Author response to the review of Oyabu et al. "Fractionation of O_2/N_2 and Ar/N_2 in the Antarctic ice sheet during bubble formation and bubble-clathrate hydrate transition from precise gas measurements of the Dome Fuji ice core"

We would like to thank Dr. Nanna B. Karlsson and the anonymous reviewer for comments to improve and clarify the manuscript. We revised the manuscript following the comments. Our replies are in blue, the changes we made in red and the editor and reviewer comments are written in *black italic letters*.

Reply to the Editor

Line 90: "(corresponding to \sim 10 years)" - is this correct? Presumably the age span varies greatly with depth so the 10 years is not correct for the entire ice core.

We changed to "(corresponding to ~ 3 to 30 years)" (Line 89).

Line 600: "Finally, our constraints on the permeation coefficients of the gases in ice might be useful for predicting the magnitude of diffusive smoothing of air composition in the ice sheet more than 1 million years old to be drilled in the future."

Is there a word missing? This sentence is difficult to read, I suggest splitting it up and referencing the Oldest Ice effort, e.g., "Finally, the International Partnership for Ice Core Sciences regards the retrieval of an ice core containing ice older than 1 million years as highest priority (Fisher et al., 2013). Our constraints on the permeation coefficients of the gases in ice might be useful for predicting the magnitude of diffusive smoothing of air composition in such an ice core."

You could also add (if appropriate) a reference the manuscript by Tsutaki (in review https://tc.copernicus.org/preprints/tc-2021-266/) for a specific Dome Fuji reference.

Fisher et al., 2013: Clim. Past, 9, 2489–2505, 2013, www.clim-past.net/9/2489/2013/, doi:10.5194/cp-9-2489-2013

We revised as the editor suggested (Line 520, added Fischer et al., 2013 and Tsutaki et al., in review).

In addition to the above corrections, we added information about data citation in the Data availability "(https://ads.nipr.ac.jp/dataset/A20210430-001; https://doi.org/10.17592/001.2021043001, Oyabu et al., 2021)" and added the corresponding reference in the reference list.

Oyabu, I., Kawamura, K., Kitamura, K., Hirabayashi, M., Aoki, S., Morimoto, S., and Nakazawa, T.: Dome Fuji ice core gas data (112 - 2001m, ¹⁵N, O₂/N₂, Ar/N₂), 1.00, Arctic Data archive System (ADS), Japan, http://doi.org/10.17592/001.2021043001, 2021.

Reply to Referee #2

Lines 390ff:

"For example, if a sample coincidentally includes a thin (e.g., 1-mm-thick) layer with $\delta O2/N2$ of +1000 % at the top or bottom of a 10-cm-long ice, an anomaly of ~ 10 % from the average $\delta O2/N2$ (excluding the anomalous layer) should result."

I think that sentence is confusing.

We revised the sentence as follows (Line 356):

For example, if a 100-mm-long ice sample coincidentally includes a 1-mm-thick anomalous layer with $\delta O_2/N_2$ of ± 1000 %, the $\delta O_2/N_2$ of the bulk sample should be elevated by ~ 10 % relative to the value without the anomalous layer.

Line 482ff:

"We thus disfavor the possibility that calls for a failure of the recording mechanism of insolation variations during the past firn-ice transition to generate the high scatters of $\delta O2/N2$ and $\delta Ar/N2$ in and below the BCTZ."

I think that sentence is confusing. Please remove double negative phrase.

We revised the sentence as follows (Line 430):

Thus, we favor the possibility that the recording mechanism of insolation variations at Dome Fuji was intact when the layers in the modern BCTZ and below were initially formed at the past firn-ice transition.

Line 594:

"More observational and theoretical works are still needed for advancing our understanding of the mechanisms of gas movements in different zones in the ice sheet."

Change "works are" to "work is"

Corrected (Line 515).

Eq. A9 to A12: There is something wrong with the indices in the brackets, the brackets do not line up with the indices.

It might be a problem with the software that displays the PDF file. On my laptop, it looks like the picture that I attached. I will carefully check the brackets in the proof.

Discretization

The model uses the central differencing scheme. The downward diffusive flux (f_m) of m-molecule per unit area at the top boundary of i-th box is the product of the diffusivity and concentration gradient:

$$f_{m(i)} = D_m \frac{c_{m(i-1)}^h - c_{m(i)}^h}{\Delta z \tau_r}.$$
 (A9)

where Δz is initial box height (0.5 mm) and τ_r is relative thinning function (thinning function divided by the initial value at 1258 m). By substituting eq. 3 into eq. A9, f_m is expressed as

$$f_{m(i)} = \frac{D_m S_m P_m^d}{\Delta z \tau_r} \big(X_{m(i-1)} - X_{m(i)} \big). \tag{A10} \label{eq:angles}$$

The net flux of m-molecule for i-th box is

$$F_{m(i)} = \frac{f_{m(i)} - f_{m(i+1)}}{\Delta z \tau_r}, \tag{A11}$$

and the concentration change of m-molecule in total air content becomes

$$\Delta U_{m(i)} = F_{m(i)} \Delta t, \tag{A12}$$

where Δt is time step (~11.6 days).