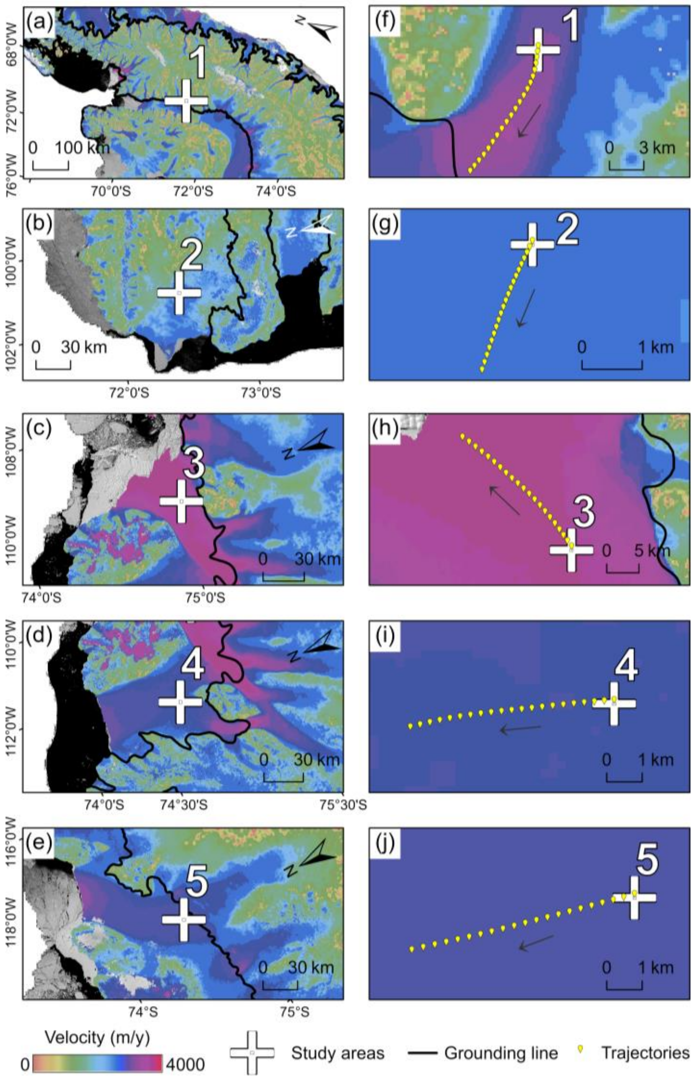

Figure R1-4. (a-e) locations of trajectories in 5 glaciers with complex geometry, including

George VI, Abbott, Dotson, Crosson, and Getz. (f-j) 20-year span trajectories in 5 glaciers.

It is shown in Table R1-1 that in these 5 glaciers three glaciers (Abbott, Dotson and Getz) have very small OEs in the 20-year span L-velocities (< $\sigma$ = 20 m a$^{-1}$), and their "curvature" induced differences $\Delta V_{20\,y}^{L-E}$ are 0 m a$^{-1}$. The other two glaciers (George VI and Crosson) have OEs of 114 m a$^{-1}$ and 143 m a$^{-1}$, respectively. However, the "curvature"-induced differences are only 11 m a$^{-1}$ and 6 m a$^{-1}$, both of which are smaller than $\sigma$ and thus, negligible. So, the OEs are mainly caused by spatial acceleration here, not the curvature.

Table R1-1. Overestimations and "curvature" induced velocity differences in 5 glaciers

| ID | Name | $V_{1\,y}^{E}$ (m a$^{-1}$) | $V_{20\,y}^{L}$ (m a$^{-1}$) | OE (m a$^{-1}$) | $\Delta$t (year) | E-dist. (m) | L-dist. (m) | $\varDelta V_{20\,y}^{L-E}$ (m a$^{-1}$) |
|---|---|---|---|---|---|---|---|---|
| 1 | George VI | 507 | 621 | -114 | 20 | 12412 | 12633 | 11 |
| 2 | Abbott | 110 | 115 | -6 | 20 | 2306 | 2308 | 0 |
| 3 | Crosson | 1054 | 1197 | -143 | 20 | 23942 | 24064 | 6 |
| 4 | Dotson | 280 | 290 | -10 | 20 | 5803 | 5809 | 0 |
| 5 | Getz | 315 | 324 | -9 | 20 | 6481 | 6482 | 0 |

In the Totten Glacier flow lines are less curved and velocity is higher (up to ~1,400 m a$^{-1}$). The E- and L-velocity differences of the 7-year trajectories in all five areas ($V^{L} - V^{E}$ in Table 1) are within 2$\sigma$ (40 m a$^{-1}$, < 2% of their velocities). Thus, the flow line curvature does not cause a significant E- and L-velocity difference, and the conditions in *Premise I* are well met.

We further performed an in-depth experiment in *Area 2* of PIG (in Fig. 3a), which is located in the most curved section along the main trunk of PIG (Fig. R1-5). We tracked positions of point $P$ from 2013 ($P$) to 2020 ($P'$) consecutively using 7 Landsat-8 images with a 1-year interval, resulting in 7 trajectory segments. The straight distance *PP'* is 19,720 m and the curved distance is 21,161 m. Accordingly, the E-and L-velocities of the 7-year span are 2,817 m a$^{-1}$ and 3,023 m a$^{-1}$, respectively. That means that the curvature of the 7-year trajectory (a deviation $b$ of 1,305 m from the straight line) created a difference of 206 m a$^{-1}$ at the trajectory end, among which 195 m a$^{-1}$ (95%) was corrected by our method (Table 3).

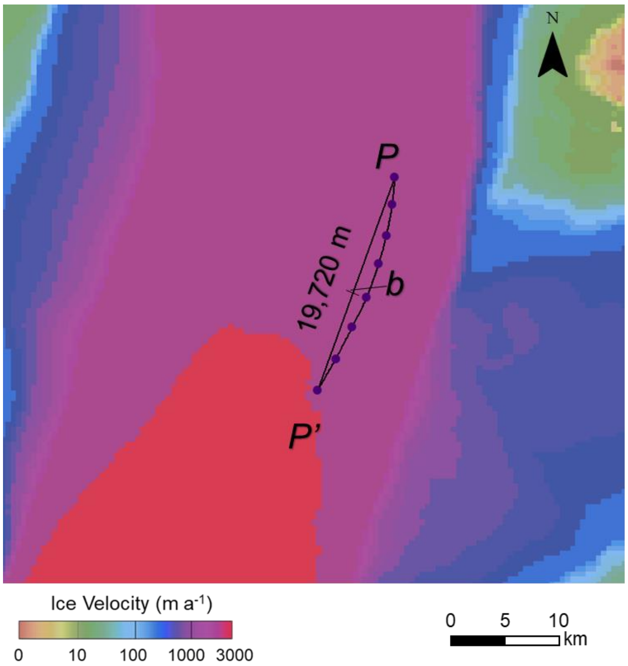

Figure R1-5. Illustration of difference between E- and L-velocities along a 7-year trajectory from *P* to *P'* in the most curved section in PIG. The background is the one-year span velocity map of 2013 from ITS_LIVE.

4.3 OE estimation in fast flowing glaciers

We estimate OE corrections assuming that within a longer time span (e.g., over 5 - 10 years) in *Premise II* the acceleration trend would not change significantly, $\left|\bar{a}\left(V^{L(n)}\right) - \bar{a}\left(V^{L}\right)\right| \leq k' \frac{\sigma}{\Delta t_n}$. As shown in *Experiments 1*, this trend change is under $1 \frac{\sigma}{\Delta t_n}$ (the acceleration equivalent of velocity mapping uncertainty σ) for the Totten Glacier, one of the fast-flowing glaciers in East Antarctica. Since the velocity requirements in *Premise I* and acceleration requirement in *Premise II* were met properly, we were able to correct in average 88% of the OEs.

The extremely high ice dynamics exists in fast flowing glaciers in West Antarctica or Greenland due to impact of climate warming. Here we evaluate the influence of such high dynamics on OE corrections in PIG. The acceleration trend differences $\left|\bar{a}\left(V^{L(n)}\right) - \bar{a}\left(V^{L}\right)\right|$ in all 5 areas (Table A4) are in average less than $3 \frac{\sigma}{\Delta t_n}$. Correspondingly, the black and blue lines of all 5 areas appear parallel (Panels 1c-5c in Fig. A3), indicating that the acceleration condition in *Premise II* is properly met. Consequently, the proposed method corrected in average 97 m a$^{-1}$ (~40%) of the total OE (245 m a$^{-1}$), leaving the residuals (60%) in the adjusted velocities. The residuals represent the velocity change signature over the time span caused by the continuous basal melting and drastic calving activities in and after 2017 in PIG (*Experiment 3*).

We performed an experiment for Jakobshavn Isbrae (JI), Greenland. We used a baseline velocity map of 2013 (one year span) from ITS_LIVE, based on which we calculated L-velocities of 2-, 3-, and 10-year time spans along the centerline of the main trunk. Subsequently, we estimate the L-accelerations and OEs (Fig. R1-6).

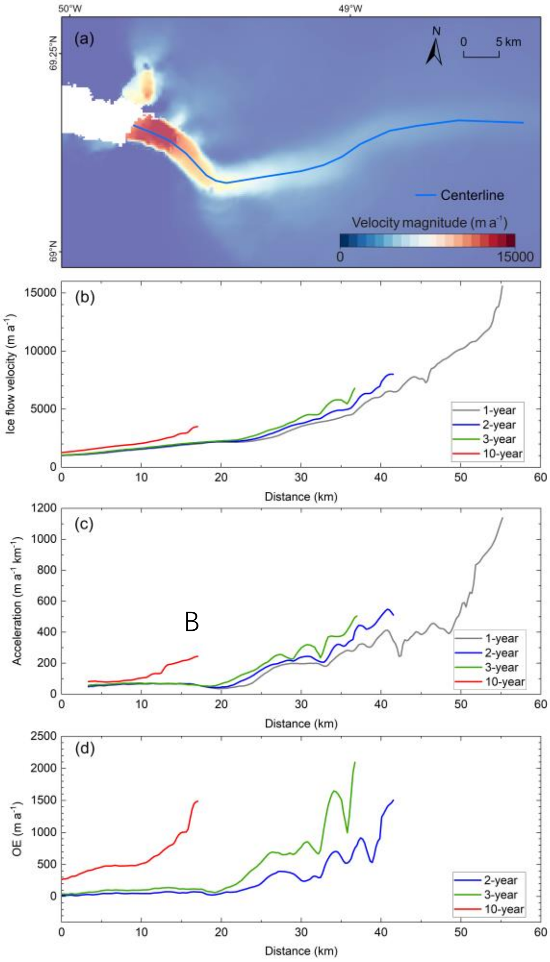

Figure R1-6. (a) Velocity, grounding line, and profile position along the main trunk of Jakobshavn Isbrae; (b) E-velocity of 1-year span along the profile from ITS_LIVE, and L-velocities of 2-, 3-, and 10-year spans calculated from the 1-year span E-velocity map; (c) L-accelerations of the corresponding time spans along the profile, and (d) Estimated OEs caused by the L-velocities of 3 time spans.

Jakobshavn Isbrae is about ~60 km from the inland interior to the marine terminus (Figs. R1-6a), over which the ice mass picks up velocity from ~1000 m a$^{-1}$ to over ~15000 m a$^{-1}$. The maximum velocity is ~3 times higher, while the main trunk is ~4 times shorter than in PIG. Consequently, the ice shelf part can only be covered by the 1-year span E-velocity map (Fig. R1-6b); similarly, only 16 km long inland interior along the profile is covered by the 10-year span L-velocity. This makes it difficult to map velocities in the grounding zone and ice shelf region using images of over 1-year span, i.e., lost opportunities for historical velocity recovery. The estimated OEs of three spans reached ~1,500 m $a^{-1}$ (up to 38% of the 10-year span L-velocity, Figs. R1-6b and d). This is significantly higher than 19% in PIG (Table 3).

In comparison to the reported velocity changes of ~125 m $a^{-1}$ from 2001-2008 in TG (Li et al., 2015) and ~500 m $a^{-1}$ from 2018-2020 in PIG (Joughin et al., 2021), the estimated average OE corrections of up to ~20 m $a^{-1}$ in TG and DG (Tables 1 and 2), and ~97 m $a^{-1}$ in PIG (Table 3) are not significant with respect to the natural variability of the glacier flow. In addition, the proposed method is applied for longer span velocities (a few years to over 5-10 years), seasonal variations should be averaged out. Therefore, the applicability of this method should not be affected by seasonal velocity changes and natural variability of glacier flow in Antarctica.

The estimated OEs in Jakobshavn Isbrae reached ~1,500 m $a^{-1}$ (up to 38% of the 10-year span L-velocity, Figs. R1-6b and d). The reported seasonal change can go as high as ~5,000 m $a^{-1}$ (50%) in last decade (Joughin et al., 2020). We believe that more comprehensive studies are needed in applicability of our method in Jakobshavn Isbrae and other fast flowing glaciers in Greenland.

Li, X., E. Rignot., M. Morlighem., J. Mouginot., & B. Scheuchl. (2015). Grounding line retreat of Totten Glacier, East Antarctica, 1996 to 2013. Geophysical Research Letters, 42(19), 8049-407. http://doi.org/10.1002/2015GL065701.

Joughin, I., Shean, D. E., Smith, B. E., & Floricioiu, D. (2020). A decade of variability on Jakobshavn Isbræ: ocean temperatures pace speed through influence on mélange rigidity. The Cryosphere, 14(1), 211-227.

Joughin, I., D. Shapero, B. Smith, P. Dutrieux, M. Barham (2021). Ice-shelf retreat drives recent Pine Island Glacier speedup. Sci. Adv. 7, eabg3080.

**Finally, while the authors are discussing the large overestimation error on Pine Island glacier (36%), they are presenting a first application of the method on Totten glacier. Hence, I think that it would increase the paper's logic and readability to keep this example for the application part (Totten could be put in the supplementary material). With such a high overestimation, I expect the results to be spectacular.**

Response:
As suggested, we carried out an experiment in PIG, *Experiment 3*. The results are presented as a new section:

(Line 391) "***3.3 Experiment 3: Velocity overestimation correction at Pine Island Glacier, West Antarctica***"

**Comments**

**L45. This is a citation for the Landsat-8 program. Not appropriate here.**

Response:

(Line 53) Agreed. We replaced Wulder et al. 2019 with Chander et al. (2009) that is more relevant to the historical Landsat programs (Landsat MSS, TM, etc.). It is also added in references.

Chander, G., Markham, B. L., & Helder, D. L. (2009). Summary of current radiometric calibration coefficients for Landsat MSS, TM, ETM+, and EO-1 ALI sensors. Remote sensing of environment, 113(5), 893-903.

**L46. "3 to 15 years", is not accurate. Bindschadler and Scambos., 1991 used a cross-correlation algorithm on two images separated by roughly 1 year. Similarly, Bindschadler et al., 1996 also uses 1 year image-pairs (see Table 1 of their paper). Wulder et al does not contain ice velocity maps prior to the 1990s.**

Response:

(Line 54) We revised the text to "*... ... have been used to create regional velocity maps with a time span ranging from 1 to 23 years (Bindschadler & Scambos, 1991; Bindschadler et al., 1996; Wang et al., 2016; Cheng et al., 2019; Rignot et al., 2019)*". Here we deleted Wulder et al. 2019.

The time spans for the maps in the cited papers are: ~ 1 year in Bindschadler & Scambos (1991), 1 to 7 years in Bindschadler et al. (1996), 2 to 23 years in Wang et al. (2016), 1 to 15 years in Cheng et al. (2019), and 1 to 15 years in Rignot et al. (2019).

**L46. Can you define after which time span the overestimation is significant? (2 yr, 3yr ?). I found the use of images acquired more than 2 years apart quite rare, or limited to few points (large rifts for example).**

Response:

(Line 425) In Discussion we added: "……*Our experiment results show $\Delta t_{Ref}$ as 3.2 years, 3.0 months, and 1.4 months for TG, PIG, and Jakobshavn Isbrae (JI), Greenland, respectively. Thus, the estimated OE-free time spans appear to be related to ice flow dynamics of the glaciers. In Experiments 1 and 2, we used 1 year for TG and 3 months for PIG. We suggest that a linear regression for $\Delta t_{Ref}$ estimation be performed before extensive historical velocity mapping would be carried out.*" The detailed reasoning is given in the responses to General Comments (4.1).

**L49-51. From these lines it is a bit difficult to understand the overestimation issue. Please, extend a bit this description with more details, and split the sentence in two or**

**three parts.**

Response:

(Line 57) We explained it in a more mathematical or physics way. "*For example, at time1 a feature, with an initial velocity $v_0$ at the first location, is taken in the first image. The same feature is tracked in the second image taken at time2 after traveling at the velocity $v_0$ and an acceleration a for a time span of $\Delta t$ (time2-time1). Thus, the velocity $v=v_0+a\Delta t$ increases along with the time span $\Delta t$ if acceleration a exists. Given a constant acceleration, the velocity can be overestimated if the time span is long. Or the velocity overestimation is proportional to the time span.*"

**L52. Greene et al., 2020b; the reference list just says (Personal communication, comments on a manuscript), which I found a bit weak for a reference of a concept that is the base of this paper.**

Response:

(Line 691) We understand the concern. Chad Greene is now Referee #2 who volunteered to make the referee information open. We added Greene's Figures in Appendix as Figure A1 and quoted his text.

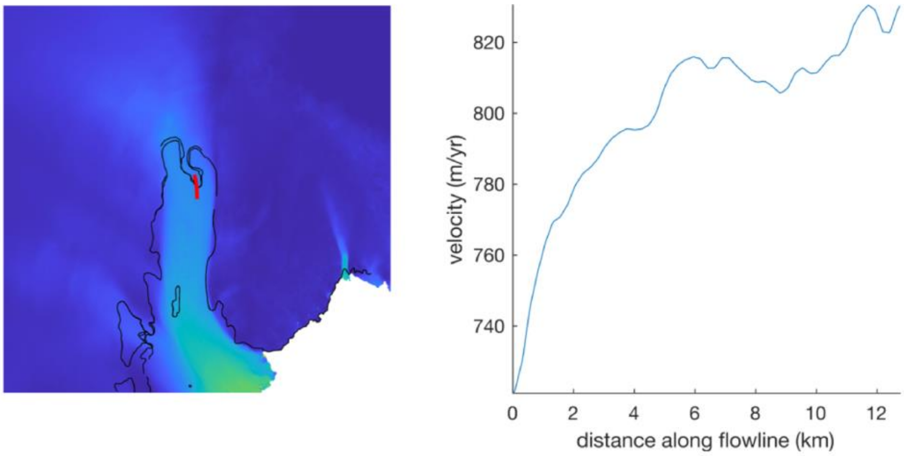

Figure A1. Velocity map of the Totten Glacier from ITS_LIVE (left) used to explain the concept of velocity overestimation caused by acceleration (right): "*Over 16 years, that parcel of ice travels about 13 km downstream (red path). It begins at a velocity of about 720 m/yr, and in the first 8 months it travels at an average rate very close to 720 m/yr. But then the ice picks up speed as it moves downstream, so in the first 10 years it does not travel just 7200 m—it actually travels about 7900 m, or an average speed of 790 m/yr……*" (Greene, 2020b).

**L53. The overestimation calculation over Pine Island Glacier is derived later in the manuscript, hence remove this part of the sentence.**

Response:

The "Pine Island" part of the sentence is deleted.

**L54. I would like to see a complete comparison of the simple method from Berthier et al., 2003, with the approach proposed here in the discussion section.**

Response:

(Line 64) Here we first present an analytical proof, and then we added a section of experiment results of TG in Discussion.

Assume that a tracked feature flows from A to B over a period of n years, with the middle point of AB denoted as M (arriving in m years); $v_0$ is the initial velocity at A; $a$ is acceleration that is constant both spatially and temporally (not a requirement in this paper). The following figure is a simplified situation (straight flow line, otherwise Lagrangian path and L-velocity have to be used).

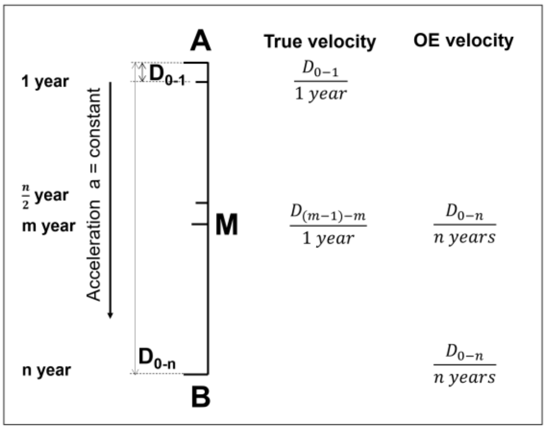

Figure R1-10. Analytical description of the "midpoint" method for OE correction.

The one year (or shorter) "true" velocity at A (year 0 - 1) is $V_{0-1} = \frac{D_{0-1}}{1\,year}$; the one year (or shorter) "true" velocity at M (year (m-1) – m) is $V_{(m-1)-m)} = \frac{D_{(m-1)-m}}{1\,year}$; the overestimated velocity at A using the n year segment AB (year 0 - n) is $V_{0-n} = \frac{D_{0-n}}{n\,year}$. The math or physics problem is

$$V_{(m-1)-m)} = \frac{D_{(m-1)-m}}{1\,year} \overset{?}{=} V_{0-n} = \frac{D_{0-n}}{n\,year} \qquad \text{(R1-1)}$$

We further calculate the one year "true" velocity at M:

$$V_{(m-1)-m} = \frac{D_{(m-1)-m}}{1\,year} = \frac{V_{(m-1)} \cdot 1\,year + \frac{1}{2} \cdot a \cdot 1\,year^2}{1\,year} = V_{(m-1)} + \frac{1}{2} \cdot a \cdot 1\,year = V_0 + a \cdot (m-1)\,years + \frac{1}{2} \cdot a \cdot 1\,year = V_0 + a \cdot \left(m - \frac{1}{2}\right)\,years.$$

On the other hand, the overestimated velocity is

$$V_{0-n} = \frac{D_{0-n}}{n\,years} = \frac{V_0 \cdot n\,years + \frac{1}{2} \cdot a \cdot (n\,years)^2}{n\,years} = V_0 + \frac{1}{2} \cdot a \cdot n\,years$$

Therefore, if Equation (R1-1) holds we must have

$$\left(m - \frac{1}{2}\right) = \frac{n}{2};\ \mathrm{m} = \frac{n+1}{2}. \qquad \text{(R1-2)}$$

However, given that it is a simplified uniformly accelerated motion, the tracked feature arrives at the halfway (M) in m years, which must be greater than half time $\frac{n}{2}$, namely

$$m > \frac{n}{2}. \qquad \text{(R1-3)}$$

Therefore, we have

$$V_{(m-1)-m} = V_0 + a \cdot \left(m - \frac{1}{2}\right) years > V_0 + a \cdot \left(\frac{n}{2} - \frac{1}{2}\right) years = V_{0-n} - a\frac{1}{2} year.$$

We further have

$$V_{(m-1)-m} > V_{0-n} - a\frac{1}{2} year. \qquad \text{(R1-4)}$$

If a = 0, $V_{(m-1)-m}$ and $V_{0-n}$ are the same. Otherwise, they may be different, depending on acceleration “a”. Thus, for glaciers with low to median range acceleration (weak spatial gradient), the overestimation may have been corrected to a good percentage by “assigning velocities at middle points of segments”. But if $a \neq 0$, the velocity at midpoint M is different from $V_{0-n}$.

We added a new section in Discussion:

(Line 484) “***4.4 Comparison with the “Midpoint” method***

*The “Midpoint” method presented in Berthier et al. (2003) compensates overestimations by assigning the overestimated velocities to middle points of the trajectories. We use the velocity measurements in Experiment 1 to compare the performances of these two OE correction methods. Since Area 4 was affected by a calving event during the time span, here we use other four areas (Areas 1, 2, 3, and 5, Fig. 4a). We estimated the E-velocities of 7 years (2013-2020) $V^E_{2013-2020}$ and assigned them to the midpoints of the trajectories in the four areas (Table A4). They were then compared to the one-year span velocity $V^E_{2013-2014}$ at the midpoints to calculate the bias $\varepsilon_M$. Similarly, the same overestimated 7-year span E-velocities $V^E_{2013-2020}$ were corrected using the OE correction method of this paper and assigned to the start points of the trajectories as $V_{corrected}$ (Table A4). They were then compared to the one-year span $V^E_{2013-2014}$ also, but at the start points to calculate another set of bias $\varepsilon_S$. As shown in Table A4, the proposed OE correction method achieved a higher overall accuracy of 4 ±10 m $a^{-1}$, compared to 12 ±14 m $a^{-1}$ of the “midpoint” method.*”

**Table A4. Comparison of the proposed OE correction method with the “Midpoint” method in Berthier et al. (2003)**

| *Area ID* | *OE velocity assigned to midpoint* | *1-year map at midpoint* | *Bias* | *OE-corrected velocity assigned to start point* | *1-year map at start point* | *Bias* |
|---|---|---|---|---|---|---|
| | $V^E_{2013-2020}$ (m $a^{-1}$) | $V^E_{2013-2014}$ (m $a^{-1}$) | $\varepsilon_M$ (m $a^{-1}$) | $V_{corrected}$ (m $a^{-1}$) | $V^E_{2013-2014}$ (m $a^{-1}$) | $\varepsilon_S$ (m $a^{-1}$) |
| **1** | 843 | 860 | 17 | 824 | 824 | 0 |
| **2** | 1008 | 1007 | -1 | 999 | 1007 | 8 |
| **3** | 1317 | 1332 | 15 | 1297 | 1280 | -17 |
| **5** | 789 | 807 | 18 | 759 | 754 | -5 |
| **MEAN** | **989** | **1001** | **12** | **970** | **966** | **-4** |

| | | |
|---|---|---|
| **RMSE** | **14** | **10** |

**L108. Please add reference to Figure 1a,b,c to help the reader's understanding of the whole concept.**

Response:

(Line 116) Wherever appropriate, we added references to these figures in the text for more clarity. "*As the time span increases at a fixed rate of 1 year, the traversed straight-line distance* $D_{0\text{–}i}$ *(red lines in Figure 1b), correspondingly E-velocity* $V_{0-i}^{E}$*, increases rapidly because of the acceleration over the traverse (Fig. 1c). In principle, every* $V_{0-i}^{E}$ *(i=1, 2, ...n) value represents the velocity at the same point* $P_o(x_o, y_o)$ *(Figure 1a) in these n velocity maps. In the cases where* $Image_i$ *were not available and thus the maps* $V_{0\text{–}i}$ *(i=1, ... n-1) were not produced, we only had the map* $V_{0\text{–}n}$ *with the longest span of n years. It is obvious that at* $P_o(x_o, y_o)$ *its n-year velocity* $V_{0-n}^{E}$ *is significantly larger than the 1-year velocity* $V_{0-1}^{E}$ *(Figure 1c).*

**L110. Here and in the remaining of the manuscript you use 1 year ice velocity as a reference map. But does your method still apply for very fast glaciers? For example Jakobshavn Isbrae (Greenland), or Penguin gl. (Patagonia) are flowing at speeds that are exceeding 10 km/yr, hence there is good chances of acceleration along flowlines within that year. Can you please discuss this point here? And better specify the use of a 1 year ice velocity map as a reference for your method.**

Response:

(Line 123) We agree with you that the time span of the reference (or baseline) velocity map may be different for different glaciers. We changed the sentences: "*Here we use a velocity map of a 1-year span as a baseline ("overestimation free") throughout the paper for simplicity, which can be changed for glacier regions of different ice flow dynamics (spatial acceleration, mainly caused by bed topography and slopes). For example the baseline span is one year for TG in Experiment 1 and three months for PIG in Experiment 3. We require that the overestimation of the baseline map is negligible, or smaller than* $\sigma$ *(velocity mapping uncertainty).*"

(Line 419) We also added a section in Discussion to introduce an analytical method for determining the threshold of an "OE-free" time span: "*4.1 Threshold of the overestimation-free time span for trajectory segments*"

**L114-115. Does it depends on the speed of the glacier? i.e this assumption still hold for Jakobshavn Isbrae flowing at more than 15 km/yr ? Or Penguin gl. In Patagonia (12 km/yr) ?**

Response:

(Line 123) Yes, it does depend on speed/acceleration of the glaciers. Our new experimental results proved that also. In the earlier part of the paper we added: "*Here we use a velocity map of a 1-year span as a baseline ("overestimation free") throughout the paper for simplicity, which can be changed for glacier regions of different ice flow dynamics (spatial acceleration, mainly caused by bed topography and slopes). For example the baseline span is one year for TG in Experiment 1 and three months for PIG in Experiment 3. We require that the overestimation of the baseline map is negligible, or smaller than $\sigma$ (velocity mapping uncertainty).*"

(Line 425) In Discussion we added: "*……Our experiment results show $\Delta t_{Ref}$ as 3.2 years, 3.0 months, and 1.4 months for TG, PIG, and Jakobshavn Isbrae (JI), Greenland, respectively. Thus, the estimated OE-free time spans appear to be related to ice flow dynamics of the glaciers. In Experiments 1 and 2, we used 1 year for TG and 3 months for PIG, respectively. We suggest that a linear regression for $\Delta t_{Ref}$ estimation be performed before extensive historical velocity mapping would be carried out.*"

**Section 2.2. I am getting lost with the notation, between the U, S, Map, V…. What do you mean by Maps? Maps of ice velocity I guess, than why introducing Maps if you have later V? Why not just using V, and add E and L for Eulerian and Lagrangian as indice (VE and VL).**

Response:

We accepted your suggestion and used $V$ for map, $V^E$ for Eulerian and $V^L$ for Lagrangian velocity throughout the manuscript.

**Figure 2. Please give a more comprehensive caption of Figure2. This one is just not enough to understand what is there. What the difference between the two Lagrangian lines mean? See earlier comment on the writing of equation to simplify the text and improve the understanding the paper. I guess you have Map$_{0\text{-}1}$ to specify that the Lagrangian is only calculated with Map$_{0\text{-}1}$ ? This should be specified in the caption.**

Response:

(Line 136) The caption of Figure 2 is extended according to your suggestion. "***Figure 2: Derivation of equation for overestimation correction using L-velocity. Eulerian velocities $V_{0-i}^{E}$ (i=1, 2, … n) are represented as bars. The red line is the average Eulerian velocity $\overline{\boldsymbol{V}}_{\boldsymbol{0-i}}^{\boldsymbol{E}}$ of $V_{0-1}^{E}$ and $V_{0-n}^{E}$ . The blue line is the average Lagrangian velocity $\overline{\boldsymbol{V}}_{\boldsymbol{0-i}}^{\boldsymbol{L}}$ of $V_{0-1}^{L}$ and $V_{0-n}^{L}$ derived from $V_{0-1}^{E}$. The black line is the average Lagrangian velocity $\overline{\boldsymbol{V}}_{\boldsymbol{0-i}}^{\boldsymbol{L(n)}}$ of $V_{0-1}^{L(n)}$ and $V_{0-n}^{L(n)}$ derived from $V_{0-n}^{E}$.***"

**L125-155. I think that the choice of hyperscript and subscript in equations could be simplified for the seek of the reader's understanding. First, ice velocity maps are defined as Map$_{0\text{-}i}$, V is used for Eulerian ice velocity and U is used for Lagrangian ice velocity. All of these are referring to ice velocities, so I suggest you switch to V for the velocity maps, V$_{E(0\text{-}i)}$ for the Eulerian speeds and V$_{L(0\text{-}i)}$ for the Lagrangian ones. You could also do V$^{L}_{0\text{-}I}$ and V$^{E}_{0\text{-}I}$, since I don't think that the use of J at line 240 is necessary for**

**understanding (you could just say in the text that you calculate the overestimation of all sub-images).**

Response:
We accepted your suggestion and used *V* for map, $V^{E}_{0-i}$ for Eulerian and $V^{L}_{0-i}$ for Lagrangian velocity throughout the manuscript.

**L 133. What do you define as a "short" lagrangian trajectory? This should depend on the glacier speed (see earlier comments on fast flowing glaciers), hence the distance where this premise holds decreases when the glacier speed increases (which is in part linked to the local bedrock slope). Furthermore, this premise holds if you assume that the point moved on a straight line within this short time span.**
**What do you define as short time span? If I assume this is 1 year, this premise might be true for some ice shelves, but what happens if the flow changes direction? This might happen within 1 year for example for George VI, Abbott, Dotson/Crosson or the Getz ice shelves in Antarctica.**

Response:
(Line 152) We added a section in Discussion to introduce a linear regression method for determine the threshold of an "OE-free" time span $\Delta t_{Ref}$ (see responses to "General Comments" above). The result shows that $\Delta t_{Ref}$ is ~3.2 years ($R^2$=0.96) for TG. Thus, $\Delta t_{Ref}$=1 year would be appropriate for a large number of glaciers in Antarctica, including TG. However, a reduced time span of ~3.0 months ($R^2$=0.99) for PIG and ~1.4 months ($R^2$=0.86) for Jakobshan Isbrae in Greenland should be used. As shown in the proposed regression equation, this threshold changes with how "fast" the ice flows in a glacier.
We changed the text for *Premise I* to: *"**Premise I:** Within a baseline time span (e.g., 1 year or shorter) each segment (from $P_{i-1}$ to $P_i$ in Figure 1b) is relatively short and the E- and L-velocity difference is smaller than σ. Furthermore, over the map span of n years (e.g., 5-10 years or longer) the accumulated E- and L-distances along the entire trajectory do not deviate significantly from each other, so that the maximum velocity difference in the Lagrangian and Eulerian frameworks (end points of blue and red lines in Fig. 2) is limited within a threshold ($V^{L}_{0-n} - V^{E}_{0-n} \le k\ \sigma$), where k is a constant and σ is the velocity mapping uncertainty."*

**L134-136. Here the use of the i=1,2,…,n is confusing. You are describing the case of a short time span, hence why not just using the $V_{0-1}$ and $U_{0-1}$ (as you just said in the previous lines)? Or $V_{i-i+1}$ ? What is a short time span on Figure 2 ? All of this Premise holds in what you define as a "limited time span" and "short L trajectory" (which should be straight). Please clarify these points.**

Response:
(Line 152) Agreed. We now use "$P_{i-1}$ to $P_i$". Thus, we changed *Premise I*. See text above.

**The author choose to make a clear distinction between the theory vs the application,**

**which I think was a good idea, but here, it would help the readers to have some more self-explanatory examples, as it is done in section 2.3.**

Response:

(Lines 129-158) We revised the text of *Premises* and other paragraphs to make it more self-explanatory. For example, figures 1 and 2 are used in the text to explain the concept. That way we link the theory with “application” scenarios for better understanding.

**L 137. This could be reformulated, for the more clarity, to “In reality, the available historical images only allow us to produce eulerian velocity maps with a long timespan, i.e Map$_{0\text{-}n}$ which leads to the maximum overestimation value as defined in equation 2”**

Response:

(Line 159) Accepted. The sentence is rewritten accordingly.

**L138. “As we can only use Map$_{0\text{-}n}$ , the lagrangian velocity, for a long time span, is defined as follow”. Please also add a reference to the line in Figure 2.**

Response:

(Line 162) Yes, the sentence is changed to: “… … *Based on this map* $V_{0\text{-}n}$ *of n-year span, the i-year span L-velocity (black line in Figure 2) is defined as follow:*”

We added the text to the black line in Figure 2: “*Average L-velocity* $\overline{V}_{0-i}^{L(n)}$ *from* $V_{0-n}^{E}$”.

**L 141. “Consequently, the 1 year L-velocity U’$_{0\text{-}1}$…”**

Response:

Thanks. It is so changed.

**L145. What do you define again as a limited time span? If you compare Map$_{0\text{-}1}$ and Map$_{0\text{-}n}$, then you are comparing the smallest and largest time span, hence the use of the term “short timespan” is a bit confusing**

Response:

Now we clearly distinguish the baseline (or segment) span (e.g., 1 year or shorter) with the n year (max., entire trajectory) span (e.g., over 5-10 years) in *Premises I* and *II*.

(Line 152 in the marked-up manuscript) ***“Premise I:*** *Within a baseline time span (e.g., 1 year or shorter) each segment (from* $P_{i-1}$ *to* $P_i$ *in Figure 1b) is relatively short and the E- and L-velocity difference is smaller than* $\sigma$*. Furthermore, over the map span of n years (e.g., 5-10 years or longer) the accumulated E- and L-distances along the entire trajectory do not deviate significantly from each other, so that the maximum velocity difference in the Lagrangian and Eulerian frameworks (end points of blue and red lines in Fig. 2) is limited within a threshold (*$V_{0-n}^{L} - V_{0-n}^{E} \leq k\ \sigma$*), where k is a constant and* $\sigma$ *is the velocity mapping uncertainty.”*

(Line 169) ***“Premise II:*** *Within the time span of n years (e.g., over 5-10 years), the velocity field described by* $V_{0-1}^{E}$ *and* $V_{0-n}^{E}$ *does not change significantly, so that the line between* $V_{0-n}^{L(n)}$ *and* $V_{0-1}^{L(n)}$

*(black line in Fig. 2) and that between* $V_{0-n}^{L}$ *and* $V_{0-1}^{L}$ *(blue line in Fig. 2) are approximately parallel to each other. Accordingly, the difference between their simple averaged accelerations is within a threshold (*$\left|\overline{a}\left(V^{L(n)}\right) - \bar{a}(V^{L})\right| \leq k' \frac{\sigma}{\Delta t_n}$*), where* $k'$ *is a constant and* $\sigma$ *is the velocity mapping uncertainty and*

$$\overline{a}\left(V^{L(n)}\right) = \frac{V_{0-n}^{L(n)} - V_{0-1}^{L(n)}}{\Delta t_n},\ \overline{a}(V^{L}) = \frac{V_{0-n}^{L} - V_{0-1}^{L}}{\Delta t_n}. \qquad (7)”$$

**L145. I guess that the magnitude of the velocity Map$_{0\text{-}n}$ should be larger than Map0-1 , but the pattern is similar ? Can you provide a figure example with velocity direction to illustrate this point?**

Response:

We agree that the text along the three lines in Figure 2 is a bit confusing. The blue is the average L-velocity calculated from map $V_{0-1}$, and red line is the average E-velocity calculated from 1 year and n year E-velocities. Generally, the L-velocities are greater than E-velocities (see blue and red examples in Figure 5). We revised text in Figure 2 as follows:

*“Average L-velocity* $\overline{V}_{0-i}^{L(n)}$ *from* $V_{0-n}^{E}$ *(for black line)*

*Average L-velocity* $\overline{V}_{0-i}^{L}$ *from* $V_{0-1}^{E}$ *(for blue line)*

*Average E-velocity* $\overline{V}_{0-i}^{E}$ *from* $V_{0-1}^{E}$ *and* $V_{0-n}^{E}$ *(for red line)”*

**L148. The first part of the sentence can be removed since it has been described earlier, before Premise II. Then, you can just start with: “Hence, based on Premise II, we have….”**

Response:

Thanks. The text is revised accordingly.

**L150. Again, the U$_{0\text{-}n}$=V$_{0\text{-}n}$ is based on the fact that you are considering only short timespan. But is that the case if you use “0-n” ? (see earlier comment)**

Response:

If there is only spatial acceleration and ($V_{0-n}^{L} - V_{0-n}^{E} \leq k\ \sigma$) in *Premise I*, $V_{0-n}^{L} \approx = V_{0-n}^{E}$ (or $U_{0-n} \approx V_{0-n}$), OE can be corrected; otherwise there is also temporal acceleration, we correct spatial acceleration induced OE, but preserve temporal acceleration – induced OE in residuals. We added the following:

(Line 190) “***Overestimation Correction Theorem****: Assume that the necessary condition in Premise II is met, spatial acceleration - induced overestimations in long time span velocities* $V_{0-n}^{E}$ *can be corrected or reduced using the following Correction term, regardless of temporal acceleration:*

$$V_{0-1}^{E} = V_{0-n}^{E} + Correction + \varepsilon\,, \qquad (9)$$

$$Correction = V_{0-n}^{E} - V_{0-n}^{L(n)}. \qquad (10)$$

*If Premise I holds, Correction* $\approx -OE_{0-n}$*; otherwise,* $|Correction|<|OE_{0-n}|$*, preserving the velocity increases induced by temporal accelerations in the residual term ε (Fig. A2).*"

**L 151. I would reformulate this sentence to remind the reader about the aim of this paper : "Consequently, using the map with the longest timespan, we can go back to $V_{0-1}$ using a Correction term defined as Correction=$V_{0-n}$-$U'_{0-n}$".**

Response:

(Line 182) Thanks. The sentence is revised accordingly: "*Consequently, using the map with the longest time span, we can go back to* $V^{E}_{0-1}$ *using a correction term defined as Correction=*$V^{E}_{0-n} - V^{L(n)}_{0-n}$*:* "

**L 164. What does it mean to interpolate the positions to the sub grid-level ? Does it make any sense to interpolate the position at a higher level of resolution than the velocity field?**

Response:

(Line 215) The interpolation is not used to make a new higher resolution velocity map, but to determine the positions of the distance segments for L-distance integration. Thus, the intermediated sub-grid positions are used for a continuous distance integration. The sentence is changed to: "*The sub-grid positions of the monthly segments are interpolated for integration of the overall L-distance.*"

**Figure 3. Please add a general Figure of the entire Pine Island glacier, to check out where the location of your flowline is (similar as Figure 4).**

Response:

It is done. Thanks.

**L179-180. What about orthorectification errors in historical images ?**

Response:

(Line 232) We added it to the sentence: "*Despite the sub-pixel accuracy of the orthorectification of historical images and ……*"

**Acceleration computation: this has already been described L147. I would suggest to move this part earlier (or remove it).**

Response:

(Line 198) This section is now moved to the earlier part.

**Section 3.1. Since section 2.3 shows an example over Pine Island glacier, I don't know why the author didn't continue using this example. Since the overestimation is quite spectacular, I would strongly suggest to use Pine Island instead of Totten here.**

Response:

(Line 391) Yes, we accepted your suggestion and added the PIG results as a new section *Experiment 3.*

**L213. Why did you choose a 7 year trajectory ?**

Response:

(Line 266) We found the earliest available high-quality Landsat 8 images in 2013 and latest ones in 2020 (7 years). We changed the sentence: "*To avoid lower quality historical velocity maps that may influence the effectiveness of the validation, we use the earliest available high-quality Landsat 8 images from 2013 to 2020 to produce velocity maps* $V_{2013-i}$ *(i=2014, ... 2020)……*"

**L217. See previous comment on the choice of symbols in equations.**

Response:

They are all fixed throughout the manuscript (see responses to previous comments)

**L215. Do you generate the map separately or over the entire glacier directly ?**

Response:

Because we have to generate maps for 7 time spans for validating *Premises I* and *II*, it is a lot of work to produce them all. Therefore, we only mapped areas of 5 trajectories separately, instead of the entire glacier directly.

**L225. I am surprised about the error estimation here. Millan et al., 2019; had some smaller number for 1 year map of ice velocity using Landsat-8. Can you discuss why is that? How does your map compare with available NSIDC data ? What is the difference with recent map assembled from sar interferometry? (see Mouginot et al., 2019)**

Response:

The highest accuracy of less than 1 m/year was achieved by using **InSAR** technique (Mouginot et al., 2019), which used the InSAR phase information and the data requirement is generally high. An accuracy of 10 m/year was reported by Millan et al. (2019) for average annual velocities from **multiple individual velocities** derived from Sentinel-2 (10 m resolution) and Landsat-8 (15 m resolution) and other images. Similarly, Gardner et al. (2018) also achieved an accuracy of 10 m/year of annual velocities by averaging velocities derived from **multiple Lansat-7 and -8 image pairs** (15 m resolution). However, our velocity sub-maps (5 areas) were built from **only one Landsat-8 image pair** (15 m resolution) for each map. Given the accuracy of individual maps as $\sigma^2_i$, the accuracy of the averaged velocity is $= \frac{\sqrt{\sum \sigma_i^2}}{n}$. In general, the accuracy of the average annual velocity should be smaller than that of the velocity of an individual pair. Therefore, our accuracy of 20 m/year for 1-year (individual pair) and 3 m/year for 7-year (individual pair) velocities are reasonable values.

References

Mouginot, J., Rignot, E., & Scheuchl, B.(2019). Continent wide, interferometricSAR phase, mapping of Antarctic icevelocity. Geophysical Research Letters,46, 9710–9718.

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Gardner, A. S., Moholdt, G., Scambos, T., Fahnstock, M., Ligtenberg, S., van den Broeke, M., and Nilsson, J.: Increased West Antarctic and unchanged East Antarctic ice discharge over the last 7 years, The Cryosphere, 12, 521–547, https://doi.org/10.5194/tc-12-521-2018, 2018.

**L239. Is it "faster than" or "close" ?**

Response:

(Line 302) We deleted "close to".

**L240. I think that the use of "J" in exponent is adding to much complication (see earlier comment). You can just specify that you do the calculation for all sub-images.**

Response:

We accepted your suggestion and removed "J" throughout the manuscript.

**L257. The "apparent parellelity" is quite subjective I think. Is the premise II validated for case 2c and 2a?**

Response:

(Line 320) Cases 2c and 2a are ok (average difference of 1.2 m a$^{-2}$). Cases 3c and 3a are 2.1 m a$^{-2}$ (max) that is still within the allowable uncertainty of 3 m a$^{-2}$ determined based on the velocity mapping uncertainty of 20 m a$^{-1}$. To make it objective, we changed "*apparent parellelity*" to "*relatively well-maintained parellelity*".

**Figure 5. Add the direction of the flow in the sub-images. You could also consider using different symbols for the 7 yr and 1 yr trajectory and use a color gradient for the position of the points that changes with the year. This "year" color could then also be used in the scatter plots.**

Response:

We revised Fig. 5 accordingly.

**Table 1. Please add more details on the caption of the Table, ie, the content of each column.**

Response:

(Line 326) The caption is revised: "***Table 1. Velocity and acceleration in Eulerian and Lagrangian frameworks used for validation of the overestimation correction method. "Actual E-velocity and OE" lists actually mapped 1-year and 7-year E-velocities and their differences as overestimations in all five areas. "Premise I" contains 7-year L-velocities computed from the 1-year velocity map and corresponding L- and E-velocity differences, which are used for validating Premise I. "Premise II" illustrates averaged L-accelerations computed from the 7-year and 1-year velocity maps, respectively, as well as their differences, which are used for validating Premise II. "Overestimation correction" presents 7-year L-velocities computed from the 7-year map, overestimation corrections, and E-velocities and residuals (or errors) after correction.***"

**L302. The acronym OE has been defined before.**

Response:
We deleted "(OE)".

**Section 4 Discussion. Can you discuss the performance of your method, with the relatively simple approach defined by Berthier et al., 2003? I think that the section is missing some discussion on the applicability of the method to 1) fast glacier, 2) the glacier geometry, which can be much more complex than the glaciers that were used here to validate the method (not straight) (see earlier comment). An additional discussion about the significance of the correction, with respect to the seasonal variation in ice flow velocity of the glacier should also be discuss, if the method is expected to be applicable in Greenland. Specifically, does the magnitude of the correction could exceed the natural variability of the glacier? I guess that the amount of acceleration with a flowline would need to be significant in order to induce a correction that would exceed the variability of the seasonal signal?**

Response:
The comparison results with the "Midpoint" method are presented in the response to comment L54. (Line 484) We also added a section "*4.4 Comparison with the "Midpoint" method*" in Discussion.
The responses related to fast glaciers, complex geometry, seasonal variability, applicability in Greenland etc. are presented in responses to General Comments and in Discussion.

**Figure 5-6. Can you provide a figure of the corrected ice velocity? Maybe a difference map.**

Response:
We produced the map of DG after correction (Fig. R1-7b). It does not appear distinctly different from the map before correction (Fig. R1-7a = Fig. 7b). Thus, we will not include this corrected map in the main text. The map with the OEs (Fig. 7c) is actually the difference map. Similarly, the corrected submaps in TG are also visually not distinct from those before correction. Similarly we will not add the corrected or difference map to Fig. 5.

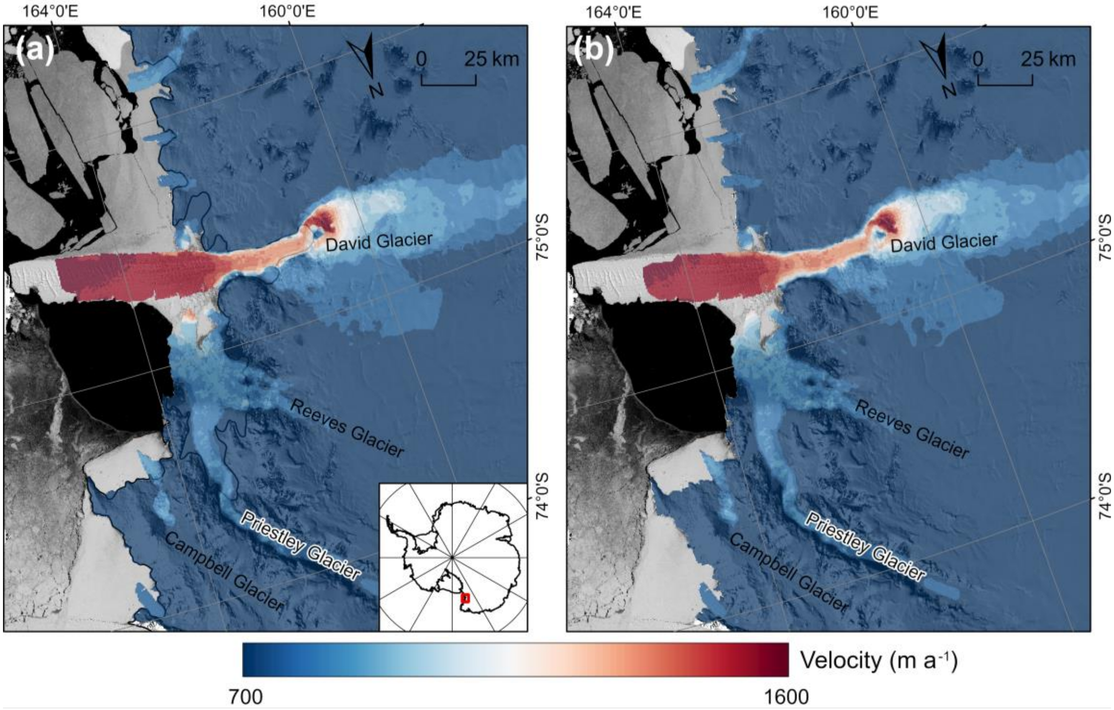

Figure R1-7. E-velocity in TG: (a) before OE correction, and (b) after OE correction.

We appreciate the constructive comments from two reviewers and the community. Our manuscript will be much improved by their input. We have made changes to our manuscript. In the following responses, we use "**bold**" text for comments, "non-bold" text for our responses, and "*italic*" for changed text in the manuscript.

**Referee #2 (Chad A. Greene):**

**This paper identifies three key shortcomings of a common velocity measurement technique, and provides a solution that addresses all three. At issue are 1. the true location of a feature tracked velocity measurement, 2. the acceleration that a parcel of ice may experience between image acquisition times, and 3. the fact that ice does not always move in a perfectly straight line between image acquisition times. The problem and solution are described well in this paper, and the authors demonstrate that they have a good handle on the data and how velocities are interpreted from a glaciological standpoint.**

**This work will be of value to the community, both to raise awareness of the overestimation issue, and to provide a solution to it. I recommend publication, with just a few suggestions that may help readers understand the impact of overestimation and how it should affect our interpretation of previous studies.**

## Main Issues

**The paper does a good job of describing the problem and solution from a technical standpoint, and anyone who has written feature-tracking algorithms will benefit from reading the paper. However, there are many readers who don't write their own algorithms, but will nonetheless want to understand how overestimation might affect their scientific results. Some work could be done in this paper to better communicate the overall impact of how overestimation impacts long-term studies.**

**Here's a type of analysis that I would find much more insightful than the stats for PIG, Totten, and David Gl that are currently presented in the abstract: I would like to see a figure showing Eulerian grounding line flux calculations as a function of dt, where dt might range from a day to 20 years. This would provide readers with some intuition for a threshold value of dt, beyond which Eulerian measurements produce significantly different estimates of ice flux. It's possible that the percentage reduction in GL flux as a function of dt might vary regionally, and that diversity could be interesting to show as well.**

Response:

We performed an experiment of "GL flux vs. time span" for PIG. We used a baseline velocity map of PIG 2013 (Fig. R2-1a) from ITS_LIVE (Gardner et al., 2019). The flux gate (red line) is set along the grounding line (GL, black line) and separated into flux nodes every 240 m where the ice flow velocity and ice thickness data (BedMachine) are used to calculate ice flux. We calculated L-velocities along GL with time spans of 1-15 years based on the 2013 E-velocity map (Fig. R2-1b). Instead of suggested 20 years, we used 15-year time span mainly because the 20-year tracking distance from GL would run beyond the ice shelf front.

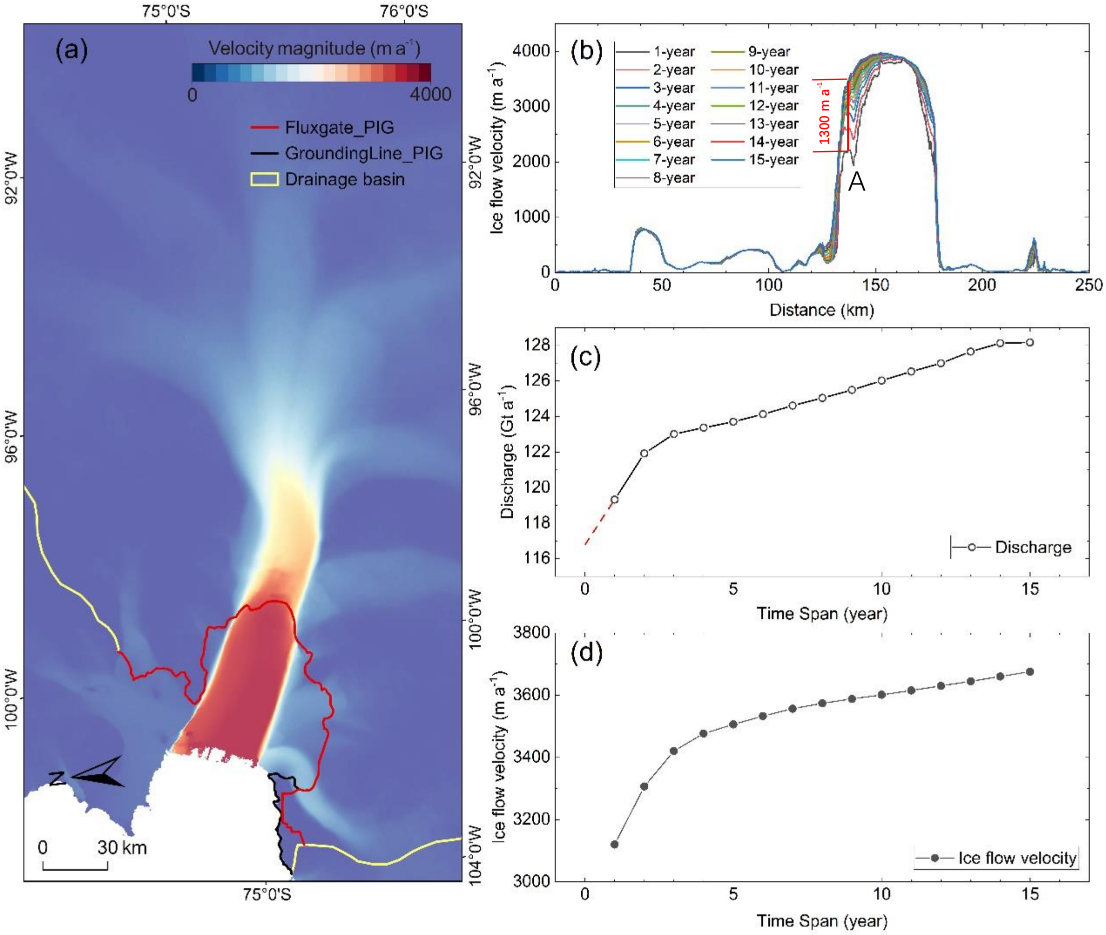

Figure R2-1: (a) Annual ice velocity map of PIG (2013) from ITS_LIVE (Gardner et al., 2019) with the flux gate (red line) set on GL (black line), (b) L-velocities of 1- to 15-year span $V_{0-i}^{L}$ ($i=1, 2, \dots 15$) along GL calculated from the 2003 annual map, (c) GL ice flux (estimated from $V_{0-i}^{L}$) vs. time span, and (d) L-velocities $\bar{V}_{0-i}^{L}$ (averaged over the GL portion across main trunk, over 1500 m a$^{-1}$) vs. time span.

The L-velocity along GL increases mainly in the margin areas of the main trunk as the time span increases (Fig. R2-1b). The maximum velocity OE reached ~1300 m a$^{-1}$ for the 15-year span (marked “A” in Fig. R2-1b). In this place, the annual maximum OE rate started at ~461 m a$^{-1}$ and decreases to ~31 m a$^{-1}$ as the time span increases, because the later part of the trajectory reached the flat part of the ice shelf front (high velocity, but low acceleration).

Consequently, the OE induced GL flux increase (flux overestimation) speeds up quickly at an annual rate of ~2.1 Gt a$^{-1}$ for the first 4 annual spans, by ~6.3 Gt a$^{-1}$ from ~116.7 Gt a$^{-1}$ to ~123.0

Gt $a^{-1}$. Thereafter the annual rate is maintained at 0.4 Gt $a^{-1}$ until the 15-year span, reaching the maximum flux OE of 11.5 Gt $a^{-1}$. This flux OE has the same trend pattern as the average L-velocity over the GL portion across main trunk (Fig. R2-1d), indicating that the main flux OE "came" across GL from the main trunk portion.

Due to the limited time and amount of work, we will investigate the same issue in other glacier regions (the Totten and David glaciers) in the future. We added the following text in Discussion:

(Line 471 in the marked-up manuscript) "*……In addition, based on an annual E-velocity map of 2013 in PIG from ITS_LIVE, L-velocities along grounding line (GL) with time spans of 1 to 15 years and the associated GL ice flux were computed. The results show that the velocity OEs of the 15-year span can reach up to ~1,300 m $a^{-1}$ in the GL region. Such OEs in velocity can further cause an overestimation in GL flux, which is negligible within a 3-year span ($\leq \sigma_{Flux}$). We used $\sigma_{Flux}$=5.8 Gt $a^{-1}$ based on the flux uncertainty in PIG reported by Rignot et al. (2019). The GL flux OE increases rapidly by ~6.3 Gt $a^{-1}$ within the first 4-year span; thereafter it slows down until a maximum of 11.5 Gt $a^{-1}$ is reached at the 15-year span. Therefore, the influence of the velocity OEs on the GL flux appears to be not very significant (11.5 Gt $a^{-1}$ $\leq 2\ \sigma_{Flux}$).*"

**In addition to a figure showing how dt affects GL flux in Eulerian measurements, I'd like some clear guidance in the abstract for when the Eulerian approximation is sufficient or insufficient.**

The above simulation result shows that a GL flux OE of ~11.5 Gt $a^{-1}$ in PIG would be induced by a 15-year span L-velocity map, which is significant in comparison to the flux uncertainty $\sigma_{\text{Flux}}$ of 5.8 Gt $a^{-1}$ in PIG given by Rignot et al. (2019). Therefore, assuming $\sigma_{\text{Flux}}$ = 5.8 Gt $a^{-1}$, we estimated a "flux-OE-free" time span of ~3 years using the curve in Fig. R2-1c.

Please see the added text in Discussion (Line 476; above).

(Line 26) We added a statement in Abstract: "*……Our experiment results in PIG show that, if not corrected, the OEs can further cause an overestimated grounding line flux that is negligible within a 3-year span, but reaches the maximum of 11.5 Gt $a^{-1}$ with a 15-year span……*"

**In the abstract and/or discussion, I suggest flipping the logic/wording around at least once to make it clear that the overestimation of historical velocities could mean that previous papers have underestimated the magnitude of glacier acceleration over the past few decades. It's only a minor change in wording, but I think it's an important take-home message of this paper that should be stated directly.**

Response:

(Line 27) We added a statement in Abstract: "…… *The implication is that, when using newer velocity maps of short spans along with historical maps of long spans produced in previous studies over the past few decades, the overestimation of historical velocities could have caused an underestimation* *in the long-term acceleration magnitude. We recommend that overestimations of more than the velocity mapping uncertainty (1σ) be corrected……*"

(Line 498) We added a statement in the discussion section: "*……The implication is that, when using newer velocity maps of short spans along with historical maps of long spans produced in previous studies over the past few decades, the overestimation of historical velocities could have caused an underestimation in the long-term acceleration magnitude. On the other hand, new efforts in historical velocity mapping at an ice sheet – wide or large regional scale should be made with a full consideration of OE corrections……*"

## Minor comments

**Abstract: The case studies of PIG, Totten, and David Glacier provide decent testing grounds for the methods presented in this paper, but the details of these studies feel somewhat anecdotal and very specific to the exact images that were used in these particular cases. I recommend generalizing the results in the abstract to give readers a better overview of the problem. Only after discussing the overall impact of the overestimation, then it may be helpful to mention a specific case of PIG, Totten, or David to as a tangible example.**

Response:

(Line 11) Accepted. We revised Abstract accordingly: "*…… In comparison to velocity maps derived from recent satellite images of monthly to weekly time spans, historical maps, from before the 1990s, generally cover longer time spans, e.g., over 10 years, due to the scarce spatial and temporal coverage of earlier satellite image data. We found velocity overestimations (OEs) in such long-span maps that can be mainly attributed to ice flow acceleration and time span of the images used. If used for long-term change studies, these OEs in historical velocities may further affect the estimated trends of ice flow dynamics and mass balance. For example, the OEs can reach from ~69 m* $a^{-1}$ *(7-year span) in Totten Glacier (TG), East Antarctica, up to ~930 m* $a^{-1}$ *(10-year span) in Pine Island Glacier (PIG), West Antarctica……*"

**L29: This line mentions "the input-output method" and some good references are provided for it, but some readers may be unfamiliar with the term. If the term is necessary for some point that's being made, then I recommend briefly describing what is meant by "the input-output method" here. If the term is not important for this paper, then consider removing it.**

Response:

The phrase "using the input–output method" is removed.

**L69: Recommend changing "It is proven that…" to "We show that…" to make it clear that the correction is original work that is presented in this paper.**

Response:

We changed the sentence accordingly.

**L80: I'm not entirely sure what "descending passages" means. Consider rewording.**

Response:

(Line 91) The sentence is revised: "*We describe an acceleration - induced overestimation using a typical scenario in AIS (Fig. 1a) where ice flow accelerates over a long slope from several glaciers originated from the inland interior, running through the main trunk, and discharging to the ocean*."

**L160: "At each grid…" I think this should be "At each grid cell…" or "At each pixel…"**

Response:

We changed it to "*At each grid cell* ……"

**Figure 5 is very compelling, and I want to make sure I understand it. Unfortunately, the labels and caption are somewhat cryptic, so I'm not sure if I'm even getting the main message right. The caption contains a list of the data labels that are mostly redundant with labels that are presented directly in the figure. What's missing is physical interpretation or any direct take home message. For example, the variables U, U', and V are labeled in the figure and in the caption, but there's no physical definition of what U, U', or V mean. Help readers by providing a sentence or two in the caption that directly states the main point and any secondary point(s) that may be worth noticing. The main point, I assume, is that the black line is consistently higher than the red and blue curves. State that in the caption, in terms of what it means physically. What causes the red and blue curves to cluster together or spread apart from each other? Mention the underlying mechanism in the caption. Most of these points are described in detail on page 11, but most readers will appreciate having the main points stated directly in the figure caption.**

Response:

(Line 292) The caption is revised according to the suggestions: "*Figure 5: Velocities in five areas (rectangles in Fig. 4) of the Totten Glacier are used to validate Premises I and II. (a) Panels 1a–5a show the reconstructed 1-year velocity maps* $V_{2013\text{-}2014}$ *in the areas; matched points (red triangles) are used to map E-velocity* $V_{2013-i}^{E}$ *(i=2014, …, 2020) (red lines in Panels 1c-5c); points along the flow line (blue dots) are tracked from the 1-year maps and used to calculate L-velocity* $V_{2013-i}^{L}$ *(blue lines in Panels 1c-5c). (b) Similarly, Panels 1b–5b illustrate the reconstructed 7-year velocity maps* $V_{2013\text{-}2020}$ *in the areas with the matched points (red triangles) for E-velocity* $V_{2013-i}^{E(7)}$*; points along the flow line (black crosses) are tracked from the 7-year span velocity map and used to calculate L-velocity* $V_{2013-i}^{L(7)}$ *(black lines in Panels 1c-5c). (c) In each area (Panels 1c–5c) the difference between red and blue lines increases with time, but is limited within k* $\sigma$ *at the end (Premise I), except a large k in Area 4 because of effect of a calving event; the black line is above blue line due to the spatial acceleration - induced OE over 7-year span; but they are relatively parallel (Premise II) and thus, a correction can be estimated.*"

We appreciate the constructive comments from two reviewers and the community. Our manuscript will be much improved by their input. We have made changes to our manuscript. In the following responses, we use "**bold**" text for comments, "non-bold" text for our responses, and "*italic*" for changed text in the manuscript.

**Community comment (Massimo Frezzotti):**

**Very interesting paper, the manuscript does not take in account the previous studies on glacier analyzed. To improve the results of their paper I suggest the authors to compare their result with previous ice velocity analysis using satellite image and also with GPS measurements.**

**For David, Reeves, Priestley several papers and measurements are available since 1998:**

**Frezzotti M., Capra A. & Vittuari L. (1998) Comparison between glacier ice velocities inferred from GPS and sequential satellite images. Ann. Glaciology, 27, 54-60,**

**Frezzotti M., I. Tabacco and A. Zirizzotti (2000) Ice discharge of eastern Dome C drainage area, Antarctica, determined from airborne radar survey and satellite image analysis. J. of Glaciology, Vol 46 (153), 253-273, DOI: 10.3189/172756500781832855**

**Danesi, S., Dubbini, M., Morelli, A., Vittuari, L., & Bannister, S. (2008). Joint geophysical observations of ice stream dynamics. In Geodetic and Geophysical Observations in Antarctica (pp. 281-298). Springer, Berlin, Heidelberg.**

**Rignot E, Mouginot J, Scheuchl B (2011) Ice flow of the Antarctic Ice Sheet. Science 333:1427–1430.**

**Stearns, L. A. (2011). Dynamics and mass balance of four large East Antarctic outlet glaciers. Annals of Glaciology, 52(59), 116-126.**

**Mouginot J, Rignot E, Scheuchl B, Millan R (2017) Comprehensive annual ice sheet velocity mapping using Landsat-8, Sentinel-1, and RADARSAT-2 data. Remote Sens 9:364–1370.**

**https://earthdata.nasa.gov/esds/competitive-programs/measures/ice-velocity-mapping-of-the-great-ice-sheets-antarctica**

**Moon, J., Cho, Y., & Lee, H. (2021). Flow Velocity Change of David Glacier, East Antarctica, from 2016 to 2020 Observed by Sentinel-1A SAR Offset Tracking Method. Korean Journal of Remote Sensing, 37(1), 1-11.**

**Your Sincerely**
**Massimo Frezzotti**

Response:

(Line 45 in the marked-up manuscript) Thank you for the comments, well received. We added some of your suggested references in Introduction: "*……, regional velocity maps at a seasonal or monthly scale have been generated from optical and SAR images (e.g., Landsat and Sentinel; Frezzotti et al., 1998, 2000; Nakamura et al., 2010; Zhou et al., 2014; Greene et al., 2017, 2018, 2020a; Moon et al., 2021).*"

Frezzotti M., Capra A. & Vittuari L. (1998). Comparison between glacier ice velocities inferred from GPS and sequential satellite images. Ann. Glaciology, 27, 54-60.

Frezzotti M., I. Tabacco and A. Zirizzotti (2000). Ice discharge of eastern Dome C drainage area, Antarctica, determined from airborne radar survey and satellite image analysis. J. of Glaciology, Vol 46 (153), 253-273, DOI: 10.3189/172756500781832855.

Moon, J., Cho, Y., & Lee, H. (2021). Flow Velocity Change of David Glacier, East Antarctica, from 2016 to 2020 Observed by Sentinel-1A SAR Offset Tracking Method. Korean Journal of Remote Sensing, 37(1), 1-11.

(Line 349) In *Experiment 2* where the David Glacier region is used for demonstration of the proposed method, we added the following sentences to recognize the previous work and comparative coverages of the velocity maps: "*…… Velocities in this region from 1988 to 1992 were mapped by using GPS and image feature tracking techniques (Frezzotti et al., 1998, 2000). A new GPS campaign was carried out in the region during 2005-2006 (Danesi et al., 2008). Velocity changes from 2016 to 2020 were detected using Sentinel-1A SAR images (Moon et al., 2021). In this experiment we produced a velocity map of the region from 64 Landsat images collected from 1972 to 1989 (Table A3)……*"