We appreciate the constructive comments from two reviewers and the community. Our manuscript will be much improved by their input. We have made changes to our manuscript. In the following responses, we use "**bold**" text for comments, "non-bold" text for our responses, and "*italic*" for changed text in the manuscript.

Referee #1

General comments

The paper by Li et al., 2021 presents an innovative method that aims at correcting glacier velocity overestimation, that are due to accelerations, when using long timespan. The paper is well presented with a clear structure, well written and the Figures are relatively clear.

A large share of the paper is dedicated to the description of the method, which is simple in principle, but that could be ambiguous to understand clearly. Consequently, I have few comments that I hope, will help to make the paper more understandable.

Among those comments, the definition of the different Premises needs a bit of clarification, and particularly on their area of validity (see below)……

Response:
In *Premises I and II* we clarified the correspondence between the short span (months to year) and "OE-free" (< σ) trajectory segments, as well as that between the longer span (5 to over 10 years) and difference of E- and L-velocities (< *k* σ) at the end of a trajectory.

"Premise I: *Within a period of n short time spans (e.g., 1 year or shorter) each segment (from* P_{i-1} *to* P_i *in Figure 1b) along a flow line is relatively short and straight, so that their accumulated curved L-distance* S_{0-i} *over a longer span (e.g., over 5-10 years) is not significantly different from the corresponding straight E-distance* D_{0-i}*; furthermore, their averaged velocity trends of* V_{0-i}^{L} *in the Lagrangian framework and* V_{0-i}^{E} *in the Eulerian framework (blue and red lines in Fig. 2) do not deviate significantly from each other, and their maximum difference is limited within a threshold (*$V_{0-n}^{L} - V_{0-n}^{E} \leq k\ \sigma$*), where k is a constant and* σ *is the velocity mapping uncertainty."*

"Premise II: *Within the time span of n years (e.g., over 5-10 years), the velocity field described by* V_{0-1}^{E} *and* V_{0-n}^{E} *does not change significantly, so that the line between* $V_{0-n}^{L(n)}$ *and* $V_{0-1}^{L(n)}$ *(black line in Fig. 2) and that between* V_{0-n}^{L} *and* V_{0-1}^{L} *(blue line in Fig. 2) are approximately parallel to each other. Accordingly, the difference between their simple averaged accelerations is within a threshold (*$\left|\bar{a}\left(V^{L(10)}\right) - \bar{a}\left(V^{L}\right)\right| \leq k' \frac{\sigma}{\Delta t_n}$*), where* k' *is a constant and* σ *is the velocity mapping uncertainty and*

$\bar{a}\left(V^{L(10)}\right) = \frac{V_{0-n}^{L(n)} - V_{0-1}^{L(n)}}{\Delta t_n}$, $\bar{a}\left(V^{L}\right) = \frac{V_{0-n}^{L} - V_{0-1}^{L}}{\Delta t_n}$ *."*

Their validity in different types of glaciers is introduced in the Discussion section (see following responses).

…… The discussion also needs to be supplemented with an overview of the method applicability to different glacier types, and specifically fast flowing glaciers (e.g Jakobshavn Isbrae), or with a more complex geometry (e.g Zachariae Isstrøm ice shelf, Getz Ice Shelf or George VI ice shelf). Another interesting point of discussion is the impact of the glacier seasonal variability. Are the corrections significant with respect to the natural variability of glacier flow? While, seasonal signals are not really pronounced in Antarctica, variability can be much greater in Greenland (cf. Joughine et al., 2020).

For example, using a 1-year velocity reference for a glacier like Jakobshavn Isbrae, might not be ideal, as the glacier is flowing at more than 15 km/yr (which increases the chances of acceleration along a flowline). Similarly does the premises still holds, for glaciers that are changing directions and not flowing in a straight line (for example the ice shelf of Zachariae Isttrøm before 2000)?

Response:
The Discussion section is restructured. At the beginning we added an overview statement: "*In this section we discuss the applicability of the proposed method in terms of overestimation-free time span, influence of complex glacier geometry, overestimation in fast flowing glaciers, and preserve of historical glacier change signature.*"

Then we added three new subsections to make a strong Discussion section according to the comments:

"4.1 Threshold of the overestimation-free time span for trajectory segments

The choice of a short time span Δt_{Ref} *for the "overestimation-free" segments along a trajectory in Premises I makes sure that the difference between the E-and L-velocities within the span (e.g., a few months to one year) is negligible, or less than* σ *(velocity mapping uncertainty). It is also the time span of the initial "OE-free" E-velocity map that is used for ice mass tracking and L-velocity computation in premise validation. Determination of this threshold has an implication on validation of Premise I, as well as the integration period of the trajectory segments from* P_i *to* P_{i+1} *(Fig. 3b). Estimation of the OE-free time span* Δt_{Ref} *can be performed by a linear regression between the E- or L-velocity* V *and time span* Δt, $V = K\,\Delta t + b$. *Given* σ *(20 m* a^{-1}*),* Δt_{Ref} *can be calculated as* $\Delta t_{Ref} = \frac{\sigma - b}{K}$. *Our experiment results show* Δt_{Ref} *as 3.2 years, 3.0 months, and 1.4 months for TG, PIG, and Jakobshavn Isbrae (JI), Greenland, respectively. Thus, the estimated OE-free time spans appear to be related to ice flow dynamics of the glaciers. In Experiments 1 and 3, we used 1 year for TG* *and 3 months for PIG, respectively. We suggest that a linear regression for Δt_{Ref} estimation be performed before extensive historical velocity mapping would be carried out.*

4.2 Glaciers with complex geometry and E- and L-velocity difference along trajectory

Furthermore, within a longer time span (e.g., over 5 - 10 years) in Premise I the difference between E- and L-velocities accumulated over all segments along the entire trajectory ΔV^{L-E}, i.e., the end-point deviation between the red and blue lines in Fig. 2, is measured with a more tolerable threshold of k times of σ (k σ). Although the OEs of the trajectory segments are controlled by Δt_{Ref}, ice mass moving along a curved flow line over this long span may result in an accumulative discrepancy. In Experiment 1 we showed that the E- and L-velocity difference ΔV^{L-E} in TG are within 2σ. Our further experiment in five smaller Antarctic glaciers with complex geometry, including the George VI, Abbott, Dotson, Crosson, and Getz glaciers, resulted in ΔV^{L-E} values that are negligible (smaller than 1σ) in all five glaciers. Thus, if Premise I is met (e.g., $k \leq 2$) there are mainly spatial acceleration-induced OEs, which can be effectively corrected. We found that in places, such as PIG in Experiment 3 and Area 4 in TG in Experiment 1, where temporal accelerations caused by basal melting and calving activities (Li et al., 2015; Joughin et al., 2020, 2021) exist the threshold k exceeded 2. However, the computed OE corrections can still remove the spatial acceleration-induced OE portion and leave the temporal acceleration-induced OE portion as a signature for long-term ice dynamics studies. Hence, Premise I is not a necessary condition, but a sufficient condition. Furthermore, the largest curvature-induced OE in PIG is 206 m a^{-1} over a 7-year span trajectory, among which 195 m a^{-1} (95%) was corrected.

4.3 OE estimation in fast flowing glaciers

We estimate OE corrections assuming that within a longer time span in Premise II the acceleration trend would not change significantly during the time span, $\left|\bar{a}\left(V^{L(n)}\right) - \bar{a}(V^L)\right| \leq k' \frac{\sigma}{\Delta t_n}$. As shown in Experiments 1 and 2, the acceleration trend difference is under $1 \frac{\sigma}{\Delta t_n}$ for both David Glacier and Totten Glacier that is one of the most dynamic glaciers in East Antarctica. Since the velocity requirements in Premise I and acceleration requirement in Premise II were met properly, we were able to correct in average 88% of the OEs in TG (up to 69 m a^{-1}, Table 1). Furthermore, in the fast-flowing PIG the acceleration trend differences $\left|\bar{a}\left(V^{L(n)}\right) - \bar{a}(V^L)\right|$ in all 5 areas are in average less than $3\frac{\sigma}{\Delta t_n}$. Correspondingly, the black and blue lines of all 5 areas appear parallel (Panels 1c-5c in Fig. R1-8), indicating that the acceleration condition in Premise II is properly met. Consequently, out of the total average OE of 245 m a^{-1} the proposed method effectively corrected the spatial acceleration-induced *portion of 97 m a^{-1} (~40%), leaving the temporal acceleration-induced portion of 148 m a^{-1} (60%) in residuals. The uncorrected portion of OEs represent the velocity change signature over the time span caused by the continuous basal melting and drastic calving activities in and after 2017 in PIG (Joughin et al., 2021). Finally, our experiment results show that in Jakobshavn Isbrae the grounding zone and floating ice part cannot be covered by multi-year span maps (over 2-3 years) because of the relatively short main trunk (~60 km) and extremely high velocity (~15000 m a^{-1}), i.e., lost opportunities for historical velocity recovery. A comprehensive study is needed to investigate the influence of the extremely high ice flow dynamics on the proposed OE correction method in Greenland.*"

The above revision of the manuscript is supported by the results of a number of new experiments and data analysis. In the following we provide a detailed version:

4.1 Threshold of the overestimation-free time span for trajectory segments

The choice of a short time span Δt_{Ref} (e.g., a few months to a year) for the "overestimation-free" segments along a trajectory in *Premises I* makes sure that the difference between E-and L-velocities within the span is negligible, or less than σ (velocity mapping uncertainty, σ=20 m a^{-1} in this study). It is also the time span of the initial "OE-free" E-velocity map that is used for ice mass tracking and L-velocity computation in premise validation. Determination of this threshold has an implication on validation of *Premise I*, as well as the integration period of the trajectory segments from P_i to P_{i+1} (Fig. 3b). Estimation of Δt_{Ref} can be performed in a systematic way. An area of the highest acceleration in a glacier should be selected. Within the area a multi-span E-velocity series V^E_{0-i} ($i=1, 2, \ldots n$) can be used to establish a linear relationship between the E-velocity V^E and time span Δt, $V^E = K\,\Delta t + b$, by a linear regression (red line in Fig. 2). With the known parameters of b and K, Δt_{Ref} can be calculated as $\Delta t_{Ref} = \frac{\sigma - b}{K}$. For example, *Area 5* in TG in *Experiment 1* has the highest acceleration (Table 1, Fig. 5c). After a regression using the E-velocities in *Area 5*, Δt_{Ref} is calculated as 3.2 years (R^2=0.96, Fig. R1-1a). Thus, the selected Δt_{Ref} of one year for TG in *Experiment 1* is justified.

Alternatively, if the E-velocities are not available, multi-span L-velocities along a profile on the main trunk of a glacier may be established from an available short span E-velocity map (e.g., 1-year map in PIG, Fig. R1-2a). Along the profile the highest acceleration (location "A" in Fig. R1-2a) is localized where a multi-span L-velocity series can be computed (Fig. R1-2b). Using this L-velocity series and the above regression method, the "OE-free" time span Δt_{Ref} can also be estimated. For example, given σ=20 m a^{-1} and a series of computed multi-span L-velocities from 1- to 10-years at location "A" in PIG (Fig. R1-2) and "B" in Jakobshavn Isbrae (JI, Fig. R1-6), we estimated Δt_{Ref} as ~3.0 months (R^2=0.99, Fig. R1-1b) and ~1.4 months (R^2=0.86, Fig. R1-1c) for PIG and JI, respectively.

Based on the above analysis results of Δt_{Ref} in TG (3.2 years), PIG (3.0 months) and JI (1.4 months), it appears that the threshold of an "OE-free" time span is strongly related to the ice flow dynamics of the glaciers. Given a known σ, shorter spans of Δt_{Ref} should be selected for trajectory segments in validation of *Premise I* and L-velocity integration in faster flow glaciers. We suggest that an analysis of multi-span L-velocities and a regression for Δt_{Ref} be performed before extensive historical velocity mapping would be carried out.

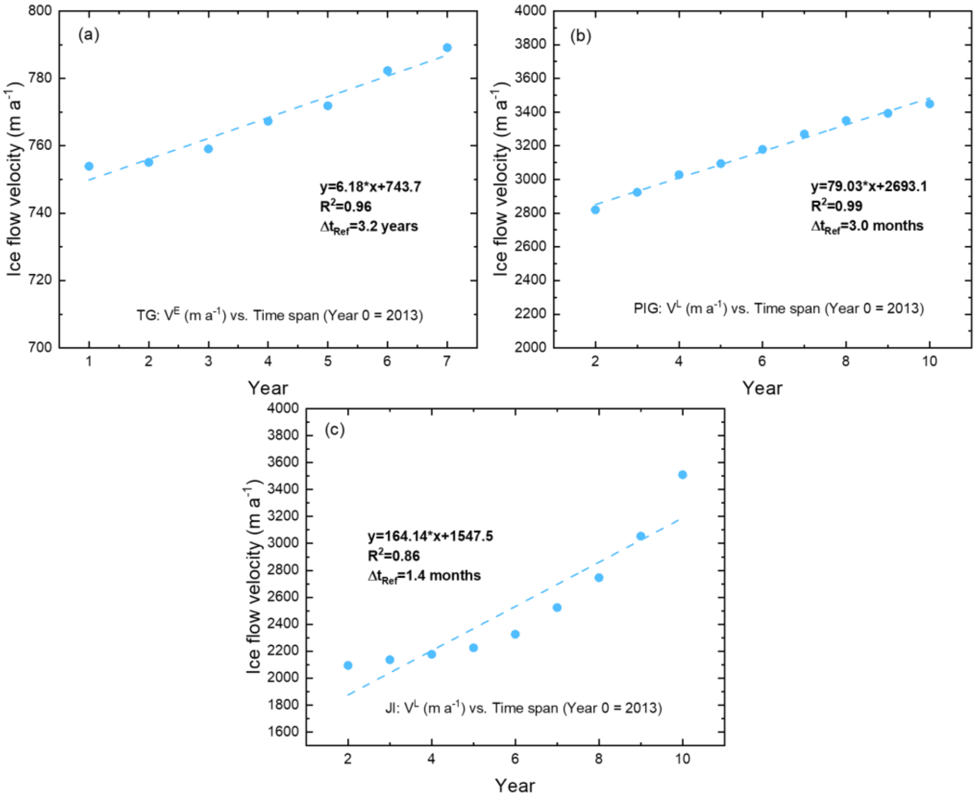

Figure R1-1. Linear regression of multi-span velocities vs. time span is performed to estimate the "OE-free" time span. (a) Totten Glacier (TG): 7-years of E-velocities in *Area 5* (Fig. 4), Δt_{Ref}=3.2 years; (b) Pine Island Glacier (PIG): 10-years of L-velocities at location "A" near grounding line (Fig. R1-2), Δt_{Ref}=3.0 months; and (c) Jakobshavn Isbrae (JI): 10-years of L-velocities at location "B" along the main trunk profile (Fig. R1-6), Δt_{Ref}=1.4 months.

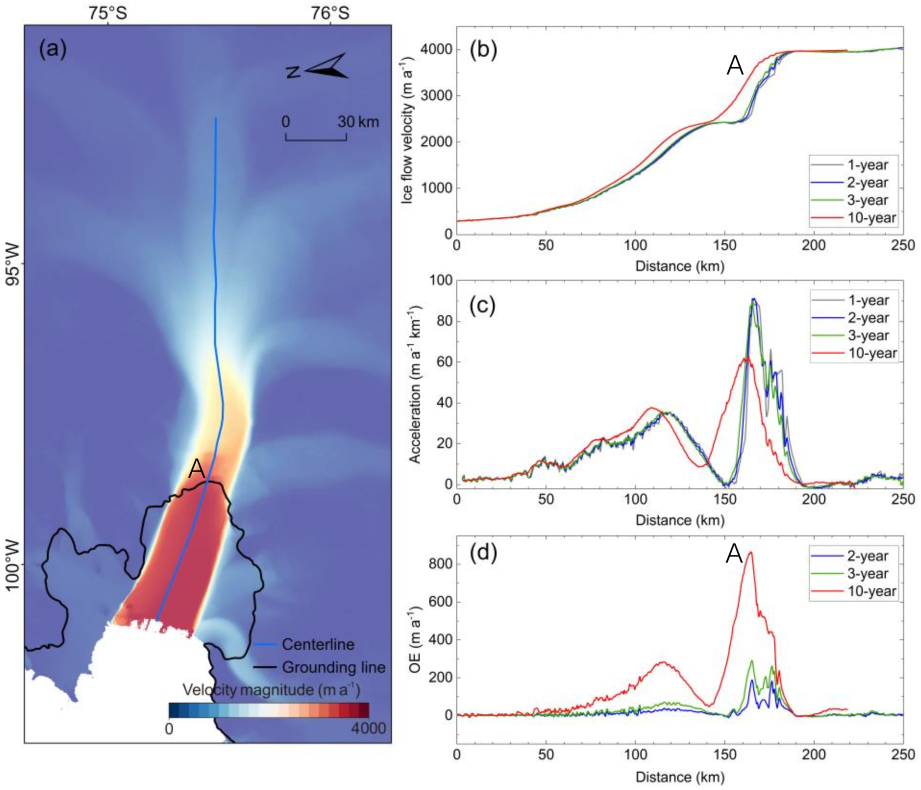

Figure R1-2. (a) Velocity, grounding line, and profile position along the main trunk of PIG; (b) E-velocity of 1-year span along the profile from ITS_LIVE, and L-velocities of 2-, 3-, and 10-year spans calculated from the 1-year span E-velocity map; (c) L-acceleration of the corresponding time spans along the profile, and (d) Estimated OEs caused by the L-velocities of different time spans. Please note that the L-velocities are only used for simulation estimation of time span Δt_{Ref}. In validation of *Premises I* and *II*, we used actual measurements of multi-year span E-velocities.

4.2 Glaciers with complex geometry and E- and L-velocity difference along trajectory

Furthermore, within a longer time span (e.g., over 5 - 10 years) in *Premise I* the difference between the E- and L-velocities accumulated over all segments along the entire trajectory, i.e. the end-point deviation between the red and blue lines in Fig. 2, is measured with a more tolerable threshold of k times of σ (k σ). Although the OEs of the trajectory segments are controlled by Δt_{Ref}, ice mass moving along a curved flow line over this long span may result in an additional discrepancy along the entire trajectory.

As suggested, we examined the complex geometry issue in 5 Antarctic glaciers, including George VI, Abbott, Dotson, Crosson and Getz (Fig. R1-3a) where the velocity ranges from ~100 m a^{-1} to 1,000 m a^{-1}. We used one-year velocity maps of 2013 of 5 glaciers from ITS_LIVE to derive 20-year span L-velocities along trajectories in the significantly curved sections of the glaciers (Fig. R1-4). The computed straight and curved lengths vs. time span in the 5 glaciers are illustrated in Fig. R1-3b. The statistics of the corresponding E- and L-velocities are given in Table R1-1.

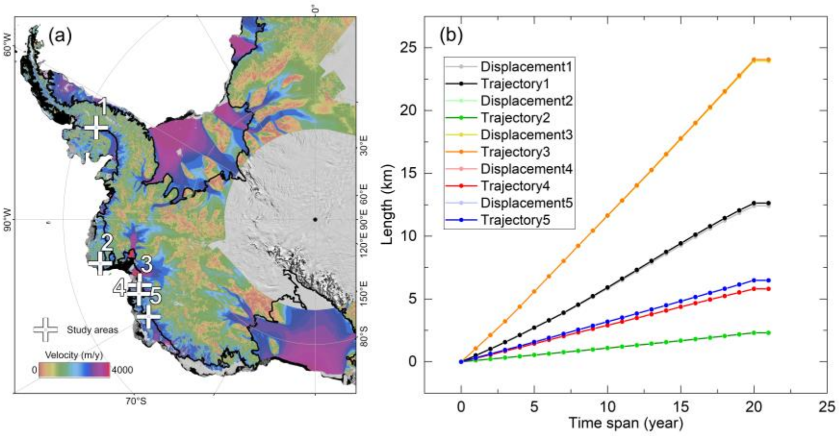

Figure R1-3. (a) Locations of 5 glaciers with complex geometry, including George VI, Abbott, Dotson, Crosson, and Getz. (b) Computed straight (displacement) and curved (trajectory) lengths vs. time span in 5 glaciers.

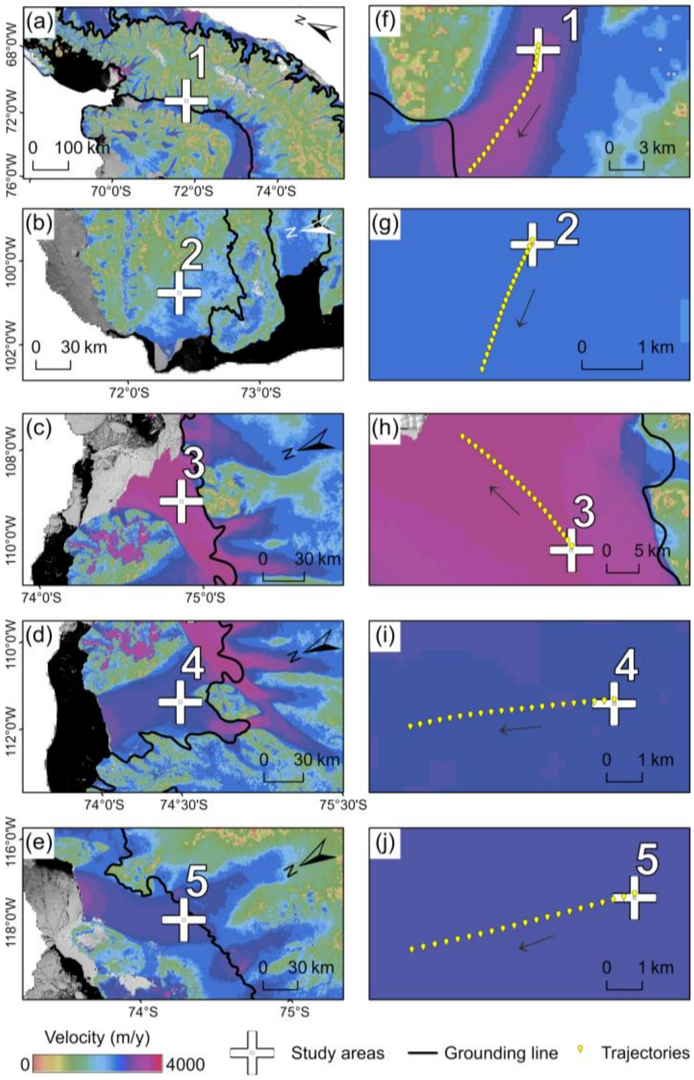

Figure R1-4. (a-e) locations of trajectories in 5 glaciers with complex geometry, including

George VI, Abbott, Dotson, Crosson, and Getz. (f-j) 20-year span trajectories in 5 glaciers.

It is shown in Table R1-1 that in these 5 glaciers three glaciers (Abbott, Dotson and Getz) have very small OEs in the 20-year span L-velocities ($< \sigma = 20$ m a^{-1}), and their "curvature" induced differences $\Delta V_{20\,y}^{L-E}$ are 0 m a^{-1}. The other two glaciers (George VI and Crosson) have OEs of 114 m a^{-1} and 143 m a^{-1}, respectively. However, the "curvature"-induced differences are only 11 m a^{-1} and 6 m a^{-1}, both of which are smaller than σ and thus, negligible. So, the OEs are mainly caused by spatial acceleration here, not the curvature.

Table R1-1. Overestimations and "curvature" induced velocity differences in 5 glaciers

ID	Name	$V_{1\,y}^{E}$ (m a^{-1})	$V_{20\,y}^{L}$ (m a^{-1})	OE (m a^{-1})	Δt (year)	E-dist. (m)	L-dist. (m)	$\Delta V_{20\,y}^{L-E}$ (m a^{-1})
1	George VI	507	621	-114	20	12412	12633	11
2	Abbott	110	115	-6	20	2306	2308	0
3	Crosson	1054	1197	-143	20	23942	24064	6
4	Dotson	280	290	-10	20	5803	5809	0
5	Getz	315	324	-9	20	6481	6482	0

In the Totten Glacier flow lines are less curved and velocity is higher (up to ~1,400 m a^{-1}). The E- and L-velocity differences of the 7-year trajectories $\Delta V_{7\,years}^{L-E}$ in all five areas ($V^L - V^E$ in Table 1) are within 2σ (40 m a^{-1}, < 2% of their velocities). Thus, the flow line curvature does not cause a significant E- and L-velocity difference, and the velocity requirement in *Premise I* is well met.

Moving to PIG (Fig. R1-7), the most dynamic glacier in Antarctica (velocity up to ~4,000 m a^{-1}), there exist a high level of temporal accelerations caused by basal melting and calving activities (Joughin et al., 2020, 2021), similar to what happened in *Area 4* of TG in *Experiment 1* (Li et al., 2015). In this case the threshold k for the average E- and L-velocity difference is 5 (> 2). However, the computed OE corrections can still remove the spatial acceleration-induced OE portion and leave the temporal acceleration-induced OE portion as a signature for long-term ice dynamics studies. Hence, *Premise I* is not a necessary condition, but a sufficient condition.

We further performed an in-depth experiment in *Area 2* of PIG (in Fig. R1-7), which is located in the most curved section along the main trunk of PIG (Fig. R1-5). We tracked point P in 2013 to point P' in 2020 consecutively using 7 Landsat-8 images with a 1-year interval, resulting in 7 trajectory segments. The straight distance PP' is 19,720 m and the curved distance is 21,161 m. Accordingly, the E-and L-velocities of the 7-year span are 2,817 m a^{-1} and 3,023 m a^{-1}, respectively. That means that the curvature of the 7-year trajectory (a deviation b of 1,305 m from the straight line) created a difference $\Delta V_{7\,years}^{L-E}$ of 206 m a^{-1} at the trajectory end, among which 195 m a^{-1} (95%) was corrected by our method (Table R1-2).

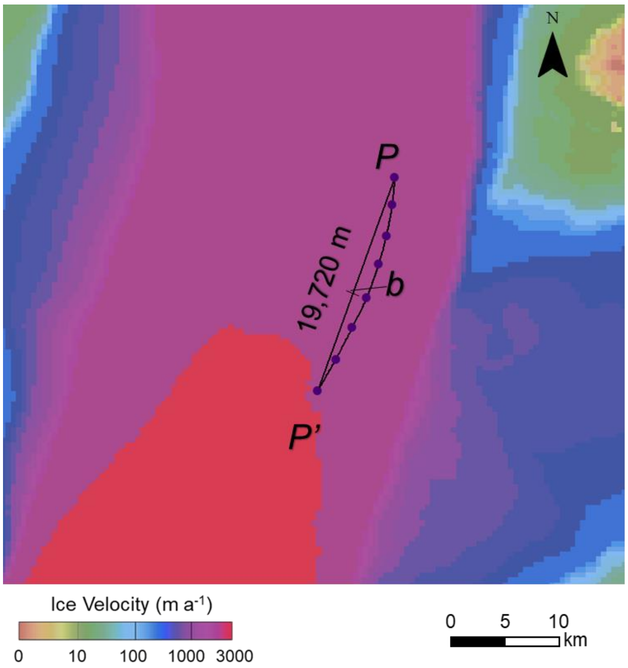

Figure R1-5. Illustration of difference between E- and L-velocities along a 7-year trajectory from *P* to *P'* in the most curved section in PIG. The background is the one-year span velocity map of 2013 from ITS_LIVE.

4.3 OE estimation in fast flowing glaciers

We estimate OE corrections assuming that within a longer time span (e.g., over 5 - 10 years) in *Premise II* the acceleration trend would not change significantly during the time span, $\left|\bar{a}\left(V^{L(n)}\right) - \bar{a}(V^{L})\right| \leq k' \frac{\sigma}{\Delta t_n}$. As shown in *Experiments 1*, this trend change is under $1 \frac{\sigma}{\Delta t_n}$ (the acceleration equivalent of velocity mapping uncertainty σ) for the Totten Glacier, one of the fast-flowing glaciers in East Antarctica. Since the velocity requirements in *Premise I* and acceleration requirement in *Premise II* were met properly, we were able to correct in average 88% of the OEs.

The extremely high ice dynamics exists mostly in fast flowing glaciers in West Antarctica or Greenland due to impact of climate warming. Here we evaluate the influence of such high dynamics on OE corrections in PIG. The acceleration trend differences $\left|\bar{a}\left(V^{L(n)}\right) - \bar{a}(V^{L})\right|$ in all 5 areas (Table R1-2) are in average less than $3 \frac{\sigma}{\Delta t_n}$. Correspondingly, the black and blue lines of all 5 areas appear parallel (Panels 1c-5c in Fig. R1-8), indicating that the acceleration condition in *Premise II* is properly met. Consequently, the proposed method corrected in average 97 m a^{-1} (~40%) of the total OE (245 m a^{-1}), leaving the residuals (60%) in the adjusted velocities. The residuals represent the velocity change signature over the time span caused by the continuous basal melting and drastic calving activities in and after 2017 in PIG (*Experiment 3*).

We performed an experiment for Jakobshavn Isbrae (JI), Greenland. We used a baseline velocity map of 2013 (one year span) from ITS_LIVE, based on which we calculated L-velocities of 2-, 3-, and 10-year time spans along the centerline of the main trunk. Subsequently, we estimate the L-accelerations and OEs (Fig. R1-6).

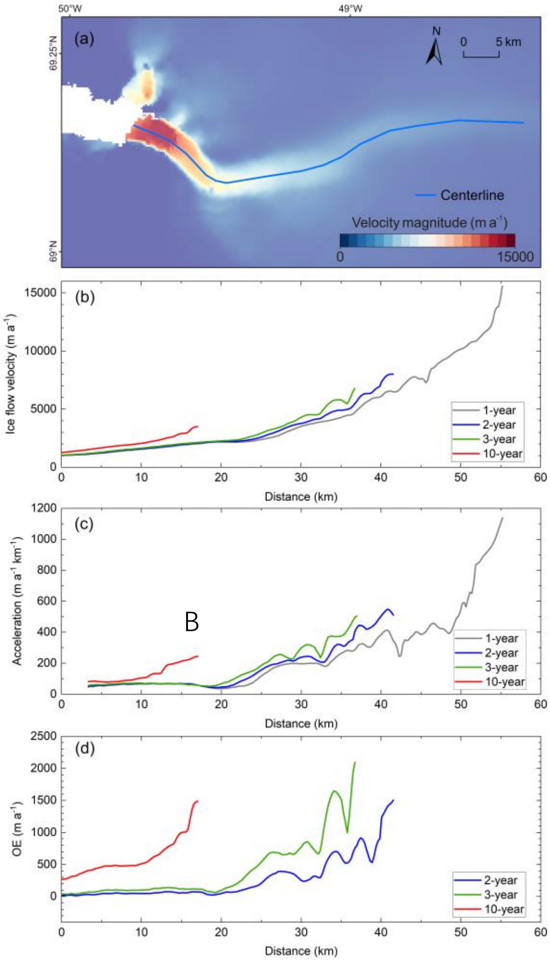

Figure R1-6. (a) Velocity, grounding line, and profile position along the main trunk of Jakobshavn Isbrae; (b) E-velocity of 1-year span along the profile from ITS_LIVE, and L-velocities of 2-, 3-, and 10-year spans calculated from the 1-year span E-velocity map; (c) L-accelerations of the corresponding time spans along the profile, and (d) Estimated OEs caused by the L-velocities of 3 time spans.

Jakobshavn Isbrae is about ~60 km from the inland interior to the marine terminus (Figs. R1-6a), over which the ice mass picks up velocity from ~1000 m a^{-1} to over ~15000 m a^{-1}. The maximum velocity is ~3 times higher, while the main trunk is ~4 times shorter than in PIG. Consequently, the ice shelf part can only be covered by the 1-year span E-velocity map (Fig. R1-6b); similarly, only 16 km long inland interior along the profile is covered by the 10-year span L-velocity. This makes it difficult to map velocities in the grounding zone and ice shelf region using images of over 1-year span, i.e., lost opportunities for historical velocity recovery. The estimated OEs of three spans reached ~1,500 m a^{-1} (up to 38% of the 10-year span L-velocity, Figs. R1-6b and d). This is significantly higher than 19% in PIG (Table R1-2).

In comparison to the reported velocity changes of ~125 m a^{-1} from 2001-2008 in TG (Li et al., 2015) and ~500 m a^{-1} from 2018-2020 in PIG (Joughin et al., 2021), the estimated average OE corrections of up to ~20 m a^{-1} in TG and DG (Tables 1 and 2), and ~97 m a^{-1} in PIG (Table R1-2) are not significant with respect to the natural variability of the glacier flow. In addition, the proposed method is applied for longer span velocities (a few years to over 5-10 years), seasonal variations should be averaged out. Therefore, the applicability of this method should not be affected by seasonal velocity changes and natural variability of glacier flow in Antarctica.

The estimated OEs in Jakobshavn Isbrae reached ~1,500 m a^{-1} (up to 38% of the 10-year span L-velocity, Figs. R1-6b and d). The reported seasonal change can go as high as ~5,000 m a^{-1} (50%) in last decade (Joughin et al., 2020). We believe that more comprehensive studies are needed in applicability of our method in Jakobshavn Isbrae and other fast flowing glaciers in Greenland.

Li, X., E. Rignot., M. Morlighem., J. Mouginot., & B. Scheuchl. (2015). Grounding line retreat of Totten Glacier, East Antarctica, 1996 to 2013. Geophysical Research Letters, 42(19), 8049-407. http://doi.org/10.1002/2015GL065701.

Joughin, I., Shean, D. E., Smith, B. E., & Floricioiu, D. (2020). A decade of variability on Jakobshavn Isbræ: ocean temperatures pace speed through influence on mélange rigidity. The Cryosphere, 14(1), 211-227.

Joughin, I., D. Shapero, B. Smith, P. Dutrieux, M. Barham (2021). Ice-shelf retreat drives recent Pine Island Glacier speedup. Sci. Adv. 7, eabg3080.

Finally, while the authors are discussing the large overestimation error on Pine Island glacier (36%), they are presenting a first application of the method on Totten glacier. Hence, I think that it would increase the paper's logic and readability to keep this example for the application part (Totten could be put in the supplementary material). With such a high overestimation, I expect the results to be spectacular.

Response:
As suggested, we carried out an experiment in PIG, *Experiment 3*. We present the detailed results in the following.

Experiment 3. Velocity overestimation correction in the Pine Island Glacier, West Antarctica

Five areas along the centerline and in the margin in PIG (Fig. R1-7) are selected to show the applicability of the OE correction method. Limited by the fast velocity, lost image features over long time spans, and availability of images, the maximum span is 7 years and most of the ice shelf cannot be covered. Since TG has a complete coverage of 7-year span and more systematic results, its results in *Experiment 1* are presented in the main text of the paper. And the PIG results in *Experiment 3* will be presented in Appendix.

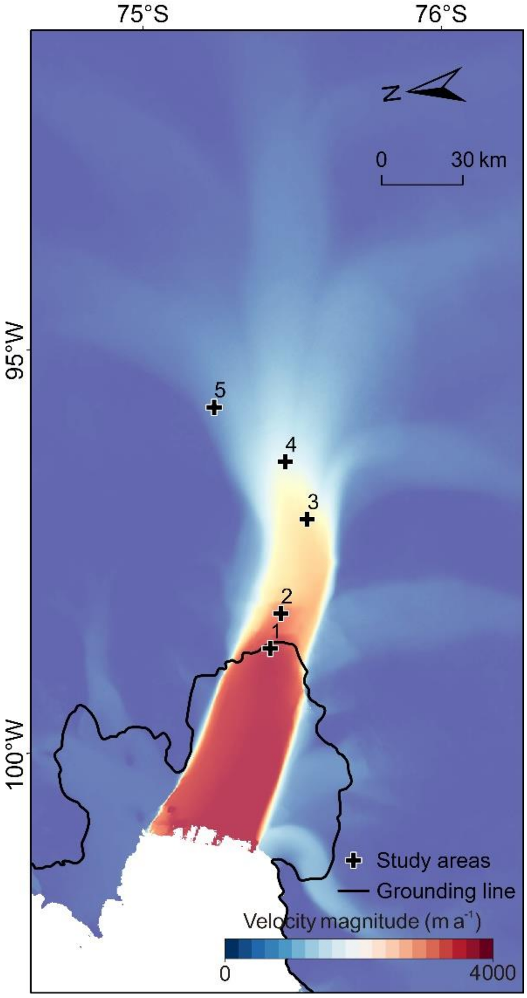

Figure R1-7. Application of the OE correction method in PIG. Five areas are selected to map E-velocities and calculate L-velocities. The OEs are estimated and adjusted. Their effect on grounding line flus estimation is also analyzed. The background is the 1-year velocity map of PIG 2003 from ITS_LIVE.

We mapped five areas with E-velocities of 3-month (Panels 1a-5a in Fig. R1-8) and 7-year (Panels 1b-5b in Fig. R1-8) spans and used them for calculation of L-velocities (blue and black lines, Panels 1c-5c in Fig. R1-8). The multi-span E-velocities (3 months, 2, 3, … 7 years) are mapped by using image tracking to form the red lines. Overall, in *Premise II* the average acceleration trend difference is 8 m a^{-2}, less than 3 $\frac{\sigma}{\Delta t_n}$ (Avg. Δt_n=6 years, σ=20 m a^{-1}); the black lines and blue lines are approximately parallel (Panels 1c-5c in Fig. R1-8). Since there is a high level of basal melting and calving activities (Li et al., 2015; Joughin et al., 2020, 2021) in PIG over the 7 year period, the temporal acceleration-induced OE portion contributed to the average E- and L-velocity difference along the trajectories in *Premise I*, less than 5σ (4% of the average velocity). However, the computed OE corrections can still remove the spatial acceleration-induced OE portion and leave the temporal acceleration-induced OE portion as a signature for long-term ice dynamics studies. Hence, Premise I is not a necessary condition, but a sufficient condition.

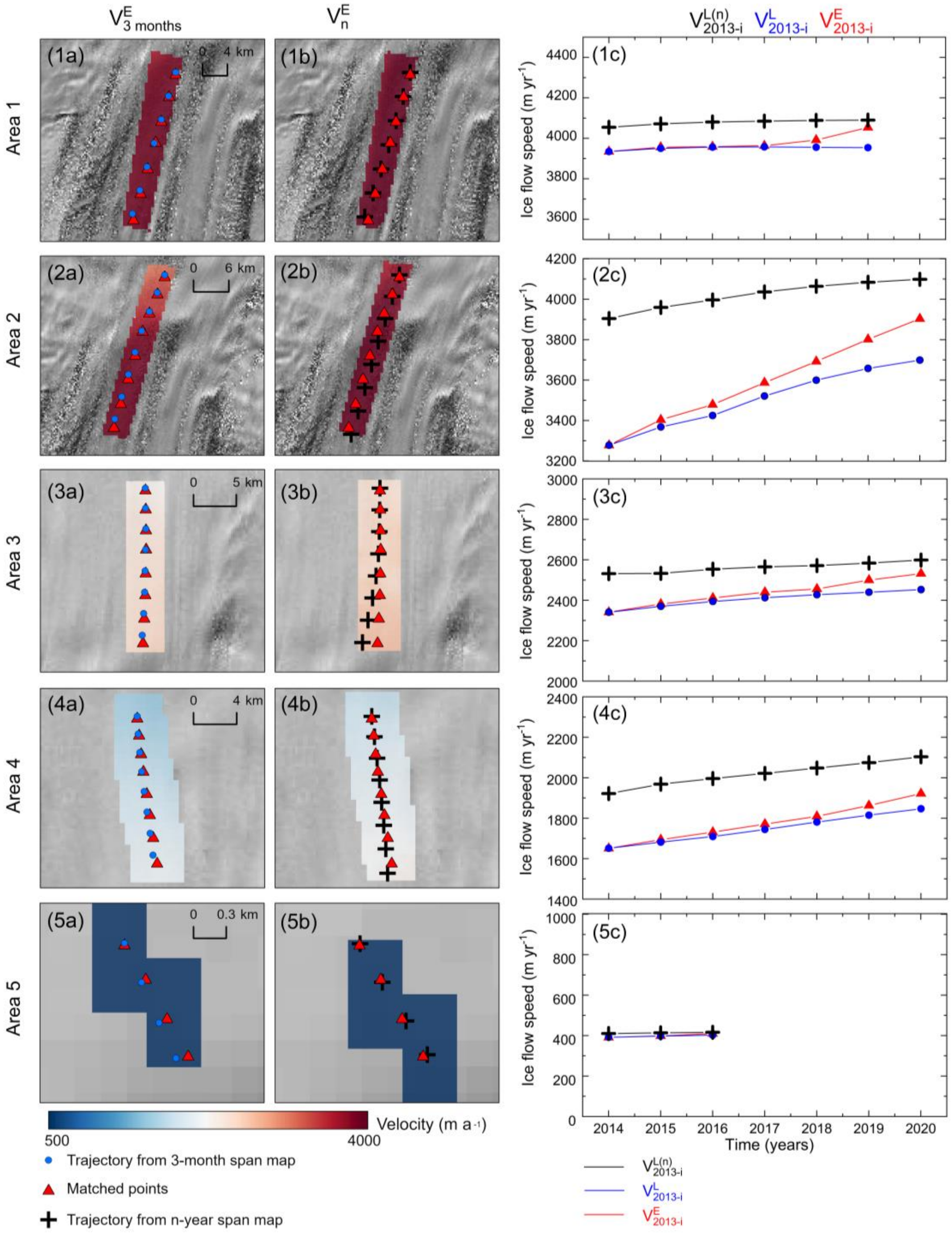

Figure R1-8. Velocities in five areas of PIG and OE corrections. (a) Panels 1a–5a show the reconstructed 3-month velocity maps $V_{3\ months}$ in the areas; matched points (red triangles) are used to map E-velocity V^{E}_{2013-i} (i=*2014, …, 2020*); points along the flow line (blue dots) are tracked from the 3-month maps and used to calculate L-velocity V^{L}_{2013-i}; they are presented as the red and blue lines in Panels 1c–5c. (b) Similarly, Panels 1b–5b illustrate the reconstructed 3, 6 and 7-year velocity maps V_{2013-n} in the areas with the matched points (red triangles) for E-velocity $V^{E(n)}_{2013-i}$; points along the flow line (blue dots) are tracked from the n-year velocity map and used to calculate L-velocity $V^{L(n)}_{2013-i}$, which are presented as the black lines in Panels 1c–5c. (c) Panels 1c–5c show E-velocity V_{2013-i}^{E} (red line), L-velocity V_{2013-i}^{L} from 3-month E-velocity map (blue line), and L-velocity $V_{2013-i}^{L(n)}$ from n-year E-velocity map (black line) in each area.

The OE in *Area 5* of the low flowing interior margin is 18 m a^{-1}, under 1σ (Fig. R1-7, Table R1-2). From *Area 1* to *Area 4* along the main trunk the OEs are higher, ranging from 119 m a^{-1} (3%) to 626 m a^{-1} (19%). Correspondingly, there is an increase in actual E-velocities over the entire span (2013-2020), making the E-velocities (red lines) mostly above the L-velocities from the 3-month span map (blue line) (Panels 1c-5c in Fig. R1-8). This temporal acceleration correlates to long-term basal melting (Joughin et al., 2021). More specifically, the OE in *Area 2* close to grounding line reached the maximum of 626 m a^{-1}, with majority of the increase occurred in and after 2017 (Panel 2c in Fig. R1-8), which may be attributed to the drastic calving activities in and after 2017 as reported in Joughin et al. (2021). Overall, the average OE of five areas in PIG is 245 m a^{-1} (11% of the average velocity 2319 m a^{-1}, Table R1-2), among which 97 m a^{-1} (40%) of the spatial acceleration induced portion is effectively corrected, leaving 148 m a^{-1} (60%) of the uncorrected and temporal acceleration induced portion. The latter portion is preserved in residuals as a significant signature of “historical” changes caused by the climate warming.

Table R1-2. Application of the OE correction method in PIG. “*Actual E-velocity and OE*” includes E-velocities of 3-month and n-year spans and their differences as OEs. “*Overestimation correction*” presents n-year span L-velocities from n-year span map, OE corrections, corrected E-velocities, and residuals (or errors) after correction.

Area ID (span)	***Actual E-velocity and OE***			***Overestimation correction***			
	$V_{3\ months}^{E}$ **(m a^{-1})**	V_{n}^{E} **(m a^{-1})**	***OE_{Actual}*** **(m a^{-1})**	$V_{n}^{L(n)}$ **(m a^{-1})**	***Corr.*** **(m a^{-1})**	$V_{Corr.}^{E}$ **(m a^{-1})**	ε **(m a^{-1})**
1 (6)	3935	4054	119	4090	-36	**4018**	83
2 (7)	3278	3904	626	4099	-195	**3709**	431
3 (7)	2341	2532	191	2599	-67	**2465**	124
4 (7)	1651	1922	271	2103	-181	**1741**	90
5 (3)	392	410	18	416	-6	**404**	12
MEAN	2319	2564	245	2661	-97	**2467**	148
STD	1387	1506	232	1538	86	**1477**	163

Finally, based on an annual E-velocity map of 2013 in PIG from ITS_LIVE, L-velocities along GL with time spans of 1-15 years and the associated GL ice flux were computed. The results show that the OEs of the 15-year span can reach up to ~1300 m a^{-1} in the GL region. These OEs in velocity can further cause an overestimation in GL flux, which rapidly increases by ~6.3 Gt a^{-1} within a 4-year span; thereafter it slows down until the 15-year span, resulting in a total flux OE of 11.5 Gt a^{-1}. Consequently, the results indicate that a velocity map of a time span within 3 years would be "flux OE-free" (< σ_{Flux}) in PIG. We used σ_{Flux}=5.8 Gt a^{-1} that is the flux uncertainty in PIG reported by Rignot et al. (2019). Overall, the influence of the OEs on the GL flux appears not very significant, with the OE of a 15-year span map (11.5 Gt a^{-1}) less than 2 σ_{Flux} (11.6 Gt a^{-1}). The details of this GL flux analysis is presented in responses to Referee #2's comments.

Comments

L45. This is a citation for the Landsat-8 program. Not appropriate here.

Response:

Agreed. We replaced Wulder et al. 2019 with Chander et al. (2009) that is more relevant to the historical Landsat programs (Landsat MSS, TM, etc.). It is also added in references.

Chander, G., Markham, B. L., & Helder, D. L. (2009). Summary of current radiometric calibration coefficients for Landsat MSS, TM, ETM+, and EO-1 ALI sensors. Remote sensing of environment, 113(5), 893-903.

L46. "3 to 15 years", is not accurate. Bindschadler and Scambos., 1991 used a cross-correlation algorithm on two images separated by roughly 1 year. Similarly, Bindschadler et al., 1996 also uses 1 year image-pairs (see Table 1 of their paper). Wulder et al does not contain ice velocity maps prior to the 1990s.

Response:

We revised the text to "*... ... have been used to create regional velocity maps with a time span ranging from 1 to 23 years (Bindschadler & Scambos, 1991; Bindschadler et al., 1996; Wang et al., 2016; Cheng et al., 2019; Rignot et al., 2019)*". Here we deleted Wulder et al. 2019.

The time spans for the maps in the cited papers are: ~ 1 year in Bindschadler & Scambos (1991), 1 to 7 years in Bindschadler et al. (1996), 2 to 23 years in Wang et al. (2016), 1 to 15 years in Cheng et al. (2019), and 1 to 15 years in Rignot et al. (2019).

L46. Can you define after which time span the overestimation is significant? (2 yr, 3yr ?). I found the use of images acquired more than 2 years apart quite rare, or limited to few points (large rifts for example).

Response:

In Discussion we added: "*……Our experiment results show Δt_{Ref} as 3.2 years, 3.0 months, and 1.4 months for TG, PIG, and Jakobshavn Isbrae (JI), Greenland, respectively. Thus, the estimated OE-free time spans appear to be related to ice flow dynamics of the glaciers. In Experiments 1 and 3, we used 1 year for TG and 3 months for PIG. We suggest that a linear* *regression for* Δt_{Ref} *estimation be performed before extensive historical velocity mapping would be carried out.*" The detailed reasoning is given in the responses to General Comments (4.1).

L49-51. From these lines it is a bit difficult to understand the overestimation issue. Please, extend a bit this description with more details, and split the sentence in two or three parts.

Response:

We explained it in a more mathematical or physics way. "*For example, at time1 a feature, with an initial velocity* v_0 *at the first location, is taken in the first image. The same feature is tracked in the second image taken at time2 after traveling at the velocity* v_0 *and an acceleration a for a time span of* Δt *(time2-time1). Thus, the velocity* $v=v_0+a\Delta t$ *increases along with the time span* Δt *if acceleration a exists. Given a constant acceleration, the velocity can be overestimated if the time span is long. Or the velocity overestimation is proportional to the time span.*"

L52. Greene et al., 2020b; the reference list just says (Personal communication, comments on a manuscript), which I found a bit weak for a reference of a concept that is the base of this paper.

Response:

We understand the concern. Chad Greene is now Referee #2 who volunteered to make the referee information open. We added Greene's Figures in Appendix as Figure A1 and quoted his text.

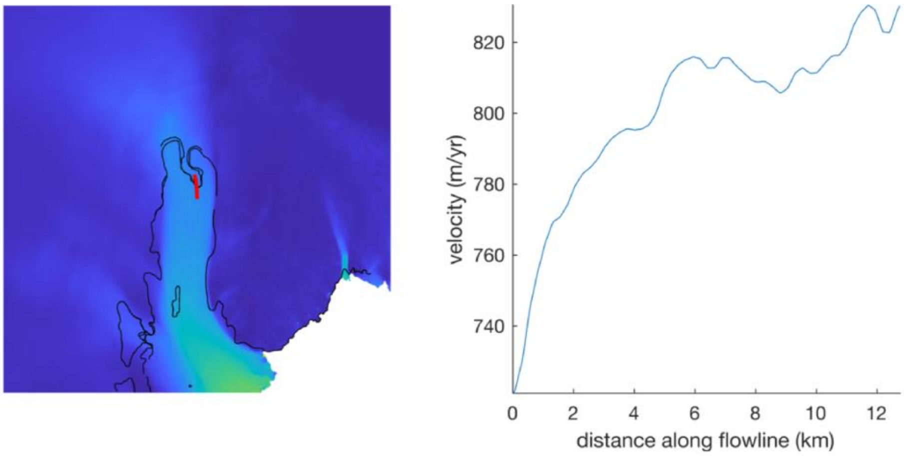

Figure R1-9. Velocity map of the Totten Glacier from ITS_LIVE (left) used to explain the concept of velocity overestimation caused by acceleration (right): "*Over 16 years, that parcel of ice travels about 13 km downstream (red path). It begins at a velocity of about 720 m/yr, and in the first 8 months it travels at an average rate very close to 720 m/yr. But then the ice picks up speed as it moves downstream, so in the first 10 years it does not travel just 7200* *m—it actually travels about 7900 m, or an average speed of 790 m/yr……*” (Greene, 2020b).

L53. The overestimation calculation over Pine Island Glacier is derived later in the manuscript, hence remove this part of the sentence.

Response:

The “Pine Island” part of the sentence is deleted.

L54. I would like to see a complete comparison of the simple method from Berthier et al., 2003, with the approach proposed here in the discussion section.

Response:

Here we first have an analytical proof and then an experiment of TG in *Experiment 1*.

Assume that a tracked feature flows from A to B over a period of n years, with the middle point of AB denoted as M (arriving in m years); v_0 is the initial velocity at A; a is acceleration that is constant both spatially and temporally (not a requirement in this paper). The following figure is a simplified situation (straight flow line, otherwise Lagrangian path and L-velocity have to be used).

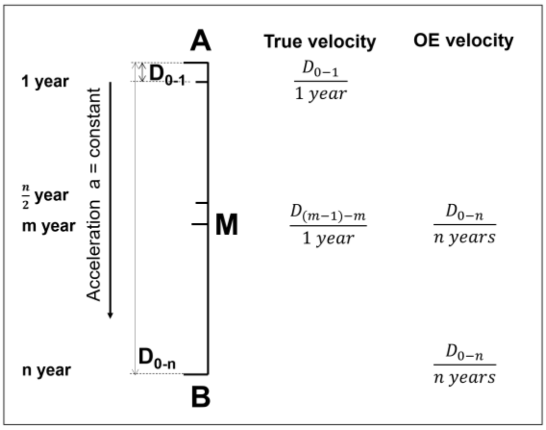

Figure R1-10. Analytical description of the “midpoint” method for OE correction.

The one year (or shorter) “true” velocity at A (year 0 - 1) is $V_{0-1} = \frac{D_{0-1}}{1\,year}$; the one year (or shorter) “true” velocity at M (year (m-1) – m) is $V_{(m-1)-m)} = \frac{D_{(m-1)-m}}{1\,year}$; the overestimated velocity at A using the n year segment AB (year 0 - n) is $V_{0-n} = \frac{D_{0-n}}{n\,year}$. The math or physics problem is

$$V_{(m-1)-m)} = \frac{D_{(m-1)-m}}{1\,year} =?\ V_{0-n} = \frac{D_{0-n}}{n\,year} \tag{R1-1}$$

We further calculate the one year “true” velocity at M:

$$V_{(m-1)-m} = \frac{D_{(m-1)-m}}{1\,year} = \frac{V_{(m-1)}\cdot 1\,year + \frac{1}{2}\cdot a\cdot 1\,year^2}{1\,year} = V_{(m-1)} + \frac{1}{2}\cdot a\cdot 1\,year = V_0 + a\cdot (m-1)\,years + \frac{1}{2}\cdot a\cdot 1\,year = V_0 + a\cdot\left(m-\frac{1}{2}\right)\,years.$$

On the other hand, the overestimated velocity is

$$V_{0-n} = \frac{D_{0-n}}{n\ years} = \frac{V_0 \cdot n\ years + \frac{1}{2} \cdot a \cdot (n\ years)^2}{n\ years} = V_0 + \frac{1}{2} \cdot a \cdot n\ years$$

Therefore, if Equation (R1-1) holds we must have

$$\left(m - \frac{1}{2}\right) = \frac{n}{2}; \mathrm{m} = \frac{n+1}{2}. \qquad \text{(R1-2)}$$

However, given that it is a simplified uniformly accelerated motion, the tracked feature arrives at the halfway (M) in m years, which must be greater than half time $\frac{n}{2}$, namely

$$m > \frac{n}{2}. \qquad \text{(R1-3)}$$

Therefore, we have

$$V_{(m-1)-m} = V_0 + a \cdot \left(m - \frac{1}{2}\right) years > V_0 + a \cdot \left(\frac{n}{2} - \frac{1}{2}\right) years = V_{0-n} - a\frac{1}{2} year.$$

We further have

$$V_{(m-1)-m} > V_{0-n} - a\frac{1}{2} year. \qquad \text{(R1-4)}$$

If a = 0, $V_{(m-1)-m}$ and V_{0-n} are the same. Otherwise, they may be different, depending on acceleration "a". Thus, for glaciers with low to median range acceleration (weak spatial gradient), the overestimation may have been corrected to a good percentage by "assigning velocities at middle points of segments". But if $a \neq 0$, the velocity at midpoint M is different from V_{0-n}.

In addition, in *Experiment 1* we estimated and corrected OEs in five areas of TG. Since *Area 4* was affected by a calving event during the time span, here we use other four areas (*Areas 1, 2, 3,* and *5*) to compare our OE correction method with the midpoint method. We estimated the E-velocities of 7 years (2013-2020) $V^E_{2013-2020}$ and assigned them to midpoints of the trajectories in the four areas (Table R1-3). They were then compared to the true velocity $V^E_{2013-2014}$ (one year span E-velocity) at the midpoints to calculate the bias. Similarly, the same overestimated 7-year span E-velocities $V^E_{2013-2020}$ were corrected using the method of this paper and assigned to the start points of the trajectories as $V_{corrected}$ (Table R1-3). They were then compared to the true velocity $V^E_{2013-2014}$ also, but at the start points to calculate another set of bias. As shown in Table R1-3, the overall accuracy (Mean±RMSE) of the proposed method is 4±10 m a^{-1} and that of the midpoint method is 12±14 m a^{-1}. Therefore, the proposed OE correction method should provide a more accurate velocity map than the "midpoint" method. We will add a statement in Discussion.

Table R1-3. Comparison of the proposed OE correction method with the "midpoint" method by Berthier et al. (2003)

Area	**Velocity assigned to the midpoint**	**1-year map at the midpoint**	**Bias**	**Velocity assigned to the start point**	**1-year map at the start point**	**Bias**
	$V^{E}_{2013-2020}$ **(m a^{-1})**	$V^{E}_{2013-2014}$ **(m a^{-1})**	ε_M **(m a^{-1})**	$V_{corrected}$ **(m a^{-1})**	$V^{E}_{2013-2014}$ **(m a^{-1})**	ε_S **(m a^{-1})**
1	843	860	17	824	824	0
2	1008	1007	-1	999	1007	8
3	1317	1332	15	1297	1280	-17
5	789	807	18	759	754	-5
MEAN	**989**	**1001**	**12**	**1069**	**1056**	**-4**
RMSE	**238**	**236**	**14**	**304**	**286**	**10**

L108. Please add reference to Figure 1a,b,c to help the reader's understanding of the whole concept.

Response:

Wherever appropriate, we added references to these figures in the text for more clarity. "*As the time span increases at a fixed rate of 1 year, the traversed straight-line distance D_{0-i} (red lines in Figure 1b), correspondingly E-velocity V^{E}_{0-i}, increases rapidly because of the acceleration over the traverse (Fig. 1c). In principle, every V^{E}_{0-i} (i=1, 2, ...n) value represents the velocity at the same point $P_o(x_o, y_o)$ (Figure 1a) in these n velocity maps. In the cases where $Image_i$ were not available and thus the maps V_{0-i} (i=1, ... n-1) were not produced, we only had the map V_{0-n} with the longest span of n years. It is obvious that at $P_o(x_o, y_o)$ its n-year velocity V^{E}_{0-n} is significantly larger than the 1-year velocity V^{E}_{0-1} (Figure 1c).*

L110. Here and in the remaining of the manuscript you use 1 year ice velocity as a reference map. But does your method still apply for very fast glaciers? For example Jakobshavn Isbrae (Greenland), or Penguin gl. (Patagonia) are flowing at speeds that are exceeding 10 km/yr, hence there is good chances of acceleration along flowlines within that year. Can you please discuss this point here? And better specify the use of a 1 year ice velocity map as a reference for your method.

Response:

We agree with you that the time span of the reference (or baseline) velocity map may be different for different glaciers. We changed the sentences: "*Here we use a velocity map of a 1-year span as a baseline throughout the paper for simplicity, which can be changed for glacier regions of different ice flow dynamics as demonstrated in Experiment 3 (3 months for PIG). We require that the overestimation of the baseline map is negligible, or smaller than σ (velocity mapping uncertainty, 20 m a^{-1}).*"

As explained in the response to the general comments (above), we also added a section in Discussion to introduce an analytical method for determining the threshold of an "OE-free" time span: "*The choice of a time span for defining an "overestimation-free" map in validation of Premises I and II can be performed in a more systematic way. An area of the highest acceleration in a glacier should be selected. Within the area the multi-span E-velocity series* V_{0-i}^{E} *(i=1, 2, ... n) can be used to establish a linear relationship between the E-velocity and time span,* $V^{E} = K\,\Delta t + b$*, by a linear regression (red line in Fig. 2). Given an achievable velocity mapping uncertainty* σ*, the time span* Δt_{Ref} *which would induce an OE that is smaller than* σ *and considered negligible can be calculated as* $\Delta t_{Ref} = \frac{\sigma - b}{K}$*. For example, Area 5 in Experiment 1 has the highest acceleration in TG (Table 1) and* σ *is 20 m* a^{-1}*. After a regression using the E-velocities in Area 5 (Fig. 5c) the "overestimation-free" time span is calculated as less than 3.2 years (*$R^2=0.96$*). Thus, the selected* Δt_{Ref} *of one year in Experiment I is justified. Alternatively, if the multi-span E-velocities are not available, a velocity profile along the main trunk of a glacier may be established from an available short span E-velocity map. Along the profile the highest acceleration is located where a multi-span L-velocity series (blue line in Fig. 2) can be computed. Using this L-velocity series and the above regression method, the "OE-free" time span* Δt_{Ref} *can also be estimated. Subsequently, we computed a series of multi-span L-velocities from 1- to 10-years and estimated an "OE-free" time span, 0.25 years (~3.0 months,* $R^2=0.99$ *and* $\sigma=20$ *m* a^{-1}*) for PIG and 0.11 years (~1.3 months,* $R^2=0.99$ *and* $\sigma=20$ *m* a^{-1}*) for Jakobshvn Isbrae (JI), Greenland, respectively. Therefore, in Experiment 3 we used the 3-month E-velocity as the "OE-free"* Δt_{Ref} *for validation of Premises I and II in PIG (Appendix). In summary the estimated* Δt_{Ref} *decreases from 3.2 years for TG, 3.0 months for PIG, and 1.3 months for JI. In general, the threshold of an "OE-free" time span for a specific glacier appears to depend on ice flow dynamics, along with others factors such as bed topography and availability of images. We suggest that a* Δt_{Ref} *analysis using multi-span L-velocities be performed before an extensive historical velocity mapping would be carried out.*"

L114-115. Does it depends on the speed of the glacier? i.e this assumption still hold for Jakobshavn Isbrae flowing at more than 15 km/yr ? Or Penguin gl. In Patagonia (12 km/yr) ?

Response:

Yes, it does depend on speed/acceleration of the glaciers. Our new experimental results proved that also. In the earlier part of the paper we added: "*Here we use a velocity map of a 1-year span as a baseline throughout the paper for simplicity, which can be changed for glacier regions of different ice flow dynamics as demonstrated in Experiment 3 (3 months for PIG).*

We require that the overestimation of the baseline map is negligible, or smaller than σ *(velocity mapping uncertainty, 20 m* a^{-1}*).*"

In Discussion we added: "…… *Our experiment results show* Δt_{Ref} *as 3.2 years, 3.0 months, and 1.4 months for TG, PIG, and Jakobshavn Isbrae (JI), Greenland, respectively. Thus, the estimated OE-free time spans appear to be related to ice flow dynamics of the glaciers. In Experiments 1 and 3, we used 1 year for TG and 3 months for PIG. We suggest that a linear regression for* Δt_{Ref} *estimation be performed before extensive historical velocity mapping would be carried out.*"

Section 2.2. I am getting lost with the notation, between the U, S, Map, V…. What do you mean by Maps? Maps of ice velocity I guess, than why introducing Maps if you have later V? Why not just using V, and add E and L for Eulerian and Lagrangian as indice (VE and VL).

Response:

We accepted your suggestion and used V for map, V^E for Eulerian and V^L for Lagrangian velocity throughout the manuscript.

Figure 2. Please give a more comprehensive caption of Figure2. This one is just not enough to understand what is there. What the difference between the two Lagrangian lines mean? See earlier comment on the writing of equation to simplify the text and improve the understanding the paper. I guess you have $\mathbf{Map_{0\text{-}1}}$ to specify that the Lagrangian is only calculated with $\mathbf{Map_{0\text{-}1}}$? This should be specified in the caption.

Response:

The caption of Figure 2 is extended according to your suggestion. *"Figure 2: Derivation of equation for overestimation correction using L-velocity. Eulerian velocities* V_{0-i}^{E} *(i=1, 2, … n) are represented as bars. The red line is the average Eulerian velocity* $\overline{V}_{0-i}^{E}$ *of* V_{0-1}^{E} *and* V_{0-n}^{E}*. The blue line is the average Lagrangian velocity* $\overline{V}_{0-i}^{L}$ *of* V_{0-1}^{L} *and* V_{0-n}^{L} *derived from* V_{0-1}^{E}*. The black line is the average Lagrangian velocity* $\overline{V}_{0-i}^{L(n)}$ *of* $V_{0-1}^{L(n)}$ *and* $V_{0-n}^{L(n)}$ *derived from* V_{0-n}^{E}*."*

L125-155. I think that the choice of hyperscript and subscript in equations could be simplified for the seek of the reader's understanding. First, ice velocity maps are defined as $\mathbf{Map_{0\text{-}i}}$, V is used for Eulerian ice velocity and U is used for Lagrangian ice velocity. All of these are referring to ice velocities, so I suggest you switch to V for the velocity maps, $\mathbf{V_{E(0\text{-}i)}}$ for the Eulerian speeds and $\mathbf{V_{L(0\text{-}i)}}$ for the Lagrangian ones. You could also do $\mathbf{V^{L}_{0\text{-}I}}$ and $\mathbf{V^{E}_{0\text{-}I}}$, since I don't think that the use of J at line 240 is necessary for understanding (you could just say in the text that you calculate the overestimation of all sub-images).

Response:
We accepted your suggestion and used V for map, $V^E_{0\text{-}i}$ for Eulerian and $V^L_{0\text{-}i}$ for Lagrangian velocity throughout the manuscript.

L 133. What do you define as a "short" lagrangian trajectory? This should depend on the glacier speed (see earlier comments on fast flowing glaciers), hence the distance where this premise holds decreases when the glacier speed increases (which is in part linked to the local bedrock slope). Furthermore, this premise holds if you assume that the point moved on a straight line within this short time span.
What do you define as short time span? If I assume this is 1 year, this premise might be true for some ice shelves, but what happens if the flow changes direction? This might happen within 1 year for example for George VI, Abbott, Dotson/Crosson or the Getz ice shelves in Antarctica.
Response:
We added a section in Discussion to introduce a linear regression method for determine the threshold of an "OE-free" time span Δt_{Ref} (see responses to "General Comments" above). The result shows that Δt_{Ref} is ~3.2 years (R^2=0.96) for TG. Thus, Δt_{Ref}=1 year would be appropriate for a large number of glaciers in Antarctica, including TG. However, a reduced time span of ~3.0 months (R^2=0.99) for PIG and ~1.4 months (R^2=0.86) for Jakobshan Isbrae in Greenland should be used. As shown in the proposed regression equation, this threshold changes with how "fast" the ice flows in a glacier.

*We changed the text for Premise I to: "***Premise I:*** Within a period of n short time spans (e.g., 1 year or shorter) each segment (from* P_{i-1} *to* P_i *in Figure 1b) along a flow line is relatively short and straight, so that their accumulated curved L-distance* $S_{0\text{-}i}$ *over a longer span (e.g., 5-10 years) is not significantly different from the corresponding straight E-distance* $D_{0\text{-}i}$*; furthermore, their averaged velocity trends of* V_{0-i}^{L} *in the Lagrangian framework and* V_{0-i}^{E} *in the Eulerian framework (blue and red lines in Fig. 2) do not deviate significantly from each other, and their maximum difference is limited within a threshold (*$V_{0-n}^{L} - V_{0-n}^{E} \leq k\ \sigma$*), where k is a constant and* σ *is the velocity mapping uncertainty."*

L134-136. Here the use of the i=1,2,…,n is confusing. You are describing the case of a short time span, hence why not just using the $V_{0\text{-}1}$ and $U_{0\text{-}1}$ (as you just said in the previous lines)? Or $V_{i\text{-}i+1}$? What is a short time span on Figure 2 ? All of this Premise holds in what you define as a "limited time span" and "short L trajectory" (which should be straight). Please clarify these points.
Response:
Agreed. We now use "*i to i+1*". Thus, we changed *Premise I*. See text above.

The author choose to make a clear distinction between the theory vs the application,

which I think was a good idea, but here, it would help the readers to have some more self-explanatory examples, as it is done in section 2.3.

Response:

We revised the text of *Premise I*. Figures 1b and 2 are used in the text to explain the concept. That way we link the theory of *Premise I* with "application" in these two figures. Hopefully, this will help the readers to better understand.

L 137. This could be reformulated, for the more clarity, to "In reality, the available historical images only allow us to produce eulerian velocity maps with a long timespan, i.e Map$_{0\text{-}n}$ which leads to the maximum overestimation value as defined in equation 2"

Response:

Accepted. The sentence is rewritten accordingly.

L138. "As we can only use Map$_{0\text{-}n}$, the lagrangian velocity, for a long time span, is defined as follow". Please also add a reference to the line in Figure 2.

Response:

Yes, the sentence is changed to: "*As we can only use the map $V_{0\text{-}n}$, the L-velocity (black line in Figure 2), for a long time span, is defined as follow:*"

We added the text to the black line in Figure 2: "*Average L-velocity* $\overline{V}_{0-i}^{L(n)}$ *from* V_{0-n}^{E}".

L 141. "Consequently, the 1 year L-velocity U'$_{0\text{-}1}$…"

Response:

Thanks. It is so changed.

L145. What do you define again as a limited time span? If you compare Map$_{0\text{-}1}$ and Map$_{0\text{-}n}$, then you are comparing the smallest and largest time span, hence the use of the term "short timespan" is a bit confusing

Response:

It is changed to "*Within the time span of n years ……*"

L145. I guess that the magnitude of the velocity Map$_{0\text{-}n}$ should be larger than Map0-1 , but the pattern is similar ? Can you provide a figure example with velocity direction to illustrate this point?

Response:

We agree that the text along the three lines in Figure 2 is a bit confusing. The blue is the average L-velocity calculated from map $V_{0\text{-}1}$, and red line is the average E-velocity calculated from 1 year and n year E-velocities. Generally, the L-velocities are greater than E-velocities (see blue and red examples in Figure 5). We revised text in Figure 2 as follows:

"Average L-velocity $\overline{V}_{0-i}^{L(n)}$ *from* V_{0-n}^{E} *(for black line)*

Average L-velocity $\overline{V}_{0-i}^{L}$ *from* V_{0-1}^{E} *(for blue line)*

Average E-velocity $\overline{V}_{0-i}^{E}$ *from* V_{0-1}^{E} *and* V_{0-n}^{E} *(for red line)"*

L148. The first part of the sentence can be removed since it has been described earlier, before Premise II. Then, you can just start with: "Hence, based on Premise II, we have…."

Response:

Thanks. The text is revised accordingly.

L150. Again, the $U_{0\text{-}n}$=$V_{0\text{-}n}$ is based on the fact that you are considering only short timespan. But is that the case if you use "0-n" ? (see earlier comment)

Response:

Whether $V_{0-n}^{L} = V_{0-n}^{E}$ (or $U_{0\text{-}n}=V_{0\text{-}n}$) in *Premise I* does depend on the time span, given acceleration and geometric complexity of a glacier. We added two subsections in Discussion to show our new experimental results.

L 151. I would reformulate this sentence to remind the reader about the aim of this paper : "Consequently, using the map with the longest timespan, we can go back to $V_{0\text{-}1}$ using a Correction term defined as Correction=$V_{0\text{-}n}$-$U'_{0\text{-}n}$".

Response:

Thanks. The sentence is revised accordingly: "*Consequently, using the map with the longest time span, we can go back to* V_{0-1}^{E} *using a correction term defined as Correction=*$V_{0-n}^{E} - V_{0-n}^{L(n)}$*.*"

L 164. What does it mean to interpolate the positions to the sub grid-level ? Does it make any sense to interpolate the position at a higher level of resolution than the velocity field?

Response:

The interpolation is not used to make a new higher resolution velocity map, but to determine the positions of the distance segments for L-distance integration. Thus, the intermediated sub-grid positions are used for a continuous distance integration. The sentence is changed to: "*The sub-grid positions of the monthly segments are interpolated for integration of the overall L-distance.*"

Figure 3. Please add a general Figure of the entire Pine Island glacier, to check out where the location of your flowline is (similar as Figure 4).

Response:

It is done. Thanks.

L179-180. What about orthorectification errors in historical images ?

Response:

We added it to the sentence: "*Despite the subpixel accuracy of the orthorectification of historical images and ……*"

Acceleration computation: this has already been described L147. I would suggest to move this part earlier (or remove it).

Response:

This section is now moved to the earlier part.

Section 3.1. Since section 2.3 shows an example over Pine Island glacier, I don't know why the author didn't continue using this example. Since the overestimation is quite spectacular, I would strongly suggest to use Pine Island instead of Totten here.

Response:

Yes, we accepted your suggestion and added the PIG results in *Experiment 3* (see responses to General Comments).

Limited by the fast velocity, lost image features over long time spans, and availability of images, the maximum span is 7 years and most of the ice shelf cannot be covered. Since TG has a complete coverage and more systematic results, its results in *Experiment 1* are presented in the main text of the paper. The PIG results in *Experiment 3* will be presented in Appendix.

L213. Why did you choose a 7 year trajectory ?

Response:

We found the earliest available high-quality Landsat 8 images in 2013 and latest ones in 2020 (7 years). We changed the sentence: "*To avoid lower quality historical velocity maps that may influence the effectiveness of the validation, we use the earliest available high-quality Landsat 8 images from 2013 to 2020 to produce velocity maps* $V_{2013\text{–}i}$ *(i=2014, ... 2020)……*"

L217. See previous comment on the choice of symbols in equations.

Response:

They are all fixed throughout the manuscript (see responses to previous comments)

L215. Do you generate the map separately or over the entire glacier directly ?

Response:

Because we have to generate maps for 7 time spans for validating *Premises I* and *II*, it is a lot of work to produce them all. Therefore, we only mapped areas of 5 trajectories, instead of the entire glacier directly.

L225. I am surprised about the error estimation here. Millan et al., 2019; had some smaller number for 1 year map of ice velocity using Landsat-8. Can you discuss why is that? How does your map compare with available NSIDC data ? What is the difference with recent map assembled from sar interferometry? (see Mouginot et al., 2019)

Response:

The highest accuracy of less than 1 m/year was achieved by using **InSAR** technique

(Mouginot et al., 2019), which used the InSAR phase information and the data requirement is generally high. An accuracy of 10 m/year was reported by Millan et al. (2019) for average annual velocities from **multiple individual velocities** derived from Sentinel-2 (10 m resolution) and Landsat-8 (15 m resolution) and other images. Similarly, Gardner et al. (2018) also achieved an accuracy of 10 m/year of annual velocities by averaging velocities derived from **multiple Lansat-7 and -8 image pairs** (15 m resolution). However, our velocity sub-maps (5 areas) were built from **only one Landsat-8 image pair** (15 m resolution) for each map. Given the accuracy of individual maps as $\sigma^2{}_i$, the accuracy of the averaged velocity is $= \frac{\sqrt{\sum \sigma_i^2}}{n}$. In general, the accuracy of the average annual velocity should be smaller than that of the velocity of an individual pair. Therefore, our accuracy of 20 m/year for 1-year (individual pair) and 3 m/year for 7-year (individual pair) velocities are reasonable values.

References

Mouginot, J., Rignot, E., & Scheuchl, B.(2019). Continent wide, interferometricSAR phase, mapping of Antarctic icevelocity. Geophysical Research Letters,46, 9710–9718. https://doi.org/10.1029/2019GL083826

Millan, R., Mouginot, J., Rabatel, A., Jeong, S., Cusicanqui, D., Derkacheva, A., & Chekki, M. (2019). Mapping surface flow velocity of glaciers at regional scale using a multiple sensors approach. Remote Sensing, 11(21). https://doi.org/10.3390/rs11212498

Gardner, A. S., Moholdt, G., Scambos, T., Fahnstock, M., Ligtenberg, S., van den Broeke, M., and Nilsson, J.: Increased West Antarctic and unchanged East Antarctic ice discharge over the last 7 years, The Cryosphere, 12, 521–547, https://doi.org/10.5194/tc-12-521-2018, 2018.

L239. Is it “faster than” or “close” ?

Response:

We deleted “close to”.

L240. I think that the use of “J” in exponent is adding to much complication (see earlier comment). You can just specify that you do the calculation for all sub-images.

Response:

We accepted your suggestion and removed “J” throughout the manuscript.

L257. The “apparent parellelity” is quite subjective I think. Is the premise II validated for case 2c and 2a?

Response:

Cases 2c and 2a are ok (average difference of 1.2 m a^{-2}). Cases 3c and 3a are 2.1 m a^{-2} (max) that is still within the allowable uncertainty of 3 m a^{-2} determined based on the velocity mapping uncertainty. To make it objective, we changed “*apparent parellelity*” to “*relatively well-maintained parellelity*”.

Figure 5. Add the direction of the flow in the sub-images. You could also consider using different symbols for the 7 yr and 1 yr trajectory and use a color gradient for the position of the points that changes with the year. This "year" color could then also be used in the scatter plots.

Response:

We revised Fig. 5 accordingly.

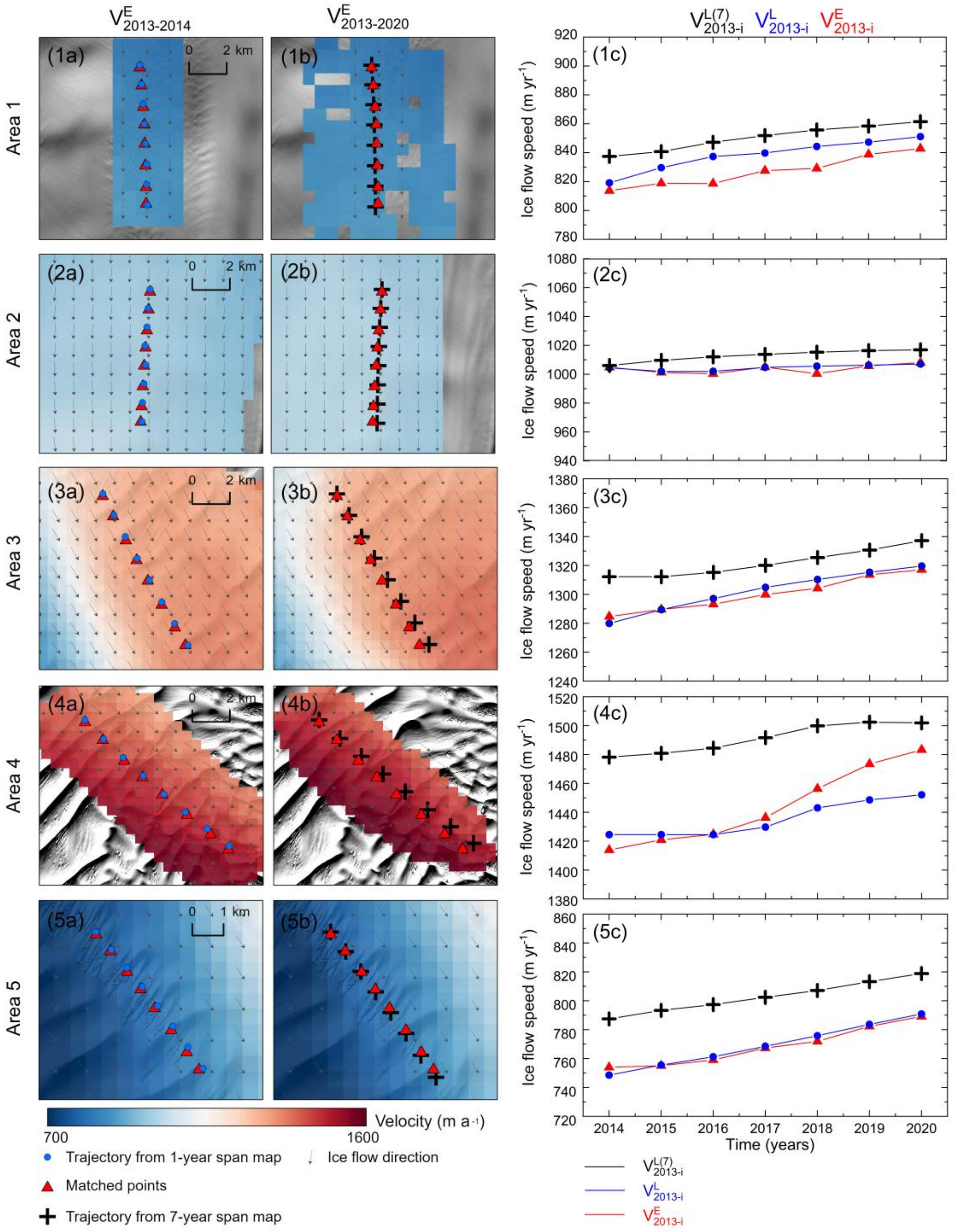

Figure 5.

Table 1. Please add more details on the caption of the Table, ie, the content of each column.

Response:

The caption is revised: "*Table 1. Velocity and acceleration in Eulerian and Lagrangian frameworks used for validation of the overestimation correction method. "Actual E-velocity and OE" lists actually mapped 1-year and 7-year E-velocities and their differences as overestimations in all five areas. "Premise I" contains 7-year L-velocities computed from the 1-year velocity map and corresponding L- and E-velocity differences, which are used for validating Premise I. "Premise II" illustrates averaged L-accelerations computed from the 7-year and 1-year velocity maps, respectively, as well as their differences, which are used for validating Premise II. "Overestimation correction" presents 7-year L-velocities computed from the 7-year map, overestimation corrections, and E-velocities and residuals (or errors) after correction.*"

L302. The acronym OE has been defined before.

Response:

We deleted "(OE)".

Section 4 Discussion. Can you discuss the performance of your method, with the relatively simple approach defined by Berthier et al., 2003? I think that the section is missing some discussion on the applicability of the method to 1) fast glacier, 2) the glacier geometry, which can be much more complex than the glaciers that were used here to validate the method (not straight) (see earlier comment). An additional discussion about the significance of the correction, with respect to the seasonal variation in ice flow velocity of the glacier should also be discuss, if the method is expected to be applicable in Greenland. Specifically, does the magnitude of the correction could exceed the natural variability of the glacier? I guess that the amount of acceleration with a flowline would need to be significant in order to induce a correction that would exceed the variability of the seasonal signal?

Response:

The comparison results with the midpoint method are presented in the response to comment L54.

The responses related to fast glaciers, complex geometry, seasonal variability, applicability in Greenland etc. are presented in responses to General Comments.

Figure 5-6. Can you provide a figure of the corrected ice velocity? Maybe a difference map.

Response:

We produced the map of DG (Fig. 6) after correction (Fig. R1-11b). It does not appear distinctly different from the map before correction (Fig. R1-11a). Thus, we will not include this corrected map in the main text. The map with the OEs (Fig. 6c) is actually the difference map. Similarly, the corrected submaps in PIG are also visually not distinct from those before correction. You may agree that we would not add the corrected or difference map to Figs. 5 and 6.

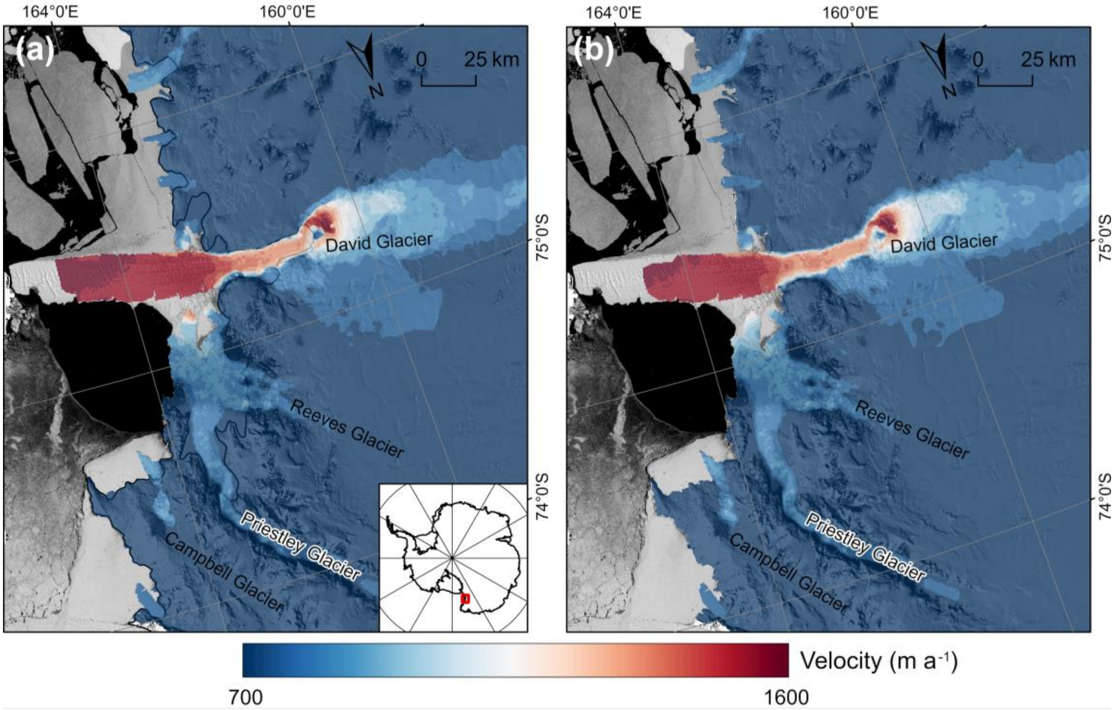

Figure R1-11. E-velocity in TG: (a) before OE correction, and (b) after OE correction.