Ice fabrics in two-dimensional flows: beyond pure and simple shear

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Abstract. Ice fabrics are key for understanding and predicting ice flow dynamics. Despite its importance, the characteristics and evolution of ice fabrics beyond pure and simple shear flow has largely been neglected. Within an ice sheet, a significant part of the deformation can be outside the regimes of pure and simple shear. We use a recently developed numerical model (SpecCAF), which has been shown to accurately reproduce experimentally observed fabrics in both compression and simple
5   shear, to classify the fabrics produced under general two-dimensional deformation regimes. Our analysis develops the first predictions for fabric patterns arising over a continuous spectrum of incompressible two-dimensional deformation regimes and temperatures, encompassing those intermediate to pure and simple shear, as well as those that are more rotational than simple shear. We find that intermediate deformation regimes between pure and simple shear result in a smooth transition between a fabric characterised by a cone-shape and a secondary cluster pattern. Highly-rotational deformation regimes are revealed to
10   produce a weak girdle fabric. In addition, we obtain predictions for the strain-scales over which fabric evolution takes place at any given temperature. The use of our model in large-scale ice flow models as well as for interpreting fabrics observed in ice cores and seismic anisotropy, will provide new tools supporting the community in predicting ice flow in a changing climate.

1   Introduction

Mass loss from ice sheets is set to be the main contributor to sea-level rise this century (e.g. Shepherd et al., 2018). Reliably
15   predicting sea-level rise depends on accurately modelling ice flow. One of the most important controls on ice flow dynamics is the ice fabric, i.e. the arrangement of ice grains in terms of their crystallographic axes. Strong alignment of the crystal lattices may cause the strain-rate response to an applied stress to vary by a factor of 9 in different directions (e.g. Pimienta and Duval, 1987). Hence, understanding the fabrics present in any flowing ice sheet, i.e. Antarctica or Greenland, is important for predicting ice-sheet flow and, in turn, the loss of ice over time.
20   To date, the analysis and discussion of ice fabrics has focused primarily on those formed under the specific deformation regimes of pure and simple shear. The primary motivation of the present paper is to use the validated fabric model SpecCAF (Richards et al., 2021) to take a step away from the isolated conditions of pure and simple shear and explore the continuous space of deformation regimes lying intermediate to these endmember cases, and also those that are more rotational than simple shear, across a unified spectrum. Ice flow is commonly modelled in the two-dimensional x-z plane (e.g Martín et al., 2009), and
25   analysis of this flow shows that it commonly resides outside the endmember regimes of pure and simple shear (we illustrate this

below in section 2.2.1 below). Therefore, an exploration of the fabric patterns arising in this generalised situation is necessary as a step towards improving our ability to interpret fabrics derived from ice cores and to predict future ice flow taking ice fabric effects into account. As a first step towards this, we seek here to address a number of open questions. First, what fabrics are produced under any given (incompressible) two-dimensional deformation regime? Second, how do fabrics evolve at the very high strains which have remained inaccessible to laboratory experiments? Third, what strains are necessary to reach steady state in the different deformation regimes? Finally, at these high strains, how are the final steady states dependent on parameters such as temperature and type of deformation regime?

2 Background

2.1 Fabric Development

2.1.1 Processes governing fabric development

The distribution of crystallographic orientations within a polycrystal is called the fabric or crystallographic preferred orientation (CPO). Basal-slip is generally dominant in ice (Duval et al., 1983), although non-basal slip systems are active, especially at high temperatures (Chauve et al., 2017). To leading order the distribution of the c-axes is the key control for the mechanical properties (e.g. viscous anisotropy) of ice (Duval, 1981). As ice flows, under deformation by crystal plasticity, the distribution of c-axes evolves to produce a fabric. The intensity of the fabric, and the pattern seen, is dependent on the conditions of deformation which will influence the relative activity of different deformation mechanisms. During crystal plasticity several deformation mechanisms may be active. These include crystal slip, grain boundary migration (also termed migration recrystallization), rotational recrystallization and rigid-body rotation. Crystal slip occurs predominantly by slip along the basal plane, which results in a systematic change in orientation of the c-axis (Steinemann, 1958; Hondoh, Takeo, 2000), even though other slip systems are activated to a minor degree if deformed at high temperatures (Chauve et al., 2017) and in areas of local stress heterogeneities (Piazolo et al., 2015).

Migration recrystallization in a deforming crystalline material that deforms by crystal plasticity is mainly driven by difference in strain energy i.e. energy related to unbound dislocation density either side of a grain boundary (e.g. Humphreys and Hatherly, 2004). Hence, in the case of such strain induced migration recrystallization (Drury et al., 1985; Drury and Urai, 1990) the less strained grain grows at the expense of the more strained grain resulting in an overall decrease in the strain energy of the system. As the strain energy stored in a grain (i.e. dislocation density) is directly related to the imposed stress (Gottstein and Shvindlerman, 1999) migration recrystallization is inherently related to stress as well. The dislocation density accumulated within a certain grain is primarily a response to its orientation relative to the main deviatoric stress axes. For example, a grain favourably oriented for basal slip will accumulate less dislocations than a grain that is oriented unfavourably. Consequently, depending on the deformation regime, grains of certain orientation will grow at the expense of grains of another unfavourable orientation. As a result, the effect of migration recrystallization is to produce c-axes clustered towards certain orientations in a

polycrystal. Even though some studies suggest that fabric is stress-dependent (Harte et al., 2020; Hudleston, 2015), the effect of migration recrystallization will be the same on the resulting fabric.

Rotational recrystallization occurs when dislocations recover into subgrain boundaries which, with increasing strain, will develop into grains (Drury et al., 1985). As dislocations are concentrated close to grain boundaries due to stress heterogeneities (Piazolo et al., 2015), subgrains, and therefore new grains developing from subgrains, dominantly occur near grain boundaries. The orientation of these new grains is random to leading order. This acts to diffuse the c-axes of the original grain (Gödert, 2003). Rigid-body rotation acts to rotate any grain and therefore its c-axis in accordance with the rotational characteristics of the deformation regime that the ice grains are subjected to.

2.1.2 Observed fabrics

Ice fabrics can be observed through laboratory experiments, through ice cores from real-world locations, and, more recently, inferred through seismic measurements.

In the laboratory, the majority of experiments are performed in compression, either in pure shear (2D) or unconfined (uni-axial) compression (3D), without any rotational component (e.g. Jacka and Maccagnan, 1984; Jacka, 2000; Craw et al., 2018; Fan et al., 2020; Piazolo et al., 2013). The other endmember considered is simple-shear (Journaux et al., 2019; Qi et al., 2019). Laboratory experiments provide detailed fabric measurements in known conditions. However experiments are mostly limited to endmember deformation regimes, as well as to strains of around 0.4 for compression (Fan et al., 2020) and 2 for direct simple shear (Qi et al., 2019).

Experiments have been performed using combined compression and shear, at close to the melting point of ice (Duval, 1981; Jun et al., 1996; Budd et al., 2013). Fabrics from Duval (1981) combining unconfined compression and simple shear show a broad cluster with 3 or 4 maxima inside it. Budd et al. (2013) shows, for an experiment with mostly simple shear combined with some confined compression and at an equivalent strain to that used later in this paper of 0.75, a fabric which shows the merging of a double cluster (from pure shear) and a single-maxima (from simple shear). Experiments have also been performed with layered samples of ice (Wilson, 2000; Wilson and Peternell, 2012). In addition to the relatively few experiments that have been conducted under a combination of simple and pure shear, no analysis has been conducted of situations where fabrics are produced in deformation regimes more rotational than simple shear. We show in section 2.2.2 that deformation regimes inbetween pure and simple shear, and those more rotational than simple shear, occur widely in natural ice-sheet flows.

Fabrics can also be analysed by taking ice cores in ice sheets. An understanding of the fabrics enables us to interpret the deformation regime and temperature history of ice cores. Initial studies of ice cores have concentrated on ice domes or divides (Gow, 1961; Holtzscherer et al., 1954; Johnsen et al., 1995). These locations are deliberately chosen because they have minimal deformation, to act as a good proxy for past climate data. At domes, the ice will be deformed vertically in unconfined compression, producing either a single-maximum or a girdle shape fabric (Fig. 1). Recently, ice cores have become available in locations with more complex deformation regime histories (Stoll et al., 2018; Treverrow et al., 2010). Stoll et al. (2018) show examples of a variety of fabric shapes such as girdles and single-maximum fabrics orientated in different directions as well as
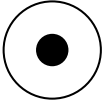
Figure 1. Illustration showing common fabrics or CPOs which develop in ice, illustrated by their pole figures, as well as the deformation regime and temperature they typically occur at. The pole figures show the distribution of c-axis orientations, with the compression axis at the centre. (a) shows a single-maximum fabric, produced in unconfined compression or simple shear at low temperatures (Qi et al., 2019). (b) shows a cone-shape fabric, produced in unconfined compression at higher temperatures, when grain-boundary migration is active (Paterson, 1999). This can also be considered a girdle fabric when the cone-angle approaches $90°$. (c) shows a double maxima fabric produced in confined compression (pure shear) (Budd et al., 2013). (d) shows a single-maximum with a secondary cluster, produced in simple shear at higher temperatures (Kamb, 1972; Qi et al., 2019).

90   relatively faster fabric development with depth compared to ice cores at domes. Fabrics can also be measured from boreholes using sonic and optical techniques (Gusmeroli et al., 2012; Kluskiewicz et al., 2017).

Recently, data from radar and seismics has also been used to infer fabric properties (Matsuoka et al., 2003; Fujita et al., 2006; Booth et al., 2020). These methods can capture natural ice fabrics without expensive drilling, allowing data to be collected at more active locations such as ice streams (Jordan et al., 2020).

2.1.3 Fabric development

Figure 1 illustrates schematically some commonly observed CPO patterns seen in laboratory experiments and from ice cores. These are illustrated by pole figures which show half of the of c-axis orientations (which cover the space of a unit sphere) projected onto a plane (since the c-axes are antipodally symmetric, this is sufficient to show all the information). Laboratory experiments studying CPO evolution have been performed mostly in uniaxial compression and this produces either a single maximum at low temperatures (Fig. 1a) or a cone-shape fabric at high temperatures (Fig. 1b). At low temperatures ($T \approx -30°$ C) basal-slip deformation regime dominates and this causes c-axes to rotate towards the axis of compression, producing the single-maximum shown in Fig. 1a. The cone-shape fabric shown in Fig. 1b, seen in experiments such as Kamb (1972). is produced by the balance of basal-slip deformation regime and migration recrystallization. The process of migration recrystallization acts to consume grains orientated towards the compression axis, and grow grains orientated in a ring $45°$ away from the compression axis. Therefore, the balance of basal-slip deformation regime and migration recrystallization produces a ring or cone-shape pattern in the pole figure. In pure shear, which is a 2D compression, the grains produced by migration recrystallization instead form two clusters at $45°$, as shown in Fig. 1c is produced rather than a cone-shape fabric. This has been seen in experiments such as Budd et al. (2013).

Recent simple shear experiments produce either a single-maximum at low temperatures, or a single-maximum with an offset secondary cluster (Fig. 1d) at intermediate strains and high temperatures (Qi et al., 2019; Journaux et al., 2019). This pattern is similar to a double-maxima but the presence of vorticity in simple shear causes the cluster that is not aligned with the shear plane normal to be advected towards the primary cluster. As the secondary cluster's orientation changes, grains of that orientation accumulate more dislocations so start to be consumed by migration recrystallization. This results in the imbalance in cluster strengths seen in Fig. 1d.

2.2 Classifying flow regimes

2.2.1 General deformation regimes

There exists a significant variety of deformation regimes in the natural world. One way to classify a deformation regime is by the vorticity number (Passchier, 1991), which measures the ratio of vorticity magnitude to strain-rate magnitude:

$$\mathcal{W} = \sqrt{\frac{W_{ij}W_{ij}}{D_{ij}D_{ij}}}, \tag{1}$$

where $\mathbf{W} = \frac{1}{2}(\nabla \boldsymbol{u} - \nabla \boldsymbol{u}^T)$ is the anti-symmetric part of the velocity gradient (the spin-rate tensor) and $\mathbf{D} = \frac{1}{2}(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T)$ is the symmetric part of the velocity gradient (the strain-rate tensor). Figure 2 illustrates different flow regimes and the corresponding vorticity number $\mathcal{W}$ for each. The vorticity number is 0 for pure shear or uniaxial compression, 1 for simple shear and ∞ for rigid-body rotation. Ice in the natural world will experience deformation regimes with vorticity numbers from 0 to ∞, however to date fabrics produced for $\mathcal{W} = 0$ and $\mathcal{W} = 1$ are the only areas which have been extensively explored due to the limitations of possible deformation regimes in experiments.

In the late 1990s and early 2000s it was recognised in the geological community that flow in rocks cannot be approximated by endmember plane strain flow models alone (Jiang, 1994; Bailey and Eyster, 2003). There is now an extensive literature within structural geology which developed conceptual models and analytical techniques to predict and recognise natural geological flows with vorticity numbers between 0 and 1 (Fossen and Tikoff, 1993; Tikoff and Fossen, 1995; Piazolo et al., 2002, 2004; ten Grotenhuis et al., 2002) In contrast, such analysis is less common in discussions surrounding ice core interpretation; pure shear and simple shear tend to dominate discussions regarding both the interpretation of ice-sheet flow and experimental analysis. This may be mainly due to the fact that such endmember scenarios are a) experimentally straightforward to achieve, b) the two endmembers can – as a first approximation - be associated with different ice flow scenarios of an ice divide and the shallow ice approximation (although the shallow ice approximation is only in simple shear towards the base).

2.2.2 Two-dimensional deformation regimes in natural ice flow

As a first step towards exploring the fabrics produced by all possible deformation regimes, we will focus here on general incompressible two-dimensional deformations. Although deformation regimes in the natural world will be three-dimensional, exploring fabrics produced by two-dimensional deformation regimes is a natural first step away from the canonical regimes of pure and simple shear. It is also common to limit the modelling of ice-sheet to two dimensions, in the vertical cross-section
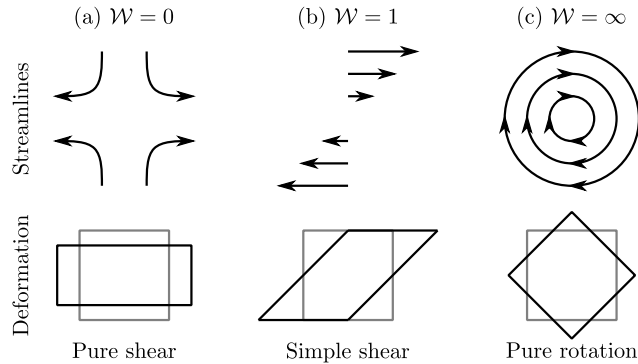
Figure 2. Schematics illustrating two-dimensional flow regimes at different vorticity numbers $\mathcal{W}$ (Eq. (1)): (a) Pure shear ($\mathcal{W} = 0$), (b) Simple shear ($\mathcal{W} = 1$), (c) Pure rotation ($\mathcal{W} = \infty$). For each flow the streamlines and deformation regime produced are shown.

140 (Pattyn et al., 2008; Martín et al., 2009). In Fig. 3, we show the vorticity number from a 2D simulation of flow at an ice divide and its horizontal transition towards a flow dominated by vertical shear stresses (the shallow ice approximation). Due to the vanishing of horizontal velocity at the divide, the vorticity number is 0 there, corresponding to the regime of pure shear along $x = 0$. Away from the divide, the flow dominated by vertical shear stresses exhibits a transition in the vorticity number from 1 at the base, corresponding to simple shear, towards close to 0 at the surface. In summary, the figure shows vorticity numbers

145 between 0 and 1 are the norm.

It is not only vorticity numbers between 0 and 1 which can occur. Vorticity numbers greater than 1, corresponding to deformation regimes more rotational than simple shear, are predicted to occur in ice sheets. As an example consider the flow of ice over a Gaussian bump at the base shown in Fig. 4. This is modification of the ISMIP-HOM experiment F benchmark (Pattyn et al., 2008), representing ice flowing downhill over a Gaussian bump. Compared to Pattyn et al. (2008), the bump is 4

150 times higher and 31.6 times sharper. This shows even in a simple configuration, vorticity numbers greater than 1 can potentially occur in the vertical velocity profile. Fig. 4 shows vorticity numbers up to 2. To-date, the fabrics produced for vorticity numbers above 1 have not been analysed.

Further, we have calculated an estimate of the vorticity number near the surface of Antarctica, in order to illustrate the natural deviation from endmember regimes. To do this we have used surface velocity data from Antarctica (Mouginot et al., 2019) to

155 calculate the vorticity number, shown in Fig. 5. We have calculated the surface velocity gradients $\partial u/\partial x, \partial u/\partial y, \partial v/\partial x, \partial v/\partial y$ using central differencing. To evaluate the vorticity number, we use these estimates of horizontal velocity gradients combined with an estimate of the maximum vertical shear ($\partial u/\partial z, \partial v/\partial z$) in the top 25% of the depth of the ice-sheet determined using the shallow-ice approximation assuming no basal-slip. The derivative $\partial w/\partial z$ is calculated using mass continuity and we have neglected the higher order contributions $\partial w/\partial x$ and $\partial w/\partial y$.

160 To reduce the errors in the data for Fig. 5 we have taken the mean value from a 10×10 block (covering a 4.5 km square). We have defined the standard deviation of any variables, such as velocity components, as the variance from the mean of the 100 samples in each 10×10 block. The relative standard deviation controls the transparency: if this is $> 100\%$ this location
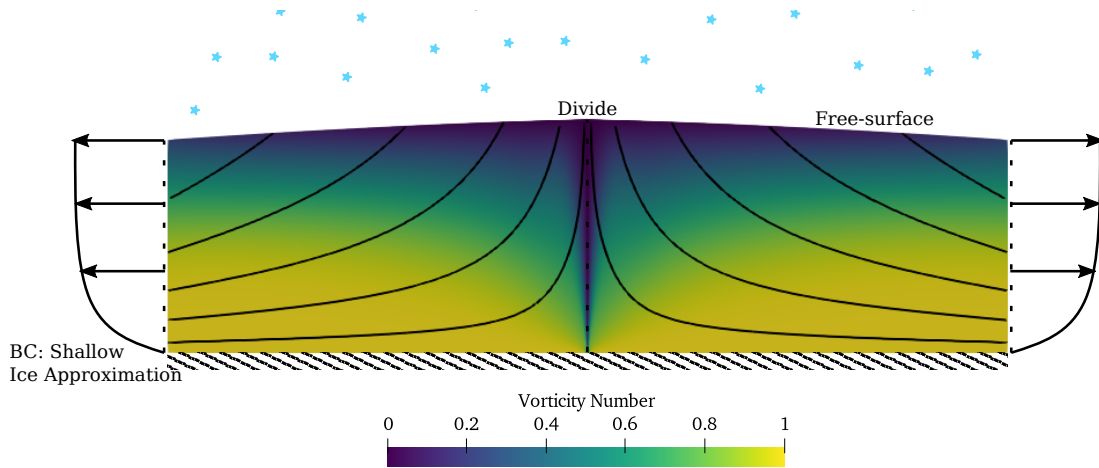
Figure 3. Illustrative figure showing the vorticity number for ice flowing at a divide, showing a range from 0 to 1. The problem setup is from Martín et al. (2009) with isotropic ice. The simulation has an aspect ratio of 20, but only the region from $x = 0$ to $x = 10H$, where H is the height of model domain, is shown. The domain has the velocity from the shallow-ice-approximation imposed at the left and right boundaries, and the surface accumulation is set to match the outflow, corresponding to a steady state. No-slip is imposed at the base and a free surface is assumed at the top. The vorticity number is shown, alongside streamlines. This flow was computed using a full-Stokes solver written in FEniCS (Martin Alnæs et al., 2015) with $n = 3$ and solved using Taylor-Hood elements.
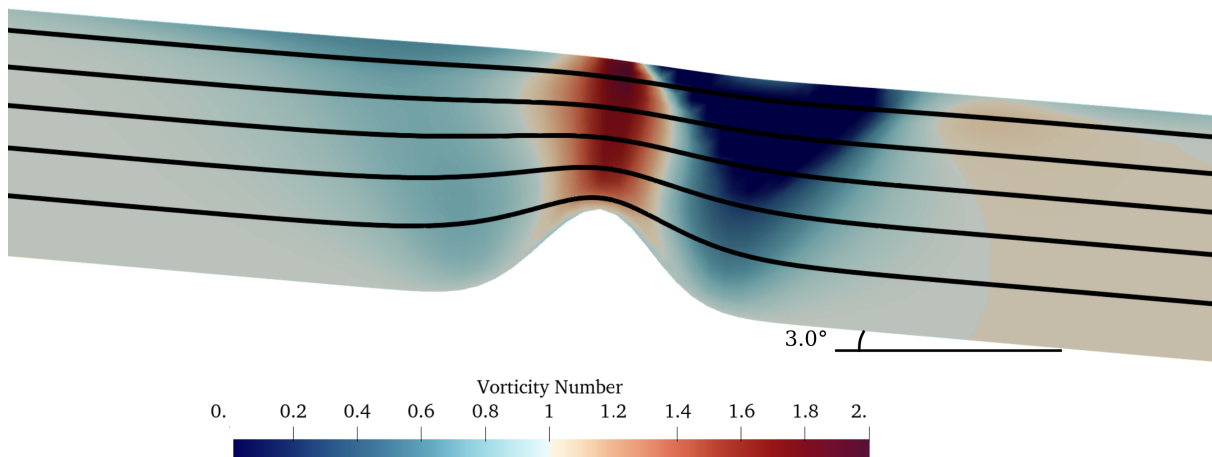
Figure 4. The vorticity number for a 2D slice from a modified version of the ISMIP-HOM Experiment F benchmark (Pattyn et al., 2008). The ice is flowing from left to right, down a hill with angle $3.0°$, and encounters a Gaussian bump at the base. Compared to Pattyn et al. (2008), the Gaussian bump is 4 times higher and 31.6 times sharper. Over the bump the flow accelerates leading to vorticity numbers greater than 1. This simulation was performed using Elmer/ICE (Gagliardini et al., 2013) with $n = 1$.

is plotted as white, and a relative standard deviation of 0% is plotted as the full colour. From this figure we can see that there are many regions of Antarctica where the vorticity number near the surface is both at intermediate values between 0 and 1, and greater than 1. The regions characterised by high vorticity numbers ($\mathcal{W} > 1$) occur typically in highly dynamic regions such as ice streams. Closer to the base of an ice-sheet, the vorticity number tends to 1 as the vertical gradients dominate. In regions with no basal slip, these vorticity numbers are potentially valid to approximately 25% into the ice-sheet. In regions with basal slip, the vorticity number will be valid deeper into the ice-sheet.

2.3 Hierarchy of ice modelling spanning the microscale to the macroscale

The effects of deformation on the dynamics of ice covers a vast range of scales from the order of microns for studying grain-grain interactions, to continental scales of 1000s of kilometres when studying ice-sheets. Consequently, different approaches must be used depending on the scale one seeks to work on, with microscale models serving to provide parametrisations of small-scale processes for use in larger scale models. At the scale of micrometres and millimetres, there exist several approaches for modelling the microstructure directly (e.g. Llorens et al., 2016; Kennedy and Pettit, 2015). This involves simulating grain-grain interactions with deformation regimes imposed via stress or velocity boundary conditions at the edges of the numerical domain. This is useful for improving our understanding of ice microstructure and fabric evolution. However, it cannot be scaled-up to be incorporated into ice-sheet model due to the numerical cost. At the largest scale, ice-sheet models typically neglect the effect of fabric entirely. The state-of-the-art for incorporating fabric evolution into large-scale models is to track the evolution of a 2nd-rank tensor representing the second moment of the orientation density function (to be defined below in Eq. (2)). A key omission in utilising the second moment alone is that it cannot incorporate the important process of migration recrystallization (Gillet-Chaulet et al., 2006; Gagliardini et al., 2013), which is key in terms of producing the details of differing fabric patterns.

In this paper, we use the SpecCAF model from Richards et al. (2021), which forms a continuum model of the full orientation density function with parameterisations of the underlying processes calibrated using laboratory experiments. In the hierarchy of ice modelling, SpecCAF seeks to model ice fabrics at a larger scale than models by Llorens et al. (2017) for example. SpecCAF acts as a continuum model for fabric evolution which, through use of a spectral method, is sufficiently efficient computationally that it can be incorporated into ice-sheet models, while retaining all key processes. In this continuum approach, grain-to-grain interactions are incorporated statistically through parameterisations of an orientation density function representing a distribution function of c-axes averaged over a polycrystal. The model thereby describes the evolution of the orientation density function. Since it incorporates parameterisations of the mean effect of grain-grain interactions, there is no need to represent the strain and stress fields explicitly within the microstructure. However, these interactions are incorporated statistically within the model by parameterisations of the net effect of these interactions calibrated using laboratory experiments. This model (SpecCAF) has been shown to predict fabrics accurately in both the endmember limits of compression and simple shear. The model can predict fabric evolution with only the velocity gradient and temperature as inputs.

We note that, by itself, SpecCAF models fabric evolution only, it does not include a viscosity formulation like the CAFFE model of Placidi et al. (2010). Thus, we take the velocity gradient as being prescribed, and consider the evolution of fabrics
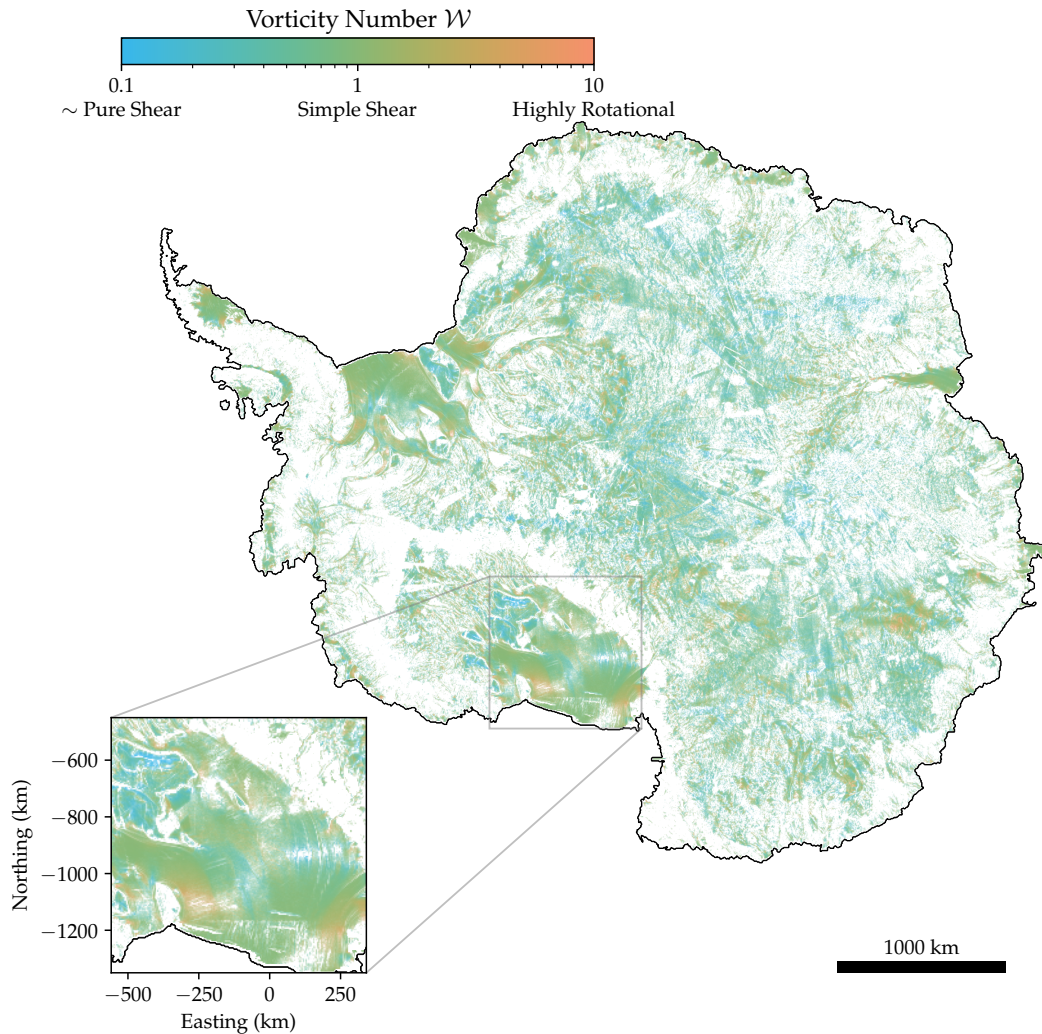
Figure 5. Vorticity number (defined in Eq. (1)) calculated from the surface velocity data of Antarctica (Mouginot et al., 2019) after averaging over a 10×10 block and taking the mean value within each block. The calculation uses the horizontal velocity fields from the observed surface velocity combined with an estimate of the average vertical shear predicted to occur in the top 25% of the ice sheet using the shallow-ice-approximation. The colour shows the vorticity number on a log scale. The transparency shows the relative error: an error in $\mathcal{W}$ of 100% or greater is plotted as white and an error of 0% is plotted as the full colour. The inset shows the Ross Ice Shelf, with the easting and northing in Antarctic polar stereographic coordinates. This shows considerable variation across the continent, including deformation regimes not accessible in the laboratory.

that occur as a result. In principle, the model could be coupled to a flow law (viscosity formulation) describing the effect of the fabric on the anisotropy. This could be considered in future.

2.4 Open questions addressed here

In this paper we use the SpecCAF model to address the open questions highlighted in the introduction: what fabrics are produced under any given (incompressible) two-dimensional deformation regime, how do fabrics evolve at the very high strains, what strains are necessary to reach steady state, and how are these final steady states dependent on the parameters?

In section 3 we review the SpecCAF model and clarify its underlying modelling assumptions and comparison with other modelling approaches. In section 4.1 we determine for the first time the evolution of fabrics in general 2D deformation regimes, bridging the complete spectrum from pure shear to rigid-body rotation, across the range of temperatures seen in ice sheets. In section 4.2 we construct a complete regime diagram for two-dimensional deformation regimes documenting fabrics that arise over the space of temperature, deformation regime, and strain, and explain the physical balances leading to these fabrics. Finally, in section 4.3 we investigate the time or strain scales over which ice fabric evolution takes place, as well as investigating the steady-state strength of ice fabrics, across the space of deformation regime and temperature. We then discuss the implication of these results for the interpretation of ice cores (section 5.3) and ice flow (sec 5.4).

3 Methods

3.1 The continuum approach

The model described in this section is based on Placidi et al. (2010) has already been described in Richards et al. (2021). For the sake of clarity we include an explanation here as well. The SpecCAF model uses a continuum approach to represent the mass distribution of c-axes within a polycrystal $\rho^*(\boldsymbol{x}, t, \boldsymbol{n})$, termed the *orientation mass density*, defined according to

$$\rho(\boldsymbol{x}, t) = \int_{S^2} \rho^*(\boldsymbol{x}, t, \boldsymbol{n}) \, \mathrm{d}\boldsymbol{n}, \tag{2}$$

where $\rho(\boldsymbol{x}, t)$ is the mass density of ice, and $\boldsymbol{n}$ is a unit vector representing the direction of a c-axis. Here, $\rho^*(\boldsymbol{x}, t, \boldsymbol{n}) \, \mathrm{d}\boldsymbol{n}$ is the mass fraction of grains with orientations directed towards $\boldsymbol{n}$ within the solid angle $\mathrm{d}\boldsymbol{n}$. In accordance with the equation above, integrating ρ^* over the space of possible orientations (the surface of a unit sphere S^2) gives the mass density of ice at that particular point in physical space, $\rho(\boldsymbol{x}, t)$.

SpecCAF incorporates the effect of basal-slip deformation, migration recrystallization and rotation recrystallization (all as functions of temperature), as well as rigid-body rotation, to develop an evolution equation for the orientation mass density ρ^*. As discussed above, the model evolves ρ^* as the dependent variable, with the net effects of grain-to-grain interactions incorporated through parametrisations. This is similar to other continuum approaches, for example how explicit descriptions of particle interactions are not included in the Stokes equations, yet are satisfactorily modelled statistically through parameterisation in the form of a constitutive relation. In the present case, rotation recrystallization is modelled as a diffusion of concentrations of ρ^*, originally the idea of Gödert (2003). Migration recrystallization is modelled through an orientation dependent source-term producing ρ^* preferentially at orientations where grains would be likely to have a large basal shear-stress, and vice-versa. Basal-slip deformation is incorporated in accordance with the 'deck-of-cards' analogy, and assuming linear dependence on the

strain-rate tensor (Placidi et al., 2010). The resulting continuum model has been shown to reproduce all detailed features of the distribution functions calculated from experimental samples (Richards et al., 2021).

The essential continuum approach was proposed previously by Faria (2001, 2006). Grains with the same orientations are called a *species*. There has been discussion in the literature (Gagliardini, 2008; Faria et al., 2008) on whether this theory implicitly includes a *Taylor assumption*, namely, that all ice grains forming the polycrystal experience the same strain-rate. The Taylor assumption has been shown not to be valid for ice (Castelnau et al., 1998). To summarise this debate, Gagliardini (2008) suggests that the assumption in Faria (2006) that the strain-rate of a species is independent of orientation is equivalent to every grain undergoing the same deformation (a Taylor assumption). However, Faria et al. (2008) rejected this assertion and replied that this assumption only requires that grains move with the surrounding material, with no direct constraint on the individual deformation of grains. In accordance with the continuum approach, the net effect of deformations on individual grains, which can vary from grain to grain, is incorporated via parametrisations of the overall net effect of these interactions. Since these parametrisations are calibrated using fully resolved experiments (Richards et al., 2021), they represent net effects relevant to real samples in which grains do not all experience the same strain. Therefore, the continuum model does not impose a Taylor assumption on the grain deformation. Furthermore, care should be taken in attributing our calibrated parameters as applying specifically to grain-grain interactions rather than the bulk interactions representing their net statistical effects in the model.

Despite the model not including the Taylor hypothesis, the term for basal-slip deformation in the equation below is similar to that which would be derived from a Taylor homogenisation of ice under a simple basal-slip only model (Gagliardini et al., 2009). The only exception is that the rate of viscoplastic deformation can vary relative to rigid-body rotation.

3.2 Model specification

The evolution equation for ρ^* under the framework described above was first defined in Placidi et al. (2010):

$$\frac{\partial \rho^*}{\partial t} = -\nabla^* \cdot (\rho^* \boldsymbol{v}^*) + \lambda \nabla^{*2}(\rho^*) + \beta \big(\mathcal{D}^* - \langle \mathcal{D}^* \rangle\big)\rho^*, \tag{3}$$

where λ and β are parameters, to be defined below. Here, ∇^* is the gradient operator in orientation space defined by:

$$\nabla^* \boldsymbol{v}^* = \frac{\partial \boldsymbol{v}^*}{\partial \boldsymbol{n}} - \left(\frac{\partial \boldsymbol{v}^*}{\partial \boldsymbol{n}} \cdot \boldsymbol{n}\right)\boldsymbol{n} = \frac{\partial v_i^*}{\partial n_j} - \frac{\partial v_i^*}{\partial n_l}n_l n_j, \tag{4}$$

which is the gradient operator restricted to the surface of a sphere. The parameters λ and β represent the rates of rotational recrystallization and grain-boundary migration, respectively. The orientationally dependent term $\mathcal{D}^*$ will be defined below in Eq. (6). The term $\boldsymbol{v}^*$ defines the orientation transition rate, defined by Placidi et al. (2010) as:

$$v_i^* = W_{ij}n_j - \iota(D_{ij}n_j - n_i n_j n_k D_{jk}). \tag{5}$$

The term $W_{ij}n_j$ in Eq. (5) represents the effect of vorticity on the CPO. The second term models basal-slip deformation. The non-dimensional parameter ι represents the ratio of basal-slip deformation to rigid-body rotation.

The parameter λ (s^{-1}) represents the rate of rotational recrystallization, which can be modelled by a diffusion in orientation space (Gödert, 2003). Migration recrystallization is modelled by an orientation-dependent source term, with the rate controlled by β (s^{-1}). The orientation dependence is governed by the deformability, defined by:

$$\mathcal{D}^* = 5 \frac{(D_{ij} n_j)(D_{ik} n_k) - (D_{ij} n_j n_i)^2}{D_{mn} D_{nm}}.$$ (6)

Placidi et al. (2010) give the physical interpretation of $\mathcal{D}^*$ as the (normalised) square of the shear strain rate resolved onto the basal plane. Because ice deforms primarily by basal slip, the resolved shear rate on the basal plane also drives the accumulation of deformation energy in the physical grain, which drives migration recrystallization. The average of $\mathcal{D}^*$ is defined as:

$$\langle \mathcal{D}^* \rangle = \int_{S^2} \frac{\rho^*}{\rho} \mathcal{D}^* \, \mathrm{d}\boldsymbol{n}$$ (7)

If $\mathcal{D}^*$ is greater than the average value $\langle \mathcal{D}^* \rangle$ then $\mathcal{D}^*$ at that orientation will increase, as implied by the final term on the right hand side of Eq. (3). This corresponds to grains growing or nucleating with this orientation. Note that the total production and consumption of $\mathcal{D}^*$ always balance. The factor of 5 in Eq. (6) is a convention.

3.3 Non-dimensionalisation

To apply Eq. (3) to spatially homogeneous fabrics and to compare to fabrics deformed in the laboratory we perform a non-dimensionalisation, where we non-dimensionalise by a characteristic density ρ_0 and strain-rate, which we define as:

$$\dot{\gamma} = \sqrt{\frac{1}{2} D_{ij} D_{ji}}.$$ (8)

This is the effective strain-rate, corresponding to the second-invariant of the strain-rate tensor $\mathbf{D}$. The strain-rate we non-dimensionalise with in this paper is half the strain-rate we used in Richards et al. (2021), which was based on the experimental strain-rate. The non-dimensional variables are represented with tildes and are defined as:

$$\tilde{\rho}^* = \frac{\rho^*}{\rho_0}, \quad \tilde{\mathbf{D}} = \frac{\mathbf{D}}{\dot{\gamma}}, \quad \tilde{\mathbf{W}} = \frac{\mathbf{W}}{\dot{\gamma}}$$
$$\tilde{\lambda}(T, \dot{\gamma}) = \frac{\lambda(T, \dot{\gamma})}{\dot{\gamma}}, \quad \tilde{\beta}(T, \dot{\gamma}) = \frac{\beta(T, \dot{\gamma})}{\dot{\gamma}}$$

Recasting Eq. (3) in terms of the non-dimensional variables above, we obtain:

$$\frac{\partial \tilde{\rho}^*}{\partial \tilde{t}} = -\nabla^* \cdot [\tilde{\rho}^* \tilde{\boldsymbol{v}}^*] + \tilde{\lambda} \nabla^{*2}(\tilde{\rho}^*) + \tilde{\rho}^* \tilde{\beta}(\mathcal{D}^* - \langle \mathcal{D}^* \rangle),$$ (9)

where

$$\tilde{v}_i^* = \tilde{W}_{ij} n_j - \iota(\tilde{D}_{ij} n_j - n_i n_j n_k \tilde{D}_{jk}).$$

is the non-dimensional form of the orientation transition rate (Eq. (5)). Richards et al. (2021) constrained the non-dimensional parameters $\tilde{\lambda}, \iota, \tilde{\beta}$ as functions of temperature. This was done by finding the parameters which gave a best fit to experimental
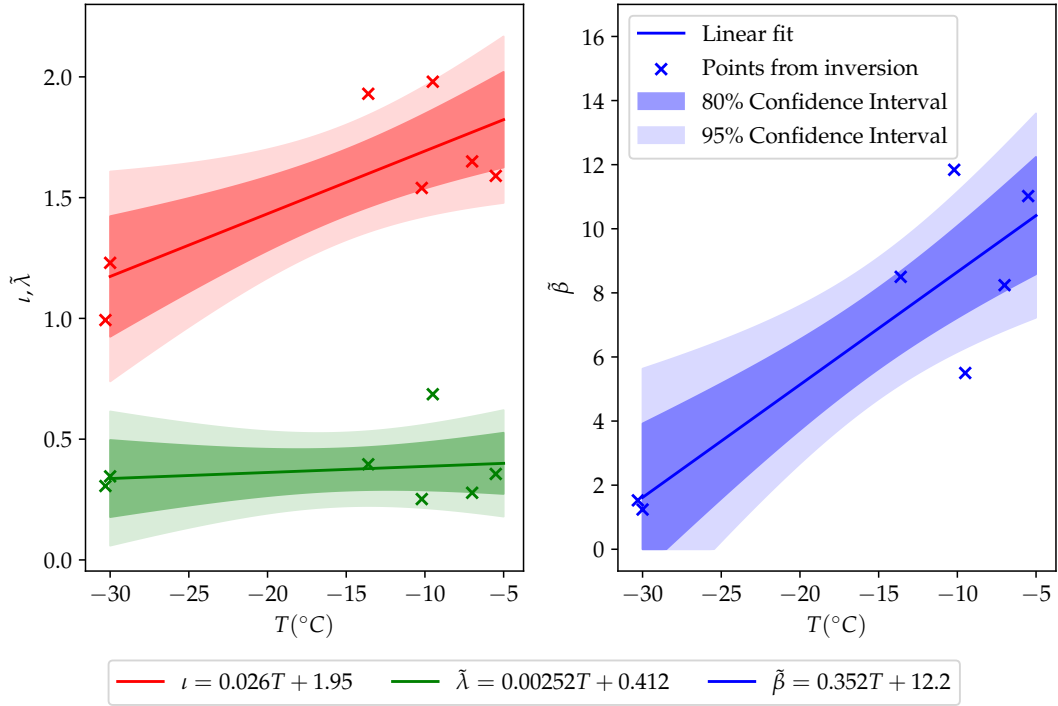
Figure 6. The model parameters $\iota, \tilde{\lambda}, \tilde{\beta}$ as functions of temperature, determined by linear regression to experimental data in both compression and simple shear, as conducted in Richards et al. (2021). The 95% and 80% confidence intervals are shown.

results in simple shear. These parameters were then found to predict well fabric produced in the other endmember deformation regime of (unconfined) compression. In this paper, we use a best fit from the entire inversion performed in Richards et al. (2021), rather than just from the inversion performed in simple shear. Furthermore, as the strain-rate we use to non-dimensionalise is half that used in Richards et al. (2021), the non-dimensional recrystallization parameters $(\tilde{\lambda}, \tilde{\beta})$ used in this paper are double those used in Richards et al. (2021). The true parameters (λ, β) are unchanged. The non-dimensional parameters as functions of temperature are shown in Fig. 6, along with the data points used in the inversion from Richards et al. (2021) and 80% and 95% confidence intervals. In the accompanying supplement to this article a parameter sensitivity study can be found, reproducing the figures below with (a) $\iota_{\max}, \tilde{\beta}_{\max}, \tilde{\lambda}_{\min}$ and (b) $\iota_{\min}, \tilde{\beta}_{\min}, \tilde{\lambda}_{\max}$, with the max and min values taken from the 80% confidence interval in Fig. 6. (a) and (b) are chosen to give the strongest and weakest fabric respectively.

Equation (9) and the parameters defined by the best fit lines in Fig. 6 combined, when solved with the spectral method defined in Richards et al. (2021), represent the SpecCAF model.

3.4 Pole figure and cross section representation

As a preliminary illustration of the model output and its representation, we show in Fig. 7 an example of model output obtained by solving the model at $T = -5°$ C, in simple shear ($\mathcal{W} = 1$). The principal axes are orientated at $\theta = \pm 45°$ directions of the
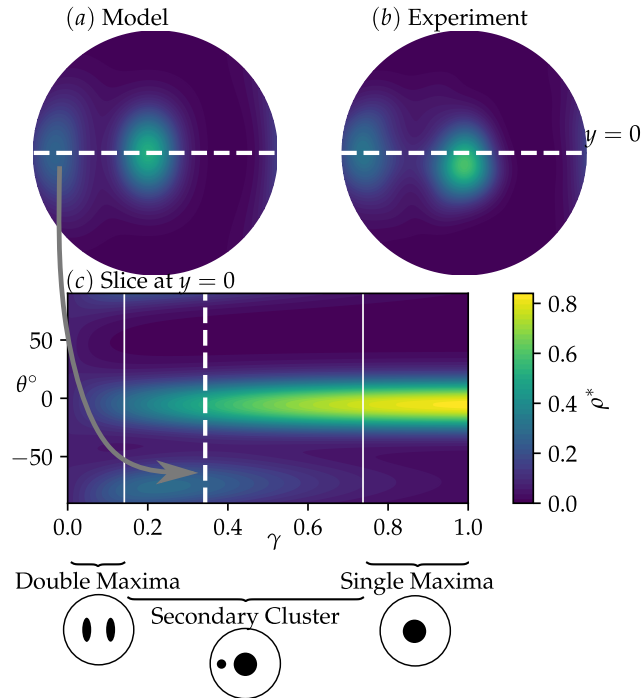
Figure 7. To illustrate the fabric we show a simulation in simple shear at $T = -5°$ C. (a) shows a pole figure from the model at an effective strain of $\gamma = 0.345$ (where the strain-rate is defined as in Eq. (8)). (b) shows a pole figure from laboratory experiments (Qi et al., 2019) at the same temperature and strain, showing good agreement. More comparisons like this can be found in Richards et al. (2021). The white dotted line in (a) and (b) shows $y = 0$. (c) shows $\tilde{\rho}^*$ at $y = 0$ against strain. Here the white dotted line highlights the strain at which the pole figure is plotted. θ is the polar angle. Also highlighted is the classification of the different fabric types at different strains; from double-maxima to secondary cluster to the steady state single maxima. For this figure only the principal axes of deformation are oriented at $\theta = \pm 45°$.

pole figure. To visualise how the fabric changes with increasing strain, we plot slices of the pole figure at $y = 0$. The example pole figure in (a) is plotted at a strain of $\gamma = 0.345$. The value of ρ^* at $y = 0$ is plotted in (c) for each strain. This shows how the CPO develops from an isotropic fabric. A secondary cluster can clearly be seen as a transient feature which has mostly disappeared by $\gamma = 0.8$.

As an example comparison of the model prediction and experimental observations, we have included a pole figure from laboratory experiments (Qi et al., 2019) in Fig. 8b. This is at the same temperature and strain as the model output in (a). There is very good agreement between the model and experiments. More experimental comparisons are detailed in Richards et al. (2021), showing that the model is able to generally capture both qualitative and quantiative features of fabrics observed in existing experiments.

4 Results

4.1 General fabric evolution: dependence on temperature and vorticity number

310 We explore fabric evolution across a complete, continuous range of vorticity numbers $\mathcal{W}$ for two-dimensional deformation regimes (spanning $\mathcal{W} = 0$ to ∞), and a continuous range of temperatures T relevant to ice-sheet flow ($T = -30$ to $-5\,^\circ$ C). For all cases, we assume an initially isotropic fabric. To make comparisons, we will limit our analysis to fabrics undergoing a constant two-dimensional deformation and at a constant temperature. The vorticity number, defined in Eq. (1), gives the ratio of vorticity to strain-rate magnitude. However, it does not fully constrain the velocity gradient. To do so, we define the

315 non-dimensional velocity gradient as:

$$\nabla \tilde{\boldsymbol{u}} = \begin{bmatrix} 1 & \mathcal{W} \\ -\mathcal{W} & -1 \end{bmatrix} \tag{10}$$

This gives pure shear for $\mathcal{W} = 0$, simple shear for $\mathcal{W} = 1$ and rigid-body rotation as $\mathcal{W} \to \infty$. The principal-axes of deformation are aligned with the coordinate axes and are unchanged as $\mathcal{W}$ varies.

With the velocity gradient fully defined, we explore the fabric dynamics produced across T-$\mathcal{W}$ space in Fig. 8. For each

320 square we show the slice through the pole figure at $y = 0$ (explained in Fig. 7) up to a finite strain of $\gamma = 1$. The temperature range is from -30° C to -10° C, temperatures typical in ice sheets (Duval et al., 2010). The vorticity number ranges from $\mathcal{W} = 0.1$, very close to pure shear, and $\mathcal{W} = 10$ representing highly vortical flow with curved streamlines. This provides a detailed picture of how the fabric evolves with increasing strain, providing insights into deformation regimes between compression and simple shear as well analysis of fabrics produced by deformation regimes more rotational than simple shear. For low vorticity

325 numbers, a single maximum can be seen at low temperatures which develops into double-maxima as strain increases. As $\mathcal{W}$ increases the clusters are moved by the rotational component of the deformation, resulting in a primary and secondary cluster. The weaker, secondary cluster is gradually consumed by migration recrystallization as strain increases, leading to a single maximum at high strains, for $\mathcal{W} \sim O(1)$.

We also show in Fig. 8 analysis of fabrics produced in highly rotational ($\mathcal{W} > 1$) deformation regimes, which as we have

330 shown are prevalent in real-world conditions (Figs. 4,5). Figure 8 shows that the fabric is strongest for $\mathcal{W} = 1$, and weakens as vorticity increases past this. For example, for $\mathcal{W} = 10$ there is only a very weak CPO produced. Furthermore, at large vorticity numbers oscillation can be seen in the fabric pattern. Figs. S1 and S2 in the supplement show that variations in the parameters from Fig. 6 affect the strength of the primary cluster primarily, but do not affect the variation with vorticity number or the transition from one fabric type to another. To further analyse the limit of very large vorticity numbers we show the fabric

335 produced as $\mathcal{W} \to \infty$ in Fig. 9. This fabric is seen for any vorticity number above $\mathcal{W} \approx 50$. To measure fabric concentration the J index is often used (Bunge, 1982), defined as:

$$J = \int_{S^2} \tilde{\rho}^{*2} \, \mathrm{d}\boldsymbol{n}. \tag{11}$$

The J index of this fabric is 1.16, very close to completely isotropic ($J = 1$). It is unlikely this weak girdle fabric would be distinguishable from an isotropic fabric in a physical sample, where the CPO is determined by sampling a limited number of grain orientations.

4.2 Fabric regime diagrams for cluster angle and fabric type

To distil all the complex information shown in Fig. 8 and make this information more easily accessible, we presents results showing the variation of the angle of the primary cluster and the regimes of different fabric patterns across the $\mathcal{W} - T$ space. The angle between the primary cluster and the closest principal axis of deformation (i.e. the axis of compression) is shown in Fig. 10 at six separate finite strain values, across $\mathcal{W}$-T space. Even at a low finite strain of $\gamma = 0.3$ there is already an established difference in angle across the parameter space (Fig. 10a). Low temperatures and low vorticity numbers have the primary cluster most closely aligned with the compression axis. The angle then increases as both temperature and especially vorticity number increase. As strain increases the variation in angle increases. However, for a finite strain greater than 0.5 the angle is mostly invariant with strain. Across the strain and temperature range, an angle of around $40°$ implies simple shear ($\mathcal{W} \approx 1$), whereas if the primary cluster and compression axis are coincident, this suggests pure shear at $T \approx -30°$ C .

Figures S3 and S4 in the supplement show Fig. 10 with the strongest and weakest possible fabric based on the 80% confidence intervals in Fig. 6. At low vorticity numbers and temperatures, the angle between the primary cluster and compression axis is slightly sensitive to variations in parameters, but outside of this space the angle is roughly unchanged.

As a means to visualise the variety of fabrics in terms of both the parameter space and across possible finite strains we show a regime diagram for fabric patterns in Fig. 11. To define whether a fabric is a double-maxima, secondary cluster or single-maxima we take the ratio of the two largest peaks in the fabric. If the 2nd largest peak is less than 10% the strength of the largest peak, it defines as a single-maxima. If the strength of the 2nd largest peak is between 10% and 90% of the largest peak, it is defined as a secondary cluster. If it is $> 90\%$ it is defined as a double-maxima. Contour lines of primary cluster angle at $20°$ and $50°$ are also shown. This shows the different fabric types (Fig. 1) across the space of temperature, vorticity number and finite strain. The figure is in the same format as 10.

Figure 11a shows the initial fabric after a finite strain of only $\gamma = 0.3$. There are three regimes at this finite strain. For approximately $\mathcal{W} < 0.7$ a double-maxima is produced. There is a small region, at high vorticity numbers and primarily at low temperatures but extending into high temperatures, at which a single-maxima is produced. Otherwise a secondary cluster is produced, this occurs for relatively high vorticity numbers and is more dominant at higher temperatures, as expected. At higher finite strain the double-maxima pattern becomes less prevalent, only occurring at lower vorticity numbers or not at all. This highlights the transient nature of this pattern. As a reminder, the double-maxima is the 2D equivalent of a cone-shape fabric (Fig. 1). The double-maxima fabric is only present up to a finite strain of about $\gamma = 0.5$. In Fig. 11c, at $\gamma = 1.0$ the parameters space is dominated by single-maxima and secondary cluster patterns. Ice fabrics which develop at higher temperatures $T > -20°$C are dominated by secondary cluster patterns, with the exception of around $\mathcal{W} \approx 3$, where a single-maxima occurs because the secondary cluster is too weak. At lower temperatures $T < -25°$C a single-maxima is produced because migration recrystallization is not active enough for multiple clusters to be produced. This balance between a single-
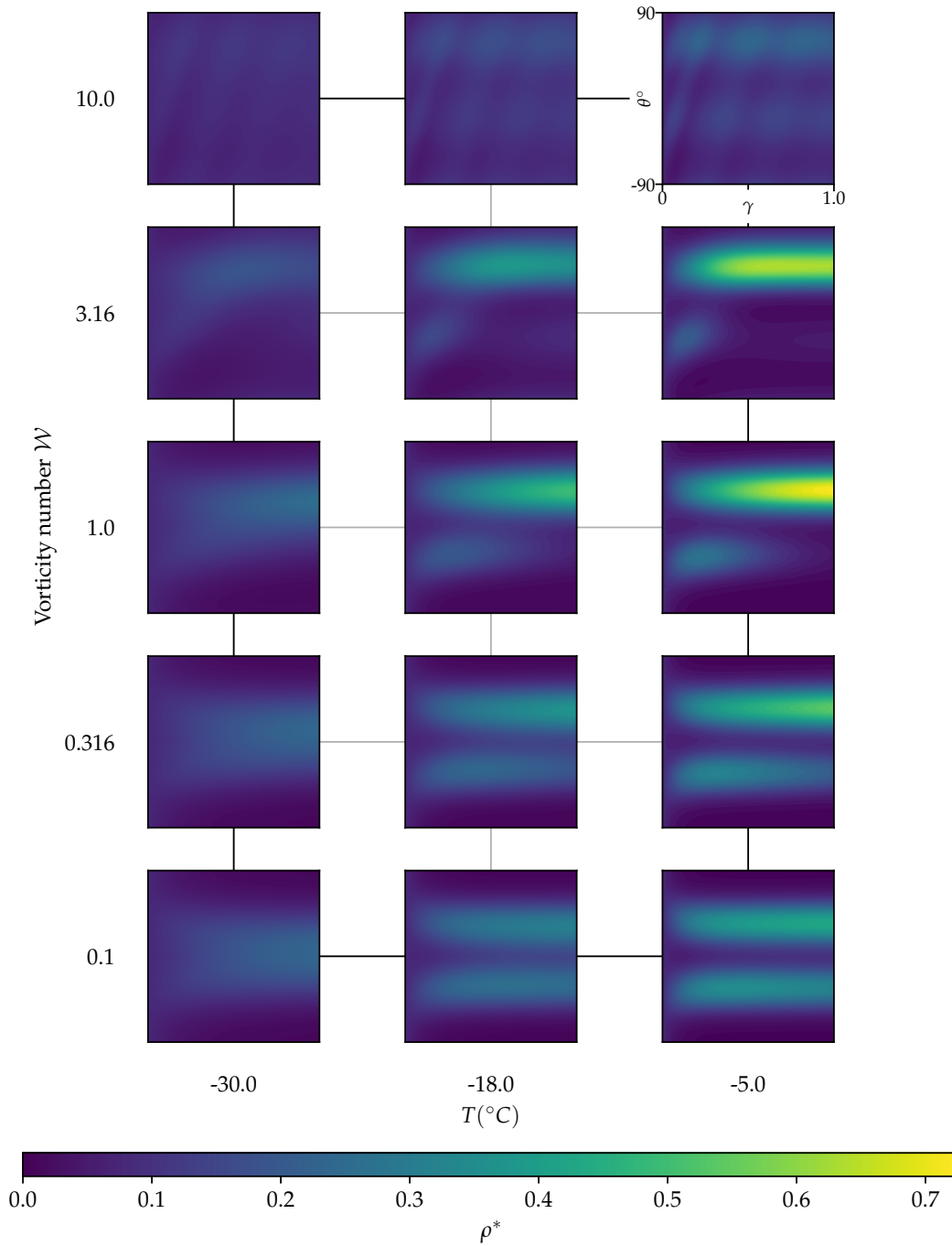
Figure 8. Slices of the pole figure showing the value of ρ^* at $y = 0$ for an array of temperatures and vorticity numbers. All $y = 0$ plots go to a strain of $\gamma = 1$. The colour limits are the same for all plots.
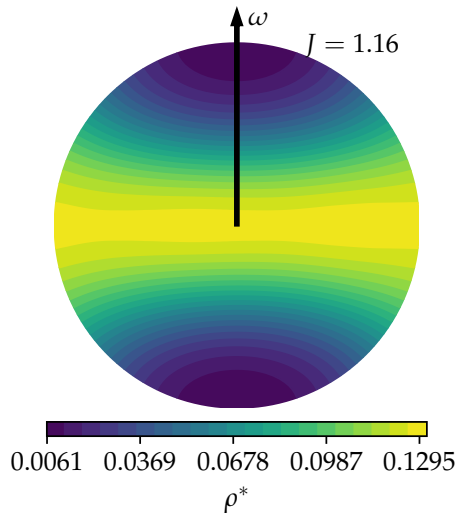
Figure 9. Pole figure for $\mathcal{W} \to \infty$ and $T = -5°\,$C at steady-state. A very weak girdle fabric is produced, with the girdle coincident with the axis of vorticity, shown. This CPO has a J index of 1.16, where $J = 1$ is an isotropic fabric.

maximum fabric and a secondary cluster fabric continues as the finite strain increases, with a single-maxima also becoming more prevalent at high temperatures for vorticity numbers around 1 (Fig. 11e and f).

Figures S5 and S6 in the supplement show Fig. 11 with the strongest and weakest possible fabric based on the 80% confidence intervals in Fig. 6. The overall picture is similar. The variation with vorticity number is approximately unchanged and boundaries between the regimes shift by roughly $\pm 7°\,$C.

To illustrate the difference in pole figure patterns at the same finite strain but different temperatures and deformation regimes we plot pole figures at a finite strain of $\gamma = 2$ (Fig. 12) overlaid onto a regime diagram of fabric patterns. The pole figures are centred at the vorticity number and temperature they are simulated at. Figure 12 highlights fabrics are still variable despite being in the same regime. The fabric at $\mathcal{W} = 1, T = -5°\,$C is much stronger than the fabric at $\mathcal{W} = 10, T = -30°\,$C or $\mathcal{W} = 0.1, T = -30°\,$C, however they are all single-maxima. We also note the difference in angle of the primary cluster across the parameter space: approximately $0°$ for $\mathcal{W} = 0.1, T = -30°\,$C but increasing as $\mathcal{W}$ and T increase, as shown in Fig. 10.

4.3 Analysis of fabric evolution timescales

A variable that is central to the interpretation of ice core fabrics is the timescale or, equivalently in the non-dimensional problem, the finite strain over which fabric evolution occurs. In Fig. 13 we explore fabric evolution timescales. Fig. 13a shows the finite strain required to reach halfway to steady-state in (a). We also show the J index when the fabric reaches steady state in (b). Both are shown across the space of $\mathcal{W}$-T. An example showing the evolution of the J index and the finite strain at which it reaches steady state is illustrated in Fig. 13c.
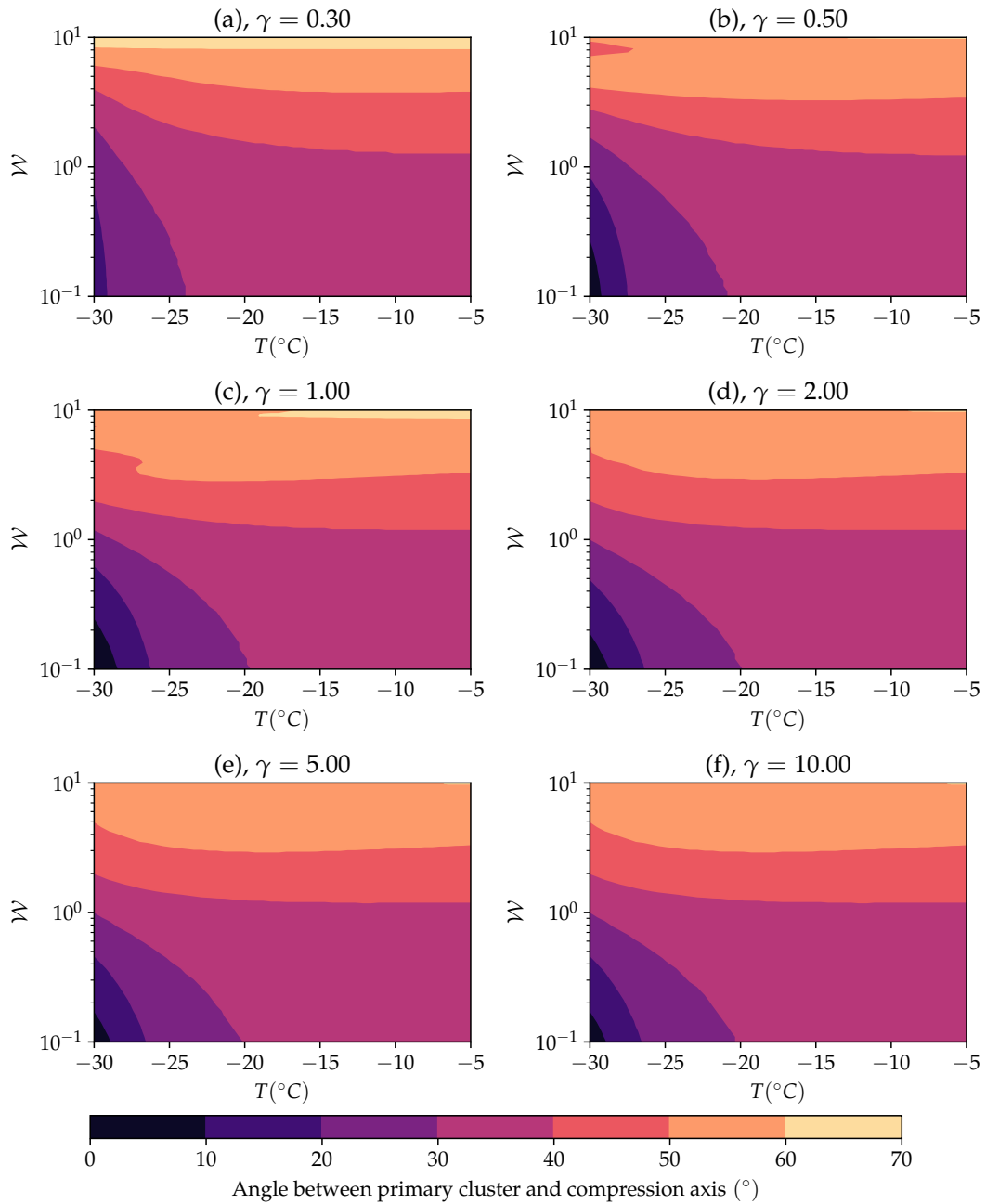
Figure 10. Contour plots showing the angle (in degrees) of the largest cluster from the compression axis. Panels are shown for progressively increasing finite strain values. This angle is fairly invariant with finite strain. The resolution of this figure is 50×50 across the parameter space.
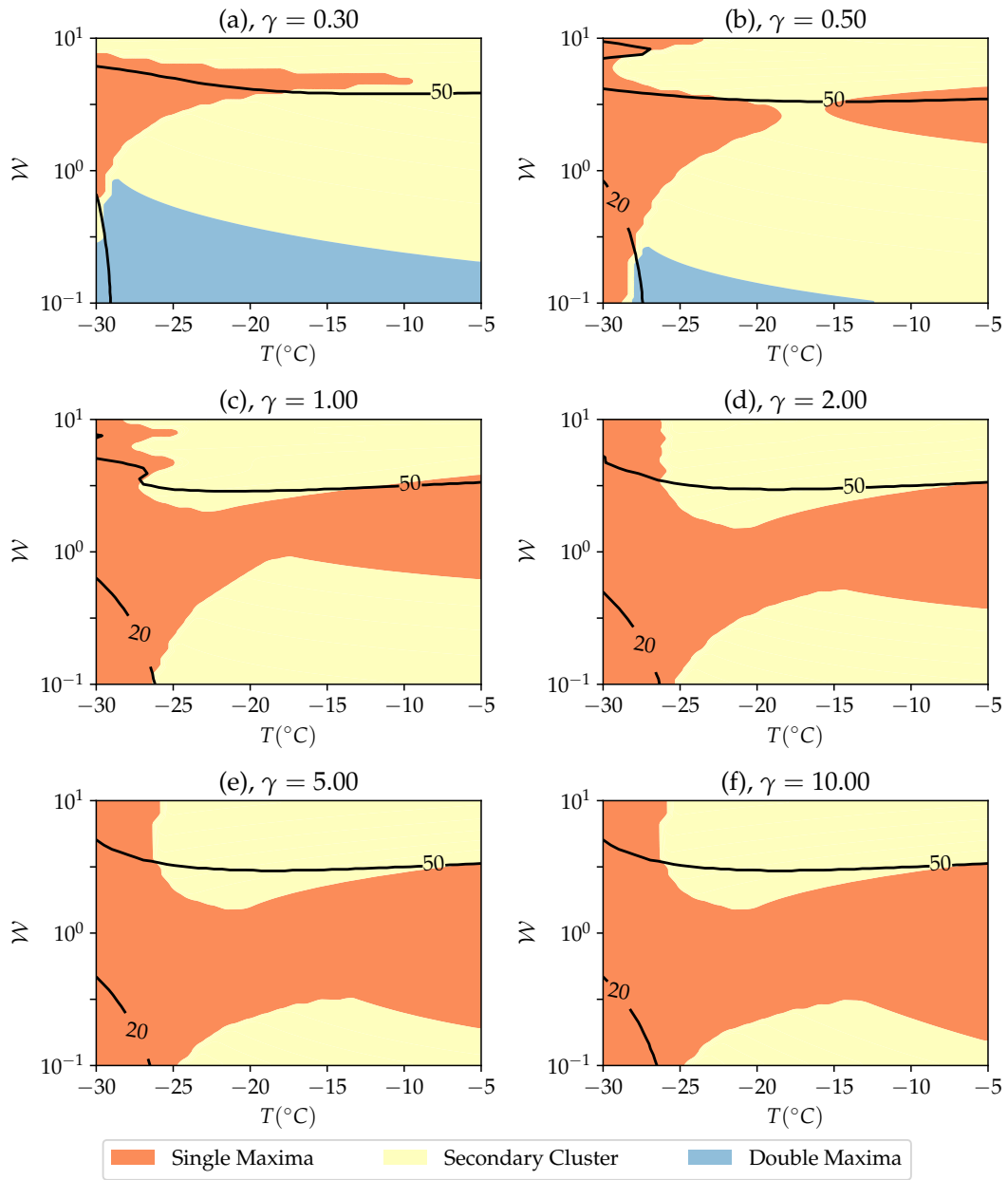
Figure 11. Regime diagram of the different fabric patterns which occur (defined in Fig. 1). The angle of the primary cluster in Fig. 10 at $20°$ and $50°$ is also overlaid. As in Fig. 10 diagrams are shown for discrete strain values. The resolution of this figure is 50×50 across the parameter space.

Figure 13a shows the finite strain at which the J-index is halfway to its steady-state value. This can be considered the half-life over which fabric evolution occurs. Measuring the strain at halfway to steady-state gives insight into the timescale over which
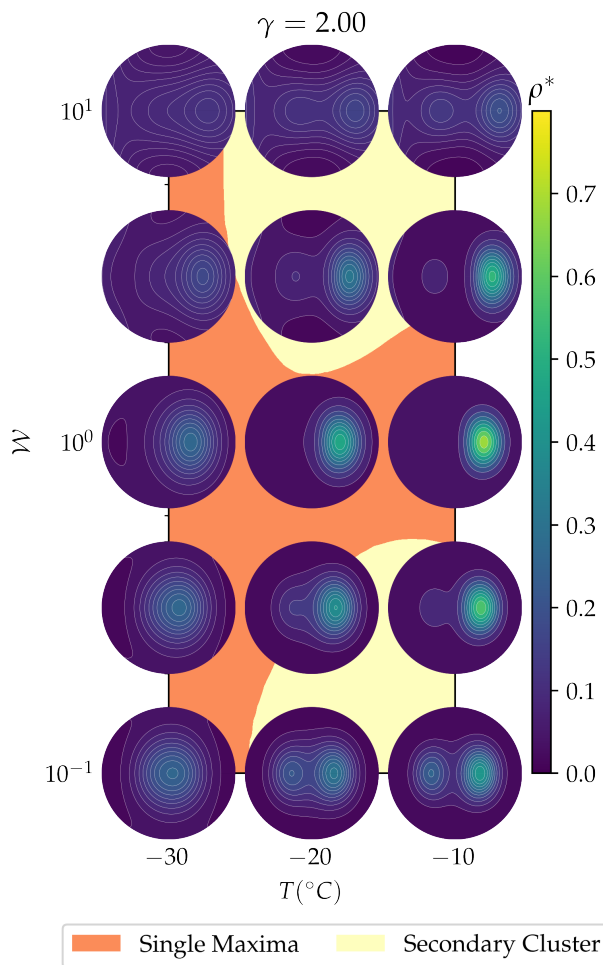
Figure 12. Pole figures overlaid onto the regimes at $\gamma = 2$. The pole figures are centred at the vorticity number and temperature they occur.

fabric development occurs and is more robust measure than estimating the strain at steady-state, which we found was sensitive to parameter variations such as those shown in the supplement. Vorticity numbers closest to 0 and the coldest temperatures have the highest halfway strain. Under these conditions, neither rigid-body rotation nor migration recrystallization are are activated to a significant degree: the fabric evolution is dominated by by basal-slip deformation. Fabrics also take longer to develop closer to a vorticity number of 0.7, due to the fact that this vorticity number has the strongest fabrics generally. For $\mathcal{W} > 1$ the strain to reach half strength decreases, as the fabrics are shown to be generally weaker as vorticity number increases. This is clearly seen in Fig. 13b (showing the J-index a steady-state). Generally, as temperature increases the steady-state fabric always increases in strength. Fabrics are strongest at a vorticity number of around 0.7 across almost the whole temperature space.

Figures S9 and S10 in the supplement show Fig. 13 with the strongest and weakest possible fabric based on the 80% confidence intervals in Fig. 6. These figures show that the halfway strain is fairly insensitive to changes in the parameters, with
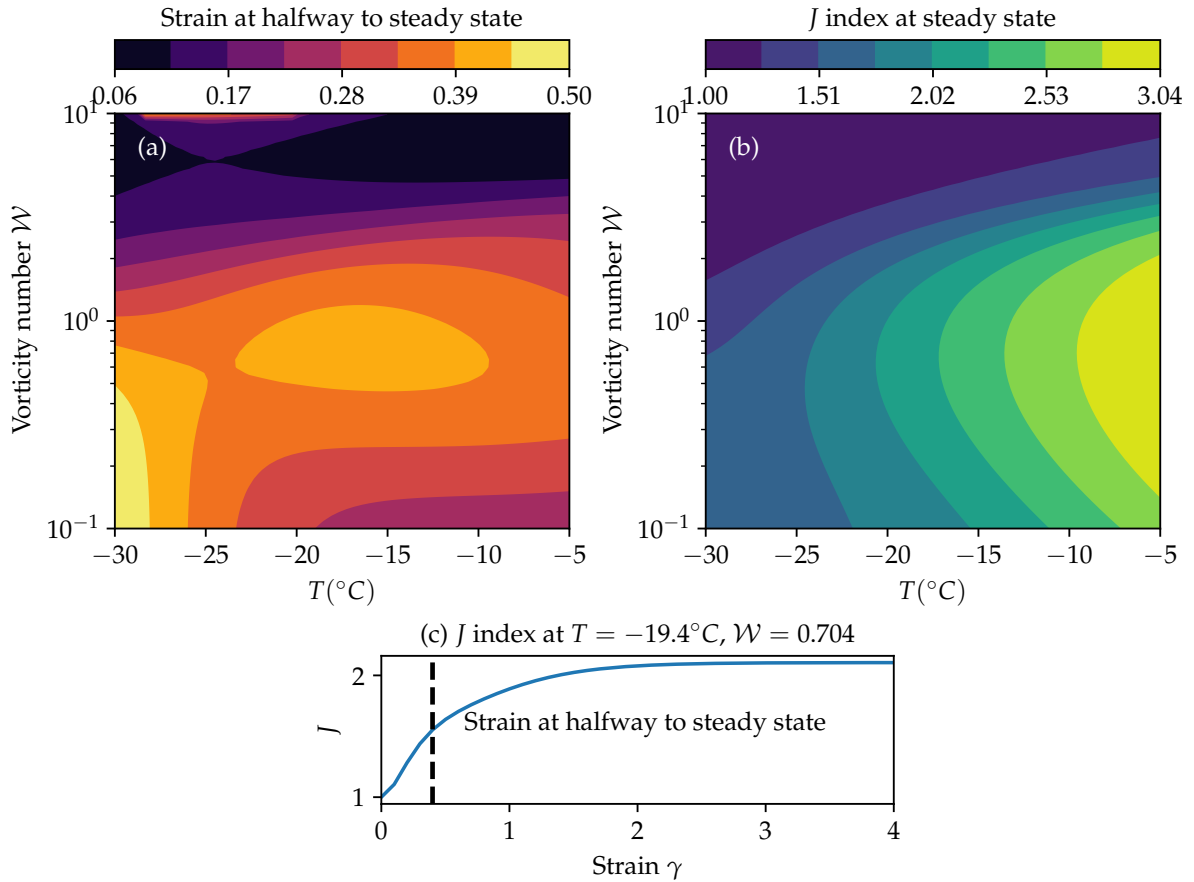
Figure 13. Properties of steady-state fabrics across the $\mathcal{W} - T$ parameter space. (a) shows the strain at which the fabric, at constant T and $\mathcal{W}$, reaches steady-state (defined as a rolling average of the J index over a strain window of 2 gets within 5% of the steady-state value). (b) shows the J index at the steady state. (c) shows the plot of J against strain γ at $T = -30°$ C, $\mathcal{W} = 0.01$, with the calculated steady-state strain

the maximum varying by around $\pm 20\%$. The maximum halfway strain remains at $\mathcal{W} = 0.1, T = -30°$ C for both Fig. S9 and S10. The J index at steady-state is more sensitive to changes in parameters but the general picture of how this variable changes across the T-$\mathcal{W}$ space is similar, with the upper bound showing generally less variation across the parameter space.

5   Discussion

405   The analysis presented here gives predictions for the fabric patterns produced over the whole range of vorticity numbers and temperatures arising for incompressible two-dimensional deformation regimes, a first for fabric modelling. We have limited the analysis here to fabrics produced under a constant deformation regime and temperature. Although ice in the natural world will

undergo changing deformation regimes, our analysis is a first step to provide insights into fabrics produced for deformation regimes away from pure and simple shear. Furthermore, the fabrics analysed here are highly relevant for ice deformed in the laboratory, which is in most cases deformed at constant temperature and vorticity number.

Ice in the real-world will undergo three-dimensional deformations yet it is common to model ice-sheets in two-dimensions along the vertical cross-section, such as in Fig 3. This has been shown to be valid around divides such as in Martín et al. (2009). In some regions, such as ice streams and regions where the ice is floating, the deformation will be more two-dimensional. Nevertheless, exploring general two-dimensional deformations is a good first step away from the isolated, two-dimensional conditions of pure and simple shear alone.

5.1 Fabric patterns across deformation regime and temperature space

Previous work has focused on modelling fabrics produced at fixed deformation regimes (Llorens et al., 2016), or modelling the deformation experienced by ice at a divide (Bargmann et al., 2012). Due to the computational efficiency of our model, we have been able to perform thousands of simulations across the parameter space of temperature and vorticity number to show how fabrics vary.

Our work generally shows a smooth transition between the two deformation regimes of pure and simple shear, as can be seen in Fig. 8. Such intermediate deformation regimes are important in the real-world (Figs. 3,4,5) and must be taken into account when considering ice fabrics, rather than focusing on the isolated cases of pure and simple shear.

The weak fabric seen for highly rotational flows has not been studied at all so far. This result is interesting as it shows for the first time the fabric produced by highly rotational deformation regimes: a weak girdle fabric. Throughout these deformation regimes we have kept the magnitude of $\mathbf{D}$, the strain-rate tensor, constant. Therefore, at high vorticity numbers the weak fabric seen in Figs. 8 and 9 is not caused by a lack of deformation. Instead it is due to the rotational component acting to quickly smear any cluster produced by basal-slip deformation or migration recrystallization to orientations where the cluster is consumed by migration recrystallization. The end result as $\mathcal{W} \to \infty$ is a very weak girdle fabric with the girdle aligned to the axis of vorticity.

The regime diagram in Fig. 11 shows the fabric patterns produced across the space of vorticity number, temperature and strain. From experimentally deformed ice, the vast majority of fabrics produced are cone-shape fabrics (Fan et al., 2020). However, ice deformed in the laboratory in compression can only reach strains of up to $\gamma = 0.5$ and Fig. 11 highlights how double-maxima fabrics are only present up to these strains. Above $\gamma = 0.5$, secondary cluster and single-maxima patterns are more prevalent. Importantly, secondary cluster fabrics, which have been to-date only seen in simple shear ($\mathcal{W} = 1$), can occur at very low vorticity numbers (Fig. 11c,d).

Results from Budd et al. (2013) show a secondary cluster fabric for $W = 0.85, \gamma = 0.34$ and $T = -2°$ C. Extrapolating Fig. 11a would suggest this agrees with our model. Budd et al. (2013) also have another laboratory data point at $\mathcal{W} = 0.52, \gamma = 0.72$ and $T = -2°$ C which shows a single-maxima pattern. Extrapolating Fig. 11 would predict a secondary-cluster fabric, however close to boundary where the fabric becomes a single-maxima. Furthermore SpecCAF has only been constrained up to $T = -5°$ C, and as ice approaches the melting point the parameters are likely to behave non-linearly, so a simple extrapolation cannot be relied upon.

5.2 Finite strains required for fabric evolution

For the interpretation of ice fabrics it is essential that we know the timescale (or in the non-dimensional case here, total finite strain) over which fabrics evolve to steady state. In this paper we have presented the first assessment of fabric timescales (Fig. 13), by examining the 'half-life' for fabrics to reach steady-state. In compression, experiments can only reach a maximum effective strain of $\gamma \approx 0.5$. It is often assumed that a strain of around $\gamma = 0.2$ represents a steady state in the mechanical properties, and consequently in fabric (Jacka, 2000). However, recent experiments show that the fabric continues to evolve past this (Piazolo et al., 2013; Qi et al., 2017). It is also known that fabrics require higher finite strain to reach steady-state in simple shear (Journaux et al., 2019). Qi et al. (2017) note that fabric data is required at higher strains than has been achieved to date by compression experiments to link to high-strain natural environments. The results in our paper fill this missing gap. The analysis of fabric regimes extends to very high strains of $\gamma = 10$. For low $\mathcal{W}$, in experiments and in Fig. 8, a CPO quickly develops at very low strain $\gamma \approx 0.2$, (also seen in Craw et al., 2018). Although the fabric pattern does not change as strain increases, it cannot be said to be in steady-state because the concentration of orientation at the clusters continues to increase. It should be noted that any change in fabric intensity will directly affect the mechanical properties such as the degree of viscous and seismic anisotropy (Duval et al., 1983; Matsuoka et al., 2003).

Figure 13a shows that, for very low temperatures ($T \approx -30°$ C) the strain to reach halfway to steady state is the highest, around $\gamma = 0.5$. as can be seen in Fig. 13c the true steady-state can not be reached to much later as the J index approaches the steady-state value slowly. From this we can remark that reaching true steady states at these temperatures in laboratory experiments will be impossible for deformations close to pure shear. Although Fig. 13a shows that, above very low temperatures ($T > -24°$ C), deformation regimes closer to simple shear take longer to reach steady-state. However these strains are more achievable in the laboratory as fabrics can be deformed in torsion, allowing very high strains such as $\gamma > 1.5$ to be reached (Journaux et al., 2019). Furthermore, the strain to reach halfway to steady state is primarily dependent on vorticity number $\mathcal{W}$, rather than temperature.

5.3 Consequences for ice core interpretation

Analysis and interpretation of ice cores remain key for understanding the processes occurring in the natural world, for both understanding the past climate history (Dansgaard et al., 1969) as well as understanding ice sheet dynamics (Schytt, 1958). The regime diagrams we have constructed (Figs. 10,11) can be used as a toolkit to interpret ice cores. For example a single maxima fabric with an angle less than $20°$ between the compression axis and primary cluster centre implies the core has undergone mostly compression at low temperature. If other constraints such as knowledge the deformation regime history or temperature has been constant to good approximation, the dominant deformation regime and temperature can be further constrained. As our work here is for constant temperature and vorticity number, any ice core fabric interpretation will inherently assume that an ice core that has been deformed primarily at a dominant temperature and deformation regime over its recent history. Since temperature varies with depth in an ice sheet (Paterson, 1999), this method is likely to be most reliable for ice cores where the ice is primarily moving horizontally i.e. far from ice divides. We also assume an initial random orientation for the fabric.

475 Although deformation regime history in the natural world is likely to be complex, this is a reasonable assumption because ice formed from surface accumulation will initially have a random distribution of orientations (Montagnat et al., 2020).

The fabric pattern is a robust way to interpret ice cores, as it requires no assumptions about the deformation regime direction. The regime diagram in Fig. 11 which is based on the fabric pattern only is complex but insights can be drawn nonetheless. For example, the presence of double-maxima fabrics (two equal strength clusters) implies that the fabric has undergone a
480 relatively low strain. A secondary cluster fabric implies that the fabric is likely to be at intermediate strains, and at temperatures $T > -20°$ C. A very weak secondary cluster fabric implies a highly rotational deformation regime $W > 3$. Furthermore, we have shown that the presence of a fabric which appears to be isotropic could be indicative not only of no deformation, but also of a highly rotational deformation regimes.

If other information about the fabric history is known, Figs. 10 and 11 can be used in combination to extend this knowledge.
485 For example, if there are independent constraints on the orientation of the deformation regime axis, then the angle of the primary cluster can be used to interpret ice cores as well. As can be seen in Fig. 10 the angle between the primary cluster and the compression axis is relatively invariant with strain. Knowing this angle can therefore give a good estimate of the vorticity number and temperature, for deformation regimes which are primarily 2D.

5.4 Implications for ice flow properties and modelling

490 Viscous anisotropy of ice is controlled by the fabric and is a key control of the flow field (e.g. Alley, 1988). This anisotropy is dependent on the pattern, direction and strength of the fabric. The current approach common in ice-sheet models is to represent anisotropy with an enhancement factor which scales the viscosity, either globally (Graham et al., 2018) or locally (Placidi et al., 2010). The model SpecCAF used in our paper can be coupled with any anisotropic viscosity formulation to include directional variation in viscosity. Martín et al. (2009) has coupled a fabric model to an anisotropic viscosity, but the fabric evolution
495 model used did not include recrystallization, which are dominant processes in controlling fabric evolutions and their approach to final steady states, as well as temperature dependence. The high sensitivity of fabric strength and patterns to temperature shown in our paper and in experimental work (Qi et al., 2019) may lead to further interesting flow features on top of those caused by anisotropy alone. This can only be captured with a coupled fabric model including a temperature dependent fabric. Temperature can also affect the flow through viscous heating (e.g Hindmarsh, 2004) and a temperature dependent fabric model
500 coupled to anisotropic viscosity such as Gillet-Chaulet et al. (2005) would allow us to understand what proportion of the effects of temperature are a consequence of viscous heating and what proportion arise through temperature induced changes to the fabric.

The fabrics we have shown here give insight into where anisotropy may be most important in an ice sheet. Areas with strong fabrics will be highly anisotropic, with the viscosity varying in different directions. Anisotropic flow is not well studied
505 but initial simulations with coupled anisotropic flow show it can explain hitherto unexplained observations such as syncline patterns observed under ice divides (Martín et al., 2009). Our analysis of fabrics produced in highly rotational deformation regimes showing an almost isotropic fabric (as seen in Figs. 8 and 13b) implies that in the regions of Antarctica where the flow is highly rotational, which as we can see in Fig. 5 does occur, the fabric will evolve towards a state with limited anisotropy

(directional variation in viscosity). However areas of approximately simple shear ($\mathcal{W} = 1$) show the strongest fabrics and hence the strongest effect of anisotropy. This means viscous anisotropic affects are expected to be widespread, further motivating the need to fully represent them in models. In future work it would be possible to explore how seismic wave velocities vary with the different fabrics modelled in this paper, again comparing to real world observations.

6 Conclusions

Prediction of fabric evolution is pivotal for the correct interpretation of ice core fabrics and reliably predicting ice losses in a changing climate using ice sheet modelling. Our work extends the ability to predict fabric evolution from the endmember deformation regimes of pure and simple shear to general two-dimensional deformation regimes. As such the presented study represents a step towards understanding fabrics in fully general conditions which is key for understanding viscous anisotropy and, in turn, large-scale ice-sheet flow modelling. We have shown that deformation regimes outside of pure and simple shear are important in common flow scenarios seen in ice-sheets. There is the potential for future work to extend the modelled fabrics to calculate to seismic anisotropic properties, enabling us to further improve our understanding of ice in natural flows.

The regime diagrams presented are a useful tool to help with the interpretation of ice core data. In combination with other information, such as the plane of deformation regime or an estimate of the temperature at which a core was deformed, these regime diagrams can be used to determine the primary deformation regime and temperature undergone by an ice core. We show that for two-dimensional deformations, the double-maxima fabric is not present at high strains when only a small amount of vorticity is present in the deformation regime $\mathcal{W} > 0.1$. This is important as many laboratory experiments are performed for $\mathcal{W} = 0$. Future work could investigate whether this conclusion extends to three-dimensional cone-shape fabrics.

Highly rotational deformation regimes were investigated for the first time and showed a weak girdle fabric aligned to the axis of vorticity. We have also shown how the timescale for fabric evolution, shown by the halfway strain to reach steady-state, changes over the parameter space. Laboratory experiments around simple shear may be able to reach the strains required to get close to steady-state, however compression experiments, especially at low temperatures cannot achieve the required strains for steady-state.

Our predictions of ice fabric evolution over a wide range of deformation regimes provide insights into how and where viscous anisotropy will be important for ice flow dynamics. Intermediate deformation regimes between pure and simple shear produce the strongest CPOs, suggesting anisotropy will be most important in these regions. Similarly, as highly rotational deformation regimes produce a weak fabric, there is likely to be a less dominant affect of anisotropy in such regions. Our understanding of these issues could be further improved by combining our model with an anisotropic viscosity formulation to model the coupled fully anisotropic flow of ice necessary to predict ice flow in a changing climate.

Code availability. A python implementation of SpecCAF, alongside code to reproduce the figures in the Results section, is available at http://doi.org/10.5281/zenodo.5607230

540 *Author contributions.* All authors designed the research and edited the manuscript. DR developed the model code, performed the analysis and visualization and wrote the draft.

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