Dear Anonymous Reviewer 3, Dear Editor,

We would like to thank the reviewer for the insightful comments and good feedback provided on our manuscript. Below we provide answers to each comment. The original comments are numbered in **red**, and written in **bold**. Our answers are provided in black, and the manuscript text including the proposed changes is provided in "*quoted italics*". For an easier comparison between this response letter and the original manuscript, we refer to the figures by their original numbering, despite a new figure being presented here (and proposed for inclusion). The new figures is presented and referenced in this letter as "new figure", but its numbering will be adjusted in the revised manuscript.

General comments:

1. Choice of model: The study addresses the issue of nunataks acting as a barrier to ice flow using an SSA model, with the authors correctly noting using a full Stokes model would be infeasible considering the long timescales involved (i.e. 20 kyr). I agree with Reviewers 1 and 2 that this warrants a robust discussion of model limitations given the 3D flow regime and steep gradients that would be present near nunataks. In addition, I think it would be useful for the authors to draw on the results of model intercomparison projects (e.g. MISMIP+, Cornford et al., 2020) to discuss whether other types of models would behave similarly or not. I'd note that SSA models are atypical for paleo-applications, with the majority of such studies using hybrid models and, more recently, higher order models, so a discussion of the model dependency of the findings in this study would be helpful.

Thank you for your comment, and for highlighting the new study by Cornford et al. (2020). Based on your comments and those provided by reviewers 1 and 2, we have expanded Sect. 2 to better justify the validity of the SSA approximation, clarify the boundary conditions used in our model, as well as its limitations compared to a Full Stokes model. We also note that Úa has been previously used to model paleo ice streams (Jones et al., 2021). The start of Sect. 2 now reads:

"To better understand how ice flow interacts with the steep topography of nunataks and what impact this interaction has on ice surface elevation patterns, we perform a suite of numerical simulations using an idealised setup. We use Úa (Gudmundsson, 2020), an ice flow model that solves the shallow shelf, also referred to as shelfy stream approximation (SSA or SStA) of the Stokes equations (Cuffey and Patterson, 2010) on a horizontal, finite element mesh. Úa has been successfully applied to model the ice flow of idealised (e.g. Gudmundsson et al., 2012), modern (e.g. Miles et al., 2021), and palaeo-ice streams (e.g. Jones et al., 2021). Úa solves the ice surface and momentum equations simultaneously, and its finite element formulation allows for an adaptive mesh refinement in areas

of particular interest, such as where the ice shallows around nunataks. For the modelled domain (which we describe below), an unstructured mesh was generated, which is refined during simulation time (including during spin up) based on a series of glaciological refinement criteria. Element size is refined according to ice thickness, from a maximum of 8 km down to 205 m around the interface between ice and nunatak, where ice thickness approaches the minimum (which we set as 1 m). The mesh is also refined (to 500 m) in a 4 km buffer zone centered at the grounding line. The mean element size after spin up was 740 m (414 m median).

In the vicinity of nunataks, where the ice is thinnest, the ice flow regime has a relevant vertical component, which is not captured by the SSA approximation. A more accurate representation of the ice dynamics for such regions can be achieved with full-Stokes models. However, such models are currently too computationally demanding when adopted for long simulation periods and multiple experiments (e.g. Schannwell et al., 2020). Over the thinnest-ice areas, the horizontal scales resolved in our model are of the order of a few hundred metres (commonly noted as $O(10^2)$ *m), while the vertical scale is of the order of a few metres (*$O(10^0)$ *m). This yields an aspect ratio of* $O(10^{-2})$*, which falls within the range where shallow approximations are applicable (*$O(\leq 10^{-2})$*, e.g. Fowler and Larson, 1978). To our knowledge, no intercomparisons between simplified and full-Stokes models exist that focus on the representation of thickness gradients. At least within the context of MISMIP+ (Cornford et al., 2020), there are little variations between SSA (including the model used in our study), L1Lx, and higher-order models regarding simulated ice retreat. Full-Stokes models that participated in the MISMIP+ experiment also agreed with the simplified-physics models, indicating that other models should behave similarly. Instead, the MISMIP+ experiments highlighted the importance of the formulation of the sliding law (i.e., Weertman versus Coulomb-limited). The two sliding laws strongly differed in their rates of grounding line retreat, but it has been shown that such differences decrease with increasing spatial resolution (i.e., model grid/mesh refinement) at the grounding line (Gladstone et al., 2017). Here, we adopt a Weertman law for basal sliding and the required refinement at the grounding line to minimise sliding law issues.*

At the free upper surface, the streamline upwind Petrov-Galerkin (SUPG) method is applied, which ensures model stability over regions of pronounced ice surface topography (Wirbel and Jarosch, 2020). In our experiments, the ice surface topography is steepest when the ice front retreats from the downstream end of the domain and in the vicinity of nunataks. When ice thins below the prescribed minimum (which we set to 1 m), the model uses the active-set method (Durand et al., 2009; Gudmundsson et al., 2012). In this method, violated ice thickness constraints are activated using the Lagrange multiplier approach (Ito and Kunisch, 2008), which is applied to the momentum equation, and ensures a better representation of the ice dynamics compared to simply resetting the thickness to the prescribed minimum value, as is commonly used in finite element models.

The model domain (Sect. 2.1) and spin up procedure (Sect. 2.2) are the same for all simulations. In a first set of simulations, we evaluate changes in ice surface elevation up and downstream of a single nunatak under three different thinning scenarios (Sect. 2.3). We then use the forcing that provides the highest ice-thinning rates to evaluate the impact of multiple nunataks and the width of glaciers between them on ice surface elevation patterns (Sect. 2.4). Finally, we repeat the last experiments for a series of regular meshes (without refinement) at horizontal resolutions commonly used in ice sheet models (Sect. 2.5). This final set of experiments assesses how well different grid resolutions resolve changes in ice surface elevation across steep marginal topography under thinning scenarios, and their implications for simulations of past ice surface elevations."

References not previously in the manuscript:

Cornford, S.L., et al. (2020). Results of the third Marine Ice Sheet Model Intercomparison Project (MISMIP+). *The Cryosphere*, *14*, 2283-2301.

Durand, G., Gagliardini, O., De Fleurian, B., Zwinger, T., & Le Meur, E. (2009). Marine ice sheet dynamics: Hysteresis and neutral equilibrium. *Journal of Geophysical Research: Earth Surface*, *114, F03009*.

Fowler, A. C., & Larson, D. A. (1978). On the flow of polythermal glaciers-I. Model and preliminary analysis. *Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences*, *363*(1713), 217-242.

Ito, K., & Kunisch, K. (2008). *Lagrange multiplier approach to variational problems and applications*. Society for Industrial and Applied Mathematics.

Wirbel, A., & Jarosch, A. H. (2020). Inequality-constrained free-surface evolution in a full Stokes ice flow model *(evolve_glacier v1.1)*. *Geoscientific Model Development*, *13*, 6425-6445.

2. Context of findings: The authors assert throughout the paper that “ice sheet models overestimate ice surface elevations and underestimate the pace of ice sheet melt contributing to sea level rise compared to empirical reconstructions.” I find this to be an oversimplification for a number of reasons. There are many examples of ice sheet models that fit empirical constraints during glacial and interglacial periods (e.g. Whitehouse et al., 2012; Golledge et al., 2013; Goezler et al., 2016; Whitehouse et al., 2017; Clark et al., 2020), as well as the rate of mass loss during deglaciations and through the Holocene (e.g. Gomez et al., 2013; Lecavalier et al., 2014; Tigchelaar et al., 2018; Cuzzone et al., 2019; Briner et al., 2020; Albrecht et al., 2020a). While two studies are cited with records from the Transantarctic Mountains (Lines 72-73), even models analysed within those studies show similar rates of ice thinning as observed in the records. I agree with the authors that datamodel mismatches occur in both Greenland and Antarctica, particularly with respect to the timing of regional ice thinning, though I’m not sure that the findings here definitively rule out other possible explanations, such as uncertainty in climate forcing (e.g. Lowry et al., 2019; Albrecht et al., 2020a), or uncertainty in the surface exposure ages themselves (e.g. Jones et al., 2019). It would be more accurate to simply point out that many previous paleo-ice sheet model studies, which use coarse resolution, do not resolve

site-specific features and this could contribute to observed timing mismatches between surface exposure records and model simulations.

We agree with the reviewer, and acknowledge that the critique of previous ice sheet modelling should be clearer. We have rephrased the last paragraph of the introduction to reflect the points made by the reviewer, and clarify that by 'empirical constraints' we were referring to cosmogenic nuclide dating:

"While ice sheet models of Greenland and Antarctica have been able to fit ice geometries reconstructed from empirical data, including the rates of ice thinning that are recorded by cosmogenic exposure ages (Whitehouse et al., 2012; Briggs et al., 2014; Albrecht et al., 2020), most models struggle to replicate the inferred timing of ice thickness change (Jones et al., 2020; Stutz et al., 2020; Johnson et al., 2021). Such data-model mismatches are likely due to a combination of factors, one of which is the spatial resolution of the models (Lowry et al, 2020; Johnson et al., 2021). When run over a glacial-interglacial cycle, ice sheet models do not typically resolve the pattern of ice flow around individual nunataks, and consequently cannot resolve the transient response of the ice surface at the sampled locations."

We also simplified our discussion in line with the reviewer's suggestion:

"Experiments using a regular grid at these resolutions do not resolve site-specific ice surface elevation responses, which contributes to mismatches relative to reconstructions from cosmogenic exposure dating (Spector et al., 2019; Stutz et al., 2020; Johnson et al., 2021)."

References not previously mentioned:

Briggs, R. D., Pollard, D., & Tarasov, L. (2014). A data-constrained large ensemble analysis of Antarctic evolution since the Eemian. *Quaternary Science Reviews*, *103*, 91-115. doi: https://doi.org/10.1016/j.quascirev.2014.09.003

Lowry, D. P., Golledge, N. R., Bertler, N. A., Jones, R. S., McKay, R., & Stutz, J. (2020). Geologic controls on ice sheet sensitivity to deglacial climate forcing in the Ross Embayment, Antarctica. *Quaternary Science Advances*, *1*, 100002. doi: https://doi.org/10.1016/j.qsa.2020.100002

Johnson, J. S., Pollard, D., Whitehouse, P. L., Roberts, S. J., Rood, D. H., & Schaefer, J. M. (2021). Comparing glacial-geological evidence and model simulations of ice sheet change since the last glacial period in the Amundsen Sea sector of Antarctica. *Journal of Geophysical Research: Earth Surface*, e2020JF005827.doi: https://doi.org/10.1029/2020JF005827

3. Influence of GIA: I am surprised that glacial isostatic adjustment was not considered in the simulations given the timescale. The calculation of surface exposure ages is highly dependent on the elevation, and correcting for GIA-driven elevation changes in non-trivial (Jones et al., 2019). Ice sheet models themselves are highly sensitive to GIA because it impacts bed elevation (e.g., Gomez et al., 2013; Kingslake et al., 2018; Colleoni et al., 2018; Albrecht et al., 2020b). While this study uses idealized experiments, in reality the basal topography would evolve through time in response to changes in ice loading. The authors do provide a useful discussion on limitations in the simplified SMB forcing for the idealized experiments (Lines 320-334), but a similar discussion with respect to GIA is currently lacking.

We have now included a discussion about the role of GIA, together with our discussion on the simplified SMB forcing. We highlight that in Antarctica, GIA is more likely to play a regional role, rather than add substantial variations between the upstream and downstream faces of a nunatak:

"We do not consider glacial isostatic adjustment (GIA) in our experiments. Bedrock topography evolves through time due to changes in ice loading, which can influence the sensitivity of the ice sheet to sea level forcing, potentially impacting the grounding line position (Whitehouse et al., 2019), and the cosmogenic production rate, possibly impacting the calculated exposure ages (Jones et al., 2019). In terms of patterns of exposure up and downstream of nunataks, the influence of differential isostatic bedrock uplift rates on each side of the nunatak are much smaller than the changes in ice surface elevation considered here, since bedrock elevation changes due to GIA happen at a spatial scale larger than the ~60 km observed in our experiments (e.g., Ivins and James, 2005)"

References not previously in the manuscript:

Ivins, E., & James, T. (2005). Antarctic glacial isostatic adjustment: A new assessment. *Antarctic Science, 17*(4), 541-553. doi:10.1017/S0954102005002968

Jones, R. S., Whitehouse, P. L., Bentley, M. J., Small, D., & Dalton, A. S. (2019). Impact of glacial isostatic adjustment on cosmogenic surface-exposure dating. Quaternary Science Reviews, 212, 206-212. doi: https://doi.org/10.1016/j.quascirev.2019.03.012

Whitehouse, P. L., Gomez, N., King, M. A., & Wiens, D. A. (2019). Solid Earth change and the evolution of the Antarctic Ice Sheet. *Nature communications*, *10*(1), 1-14. doi: https://doi.org/10.1038/s41467-018-08068-y

Specific comments:

4. Line 16-17: See above general comment on context of findings.

We have rephrased this passage:

"Such differences, however, are not typically captured by continent-wide ice sheet models due to their coarse grid resolutions. Their inability to capture site-specific surface elevation changes appears to be a key reason for the observed mismatches between the timing of ice free conditions from surface exposure dating and model simulations."

5. Fig 2: Is there a scale for the slices? The colours correspond to upstream and downstream, correct? The point is to show that samples are taken in nearly all directions?

The reviewer is correct. Following the suggestion by Reviewer 1 (comment 12), we have removed the colour coding, as it added no new information and was distracting.

6. Line 136: citation? Does SMB only change in one direction?

We have added a reference to Agosta et al. (2019, The Cryosphere), who show that most regions where nunataks are present are not suffering ablation at present. As for SMB changes, they only happen in one direction (i.e., increased ablation). This is shown in Fig. 3a,c, and we made it clearer at the beginning of Sect. 2.3:

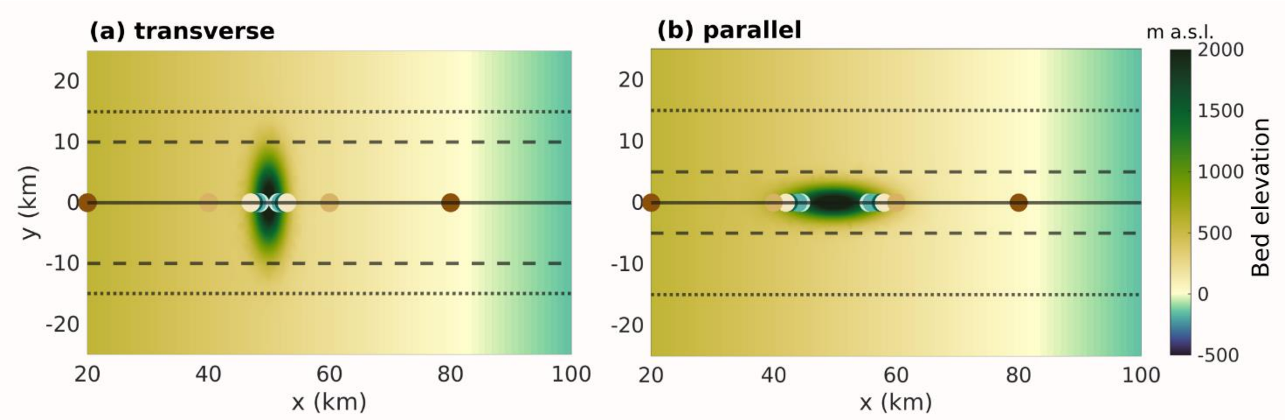
"In order to understand whether ice thinning occurs uniformly up and downstream of a nunatak, we impose three different degrees of ice thinning uniformly over the domain. The perturbations to SMB that induce ice thinning are applied through b(t) in Eq. (2), and only evolve in one direction over the entire domain, towards increased ablation."

7. Line 160: Any sensitivity tests for this value? 5-6% K-1 is appropriate for EAIS (Frieler et al. 2015). WAIS is less straightforward (Fudge et al. 2018).

We believe our text lacked clarity, which might have confused the reviewer. The percentages shown refer to the weight of the SMB factor applied in b(t). Using the 'thw' scenario as an example, the value for b(0) is 0.08*-0.75 m a^{-1}, while b(20 kyr) is 1.0*-0.75 m a^{-1}. As originally stated in L161-163, these values were chosen so that the total ice thinning matches the thinning at the respective regions in a deglacial model experiment by Golledge et al. (2014). We have added the example above to the text in Sect. 2.1 to make it clearer.

8. Fig 4: Could you make the inset larger? It is difficult to see the coloured circles that correspond to panels c and d.

Following the changes based on the suggestion by Reviewer 1 (comment 15), we have removed the inset, and instead added this information to the new figure, presented below, which will be incorporated to the manuscript:



New Figure. Bedrock elevation (in metres above sea level, m a.s.l.) used in the experiments where the nunatak is placed (a) transverse and (b) parallel to ice flow. Ice flow in this figure is from left to right. Transect lines show the position of the transects presented in Fig. 4a,b, and coloured circles show the locations for the ice surface evolution analysis presented in Fig. 4c,d following the same colours as the lines therein.

We thank the reviewer for the thorough and insightful review of our manuscript, and look forward to the opportunity to present a significantly updated manuscript based on the revisions outlined above.