Final response to RC1-George Fournier

Color Code:

Normal= Reviewer comment
Green = Reviewer comment accepted
Yellow = Reviewer comment discussed
Italics= Authors' response to the reviewer's comment

General comments

This is a well written paper that is based on a very substantial and impressive body of work by the authors. The authors developed and extensively tested a new diffuse reflectance for sea-ice. They carried out a detailed ice scattering model analysis using a complete Monte-Carlo code and experimentally validated its functioning and calibration using micro-spheres suspensions and a Mie code scattering phase function model. Their probe was then used in-situ to analyze the inherent properties of sea-ice as a function of depth from the surface. The results of this work and the resulting measurement techniques are extremely relevant and could be used as a starting point to ultimately obtain functional models of sea-ice generation and loss in natural environments. The probe and the signal analysis techniques give a glimpse of the possible performance and environment monitoring accuracy improvements obtainable from their use. This information will be extremely useful to other researchers in the field. For the reasons above I recommend publication of this paper. There are however several developments in the paper which, at the author's discretion, could, in my opinion, be improved before publication. I have noted those more serious problems and along with minor deficiencies/improvements in my comments below.

We thank the reviewer for its detailed and insightful response. The relevance of the comments regarding the structure of the theory will greatly contribute to the precision and understanding of the manuscript. Also, its propositions regarding future work are very good, smart and detailed and will certainly lead to interesting ideas.

Specific comments

Suggestions for improvements

Line 95 and following: I have had a problem in following the original theoretical introduction because of missing terms in the discussion. I would add the definition of the moments immediately after equation 2. Where

$$g_n = 2\pi \int_{\theta=0}^{\pi} P_n(\cos\theta) p(\theta) \sin\theta d\theta$$

are the Legendre polynomials and θ denotes the angle between incident photon direction and photon direction after scattering.

The first three Legendre polynomials which we will use are:

$$P_0(\cos\theta) = 1$$

$$P_1(\cos\theta) = \cos\theta$$

$$P_2(\cos\theta) = \frac{1}{2}(3\cos^2\theta - 1)$$

Equation ~~3~~2 then becomes

$$g_1 = g = 2\pi \int_{\theta=0}^{\pi} p(\theta)\cos\theta \sin\theta \, d\theta$$

Note that the solid angle element of integration has been shifted to the end of the integral to separate it from the function being integrated over the solid angle to keep the physics underlying the equation clearer.

Line 145: All the subsequent higher moments after the second moment of the modified phase function are simply

$g_n = \beta g_{HG}^n$ for $n > 2$

Since the integral of term is identically zero due to the orthogonality of the Legendre functions for any . This fact should be mentioned since at the end of the paper there is some discussion of the importance of the higher moments. The conclusion above implies that those higher moments and any of their ratios are basically controlled by the parameter which considerably limits any flexibility to model more complex situations as the behavior of the solution as a function of is already fully accounted for in the current model.

Line 232: The fact that the laser emitter cone does not have the same angular range as the NA of the fiber in ice is an indication that the fiber does not completely scramble the laser input and significant traces of the fiber input conditions remain at the fiber exit. This is not surprising for such a short fiber with a single bend. It's a known problem in diode pumped lasers. This however implies that care must be taken not to disturb the fiber by moving it after the measurement of the output beam is done. Ultimately this problem can be corrected by using a longer fiber and winding it on a mandrill or around the cavity of the probe. However, given the minimum bend radius of the fiber, you may not have enough room in the probe in which case I would recommend making sure the fiber is fixed in place by a holder or support.

Indeed, we also believe that the laser input is not completely scrambled when coming out of the fibre. We observed spiking when looking at the reflected spot coming out of the source fibre. For future work, maybe we could bend the fibre on a mandrel on top of the pole and verify the stability of the laser power.

When using the probe on the field, the user always held the pole in position for the 30 seconds interval between reference measurement and the last detecting fibre measurement. Neither the probe head nor the fibre bundle were moving during this time interval.

A probe holder is also used on the currently upgraded version to limit movements.

Line 395 The effect of the container wall of the theoretical values of reflectivity for the polystyrene sphere suspensions should be expanded as they could be a substantial portion of the errors which seem to occur predominantly at the low values of absorption and scattering. The authors mention this in the discussion and conclusions but it should be further addressed at this point to at least indicate clearly what results are significantly subject to the wall influence.

We agree that the effect of the container walls should be mentioned earlier in the manuscript as it probably accounts for an important part of the error in the validation with microspheres solutions. Indeed, the fact that the error is getting greater as b' diminishes correlates with the depth of signal origin increasing, and eventually getting greater than the depth of the container, as b' diminishes . We will elaborate this effect and clearly specify which calibration points are subject to this error in section 4.2-validation with microspheres based on our simulation of the depth of signal origin shown on figure 6.

Line 610 As a suggestion for future work and to start bridging the gap between structural and optical knowledge the researchers could use the vast and valuable simulation data base to reevaluate the behavior of the absolute and relative reflectivity as a function of different non-dimensional parameterizations to identify the significant correlations. Two parameters come to mind immediately, the backscatter coefficient and the absorption over b'.

$$\frac{b_b(\beta,g)}{b} = 2\pi\int_{\pi/2}^{\pi} p(\beta,g,\theta)sin\theta d\theta = \frac{\beta}{2g}\left[\frac{(1-g^2)}{\sqrt{(1+g^2)}} - (1-g)\right] + \frac{(1-\beta)}{2}$$

The second parameters of interest could be the asymptotic value of the mean cosine (first moment). Piskozub and McKee (see attached reference) have shown that the limit of the first moment of the radiation distribution after many collisions is given by:

$$g_\infty = \frac{g(1-\omega)}{(1-g\omega)}$$

is the first moment of the scattering function for the first collision and is the resulting radiation distribution after a large number of scattering collisions. $\omega$is the albedo

$$\omega = \frac{1}{\left(\frac{a}{b}\right)+1} = \frac{1}{\left(\frac{a}{b'}\right)(1-g)+1}$$

$$g_\infty = \frac{g(\frac{a}{b'})}{(1 - \frac{a}{b'})}$$

This indicates that the parameter is $\frac{a}{b'}$ also a candidate for which the correlations should be looked at.

These suggestions are very interesting and will certainly lead to promising future work .

-For the first equation, it seems that b_b (or b_b (1-g)?) could be inverted from spatially resolved diffuse reflectance based on our current model. It would be interesting to obtain b or b (1-g) by other means either 1) using a radiance profiler and performing an inversion or 2) providing an estimation of the size and shape distributions of the brine channel and bubbles. Using the equation, one could then provide an estimation of the relative contribution of β and g (therefore γ).

-For the second, third equation and fourth equation, maybe g_∞ could be measured either using a goniometer or a radiance profiler in the ice. a/b' could theoretically be obtained with the probe, but a measurements are very imprecise at the moment. Using g_∞ and a/b' estimations with equation 4, we might also retrieve g.

Finally, the simple scaling against b'z could be used to analyze the correspondence of the computed reflectivity at the different detectors. Detectors with identical where is the distance between the source and the detector should have the same response if $\frac{a}{b'}$ and $\frac{b_b}{b}$ are the same.

As mentioned in section 2.1, even when the source-detector distance is scaled by the reduced scattering coefficient $b'\rho$ (or b'z), other factor will affect the Reflectance. The geometry (mainly the acceptance angle of the fibre) will affect R. Maybe this first factor could be corrected normalizing R by the solid angle. The refractive index would also have an impact. Then, as mentioned in section 2.2.2, in N=2 regime, R will also be affected by γ.

Unfortunately, to our knowledge, no reflectance table provides R in function of acceptance angle and γ.

Reference [1] "Effective scattering phase functions for the multiple scattering regime" Jacek Piskozub and David McKee Optics ExpressVol. 19,Issue 5,pp. 4786-4794 https://doi.org/10.1364/OE.19.004786

Technical comments

1) Line 18 "by optical fiber" should be replaced by: "by an optical fiber"
2) Line 19 "receiving fibres" should be replaced by: "receiving fibers"

The formulation fibre(s) was preferred because The Cryosphere Journal recommend to use UK English. This formulation is also used in Canadian English, the country of origin or the work country of most authors.

3) Line 21 “allowing to analyze” should be replaced by: “allowing the analysis of”
4) Line 29 “dependent of” should be replaced by: “dependent on”
5) Line 30”this novel developed probe” should be replaced by: “this newly developed probe”
6) Line 185 “assuring them” should be replaced by: “which assured them”
7) Line 191”of optical properties” should be replaced by:”of the optical properties”
8) Line 206 “concise volume” should be replaced by ”compact volume”

Indeed, compact is more appropriate than concise. However, searching the definition of the word, the author meant ‘‘defined volume’’, meaning we can estimate the size of the volume.

9) Line 227”splitted” should be replaced by”split”
10) Line 268 “thereof” should be replaced by “therefore”
11) Line 274”inverse” should be replaced by “the inverse”
12) Line 298’’detecting fibres” should be replaced by “the detecting fibers”
13) Line 331 “uncalibrated” should be replaced by “the uncalibrated”
14) Line 390”in function” should be replaced by “as a function”
15) Line 475”induced variation” should be replaced by ”induced variations”
16) Line 536”Using Grenfell” should be replaced by “Using the Grenfell”
17) Line 543”refractive index” should be replaced by “the refractive index”
18) Line 586” Then” should be replaced by “Also”
19) Line 592”adapted” should probably be replaced by “developed and validated”
20) Line 610 “ice core” should be replaced by ”ice cores”

Only one ice core was retrieved at the snow-covered site, so ice core should stay singular. We did not include the ice core of the bare ice site because the brightness makes it difficult to compare.

21) Line 616 ”a more widespread study” could be replaced by ”more widespread and wide ranging studies”