

Dear Anonymous Reviewer 2, Dear Editor,

We would like to thank the reviewer for the insightful comments and good feedback provided on our manuscript. Below we provide answers to each comment. The original comments are numbered in **red**, and written in **bold**. Our answers are provided in black, and the manuscript text including the proposed changes is provided in *“quoted italics”*. For an easier comparison between this response letter and the original manuscript, we refer to the figures by their original numbering, despite new figures being presented here (and proposed for inclusion). New figures are presented and referenced in this letter as “new figure”, and their numbering will be adjusted in the revised manuscript.

(1) I have some doubts whether the shelfy-stream approximation (SStA) is appropriate for the tackled problem. SStA is SSA (shallow-shelf approximation) with basal drag, thus it assumes plug flow and a shallow geometry without sharp gradients. Especially when zooming in high resolution to the immediate vicinity of nunataks, these conditions are certainly violated, as the flow regime is strongly 3D and steep surface gradients occur. I think that a proper simulation of this behaviour requires nothing less than full-Stokes dynamics. There is probably not much to be done about the shortcoming at this stage, but at the minimum, the limitation should be discussed very openly and clearly.

BTW, which boundary condition is applied at the nunataks? I suppose they are treated like the ice base?

We agree with the reviewer that a Full Stokes model would yield more accurate results, and explicitly mentioned that in the original manuscript (L87). Nevertheless, based on the comments above, as well as those provided by reviewers 1 and 3, we have expanded Sect. 2 where we present the used model’s capabilities, highlighting its limitations as well as its features that also improve the representation of ice flow over our regions of interest. We also provide more information about how the boundary between ice and nunatak is handled in the model:

“To better understand how ice flow interacts with the steep topography of nunataks and what impact this interaction has on ice surface elevation patterns, we perform a suite of numerical simulations using an idealised setup. We use Úa (Gudmundsson, 2020), an ice flow model that solves the shallow shelf, also referred to as shelfy stream approximation (SSA or SStA) of the Stokes equations (Cuffey and Patterson, 2010) on a horizontal, finite element mesh. Úa has been successfully applied to model the ice flow of idealised (e.g. Gudmundsson et al., 2012), modern (e.g. Miles et al., 2021), and palaeo-ice streams (e.g. Jones et al., 2021). Úa solves the ice surface and momentum equations simultaneously, and its finite element formulation allows for an adaptive mesh refinement in areas

of particular interest, such as where the ice shallows around nunataks. For the modelled domain (which we describe below), an unstructured mesh was generated, which is refined during simulation time (including during spin up) based on a series of glaciological refinement criteria. Element size is refined according to ice thickness, from a maximum of 8 km down to 205 m around the interface between ice and nunatak, where ice thickness approaches the minimum (which we set as 1 m). The mesh is also refined (to 500 m) in a 4 km buffer zone centered at the grounding line. The mean element size after spin up was 740 m (414 m median).

In the vicinity of nunataks, where the ice is thinnest, the ice flow regime has a relevant vertical component, which is not captured by the SSA approximation. A more accurate representation of the ice dynamics for such regions can be achieved with full-Stokes models. However, such models are currently too computationally demanding when adopted for long simulation periods and multiple experiments (e.g. Schannwell et al., 2020). Over the thinnest-ice areas, the horizontal scales resolved in our model are of the order of a few hundred metres (commonly noted as $O(10^2)$ m), while the vertical scale is of the order of a few metres ($O(10^0)$ m). This yields an aspect ratio of $O(10^{-2})$, which falls within the range where shallow approximations are applicable ($O(\leq 10^{-2})$, e.g. Fowler and Larson, 1978). To our knowledge, no intercomparisons between simplified and full-Stokes models exist that focus on the representation of thickness gradients. At least within the context of MISMIP+ (Cornford et al., 2020), there are little variations between SSA (including the model used in our study), L1Lx, and higher-order models regarding simulated ice retreat. Full-Stokes models that participated in the MISMIP+ experiment also agreed with the simplified-physics models, indicating that other models should behave similarly. Instead, the MISMIP+ experiments highlighted the importance of the formulation of the sliding law (i.e., Weertman versus Coulomb-limited). The two sliding laws strongly differed in their rates of grounding line retreat, but it has been shown that such differences decrease with increasing spatial resolution (i.e., model grid/mesh refinement) at the grounding line (Gladstone et al., 2017). Here, we adopt a Weertman law for basal sliding and the required refinement at the grounding line to minimise sliding law issues.

At the free upper surface, the streamline upwind Petrov-Galerkin (SUPG) method is applied, which ensures model stability over regions of pronounced ice surface topography (Wirbel and Jarosch, 2020). In our experiments, the ice surface topography is steepest when the ice front retreats from the downstream end of the domain and in the vicinity of nunataks. When ice thins below the prescribed minimum (which we set to 1 m), the model uses the active-set method (Durand et al., 2009; Gudmundsson et al., 2012). In this method, violated ice thickness constraints are activated using the Lagrange multiplier approach (Ito and Kunisch, 2008), which is applied to the momentum equation, and ensures a better representation of the ice dynamics compared to simply resetting the thickness to the prescribed minimum value, as is commonly used in finite element models.

The model domain (Sect. 2.1) and spin up procedure (Sect. 2.2) are the same for all simulations. In a first set of simulations, we evaluate changes in ice surface elevation up and downstream of a single nunatak under three different thinning scenarios (Sect. 2.3). We then use the forcing that provides the highest ice-thinning rates to evaluate the impact of multiple nunataks and the width of glaciers between them on ice surface elevation patterns (Sect. 2.4). Finally, we repeat the last experiments for a series of regular meshes (without refinement) at horizontal resolutions commonly used in ice sheet models (Sect. 2.5). This final set of experiments assesses how well different grid resolutions resolve changes in ice surface elevation across steep marginal topography under thinning scenarios, and their implications for simulations of past ice surface elevations."

References not previously in the manuscript:

Cornford, S.L., et al. (2020). Results of the third Marine Ice Sheet Model Intercomparison Project (MISMIP+). *The Cryosphere*, 14, 2283-2301.

Durand, G., Gagliardini, O., De Fleurian, B., Zwinger, T., & Le Meur, E. (2009). Marine ice sheet dynamics: Hysteresis and neutral equilibrium. *Journal of Geophysical Research: Earth Surface*, 114, F03009.

Fowler, A. C., & Larson, D. A. (1978). On the flow of polythermal glaciers-I. Model and preliminary analysis. *Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences*, 363(1713), 217-242.

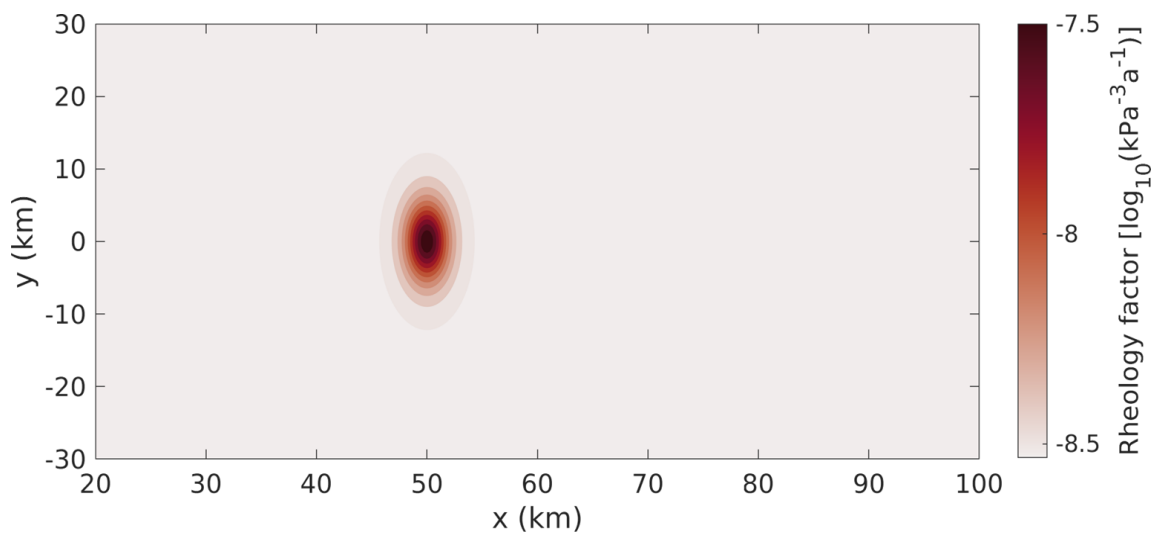
Ito, K., & Kunisch, K. (2008). *Lagrange multiplier approach to variational problems and applications*. Society for Industrial and Applied Mathematics.

Wirbel, A., & Jarosch, A. H. (2020). Inequality-constrained free-surface evolution in a full Stokes ice flow model (*evolve_glacier v1.1*). *Geoscientific Model Development*, 13, 6425-6445.

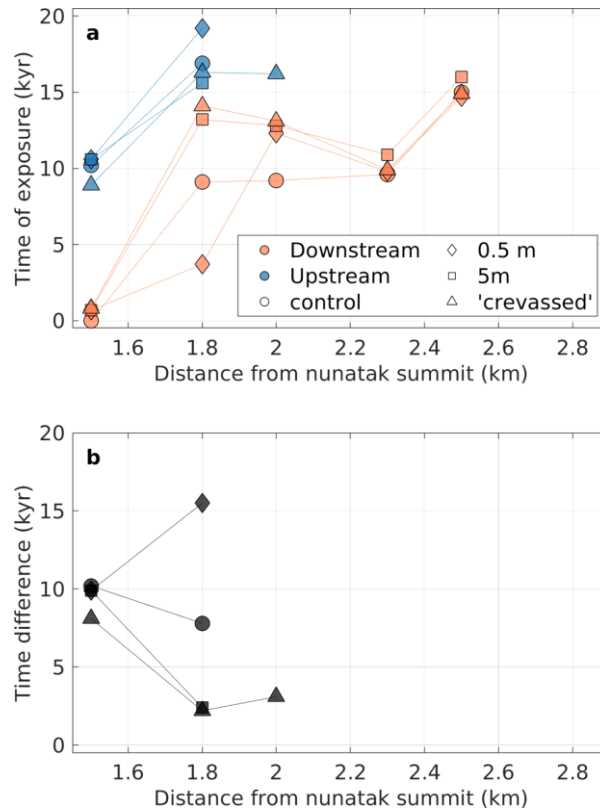
(2) What I am also missing is a discussion about the potential impact of crevassing in the vicinity of nunataks. I would expect that crevasses often occur around such flow obstacles, with the consequence that the ice is effectively softened and the large-scale flow less disturbed than under the assumption of undamaged ice that has to find its way around the obstacle. This effect has the potential to alter/weaken the described influence on the ice surface significantly. At least, a qualitative discussion about it deems appropriate. Even better would be to investigate the effect by running some tests with an assumed softening around the nunataks.

The reviewer points out a very important aspect, which we overlooked. Hence, we now include a test under the same conditions as the 'thw' experiment with one nunatak, but considering a progressively softer ice (i.e., up to the equivalent of a temperature of -5 °C) as the ice gets thinner towards the nunatak summit. We added the spatial distribution of the rheology factor, and a comparison of how the time of exposure changes to the supplement, complementing the suggestion from Reviewer 1 (comment 1) about the sensitivity of the timing of exposure to our choice of minimum thickness. We refer to these tests when discussing our results (Sect 4.1, paragraph 2):

“The ice surface steepening and consequent mismatch between the up and downstream sides of the nunatak increases as the ice thins, until the downstream side becomes exposed. Exposure happens earlier downstream, as expected due to lower ice surface elevation, and an equidistant point upstream becomes exposed (or has its thinning stabilised) up to 14 kyr later than its downstream counterpart. The rates of thinning, and consequently the timing of bedrock exposure, are dependent on the choice of the basal sliding coefficient. An increase of ca. 50 % in ice thinning was observed between the higher sliding and control experiments, as well as between the control and lower sliding experiments (not shown). This pattern is expected given the influence of basal sliding on the initial ice sheet geometry (Fig. S5), and highlights that the exposure lags between up and downstream faces of a nunatak observed in the real world will be site dependent. The choice of minimum ice thickness allowed in our model (1 m) also influences the timing of exposure at a given point on the nunatak ([ref. to the new supplementary figure below]). A lower minimum thickness increased lag times, while a higher minimum thickness reduced them. The surroundings of nunataks are often crevassed, which results in a change in ice rheology. Hence, another test was carried out where the prescribed ice rheology becomes progressively softer towards the nunataks by a factor of ten ([ref. to the new supplementary figures below]). In this test, the most notable effect from the choice of ice rheology was a delay in the timing of exposure downstream of the nunatak relative to the control case, by 0.5-5 kyr, but the lag times were still of the same magnitude as the control case (i.e., between 2 and 10 kyr). While we use stabilisation of thinning upstream to determine...”



New Supplementary Figure: Spatial distribution of the rheology factor A for the ‘crevassed’ experiment ([ref. to new figure below]). The value for A away from the nunatak is that used in all other experiments ($\log_{10}(A) \sim -8.5$).



New New Supplementary Figure: As in Fig. 5 of the main text, but for minimum ice thickness experiments where the minimum thicknesses allowed in the model are 0.5 and 5 m, and a “crevassed” experiment, where the ice is softer around the nunatak compared to the rest of the model domain (ref to Fig, above).

We thank the reviewer for the thorough and insightful review of our manuscript, and look forward to the opportunity to present a significantly updated manuscript based on the revisions outlined above.