Review of "*Estimating statistical errors in retrievals of ice velocity and deformation parameters from satellite images and buoy arrays*", Dierking et al.

By Amélie Bouchat

The manuscript presents a general mathematical background for estimating the error of sea-ice drift velocities and deformation rates obtained from buoys or synthetic aperture radar (SAR) satellite imagery. While a majority of derivations were already presented in previous studies, the authors gather and discuss here all aspects of the error analysis in a single study and also provide examples for specific cases that can help the reader understand how to apply the derivations for a variety of scenarios. Additionally, the authors present alternative methods (other than the usual boundary integral definition) for estimating the sea-ice deformation rates from observations. Given that the observed deformation fields from SAR or buoy products are recently being used in the community to evaluate sea-ice models and/or infer material properties of geophysical sea-ice, providing a complete framework for quantifying the uncertainty on the drift and deformation fields is timely and highly valuable. The paper is generally well-written and I recommend it for publication in *The Cryosphere*, after the following comments are addressed in the revisions.

<u>General comments:</u>

A lot of equations derived in Section 2 have already been derived in previous publications, but references are not always provided (e.g. Eq. (12) is already in Lindsay and Stern, 2003; and Eq. (13) is also in Griebel and Dierking, 2018;). I would also like to note that Bruno Tremblay and I have recently submitted a paper (December 2019, currently under review at the Journal of Geophysical Research: Oceans) in which we also present and discuss the same equations as the Equations (19)-(24) in Section 2.5 in the present paper, but applied to the RADARSAT Geophysical Processor System Lagrangian drift data set only. Both H. Stern and J. Hutchings are aware of this work since they have been involved in discussions or review of this work. As mentioned to me already by H. Stern, we should cite each other's work in our revisions (I will send you a copy of the pre-print once it is accepted).

Different cases and examples for specific observation products are also discussed in the manuscript, which is useful but it is hard to extract the conclusions/main points from these examples in the text. I would therefore suggest the addition of tables/figures to convey these conclusions more clearly. For example, a table presenting the geolocation, tracking, and timing errors for the different SAR and buoy products as mentioned in Sections 3.4 and 3.5 would be a very useful reference for the reader and for future studies. Below, I also suggest presenting the dependence of the error on the number of tracked points in a graphical form so that it can be used to guide future studies on choosing how many points should be considered.

Specific Comments:

P1. Line 36: *"The accuracy of deformation parameters.."*
Sea-ice deformation should be defined in the introduction (i.e. shear, divergence, etc.) before mentioning their errors. It is also not clear what "deformation parameters" are. Do you refer to the shear, convergence, divergence, etc.? In this case, I would change simply to "deformations" or "deformation rates" for consistency with previous studies.

P.1 Lines 35-37: "*For buoys, errors in drift measurements depend on [...] but also by the size and shape of buoy arrays.*"
References should be added here, e.g. Hutchings et al. (2012), Griebel and Dierking (2018).

P1. Line 41: "*The issue of error estimation was repeatedly addressed in the past, scattered in a number of publications [...]*"
Also add Bouchat and Tremblay (2020, under review).

P.2 Line 50: "*for calculating errors of drift and deformation parameters, supplemented with the derivation of general-case uncertainties of divergence, vorticity, shear, and total deformation.*"
Again, it is not clear what is the difference between "deformation parameters" and "divergence, vorticity, shear, and total deformation".

P.4 Lines 139-142: "*If, e.g., the distance between two moving objects is closer than this, the position errors cancel and $\sigma_d^2 = \sigma_{tr}^2$ for the retrieval from a SAR image pair and $\sigma_{coord}^2 = 0$ between two buoys. Hence within a circle of 10km or less in diameter, deformation can be estimated with sufficient accuracy even if geolocation errors are high.*"
I don't understand how the geolocation errors cancel given that they are squared and add up when using the propagation of error on *d* to obtain $\sigma_d^2 = 2\sigma_{coord}^2$. Can you explain?

P4. Line 157: "*considering that U=...*"
I think you mean $U = d/\Delta T$? Otherwise, you would have to use Eq. (7) to get σ_U^2.

P.6 Line 225: "*Throughout this section we assume that $\sigma_U = \sigma_u = \sigma_v$.*"
because you assume $\sigma_{\Delta T} = 0$? If so, please mention it.

P.7 Eq. (18): The term σ_{coord} should not appear in this equation since it was assumed that it is equal to zero since the beginning of this section. If not zero, then other terms should appear in Eqs. (13) & (14) and Eq. (18) to account for the error on the area and position in the strain rate definition explicitly.

P.8 Equations (23) and (24): These are the same as Eq. (15) and (16) presented earlier. Remove and refer to Eq. (15)-(16) instead?

P.8 Line 295: "*Hence, the uncertainties of divergence, vorticity, shear, and total deformation differ from one another.*"
Unless $\sigma_{u_x} \sim \sigma_{u_y} \sim \sigma_{v_x} \sim \sigma_{v_y}$, then they are equal.

P.8 Line 305: The section is titled "*Typical uncertainties of deformation parameters*" but the section does not describe uncertainties but rather a short review of observed deformation rates from previous studies. If the purpose of this section is to describe observations of shear, divergence, etc. then it should be retitled, and it should also discuss the fact that the observed deformation magnitudes are closely tied with the scale of observation given that the mean deformation rate is known to decay following a power-law with increasing spatial and temporal scale (e.g. Marsan et al., 2004; Rampal et al., 2008; Stern and Lindsay, 2009; Bouchat and Tremblay, 2020 - see at the end of this document for the references if not already in your list).

P.9 Line 322: "*The first term in Eqs. (21) and (22) is smallest if, for a given area, σ_A is at a minimum.*"
and for a given value of $u_x^2 + v_y^2$, or if $u_x^2 + v_y^2$ is also at a minimum.

P.9 Section 3.2: It would be interesting to show a graph of the ratio $\sigma_A^2/\sigma_{coord}^2$ as a function of the number of tracked points for fixed values of *A (e.g. 1 km^2, 10^2 km^2, 20^2 km^2, 100^2 km^2, etc.)* to complement the discussion. It seems like going from three points to four points (i.e. from triangle to square) increases the error contribution of this term, but then going from 4 points to 6 points (i.e. square to hexagon) reduces the contribution of this term to the global error. It could be useful to see this in a functional form to guide choosing (if possible) a reasonable number of tracked points to reduce the area error. This could be added in Section 3.6.

P.9 Line 340: "*For a given position error,...*"
and a given tracking error

P.9 Line 341: "*The third term is solely dependent on the coordinate uncertainty σ_{coord}.*"
No, the third (last) term also decreases with increasing area *A*.

P.10 Line 361: "*When ice drift is retrieved from SAR images, the contribution of those terms that depend on σ_{coord}/L can usually be neglected.*"
Lindsay and Stern (2003) report that previous estimates of the geolocation error for the RADARSAT ScanSAR images are of the order of ~200 m, hence non-negligible when compared to the tracking error (~100 m). So it is not negligible for all SAR products. It would be worth including those estimates for RADARSAT as well since this product is often used to obtain the observed sea-ice deformation fields. In fact, in Bouchat and Tremblay (2020, under review), we show that when using all the other terms except the tracking error for the RGPS data set, the resulting error on the total deformation rates can be ~1.5 times larger than Eq. (17) in Lindsay and Stern, 2003 (or the equivalent tracking error term in your Eq. 25 and 26). In this case, the terms in σ_{coord}/L cannot be neglected.

P.11 Line 395: "*with* $\sigma_{tr}^2/2\sigma_{coord}^2$ *approximately equal to* $100^2/(2\times5^2) = 200$"
Here it is assumed that σ_{coord} = 5 m for RADARSAT ScanSAR images, but Lindsay and Stern (2003) mention a geolocation error that is on the order of ~ 200 m (see also comment above). Can you indicate where the value of σ_{coord} = 5 m was taken from?

P.9-13 Sections 3.4 and 3.5: These two sections focus on estimation of errors for the divergence and vorticity, and compare the contribution of different terms to the total error on divergence and vorticity. However, shear and total deformation rates are often larger than divergence (see increased probability of larger deformation rates in PDFs of shear vs divergence in e.g. Bouchat and Tremblay 2017, or Stern et al. 1995). How does the interpretation of the importance of each term in the error formulation change when considering the error on shear and total deformation instead of divergence and vorticity? For this, it could also be useful to present the expanded version of Eq. (23) and (24).

I also found it hard/confusing to follow all the examples presented at the end of section 3.5. A lot of different cases and numbers are presented and it is easy to get lost in the conclusions that should be retained. A visual aid (such as a table or graph) that gathers the essential points that are supposed to be conveyed by these examples could be added for more clarity.

P.13 Line 489: "*Let L′ be the length of each side of the big square (Fig 5).*"
L' is not defined on Figure 5. And "*big square*" = "window" ?

P.13 Line 490: "*Because of the enclosed grid cells we can divide each side of the square window into N segments of equal length.*"
Is Eq. (30) derived here only valid when the grid cells are not moving and of equal length? For a Lagrangian grid, the cells are not necessarily of equal size and therefore Eq. (30) would not apply, and one would still need to evaluate the full expression in (12), correct?

P.13 Line 500: "*we can rewrite Eq. (30) as* $\sigma_A^2 = \sigma_{coord}^2(n-2)L^2$ *which is Eq. (16) in Lindsay and Stern (2003).*"
Equation (16) in Lindsay and Stern (2003) uses the tracking error σ_{tr}^2 instead of the geolocation error as considered here σ_{coord}^2 . It is confusing because in Section 3.4, it is mentioned (line 360) that the last term of Eq. (25) term depending on the tracking error is the same as Eq (17) in Lindsay and Stern (2003), but Eq. (17) in Lindsay and Stern is derived using their Eq. (16) which is now assumed to be using the geolocation error here… Can you please clarify? Also, please indicate if there is a mistake in Lindsay and Stern (2003).

P.13 Line 506: "*Note that* σ_A^2 *for the right triangle is* $\sigma_{coord}^2L'^2$ *for N = 1,* $1.25\sigma_{coord}^2L'^2$ *for N = 2, [...], and first with N = 4, the uncertainty can be reduced.*"
It could be worth presenting this discussion using a graphical form, for more clarity. See also previous comment for P.9 Section 3.2.

P.13 Line 513: "*Hence the uncertainty of the area increases when elementary cells are combined. However, since also the cell area increases by a factor of N^2, the single terms in Eqs. (13) – (22) that include the factor A^{-2} decrease.*"
Which one wins? Is it better in the end to aggregate cells?

P.14 Line 517: "*For buoy arrays it may be of advantage to use a larger number of buoys along the outline of a polygon.*"
Couldn't SAR drift fields also be derived using triangle cells (in principle) and the discussion regarding Eq. (31) could therefore apply to both SAR and buoy applications?

P.14 Line 535: "*To calculate the number of chords that is required to fulfill Eq. (32), we demand that n'_{sc} (1+e) = 2πr, with n'= n/2, and e the error.*"
This is unclear; "e" is the error on/of what?

P.14 Eq (33): It is not clear to me how you obtain this result. I can see that it probably involves a Taylor series expansion of u(x,y) however if I do this expansion around (x_k,y_k), e.g.:

$$u(x,y) = u(x_k,y_k)+(x-x_k)u_x+(y-y_k)u_y+\frac{1}{2}\left[(x-x_k)^2u_{xx}+2(x-x_k)(y-y_k)u_{xy}+(y-y_k)^2u_{yy}\right]+[...]$$

where the derivatives are evaluated (x_k,y_k). Then I evaluate u(x,y) at (x_{k+1},y_{k+1}) and use the same definitions of Δx_k and Δy_k as in the manuscript, and I get:

$$u_{k+1} = u_k+\Delta x_k u_x+\Delta y_k u_y+\frac{1}{2}\left[\Delta x_k^2 u_{xx}+2\Delta x_k \Delta y_k u_{xy}+\Delta y_k^2 u_{yy}\right]+[...]$$

Such that, I get:

$$\frac{1}{2}\left(u_{k+1}+u_k\right)\Delta y_k = u_k\Delta y_k+\frac{1}{2}\Delta x_k\Delta y_k u_x+\frac{1}{2}\Delta y_k^2 u_y+\frac{\Delta y_k}{4}\left[\Delta x_k^2 u_{xx}+2\Delta x_k\Delta y_k u_{xy}+\Delta y_k^2 u_{yy}\right]+[...]$$

So I see that my last term here is similar to your definition of e_k but I don't know how to get there. Can you clarify?

P.16 Line 591: "*(see above)*"
Not clear to what this is referring to.

P.16 Line 624: "*For a general configuration of points, the three methods give different estimates*."
Have you obtained numerical estimates for examples using each method? How much do they differ?

P.17 Section 4 Conclusions:
The first point has also been shown in Bouchat and Tremblay (2020, under review). The second point should also mention the exception for RGPS.

P. 19 Line 725: "*RADARSAR*" should be RADARSAT

Formatting and writing suggestions:

P.1 Line 16: "*in an array.*" → "in an array of buoys."

P.1 Line 19: "*also a tracking error has to be considered.*" → "a tracking error also has to be considered."

P.1, Line 24: "*the magnitudes of deformation parameters*" → "the magnitude of deformation parameters."?

P1. Line 33: *"sea ice mapping"* → "sea-ice mapping".

P. 2 Line 46: "*truncation error"* → change to "boundary-definition errors", or add it in parentheses to link with previously-used formulation? (or indicate why the previous formulation is incorrect).

P.4 Line 134: "*one needs to consider position and tracking uncertainties σ_{coord}^2 and σ_{tr}^2.*" → "one needs to consider position and tracking uncertainties, *i.e.* $\sigma_{\mathrm{coord}}^2$ and σ_{tr}^2 *respectively.* "

P.6 Line 208: "*The cell covers m x m square-shaped pixels.*" → "The cell covers m x m square-shaped pixels of resolution Δx."

P.6 Lines 236-237: "*the sum of variances of the left term*" → "the sum of variances of the *first* term"? And "*the sum in the right term*" → "the sum in the *second* term"?

P.8 Line 286: "*For the shear one obtains...*" → "For the shear, one obtains..."

P8. Eq. (23): x and y in σ_{u_x}, σ_{u_y}, σ_{v_x}, and σ_{v_y} should be subscripts.

P.9 Line 343: "*In the following discussion we assume that position data of all buoys are exactly synchronized but also discuss an example for which this was not the case.*"
Add reference to section 3.5 at the end of this sentence?

P.13 Line 485: "*the uncertainties have to be calculated numerically*." → "the uncertainties have to be calculated numerically using Eq. (12)"?

P.14 Line 515: "*can be considered*" → "can *also* be considered"?

Figures:

Fig. (2): The figure is blurry.

Fig. (5): Please indicate in the label what are the blue and green lines.

Additional References:

Bouchat, A., and B. Tremblay (2020), Reassessing the quality of sea-ice deformation estimates derived from the RADARSAT Geophysical Processor System and its impact on the spatio-temporal scaling statistics, *Journal of Geophysical Research: Oceans* - under review

Bouchat, A., and B. Tremblay (2017), Using sea-ice deformation fields to constrain the mechanical strength parameters of geophysical sea ice, *Journal of Geophysical Research: Oceans,* doi:10.1002/2017JC013020.

Rampal, P., J. Weiss, D. Marsan, R. Lindsay, and H. Stern (2008), Scaling properties of sea ice deformation from buoy dispersion analysis, *Journal of Geophysical Research: Oceans*, 113(C3), doi:10.1029/2007JC004143.

Stern, H. L., and R. W. Lindsay (2009), Spatial scaling of arctic sea ice deformation, *Journal of Geophysical Research: Oceans*, 114(C10), doi:10.1029/2009JC005380, c10017.

Stern, H. L., Rothrock, D. A., and Kwok, R. (1995), Open water production in Arctic sea ice: Satellite measurements and model parameterizations, *Journal of Geophysical Research*, 100(C10), 20601– 20612, doi:10.1029/95JC02306.