Journal: *The Cryosphere*
Manuscript: tc-2021-118: Ice fabrics in two-dimensional flows: beyond pure and simple shear
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Reviewer: Ed Waddington

1 Overview

I was invited to comment on this manuscript late in the review process. I have read version 3 of the manuscript, and the authors' reply to the previous reviews.

1.1 Author responses to prior reviews

Previous reviewers of this manuscript have identified several points that needed to be addressed.

- One of those points for clarification was the concept that SpecCAF does not directly model crystal-level processes such as dislocation densities, slip on basal planes, recrystallization, or crystal-crystal interactions. The authors have clarified that SpecCAF is an empirical continuum model for the evolution of $\rho^*(\mathbf{x}, t, \mathbf{n})$, the mass fraction of grains at position $\mathbf{x}$ at time t with c axes directed into a a solid angle d$\mathbf{n}$ around direction $\mathbf{n}$.
  Slip on basal planes, rotational recrystallization, and grain-boundary rotation are all incorporated in principle as continuum processes based on gradients in the continuum description, i.e. SpecCAF is essentially an empirical model with what might be called a model *shape* set by equations (3), (4) and (5), and the coefficients λ and β on the terms are set empirically by comparison with fabrics observed in samples whose deformation histories are known or understood.
  To me, this seems to be the same in principle as choosing to fit an exponential *shape* to a data set, where the data determine the prefactor and the exponent. Choosing a good *shape* and a good training data set are key to establishing a good fit over a wide range of circumstances.

- Reviewers were concerned that SpecCAF used the Taylor assumption, in which all grains experienced the same strain rate. The authors have clarified that, following Faria et al. (2008), the SpecCAF model assumes only that the material holds together such that individual grains (if they were explicitly followed, which they are not) would merely retain their position relative the surrounding continuum. There is no restriction imposed on how individual grains strain, rotate, or recrystalize, relative to their neighbors; the only restrictions are on their *species*, defined as other grains with similar orientations.

- Reviewers reminded the authors that not all previous lab tests were restricted to pure or simple shear; some previous lab experiments imposed stress and strain patterns that were combinations of pure and simple shear. The authors have incorporated the suggested references in new discussion in Section 2.1.2, and modified their claim to be the first group to study this.

- Reviewers asked how the evolving fabric was coupled to the applied strain-rate fields. The authors have clarified that they are not yet coupled; that is a goal for future work.

In my view, this paper is a commendable analysis of evoluton of fabric (CPO) in 2-D under a wide range of temperature T and flow regime as characterized by a vorticity number $\mathcal{W}$ in Equation (1). In my view, the paper is suitable for publication in *The Cryosphere*, pending minor revisions that can be negotiated with the scientific editor.

1.2 My questions

The previous reviewers are all clearly experts in anisotropic fabric development and ice-sheet flow, and I think they have done a good job of identifying technical issues and concerns. While I have some familiarity with the field, I will address mainly the likelihood that the manuscript will speak effectively to colleagues and students who are not as well-versed in the topic as the reviewers. Since these points are less germane to the scientific integrity of the manuscript, and more germane to the readability and potential readership, I expect that you can discuss with the editor the rigor with which you should follow them.

- Page 1, Line 24:
  What is meant by *a uniform spectrum*?

- Effective strain rate $\dot{\gamma}$ is introduced in Equation (8), where it is defined in terms of the strain-rate tensor, which is derived in turn from the velocity-gradient tensor. However, strain γ itself just appears without an explanation in the caption for Figure 7.

  This may be a concern, because the manuscript deals with some large finite strains that may not be simply related to the history of strain rate. It is not obvious how (or if) the strain rate is integrated over time to get the strain. I assume Lagangian or Eulerian finite-strain tensors are involved?

- Are the finite strains in SpecCAF calculated in a way that is compatible with the calculated finite strains from lab tests, and inferred from Antarctic data sets such as Figure 8?
  For example, in lab tests to large finite strains, the shape of the sample changes significantly, and even if the applied force or the applied stress is held constant, the strain rate is time-dependent. Is the strain history inferred directly from snapshots of the shape, rather than from integrating the strain rate?
  In the Antarctic Ice Sheet, the vertical strain rate is inferred from the horizontal velocity divergence through continuity, and then is assumed to be uniform through the upper 25% of the depth. How is the strain profile then calculated for ice as it moves downward?

- Equation (2)
  At first reading, it was unclear to me whether the *mass fraction* $\rho^*(\mathbf{x}, t, \mathbf{n})$ was a (dimensional) mass, or a (nondimensional) fraction. I figured out that it must be a mass, because it integrates to $\rho^*(\mathbf{x}, t)$, which appears to be a dimensional mass, rather than integrating to unity over the sphere; however, perhaps that could be made clearer to help your readers avoid an interruption in smooth reading.

- In order to help me read the paper more efficiently, I made a table of variables with definitions and notes about where they first appear. I expect that such a table of variables would be helpful for other readers, and could increase the readership of the paper.

- Exploring the full range of two-dimensional responses to two-dimensional loading is an important step, and I think the authors are making a useful contribution. However, I also expect that minor perturbations in that two-dimensional flow may create fabrics that generate instabilities causing growing nonzero strain rates and flow in the third dimension. This is a question that could also motivate further work.

1.3 Copy editorial points and clarity

- Line 44:
  The author's name is Takeo Hondoh, so the reference should be simply *Hondoh, 2000.*
  In the **References** section, at line 643, the citation should be *Hondah, T., Nature and behavior ...*, i.e. only the initial, to be consistent with all the other references.

- The manuscript uses vector notation, indicial notation, and the summation convention, but does not explain these concepts from continuum-mechanics to readers who may be unfamiliar with them. While a couple of dozen or so people in the community will understand what you are doing, this oversight is liable to dissuade other readers (such as new graduate students) from reading beyond equation (1). A couple of sentences could rectify this.

- There appears to be some oversight or misunderstanding about the difference between *maximum* and *maxima*. *Maxima* is a plural word meaning (if we were to purge the latin forms), *maximums*. Just as it makes no sense to talk about *a single maximums*, it makes no sense to talk about *a single maxima*, or *a single-maxima fabric*.
  The expression *a double maxima* is also problematic, because it could be interpreted to mean four or more peaks. A *double maximum* more clearly indicates two peaks.
  The top row in Figure 1 has it right - *single-maximum* fabric, and *double-maximum* fabric.

- Line 240:
  What is meant by *fully resolved experiments*?

- Figure 5
  To my eyes, there appears to be a slight change in the character of the vorticity number inside a ghost circle at 80 degrees South. Is this a relic of *the Pole hole* caused by polar orbits that turn at 80 South? Does this affect the quality of the data shown?

I will spare you a complete line-by-line list of other grammatical suggestions; however, I hope you will see the merit of checking throughout the text for other examples of these points. Making your text easier to read can only enhance your readership numbers.

The English language is fraught with many rules that often don't appear to make a lot of sense, and there are differences of opinion among groups who have differing communication aims, such as journalists, popular-media editors, poets, novelists, and scientists; however, some rules can eliminate ambiguities and make scientific text easier to read. The following points address recommendations on using hyphens and strings of ideas, in order to make your text more accessible to readers, and therefore helping you to create a more easily understandable, and ultimately more memorable and important paper.

- Hyphenation
  A hyphen should be used in a compound adjective (an adjective and a noun) that modifies another noun,
  e.g. line 15 and elsewhere - *ice-flow dynamics.*

  A hyphen should *not* be used between a stand-alone noun (subject or object) and an adjective that modifies the noun,
  e.g. line: 10 and elsewhere - no hyphen in *strain scales.* e.g. Figure 1 caption and elsewhere - no hyphen in *(d) shows a single maximum with ...*

  Generally there should be no hyphen after an adjective or adverb that ends in 'y' . e.g. line 9 and elsewhere - *highly rotational*

- Lists
  When comparing a string of ideas in text, the ideas are easier for readers to grasp quickly when they have equivalent and parallel grammatical structures.

  For example, in the Abstract,
  The use of our model in large-scale ice flow models as well as for interpreting fabrics observed in ice cores and seismic anisotropy,
  introduces two ideas, but the first is written as the phrase *in ice flow models*, while the second is written as the clause *for interpreting fabrics observed in ice cores and seismic anisotropy.*
  Can you rewrite both ideas as phrases, or both ideas as clauses, i.e. neither or both should contain a verb form?

- Page 1, line 4
  ... in both compression and simple shear, ... is unclear.
  Do you mean *... in both pure shear and simple shear, ...*?
  You are describing *deformational regimes* in terms of strain rather than stress. While there can be compressive stress in all directions (pressure), there can be no volumetric compressive strain for incompressible ice (neglecting elasticity). Perhaps as a community we are sloppy in our terminology, by calling it a *compression test* when we set a weight on top of an ice slab, because that slab experiences compressive deviatoric stress on one axis, but extensile deviatoric stress on other axes. (While we can't change the world, we can each make our own writing clearer.)
  It would be better to choose one wording, then stick with that throughout the manuscript. (I think the text gets it right later at line 21.)