

**Ice volume and basal topography estimation using geostatistical methods and GPR measurements: Application to the Tsanfleuron and Scex Rouge glacier, Swiss Alps.**

Author(s): Alexis Neven et al.

MS No.: tc-2021-161

MS type: Research article

Iteration: Review

*We thank the reviewer #3 for her positive evaluation and feedback, which helped us to improve the quality of the paper.*

*Here is a point-by-point answer to the suggestions and comments raised by reviewer #3.*

<b>Main remarks</b>	
<p><b>RC3</b> : The authors state that kriging cannot be used to compute ice volume uncertainty, but that is not the case. While it is true that kriging does not sample the uncertainty space in the same way that simulation does, kriging can be used to provide the variance or standard deviation of an estimate at any given location. In theory, multiple realizations produced by sequential Gaussian simulation should converge to a distribution that is represented by the kriging solution. For completeness, I recommend that the authors compute the uncertainty in ice volume for the kriging interpolation.</p>	<p>The reviewer is right when saying that kriging can be used to provide the variance or standard deviation at any given location. However, while we could easily compute the variance map associated with the kriging estimate, this map could not be used to calculate the associated uncertainty on the volume of ice. The volume is the sum of the thicknesses of ice over the map, and therefore the uncertainty is not linearly transposed.</p> <p>As stated by the reviewer, the mean SGS distribution converges to the kriging estimate. The standard deviations of the SGS are also converging to the standard deviation estimated by kriging. We decided to not represent the variance maps of the kriging estimate in the different figures or cross-sections for more clarity, since they would mainly overlap the SGS ones. Moreover, since these uncertainty maps cannot be used to calculate the ice volume uncertainty, which is one of the main research angles of the study and is also used to calculate scores between the three approaches, we decided to exclude them from the results.</p> <p>It may be confusing for non-geostatisticians readers and thus we propose to add a few sentences in the review manuscript to</p>

	<p>briefly explain this difference and the assumptions behind it.</p>
<p><b>RC3</b> : The authors fit a polynomial trend to the data in order to perform the kriging interpolation and sequential Gaussian simulation. This is a somewhat arbitrary, but often necessary, step for variogram-based methods. How was the degree of the polynomial chosen? Does it matter for your results that the synthetic examples are not detrended? I would like to see more justification for the polynomial selection and discussion on the implications of trend estimation. I also recommend mentioning in the discussion that MPS does not require trend estimation, which is another advantage of the MPS method.</p>	<p>First, we would like to clarify that the synthetic tests also use data that are “detrended”. Since the synthetic test data came from the TI, from which the variogram is calculated, we apply the same “detrending” process to these cases in order to be coherent with the rest of the approach. We will clarify that in the manuscript since it is a major point of the approach and that, thanks to the reviewer's comment, appears to be not clearly mentioned.</p> <p>Regarding the polynomial approach, we do not think that it is relevant to go more in the detail regarding the polynomial trend fitting. The identification of the trend is a common problem in geostatistics. Our approach is to try to find the simplest polynomial degree trend that brings stationarity to the dataset. A simple linear trend was not sufficient, and therefore a cubic one was defined. Choosing a higher polynomial degree may on the contrary over fit the data and remove the variations from the trends which is the variable of interest.</p>
<p><b>RC3</b> : More information is needed on the hydrological modeling method. The Pysheds package that the authors use only has examples for topography without ice. The authors do not state whether or not they account for ice overburden pressure in their hydrological modeling. If that is difficult to do with Pysheds, Chad Greene has a nice tutorial (<a href="https://www.mathworks.com/matlabcentral/fileexchange/55352-how-to-estimate-subglacial-water-routes">https://www.mathworks.com/matlabcentral/fileexchange/55352-how-to-estimate-subglacial-water-routes</a>). The synthetic examples have a flat ice surface, which could bias the hydrological models. More justification is needed for using the flow accumulation values for the synthetic examples.</p>	<p>The Pysheds package is not used directly for hydrological modeling but only for watershed determination. We only compute the number of cells connected to a cell of interest by following the gradient lines. The flow accumulation values are indeed calculated on the simulated topographic maps, without the ice coverage and without accounting for the ice pressure. The idea of this approach is certainly not to estimate a real under-glacial flow model but more to use it as an indicator to compare the structure of the topography and the connectivity of cells. We totally agree that a calculation of accumulation flow and a calculation of under-glacial flow are not the same, even if there is probably a strong link between the under-glacial topography and the flow. We will improve the terminology in the revised manuscript, to explicitly point out that we just want to use the connectivity of cells as an indicator for topography comparison.</p>

<p><b>RC3</b> : I would like to see more discussion on the implications of the hydrological findings. The authors compare the distributions of flow accumulation values for different interpolation methods, but it is unclear why this matters. Perhaps flow accumulation is important for discriminating between a channelized or distributed drainage system? It may also be helpful to refer to studies by Zuo et al., (2020) and MacKie et al., (2021) which previously investigated the impact of MPS and SGS on hydrological flow.</p>	<p>As stated in the above response, the main scope of the article is to compare the under-glacial modeled topography and the associated multi-level structures. The accumulation of flow is only a highly non-linear indicator, derived from the topography, to compare the realism of interpolation compared to the reference. Of course, we expect that a real under-glacial flow simulation will be influenced by the realism of the simulated topography, but that is not the scope of the paper. Again, we will make this more clear in the revised manuscript and will refer to relevant studies.</p>
<p><b>Specific comments</b></p>	
<p><b>RC3 Line 19</b> : No comma needed after "crucial".</p>	<p>This will be changed.</p>
<p><b>RC3 Line 28</b> : "Depending of" should be replaced by "Depending on".</p>	<p>This will be changed.</p>
<p><b>RC3 Line 30-32</b> : "the choice of the method becomes critical since the flow process is highly non-linear and is strongly linked to the morphology of the subglacial topography" I recommend clarifying that you are referring to hydrological flow, not ice flow (which is also non-linear and dependent on morphology). What is meant by a non-linear flow process? I think the authors mean that flow accumulation is not a linear function of bed elevation.</p>	<p>We thank the reviewer for pointing out this. The sentence was not clear. We indeed meant that flow accumulation is not a linear function of bed elevation, and this is why we are using this indicator. We will make it more clear in the revised version of the manuscript.</p>
<p><b>RC3 Line 35</b> : "produces by construction" Awkward wording. It would be sufficient to just say "produces".</p>	<p>This will be changed.</p>
<p><b>RC3 Line 36-38</b> : "Furthermore, even if kriging allows estimation of the local uncertainty on the elevation of the bedrock, it cannot be used to estimate the uncertainty of the global volume of ice (see e.g. Chiles and Delfiner, 2012, p. 478)."  Why can't the uncertainties from kriging be used to estimate ice volume uncertainty? What happens when you use the kriging bed uncertainties to estimate ice volume</p>	<p>It is true that the specific example given by Chiles and Delfiner on page 478 and the following is slightly different from the one of the volume computation but the key idea expressed in this part is applicable to our case. If we want to estimate the uncertainty on the volume, and if we express this uncertainty as a variance, it is clear that the variance of the estimated volume cannot be expressed as a linear function of the topography. Therefore, the kriged map</p>

<p>uncertainty?</p> <p>The reference to Chiles and Delfiner (2012, p. 478) does not support this statement. Chiles and Delfiner (2012, p. 478) describe a scenario where surface area increases with roughness. It is true that kriging underestimates the surface area of topography (if each grid cell is represented by a tilted plane), but this shouldn't affect the volume calculation.</p>	<p>(even if it provides at any point the variance of the local altitude) cannot be used in a simple manner to get the variance of the volume. Additional information is needed about the covariances between all the pairs of points in the domain. We could write a formal expression of the variance of the estimated volume and it would involve to compute a double integral on all pairs of points in the domain. It is feasible to compute it, using the covariance model or variogram model, but it's not easy and it's for sure not directly computable from the results of the kriging only.</p> <p>The general message conveyed by Chiles and Delfiner on page 478 and following is therefore applicable here: any non-linear estimation derived from the variable of interest (here the altitude) cannot be simply computed from the kriging map. Either, one has to really pose all the equations and solve them numerically (here we could compute the double integral) or one has to rely on simulations.</p>
<p><b>RC3 Line 41</b> : "Two points spatial statistics" should be "two point spatial statistics". I recommend elaborating on this sentence so that this concept is more understandable to non-geostatisticians. It might be more understandable to say that these methods are based on variance between pairs of points, and briefly state what a variogram is.</p>	<p>We will include a brief explanation of the principle of a variogram in the revised manuscript.</p>
<p><b>RC3 Line 51</b> : "that the one" should be "that the one".</p>	<p>This will be changed.</p>
<p><b>RC3 Line 55</b> : "require to define" should be changed to "requière the definition of".</p>	<p>This will be changed.</p>
<p><b>RC3 Line 55</b> : "MPS does not require to define an analytical two-point statistics model to represent the spatial variability but infers it in an implicit way".</p> <p>MPS does not define any statistical model (two-point or otherwise). It is entirely non-parametric. It would be more accurate just to say that MPS does not require the definition of a statistical model</p>	<p>MPS is of course non-parametric but in a sense, the TI is the base of an implicit statistical model. The sentence was written in that sense.</p>
<p><b>RC3 Line 57</b> : "allow to create" should be</p>	<p>This will be changed.</p>

replaced with "allow the creation of".	
<b>RC3 Line 107</b> : In the methods overview at the end of the introduction, I recommend stating that you will apply a hydrological model to the topography.	As stated before, we do not apply any hydrological model to the topography but only compute the flow accumulation map in order to compare topographies. We will make sure that the revised version explicitly states that this accumulation flow is only used as an indicator to compare topographies and not as a real flow model.
<b>RC3 Line 125</b> : "exemple" -> "example".	This will be changed.
<b>RC3 Line 139</b> : "This technique allows to co-simulate jointly several variables".  I would elaborate on this sentence for the benefit non-geostatisticians. I think it would be sufficient to say something like "this means that secondary information can be used to improve the simulations."	We will modify the text accordingly to the reviewer's suggestion.
<b>RC3 Line 142</b> : "the use of Gaussian pyramids to account for multiscale patterns" It would be helpful to provide a brief explanation of what this is and what it accomplishes.	We will add a sentence explaining that Gaussian pyramids simulate patterns at Multi-scale resolution ensuring both large and small patterns have the same quality in their reproduction.
<b>RC3 Line 154</b> : "Furthermore, a secondary variable is used during the MPS simulation". It took me a while to figure out that the secondary variable is the gradient. I would state this more clearly, and explain why it is beneficial to use the gradient.	We will explicitly introduce the secondary variable.
<b>RC3 Line 157</b> : "Two patterns that show the same relative changes even at different absolute altitudes should be considered similar". Does this mean that the TIs are detrended?	The TI is not directly detrended in this approach. However, in order to deal with non-stationary TI, when doing the pattern comparison, we subtract the mean of the pattern. By doing so, we can simulate with non-stationary TI without any trend estimation.
<b>RC3 Line 178</b> : "5'000" → "5,000" There are a few places here where the apostrophe should be replaced by a comma in numbers.	The comma is used as the decimal separator in most European countries, and to avoid any confusion we will change it to "5 000" which is the recommended notation.
<b>RC3 Figure 3</b> : I recommend changing the scale bar label on part A from "variation from the trend" to "difference from the trend" to be more precise.	This will be changed.

<p><b>RC3 Figure 4</b> : “Kriegage” → “Kriging.” Do the dashed lines represent synthetic GPR surveys? There are three lines in the figure, but in the text it says there are two.</p>	<p>This will be changed.</p>
<p><b>RC3 Line 197</b> : "SGS and ordinary kriging are applied using the same variogram model presented in section 2.3". The variogram in 2.3 is defined for detrended topography, but the synthetic examples are not detrended. How do you justify using the same variogram?</p>	<p>As explained before the synthetic cases are also “detrended”, which justify the application of the same variogram. We will make sure it is better explained in the revised version.</p>
<p><b>RC3 Lines 202-203</b> : The geostatistical methods that are used to interpolate the basal surface can be used to predict accurately certain derived quantities but not some other quantities”. This is a confusing statement that does not give the reader much information. It might be more helpful to say something along the lines of “We compare the fidelity of the different DEMs by evaluating different performance metrics.</p>	<p>We agree with the reviewer. The sentence will be changed.</p>
<p><b>RC3 Line 247</b> : The section 2.5.3 "Flow accumulation comparison" needs some motivation for why it is important to accurately represent flow accumulation.</p>	<p>The estimation of under-glacial flow is important for a wide range of applications. We can mention here for exemple prediction of water availability with glacial retreat or modeling of sediment transport. However, the flow estimation itself relies on a good under-glacial topographic estimation. We will make sure that sufficient motivation is added here.</p>
<p><b>RC3 Lines 257-258</b> : “The accumulation is calculated using the Pysheds open source code for watershed delineation.” How does this package compute flow accumulation? Does this package account for ice thickness?</p>	<p>The accumulation simply computes the gradient of the topography. Then, the connectivity of the cells is computed following this gradient. Local maximums of accumulation are obtained in the topographic minimum since multiple cells are connected to it. This package does not account for ice thickness. We do not use it to predict an accurate flow, but as a non-linear indicator to compare topographies.</p>
<p><b>RC3 Lines 299-300</b> : "As expected, the kriging estimation produces the smoothest and the SGS the roughest topography.” Why was it expected that SGS would be rougher than MPS? Is there a citation that shows this?</p>	<p>We propose to add in this part of the paper a reference to Journel and Deutsch (1993), doi: 10.1007/BF00901422. In this paper, the authors show how the multiGaussian random field model maximizes entropy.</p>

	<p>Because the multiGaussian model maximizes entropy, and because the MPS model honors in general pretty well the histogram and variogram, it means that the MPS model has the same low order statistics as the SGS. But MPS also includes higher-order correlations (less entropy) and is then more likely to produce structures that are globally more structured and connected than the equivalent multiGaussian model generated by SGS. In summary, we expect MPS to perform at least as well as SGS, or better.</p>
<p><b>RC3 Line 309</b> : "Kriging" -&gt; "kriging"</p>	<p>This will be changed.</p>
<p><b>RC3 Lines 325-326</b> : "However, the volumes estimated by kriging can over or underestimate the reference, and the method does not provide an error estimation." See main comments.</p>	<p>See main response to the RF3 general comment.</p>
<p><b>RC3 Line 340</b> : "Kriging, provides surprisingly a better distribution in these examples". This is indeed surprising. Do you think this would still be true in areas with sparser bed measurements? It may also be worth discussing the difference in the spatial patterns of the flow paths in Figure 9.</p>	<p>It is difficult to say but we expect that the kriging will tend to have a bigger difference compared to the reference data in terms of flow distribution if we use sparser conditioning data when the SGS will probably still have a similar distribution. That is probably due to the fact that kriging will tend to be smoother with fewer conditioning points, increasing the connectivity of cells and shifting the distribution up. SGS on the other hand will not produce such effects.</p> <p>We will complete the section where we comment on the flows.</p>
<p><b>RC3 Figure 9</b> : "Krigging" -&gt; "Kriging"</p>	<p>This will be changed.</p>
<p><b>RC3 Line 385</b> : "Only the SGS and MPS methods are able to estimate the uncertainties on the total volume." See main comments.</p>	<p>See the response to the main comment of the RC3.</p>
<p><b>RC3 Lines 403-404</b> : "We note that a linear extrapolation of this loss, obviously inaccurate due to all the effects that are not considered in this extrapolation, indicates that the glacier will disappear in about 30 to 40 years." As the authors have noted, ice loss cannot be accurately linearly extrapolated. As</p>	<p>We agree that such projection has to be removed and will be in the revised version.</p> <p>We will emphasize the points suggested by the reviewer, which are also an important aspect.</p>

<p>such, I recommend that they remove projections of ice sheet disappearance. Instead, I would emphasize the fact that the glacier has lost a large portion of its volume in a short period of time, and that the proposed interpolation methods could be used to improve estimates of sea level rise contributions from different glaciers</p>	
<p><b>RC3 Line 416</b> : “However, kriging cannot be used to obtain directly the uncertainty on the volume”. See main comment.</p>	<p>See the response to the main comment of the RC3.</p>
<p><b>RC3 Lines 426-427</b> : “Inded, we have shown that MPS provides a much better reproduction of the geomorphology of the simulated basal surfaces” “Inded” → “Indeed” It is interesting that SGS does so poorly. Could this be improved by choosing different simulation parameters, such as increasing the search neighborhood? For example, Herzfeld et al., (1993) found that changing the search parameters had a major impact on kriging interpolations. Herzfeld, U. C., Eriksson, M. G., &amp; Holmlund, P. (1993). On the influence of kriging parameters on the cartographic output—a study in mapping subglacial topography. <i>Mathematical Geology</i>, 25(7), 881-900.</p>	<p>The number of nodes does not influence the statistical model. Increasing the number of nodes increases the computation time and the simulation quality, but the improvement will reach a plateau. We can denote in table 2 that the difference in terms of indicators between the 12 and the 24 nodes simulations is extremely small, suggesting that increasing the number of nodes will just increase drastically the simulation time without any significant improvements in terms of indicators. We also performed some simulation using the Circular Embedding Methods (FFT) that does not depend on node number and takes all the nodes into account. The results were similar.</p> <p>Concerning the Kriging tuning, because of the “detrending” process, we are in a stationary process with a mean of zero. We are consequently much less sensitive to parameter influence since the local average estimated during the kriging is always close to zero. In addition, our search distance is almost four times the range of the variogram in order to avoid any sharp artifact when points are entering the search ellipsoid. We think that tuning the parameters will not have any positive influence on the kriging.</p>
<p><b>RC3 Line 430</b> : “highlithed” -&gt; “highlighted”</p>	<p>This will be changed.</p>
<p><b>RC3 Lines 447-448</b> : “Finally, when applying existing mass balances to our volume estimation, we were able to draft a possible evolution of the glacier in the</p>	<p>This will be changed.</p>



<p>context of global warming.” I don't think that the mass loss calculation is enough to say that you can estimate the future evolution. I would instead say that your results indicate that there has been a significant mass loss at this glacier and that these methods enable higher-accuracy ice loss estimates and could enable improvements in glacier retreat projections.</p>	
--	--

On behalf of the authors,

Alexis Neven