Review of "Ice volume and basal topography estimation using geostatistical methods and GPR measurements: Application on the Tsanfleuron and Scex Rouge glacier, Swiss Alps" by Neven et al.

General comments:

This study demonstrates different topographic interpolation techniques for two glaciers in the Swiss Alps and investigates their effect on a subglacial water routing model. The authors interpolate GPR measurements of bed topography using kriging, sequential Gaussian simulation (SGS), and multiple-point simulation (MPS) for the Tsanfleuron and Scex Rouge glaciers, as well as synthetic examples. MPS is implemented with a secondary variable, the bed gradient. The authors test different simulation parameters to determine the optimal model setup. The ice volume and hydrological flow paths are computed for each bed elevation model. The authors conclude that MPS is the most robust method for interpolating subglacial topography.

Overall, this study is clear and rigorous and addresses an important problem in glaciology. This study provides a thoughtful and thorough comparison of different interpolation methods that TC readers will find interesting. The authors demonstrate novel interpolation methods and performance metrics that are relevant to topographic interpolation, ice volume estimation, and subglacial hydrology.

Some figures require minor changes, and there are some typos and grammatical issues (see line comments below). There are some paragraphs that are only one sentence long. I recommend appending these sentences to other paragraphs. It would be helpful for general glaciology audiences to define terms such as conditional simulation, variogram, hard data, and non-stationarity. I also recommend providing a brief overview of variograms and the SGS methodology in Section 2.3. While SGS is a well-established method, TC audiences will benefit from an explanation.

Main concerns:

The authors state that kriging cannot be used to compute ice volume uncertainty, but that is not the case. While it is true that kriging does not sample the uncertainty space in the same way that simulation does, kriging can be used to provide the variance or standard deviation of an estimate at any given location. In theory, multiple realizations produced by sequential Gaussian simulation should converge to a distribution that is represented by the kriging solution. For completeness, I recommend that the authors compute the uncertainty in ice volume for the kriging interpolation.

The authors fit a polynomial trend to the data in order to perform the kriging interpolation and sequential Gaussian simulation. This is a somewhat arbitrary, but often necessary, step for variogram-based methods. How was the degree of the polynomial chosen? Does it matter for your results that the synthetic examples are not detrended? I would like to see more justification for the polynomial selection and discussion on the implications of trend estimation. I also recommend mentioning in the discussion that MPS does not require trend estimation, which is another advantage of the MPS method.

More information is needed on the hydrological modeling method. The Pysheds package that the authors use only has examples for topography without ice. The authors do not state whether or not they account for ice overburden pressure in their hydrological modeling. If that is difficult to do with Pysheds, Chad Greene has a nice tutorial (https://www.mathworks.com/matlabcentral/fileexchange/55352-how-to-estimate-subglacial-water-routes). The synthetic examples have a flat ice surface, which could bias the hydrological models. More justification is needed for using the flow accumulation values for the synthetic examples.

I would like to see more discussion on the implications of the hydrological findings. The authors compare the distributions of flow accumulation values for different interpolation methods, but it is unclear why this matters. Perhaps flow accumulation is important for discriminating between a channelized or distributed drainage system? It may also be helpful to refer to studies by Zuo et al., (2020) and MacKie et al., (2021) which previously investigated the impact of MPS and SGS on hydrological flow.

Zuo, C., Yin, Z., Pan, Z., MacKie, E. J., & Caers, J. (2020). A Tree‐Based Direct Sampling Method for Stochastic Surface and Subsurface Hydrological Modeling. *Water Resources Research*, *56*(2), e2019WR026130.

MacKie, E. J., Schroeder, D. M., Zuo, C., Yin, Z., & Caers, J. (2021). Stochastic modeling of subglacial topography exposes uncertainty in water routing at Jakobshavn Glacier. *Journal of Glaciology*, *67*(261), 75-83.

Line comments:

Lines 13-14: “significantly improve for example the precision of under-glacial flow estimation” For clarity, specify that you are referring to hydrological flow. Add commas before and after “for example”

Line 19: no comma needed after “crucial”

Line 28: “Depending of” should be “Depending on”

Lines 30-32: “the choice of the method becomes critical since the flow process is highly non-linear and is strongly linked to the morphology of the subglacial topography”

I recommend clarifying that you are referring to hydrological flow, not ice flow (which is also non-linear and dependent on morphology). What is meant by a non-linear flow process? I think the authors mean that flow accumulation is not a linear function of bed elevation.

Line 35: “produces by construction”

Awkward wording. It would be sufficient to just say “produces”

Lines 36-38: “Furthermore, even if kriging allows estimation of the local uncertainty on the elevation of the bedrock, it cannot be used to estimate the uncertainty of the global volume of ice (see e.g. Chiles and Delfiner, 2012, p. 478).”

Why can’t the uncertainties from kriging be used to estimate ice volume uncertainty? What happens when you use the kriging bed uncertainties to estimate ice volume uncertainty?

The reference to Chiles and Delfiner (2012, p. 478) does not support this statement. Chiles and Delfiner (2012, p. 478) describe a scenario where surface area increases with roughness. It is true that kriging underestimates the surface area of topography (if each grid cell is represented by a tilted plane), but this shouldn’t affect the volume calculation.

Line 41: “two points spatial statistics” should be “two point spatial statistics”

I recommend elaborating on this sentence so that this concept is more understandable to non-geostatisticians. It might be more understandable to say that these methods are based on the variance between pairs of points, and briefly state what a variogram is.

Line 51: “that the one” should be “than the one”

Line 55: “require to define” → “require the definition of”

Lines 54-55: “MPS does not require to define an analytical two-point statistics model to represent the spatial variability but instead infers it in an implicit way”

MPS does not define any statistical model (two-point or otherwise). It is entirely non-parametric. It would be more accurate just to say that MPS does not require the definition of a statistical model.

Line 57: “allow to create” → “allow the creation of”

Line 107: In the methods overview at the end of the introduction, I recommend stating that you will apply a hydrological model to the topography.

Line 125: “exemple” → “example”

Line 139: “This technique allows to co-simulate jointly several variables”

I would elaborate on this sentence for the benefit non-geostatisticians. I think it would be sufficient to say something like “this means that secondary information can be used to improve the simulations.”

Line 142: “the use of Gaussian pyramids to account for multiscale patterns”
It would be helpful to provide a brief explanation of what this is and what it accomplishes.

Line 154: “Furthermore, a secondary variable is used during the MPS simulation”

It took me a while to figure out that the secondary variable is the gradient. I would state this more clearly, and explain why it is beneficial to use the gradient.

Line 157: “Two patterns that show the same relative changes even at different absolute altitudes should be considered similar”

Does this mean that the TIs are detrended?

Line 178: “5’000” → “5,000”

There are a few places here where the apostrophe should be replaced by a comma in numbers.

Figure 3. I recommend changing the scale bar label on part A from “variation from the trend” to “difference from the trend” to be more precise.

Figure 4: “Kriegage” → “Kriging.” Do the dashed lines represent synthetic GPR surveys? There are three lines in the figure, but in the text it says there are two.

Line 197: “SGS and ordinary kriging are applied using the same variogram model presented in section 2.3”

The variogram in 2.3 is defined for detrended topography, but the synthetic examples are not detrended. How do you justify using the same variogram?

Lines 202-203: “The geostatistical methods that are used to interpolate the basal surface can be used to predict accurately certain derived quantities but not some other quantities”

This is a confusing statement that does not give the reader much information. It might be more helpful to say something along the lines of “We compare the fidelity of the different DEMs by evaluating different performance metrics.”

Line 247: “2.5.3 Flow accumulation comparison”

This section needs some motivation for why it is important to accurately represent flow accumulation.

Lines 257-258: “The accumulation is calculated using the Pysheds open source code for watershed delineation.”

How does this package compute flow accumulation? Does this package account for ice thickness?

Lines 299-300: “As expected, the kriging estimation produces the smoothest and the SGS the roughest topography.”

Why was it expected that SGS would be rougher than MPS? Is there a citation that shows this?

Line 309: “Kriging” → “kriging”

Lines 325-326: “However, the volumes estimated by kriging can over or underestimate the reference, and the method does not provide an error estimation.”

See main comments.

Line 340: “Kriging, provides surprisingly a better distribution in these examples”

This is indeed surprising. Do you think this would still be true in areas with sparser bed measurements? It may also be worth discussing the difference in the spatial patterns of the flow paths in Figure 9.

Figure 9: “Krigging” → “Kriging”

Line 385: “Only the SGS and MPS methods are able to estimate the uncertainties on the total volume.”

See main comments.

Lines 403-404: “We note that a linear extrapolation of this loss, obviously inaccurate due to all the effects that are not considered in this extrapolation, indicates that the glacier will disappear in about 30 to 40 years.”

As the authors have noted, ice loss cannot be accurately linearly extrapolated. As such, I recommend that they remove projections of ice sheet disappearance. Instead, I would emphasize the fact that the glacier has lost a large portion of its volume in a short period of time, and that the proposed interpolation methods could be used to improve estimates of sea level rise contributions from different glaciers.

Line 416: “However, kriging cannot be used to obtain directly the uncertainty on the volume.”

See main comments.

Lines 426-427: “Inded, we have shown that MPS provides a much better reproduction of the geomorphology of the simulated basal surfaces”

“Inded” → “Indeed”

It is interesting that SGS does so poorly. Could this be improved by choosing different simulation parameters, such as increasing the search neighborhood? For example, Herzfeld et al., (1993) found that changing the search parameters had a major impact on kriging interpolations.

Herzfeld, U. C., Eriksson, M. G., & Holmlund, P. (1993). On the influence of kriging parameters on the cartographic output—a study in mapping subglacial topography. *Mathematical Geology*, *25*(7), 881-900.

Line 430: “highlithed” → “highlighted”

Line 441: “However, It is” → “However, it is”

Line 447-448: “Finally, when applying existing mass balances to our volume estimation, we were able to draft a possible evolution of the glacier in the context of global warming.”

I don’t think that the mass loss calculation is enough to say that you can estimate the future evolution. I would instead say that your results indicate that there has been significant mass loss at this glacier, and that these methods enable higher-accuracy ice loss estimates and could enable improvements in glacier retreat projections.