https://doi.org/10.5194/tc-2020-79
Preprint. Discussion started: 7 April 2020

The Cryosphere
Discussions
Open Access

Sensitivity of ice flow to uncertainty in flow law parameters in an idealized one-dimensional geometry

Maria Zeitz[1,2], Anders Levermann[1,2,3], and Ricarda Winkelmann[1,2]

[1]Potsdam Institute for Climate Impact Research (PIK), Member of the Leibniz Association, P.O. Box 60 12 03, 14412 Potsdam, Germany
[2]University of Potsdam, Institute of Physics and Astronomy, Karl-Liebknecht-Str. 24-25, 14476 Potsdam, Germany
[3]LDEO, Columbia University, New York, USA.

Correspondence: Maria Zeitz (maria.zeitz@pik-potsdam.de), Ricarda Winkelmann (ricarda.winkelmann@pik-potsdam.de)

Abstract. The flow of ice drives mass losses in both, the Antarctic and the Greenland Ice Sheet. The projections of possible future sea-level rise rely on numerical ice-sheet models, which solve the physics of ice flow and melt. While a number of important uncertainties have been addressed by the ice-sheet modeling community, the flow law, which is at the center of most process-based ice-sheet models, has so far been assumed certain. Unfortunately, recent studies show that the parameters in the flow law might be uncertain and different from the widely accepted standard values. Here, we use an idealized flowline setup to investigate how uncertainties in the flow law translate into uncertainties in flow-driven mass loss given a step-wise increase of surface temperatures. We find that the measured range of flow parameters can double the flow-driven mass loss within the first centuries of warming, compared to a setting with standard parameters. The spread of ice loss due to an uncertainty in flow parameters is of the same order as the increase in mass loss due to increasing surface temperatures. While this study focuses on an idealized setting in order to disentangle the effect of the flow law from other effects, it is likely that this uncertainty carries over to realistic three-dimensional simulations of Greenland and Antarctica.

1 Introduction

Current and future sea-level rise is one of the most iconic impacts of a warming climate and affects shorelines worldwide (Hinkel et al., 2014; Strauss et al., 2015). The contribution of the large ice sheets in Greenland and Antarctica to sea-level rise sums up to 13.7 + 14.0 mm during the last four decades (Mouginot et al., 2019; Rignot et al., 2019). It is accelerating quickly and is expected to increase with sustained warming (Levermann et al., 2014, 2020; Mengel et al., 2016; Seroussi et al., 2020; Goelzer et al., 2020; Aschwanden et al., 2019; Bamber et al., 2019). Projections in the median contribution of the ice flow from Antarctica and Greenland show some uncertainty but some convergence can be observed in these numbers. Unfortunately coastal protection cannot rely on the median estimate since it has a likelihood of 50% to be exceeded. For societal purposes an estimate of the upper uncertainty range is important. The most recent IPCC special report on the ocean and the cryosphere

https://doi.org/10.5194/tc-2020-79
Preprint. Discussion started: 7 April 2020

provides projections of sea-level rise until 2100 of $0.43\,\mathrm{m}$ $(0.29-0.59\,\mathrm{m})$ and $0.84\,\mathrm{m}$ $(0.61-1.10\,\mathrm{m})$ for RCP2.6 and RCP8.5 scenarios, respectively (Pörtner et al., 2019). Other studies find wider (Levermann et al., 2020) or less wide ranges (Goelzer et al., 2011, 2016; Huybrechts et al., 2011). Such projections are typically performed with process-based ice-sheet models which represent the physics in the interior of the ice and the processes at the boundaries on the ice.

Most of these representations are associated with uncertainties and many of those uncertainties are already taken into account in state-of-the-art sea-level rise projections, in particular within the framework of current ice-sheet model intercomparison projects (Seroussi et al., 2020; Goelzer et al., 2020).

In contrast, the flow of ice, one of the key processes driving mass loss in Greenland and Antarctica, is typically not represented as an uncertainty in sea-level projections. The theoretical basis of ice flow, as implemented in ice-sheet models, has been studied in the lab and by field observations for more than half a century and is perceived as well established (Glen, 1958; Paterson and Budd, 1982; Greve and Blatter, 2009; Cuffey and Paterson, 2010). However, recent developments suggest that the parameters of the flow law are less certain then typically acknowledged in modelling approaches: A review of the original literature on experiments and field observations reveals a large spread in the flow exponent n (which describes the nonlinear response in deformation rate to a given stress) and the activation energies Q (which describe the dependence of the deformation rate on temperature) (Zeitz et al. submitted). New experimental approaches suggest a flow exponent larger than $n = 3$, which has been the most accepted value so far (Qi et al., 2017). Thorough analysis of the thickness, surface slope and velocities of the Greenland Ice Sheet, which are available through remote sensing, allow to relate the driving stress to the velocities and thus to infer the flow exponent n under realistic conditions. Bons et al. (2018) find a flow exponent $n = 4$ in regions where sliding is negligible.

Here we assess the implications of this uncertainty in simulations with the thermomechanically coupled Parallel Ice Sheet Model (the PISM authors, 2018; Bueler and Brown, 2009; Winkelmann et al., 2011), showing that variations in flow parameters have an important influence on flow-driven ice loss in an idealized flowline scenario.

This paper is structured as follows: in Section 2 we recapitulate the theoretical background of ice flow physics and describe the simulation methods used in this paper. The results of the warming experiments in a flowline setup with different flow parameters are presented in Section 3. Section 4 discusses the results and the limitations of the experimental approach and section 5 draws conclusions and suggests possible implications of these results.

2 Methods

2.1 Theoretical background of ice flow physics

The flow of ice cannot be described by the equations of fluid dynamics alone, but needs to be complemented by a material-dependent constitutive equation which relates the internal forces (stress) to the deformation rate (strain rate). Numerous laboratory experiments and field measurements show that the ice deformation rate responds to stress in a nonlinear way. Under the assumptions of isotropy, incompressibility and uni-axial stress this observation is reflected in the *Glen's flow law*, which gives

https://doi.org/10.5194/tc-2020-79
Preprint. Discussion started: 7 April 2020

the constitutive equation for ice,

$$\dot{\epsilon} = A\tau^n, \tag{1}$$

where $\dot{\epsilon}$ is the strain rate, τ the dominant shear stress, n the flow exponent and A the softness of ice (Glen, 1958).

Both, the flow exponent and the softness are important parameters which determine the flow of ice. Usually, the exponent n is assumed to be constant through space and time. Until today, there is no comprehensive understanding of all the physical processes determining the softness A. It may depend, among others, on water content, impurities, grain size and anisotropy as well as temperature of the ice. Within the scope of ice-sheet modeling A is typically expressed as a function of temperature alone

$$A = A_0 \exp\left(-\frac{Q}{RT'}\right), \tag{2}$$

where A_0 is a constant factor, Q is an activation energy, R is the universal gas constant and $T^{'}$ the temperature difference to the pressure melting point (Greve and Blatter, 2009; Cuffey and Paterson, 2010).

Due to pre-melt processes the softness responds more strongly to warming at temperatures close to the pressure melting point, which is described by a piece-wise adaption of the activation energy Q (Barnes et al., 1971; Paterson, 1991), with a larger value of Q at temperatures $T^{'} > -10°\mathrm{C}$. When using these piece-wise defined values for Q for warm and for cold ice in the functional form of the flow law, the respective factors A_0 ensure that the function is continuous at $T^{'} = -10°\mathrm{C}$. A_0 is therefore dependent on the values of the flow exponent n and both values of Q for cold and for warm ice.

The scalar form of *Glen's flow law* (Equation (1)) is only valid for uni-axial stresses, acting in only one direction. For a complete picture the stress is described as a tensor of order two. The generalized flow law reads

$$\dot{\epsilon}_{j,k} = A(T')\tau_e^{n-1}\tau_{j,k}, \tag{3}$$

where $\dot{\epsilon}_{j,k}$ are the components of the strain rate tensor and $\tau_{j,k}$ are the components of the stress deviator, τ_e is the effective stress, which is closely related to the second invariant of the deviatoric stress tensor:

$$\tau_e^2 = \frac{1}{2}\left[\tau_{xx}^2 + \tau_{yy}^2 + \tau_{zz}^2\right] + \tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2. \tag{4}$$

Each component of the strain rate tensor depends on all the components of the deviatoric stress tensor through the effective stress τ_e.

Glen's flow law (3) and the softness parametrization (2) are at the center of most numerical ice-sheet and glacier models, independently of the other approximations they might use (the PISM authors, 2018; Winkelmann et al., 2011; Greve, 1997; Pattyn, 2017; Larour et al., 2012; de Boer et al., 2013; Fürst et al., 2011; Lipscomb et al., 2018).

2.2 Ice flow model PISM

The simulations in this study were performed with the Parallel Ice Sheet Model (PISM) release stable v1.1. PISM uses shallow approximations for the discretized, physical equations: The shallow-ice approximation (SIA) (Hutter, 1983) and the shallow-shelf approximation (SSA) (Weis et al., 1999) are solved in parallel within the entire simulation domain. The shallow ice

https://doi.org/10.5194/tc-2020-79
Preprint. Discussion started: 7 April 2020

approximation is typically dominant in regions with high bottom friction, such that the vertical shear stresses dominate over horizontal shear stresses and longitudinal stresses. The shallow shelf approximation is typically dominant for ice shelves, with zero traction at the base of the ice, and for the fast flow regime in ice streams (Winkelmann et al., 2011). PISM assumes a non-sliding SIA flow and uses the results of the SSA approximations for fast flowing ice.

In PISM, the flow law enters both the SIA and the SSA part of the velocities, as detailed in Winkelmann et al. (2011). It is possible to choose different flow exponents n for the SSA and the SIA, but the softness is the same for both approximations.

The simulations performed here use the SIA mode: The geometry of a two dimensional ice sheet sitting on a flat bed and the SIA mode serve to study the effects of changes in flow parameters onto internal deformation and to separate those effects from changes in sliding etc. Including the shallow shelf approximation reproduces and even enhances the effect of changes in the activation energies Q (Supplemental Figure S3).

2.3 Range of flow exponent and activation energy values

The flow exponent n and the activation energies for warm and for cold ice, Q_w and Q_c, determine the deformation of the ice as a response to stress or temperature. A recent review (Zeitz et al. submitted) revealed that a broad range of flow parameters n, Q_w and Q_c cannot be ruled out by reported studies. The activation energy for cold ice Q_c is varied between 42 kJ/mol to 85 kJ/mol (the typical standard value is $Q_\mathrm{c} = 60$ kJ/mol). The activation energy for warm ice Q_w is varied between 120 kJ/mol to 200 kJ/mol (the standard value is $Q_\mathrm{w} = 139$ kJ/mol). For the flow exponent n, values as low as 1 have been reported, but since many experiments and observations confirm a nonlinear flow of ice, n has been varied between 2 and 4, with the standard value of $n = 3$. The standard values serve as a reference point and correspond to the default values in most ice-sheet models (the PISM authors, 2018; Greve, 1997; Pattyn, 2017; Larour et al., 2012; de Boer et al., 2013; Fürst et al., 2011; Lipscomb et al., 2018).

2.4 Adaption of the flow factor A_0

The flow factor A_0 in the flow law must be adapted to fulfill the following conditions: First, the continuity of the piece-wise defined softness $A(T^{'})$ must be ensured for all combinations of Q_w, Q_c and n. Secondly, a reference deformation rate $\dot{\epsilon}$ at a reference driving stress τ_0 and a reference temperature $T_0^{'}$ (the PISM authors, 2018) should be maintained regardless of the parameters. This is because the coefficient and the power are non-trivially linked when a power law is fitted to experimental data. These conditions give:

$$A_{0,\mathrm{old}} \cdot \exp\left(-\frac{Q_\mathrm{old}}{RT_0^{'}}\right) \cdot \tau_0^{n_\mathrm{old}} = A_{0,\mathrm{new}} \cdot \exp\left(-\frac{Q_\mathrm{new}}{RT_0^{'}}\right) \cdot \tau_0^{n_\mathrm{new}}, \tag{5}$$

$$A_{0,\mathrm{new}} = A_{0,\mathrm{old}} \cdot \exp\left(-\frac{Q_\mathrm{old} - Q_\mathrm{new}}{RT_0^{'}}\right) \cdot \tau_0^{n_\mathrm{old} - n_\mathrm{new}}. \tag{6}$$

If the reference temperature is $T_0^{'} < -10°\mathrm{C}$ the values for cold ice $A_{0,c}$ and Q_c are used in the equation above, or else $A_{0,w}$ and Q_w are used. The corresponding $A_{0,\mathrm{new}}$ for warm or cold ice respectively is calculated from the continuity condition at

https://doi.org/10.5194/tc-2020-79
Preprint. Discussion started: 7 April 2020

$T^{'} = -10°\mathrm{C}$. For e.g. $T_0^{'} < -10°\mathrm{C}$ it follows

$$A_{0,c,\mathrm{new}} = A_{0,c,\mathrm{old}} \cdot \exp\left(-\frac{Q_{c,\mathrm{old}} - Q_{c,\mathrm{new}}}{RT_0^{'}}\right) \cdot \tau_0^{n_{\mathrm{old}} - n_{\mathrm{new}}} \quad \text{and} \tag{7}$$

$$A_{0,w,\mathrm{new}} = A_{0,c,\mathrm{new}} \cdot \exp\left(-\frac{(Q_{c,\mathrm{new}} - Q_{w,\mathrm{new}})}{R \cdot 263.15\mathrm{K}}\right). \tag{8}$$

Here we choose $\tau_0 = 80\mathrm{kPa}$ as a typical stress in a glacier and $T_0^{'} = -20°\mathrm{C}$. Choosing another τ_0 in the same order of magnitude has only little effect on the differences in dynamic ice loss. Choosing another $T_0^{'}$ on the other hand influences how the softness changes with the activation energy Q, see Supplemental Figure S1. With $T_0^{'}$ closer to the melting temperature, the difference in softness at the pressure melting point decreases thus the ice loss is less sensitive to changes in the activation energy Q.

2.5 Simulation setup

The study is performed in a flowline setup, similar to (Pattyn et al., 2012), with an extent of 1000km in x-direction and 3km in y-direction (with a periodic boundary condition). The spatial horizontal resolution is 1 km. The ice rests on a flat bed of length L = 900 km with a fixed calving front at the edge of the bed, such that no ice shelves can form. In contrast to (Pattyn et al., 2012), the temperature and the enthalpy in the ice sheet are allowed to evolve freely.

The model is initialized with a spatially constant ice thickness and is run into equilibrium for each of the ensemble members. The thickness profile of the equilibrium state is similar to the *Vialov profile* (see e.g. Cuffey and Paterson (2010)). The ice surface temperature is altitude dependent, $T_s = -6°\mathrm{C/km} \cdot z - 2°\mathrm{C}$, where z is the surface elevation in km. The accumulation rate is constant in space and time for each simulation. In warming scenarios an instantaneous temperature increase of $\Delta T \in [1,2,3,4,5,6]°\mathrm{C}$ is applied.

2.6 Equilibrium states

While the extent in x-direction is given by the geometry of the setup, a flat bed with a calving boundary condition at the margin, the height and the shape of the ice sheet depend on the flow parameters n, Q_w and Q_c and the accumulation rate a. Adapting the accumulation rate for each parameter combination allows to keep the volume of the ice sheet close to the reference value with standard flow parameters (with variation of only a few percent). Simulations with high activation energies Q_w need increased accumulation rates a in order to maintain an equilibrium volume close to the reference value. For the highest combination of activation energies Q_w and Q_c, the relative differences $d_x = (x - x_0)/x_0$ of both, accumulation rates a and mean surface velocities v, increase by more than 300% (Fig. 1 a) and b)).

If the accumulation rate is fixed at $a = 0.5\mathrm{m/yr}$, the relative difference of average velocities $d_v = (\bar{v} - \bar{v}_0)/\bar{v}_0$ spans from $d_v = -7\%$ (with a corresponding relative difference in ice sheet volume of $d_{vol} = +10\%$) for the lowest combination of activation energies to $d_v = +18\%$ with a difference in volume of $d_{vol} = -15\%$ for the highest combination of values for Q_c and Q_w (Supplemental Figure S2 a) and b)).

https://doi.org/10.5194/tc-2020-79
Preprint. Discussion started: 7 April 2020

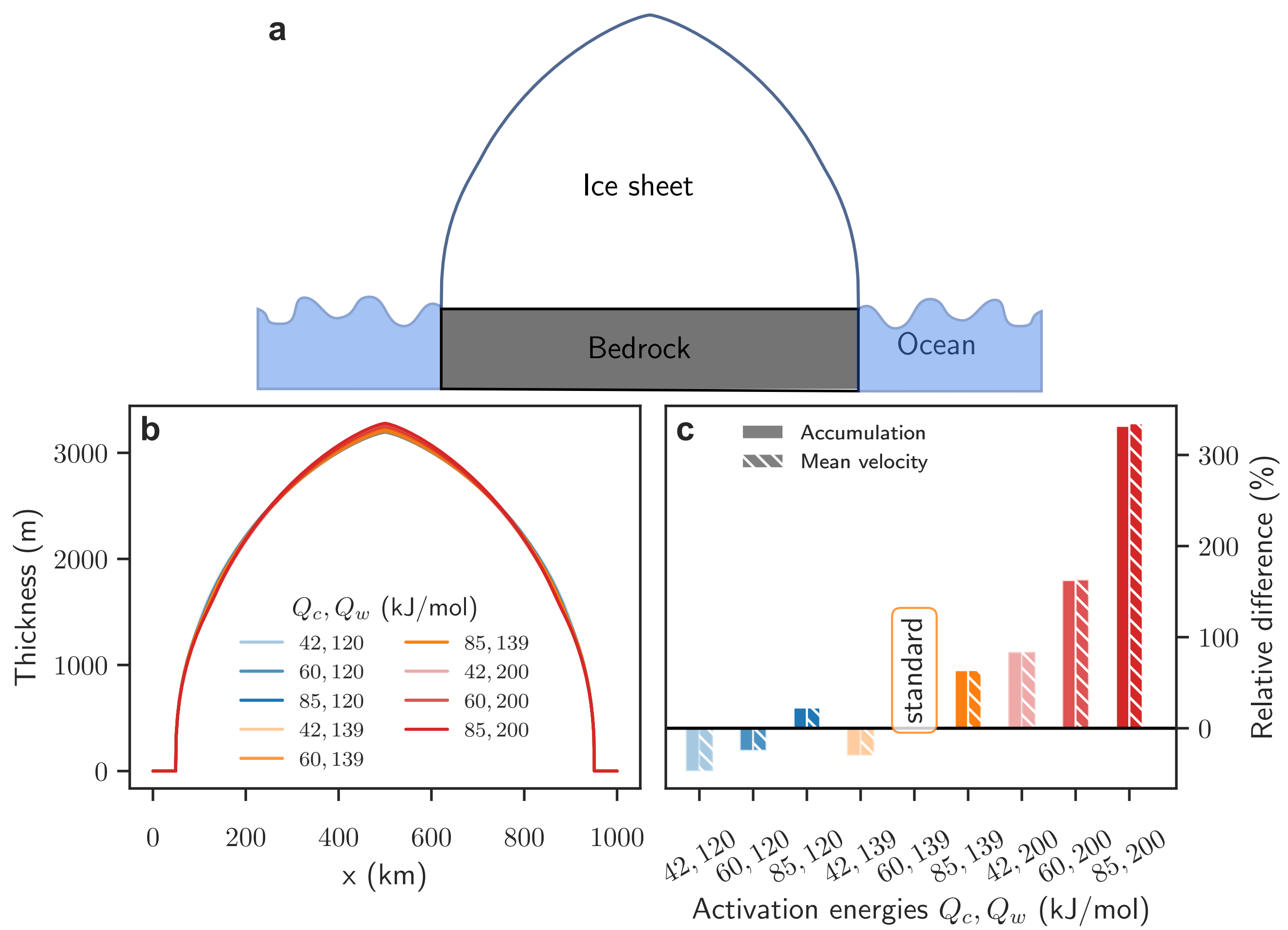

Figure 1. Effect of flow parameters on equilibrium state without warming with the flow exponent $n = 3$ **a)** Sketch of the setup. **b)** Cross sections of equilibrium states for parameter combinations of Q_w and Q_c and **c)** Relative difference of average surface velocities and accumulation rates needed to keep the ice sheet volume in equilibrium close to the reference simulations with standard flow parameters.

https://doi.org/10.5194/tc-2020-79
Preprint. Discussion started: 7 April 2020

Overall, high activation energies increase ice-flow velocities and reduce the ice-sheet volume under fixed accumulation rates. The activation energy for warm ice, Q_w, affects the volume and the velocities more strongly than the activation energy for cold ice, Q_c. The softness of the ice at the bottom of the ice sheet, where temperatures are close to the pressure melting point, is more sensitive to variations in Q_w, which might explain the greater impact (Figure **??**).

3 Results

Disentangling purely flow-driven ice losses from the influences of melting, different initial temperature profiles and variations in sliding requires several adjustments: 1) The initial volume is fixed through adjustment of the accumulation rate for all parameter combinations (Fig. 1, a) and b)). 2) The accumulation rate does not change with temperature, i.e. no melting occurs. 3) Sliding is effectively inhibited by the SIA condition.

The effect of the temperature increase is thus limited to warming at the ice surface which can propagate to the interior of the ice sheet though diffusion and advection. Increasing temperatures increase the softness of the ice (Fig. **??**) and thus increase flow and ice discharge. Warming simulations show an ice loss for all temperature increases and all parameter combinations, however, the amount and rate of the ice loss is dependent on the flow parameters.

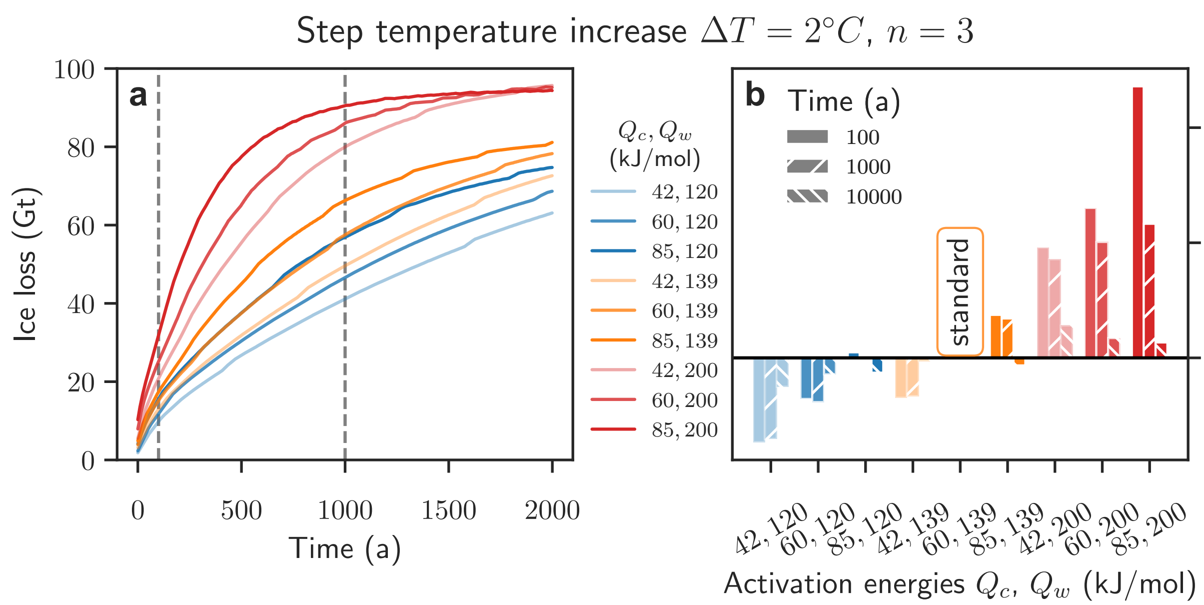

Figure 2. Effect of activation energy on flow driven ice discharge under warming: a) Time evolution of ice loss with different activation energies Q_c and Q_w and the flow exponent $n = 3$, subject to a temperature anomaly forcing of $\Delta T = 2°\mathrm{C}$. **b)** Relative difference of ice loss $d_m = (\Delta m - \Delta m_0)/\Delta m_0$ after 100, 1000 and 10000 years of warming. Ice loss with standard parameters Δm_0 serves as a reference.

https://doi.org/10.5194/tc-2020-79
Preprint. Discussion started: 7 April 2020

The Cryosphere
Discussions

A step forcing with a temperature increase of 2°C leads to immediate ice losses compared to the unforced state (Figure 2 a). The fastest ice loss is observed for the flow parameter combination of $n = 3, Q_\mathrm{c} = 85\,\mathrm{kJ/mol}$ and $Q_\mathrm{w} = 200\mathrm{kJ/mol}$ and the slowest ice loss for $n = 3, Q_\mathrm{c} = 42\,\mathrm{kJ/mol}$ and $Q_\mathrm{w} = 120\mathrm{kJ/mol}$. Simulations with $Q_\mathrm{w} = 200\mathrm{kJ/mol}$ reach a new, temperature adapted equilibrium already after 2.000 yrs, while simulations with lower Q_w continue to lose mass.

The sensitivity to variations in flow parameters is measured via the relative differences for flow-driven ice loss $d_m = (\Delta m - \Delta m_0)/\Delta m_0$, where the reference Δm_0 is always given by the simulation with standard parameters under the same temperature increase (2). The relative differences are particularly high immediately after the warming perturbation starts and decrease after approximately 300 years.

While the long term response to warming, measured after 10.000 years, is not very sensitive to the particular choice of flow parameters, the rate of flow-driven ice loss is. The largest relative differences in ice loss is seen in the first 100–300 years after the start of the temperature forcing (solid bars in Figure 4 a), indicating that high Q_w speeds up the flow-driven ice loss. Under 2 °C of warming, ice loss is enhanced more than two-fold in simulations with high activation energy for warm ice Q_w, while low Q_w may reduce the relative ice loss by 30%.

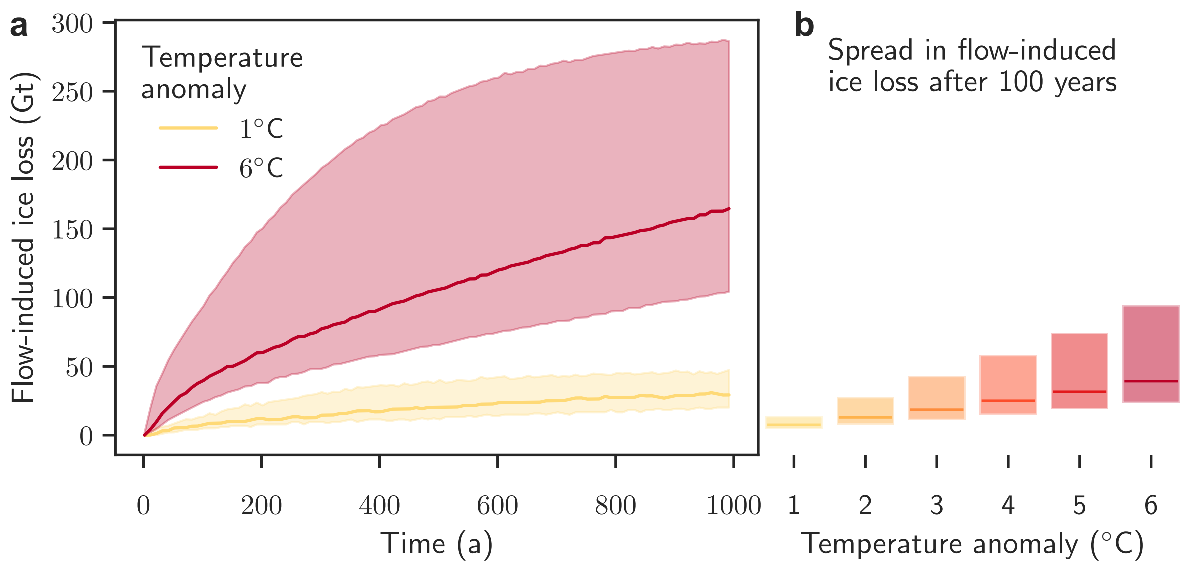

Figure 3. Effect of temperature forcing and activation energy on flow-driven ice a) Time evolution of ice loss under warming of 1 °C and 6 °C. For warming of $\Delta T = 1\,°\mathrm{C}$ the conceptual ice sheet looses 30 Gt of ice after 1000 yr for standard parameters (solid, yellow line). Variations in the activation energies Q lead to variations in ice loss on the same order of magnitude (shaded area). For a warming of ΔT=6 °C the conceptual ice sheet looses 165 Gt of ice after 1000 yr for standard parameters (solid, red line). The variations in ice loss due to different parameters for the activation energy Q (shaded area) are strongly asymmetrical and, in particular during the first 300 years, high compared to the total amount of ice loss. **b)** Uncertainty in flow induced ice loss after 100 years of simulation time over all combinations of Q_w, Q_c and temperature anomalies ΔT. The flow exponent $n = 3$ is kept fixed for all simulations.

https://doi.org/10.5194/tc-2020-79
Preprint. Discussion started: 7 April 2020

The effect of the flow parameters on flow-driven ice loss upon warming is robust for different temperature increases. Absolute ice losses as well as the absolute spread in flow-driven ice loss both increase under higher warming scenarios (see Figure 3). Variations in the flow parameters shape the ice-loss rates in particular during the first centuries of warming.

The effect of flow parameter changes onto flow-driven ice loss after 100 years is on the same order of magnitude as increasing
175 the surface temperature by several degrees. In particular, at a warming of $2\,°\mathrm{C}$ the uncertainty of ice loss overlaps with the uncertaintly of ice loss under a warming of $6\,°\mathrm{C}$ (Figure 3 b).

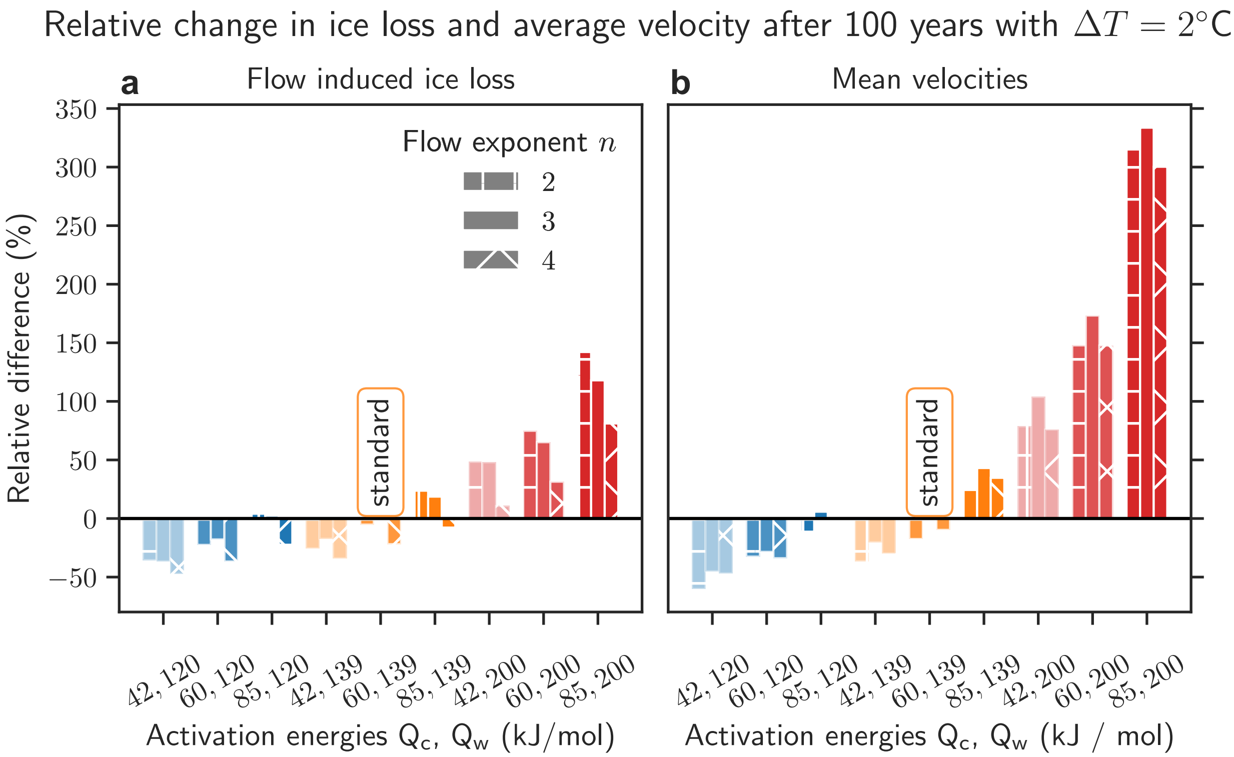

Figure 4. Effect of the flow exponent and activation energies on flow under $2°\mathrm{C}$**C of warming:** Relative difference in flow-driven ice discharge **(a)** and average surface velocity **(b)** for different combinations of the flow exponent n and activation energies Q_c and Q_w. The reference is always a simulation performed with standard parameters $n = 3$, $Q_\mathrm{c} = 60\,\mathrm{kJ/mol}$ and $Q_\mathrm{w} = 139\,\mathrm{kJ/mol}$. Variations in the flow exponent n do not significantly influence the relative difference of mean velocities after 100 years.

Variations in the flow exponent n do not change the qualitative effect of variations in activation energy Q on difference in ice loss after 100 years for a temperature anomaly of $\Delta T = 2°\mathrm{C}$ (Figure 4a). In this setup a higher n seems to mitigate the effect of the activation energy on differences in ice loss after 100 years, while a lower n seems to enhance this effect. However,
180 the effect of variations in activation energy on the averaged surface velocity is almost independent of the choice for the flow exponent n (Figure 4b).

https://doi.org/10.5194/tc-2020-79
Preprint. Discussion started: 7 April 2020

The Cryosphere Discussions

The influence of the activation energies Q_c and Q_w on ice flow is similar even with different flow exponents n. This is robust for different warming scenarios from $+1$ to $+6\,^\circ$C. A higher flow exponent n, which leads to a more pronounced nonlinearity in ice flow, does not enhance but reduce variations in dynamic ice loss. This might be linked to the details of the parametrization in the flow law: As seen in both equations (8) and (6), the factor A_0 decreases if the flow exponent n is reduced and $n_\mathrm{new} < n_\mathrm{old}$. Compared to the nonlinear stress dependency τ^n in the flow law the temperature dependent softness $A(T') = A_0 \cdot \exp(-Q/RT)$ becomes less important with increasing flow exponents n.

4 Discussion

In this study we present a first attempt to quantify the effect of uncertainties in the flow law parameters onto ice dynamics, more specifically the flow-driven ice loss under warming. Uncertainties in the flow parameters alone account for a doubling in ice loss during the first centuries of warming. The spread in flow-driven ice loss due to these uncertainties is on the same order of magnitude as the effects of increased temperature forcing.

The effect of the activation energies Q on ice dynamics (mean velocities and flow-driven ice loss) is stronger than the effect of the flow exponent n. The activation energies determine how sensitive the softness of the ice is with respect to ice temperature. Parameters which increase the ice softness close to the pressure melting point show to have the largest effect on ice dynamics.

The softness of the ice is not only dependent on the activation energies Q, but on the critical temperature where the ice changes from a cold to a warm regime. Uncertainties in the critical temperature are not easy to asses from literature. It still needs to be explored, how exactly the critical temperature influences the ice dynamics.

In this study we chose an idealistic flowline geometry, which allows to disentangle the internal flow dynamics from other effects. The ice sheet is sitting on a flat bed and its maximal extent is determined by a calving front at the borders of the bed, thus no ice-ocean interactions or impacts of the bed geography influence the ice flow. Sliding is inhibited, the ice dynamics is described by the shallow ice approximation, with zero basal velocity, no changes in sliding velocity influence the ice flow. The accumulation rate is independent of the temperature change, so that the ice loss is only driven by changes in flow and not by melting or sliding. While all those additional processes might reduce the importance of the flow parameters on computed ice loss, there are also possible positive feedbacks, if e.g. increased ice flow drives ice masses into regions with higher temperatures or into the ocean.

5 Conclusions

Our analysis explores the effects of a long-overlooked uncertainty in the ice flow parameters. New experimental and data-analysis studies need to be conducted to further determine the activation energies Q and the critical temperature for the transition between warm and cold ice and further assess their influence on ice dynamics.

https://doi.org/10.5194/tc-2020-79
Preprint. Discussion started: 7 April 2020

The Cryosphere Discussions

In this study, we try to disentangle the effect of variations in flow parameters on the internal deformation of ice and find that the flow parameters are one of the determining factors for how strongly the Greenland and the Antarctic Ice Sheet are affected by anthropogenic warming, and thus need to be taken into account in future sea-level rise projections.

Our idealized experiments in a flow-line setup reveal that the uncertainty in flow parameters can speed up flow driven ice loss up to two-fold on the timescale of centuries. For flow-driven ice loss in warming scenarios, the spread in the amount of ice loss increases with warming. The spread due to flow law parameter uncertainties can be as large or larger than the differences between different forcing temperatures.

In sea-level projections, naturally other processes such as sliding and melting are crucial in determining the overall mass balance of an ice sheet. This has strong implications for the Greenland and Antarctic ice sheets and respective sea-level projections:

First, incorporating the uncertainty in the flow-law parameters may provoke a re-evaluation of other parameters, e.g. concerning melting and basal conditions. In particular, Bons et al. (2018) thorough analysis of observational data of the Greenland Ice Sheet supports a flow exponent of $n = 4$, not the standard value of $n = 3$. This is in line with recent experiments which allow for large ice deformations in the lab, a regime similar to ice flow in glaciers, which also finds $n > 3$ (Qi et al., 2017). In turn, assuming $n = 4$ leads to a re-assessment of the basal velocity of the Greenland Ice Sheet, reducing the previously assumed area where sliding is possible significantly (MacGregor et al., 2016).

Second, both the flow exponent n and the activation energies Q feed into the grounding line flux formula (Schoof, 2007). In several ice-sheet models, this formula is used to determine the movement and the flux over the grounding lines in transient simulations (Reese et al., 2018). A change in parameters n and Q thus has additional implications for the advance and retreat of grounding lines. This is especially true for the Antarctic Ice Sheet, where the majority of the current ice loss is caused by ice flux across its grounding lines.

Third, this further affects the onset of the marine ice sheet instability, which is particularly relevant for the long-term stability of the Antarctic Ice Sheet.

Fourth, an uncertainty in flow might also mean that the ice is transported more quickly to the ice-sheet margins under global warming. In Greenland, where significant ablation happens at the margins increased ice transport might thus increase melting-induced ice loss.

Our results therefore imply that in the interplay of these processes, a potentially stronger flow response of the Antarctic and the Greenland Ice Sheet might increase their vulnerability to temperature changes.

6 Code and data availability

Data and code are available from the authors upon request.

https://doi.org/10.5194/tc-2020-79
Preprint. Discussion started: 7 April 2020

The Cryosphere Discussions
Open Access

Author contributions. R.W. and A.L conceived the study. M.Z., A.L. and R.W. designed the research and contributed to the analysis. M.Z. carried out the literature review and the analysis. M.Z., R.W. and A.L. wrote the manuscript.

Competing interests. No competing interest.

Acknowledgements. M.Z. and R.W. are supported by the Leibniz Association (project DOMINOES), Development of PISM is supported by
245 NASA grant NNX17AG65G and NSF grants PLR-1603799 and PLR-1644277. The authors gratefully acknowledge the European Regional Development Fund (ERDF), the German Federal Ministry of Education and Research and the Land Brandenburg for supporting this project by providing resources on the high performance computer system at the Potsdam Institute for Climate Impact Research. We thank G Hilmar Gudmundsson, David J. Prior and Thomas Kleiner for insightful discussions.

References

Aschwanden, A., Fahnestock, M. A., Truffer, M., Brinkerhoff, D. J., Hock, R., Khroulev, C., Mottram, R., and Khan, S. A.: Contribution of the Greenland Ice Sheet to sea level over the next millennium, Science Advances, 5, eaav9396, https://doi.org/10.1126/sciadv.aav9396, http://advances.sciencemag.org/lookup/doi/10.1126/sciadv.aav9396, 2019.

Bamber, J. L., Oppenheimer, M., Kopp, R. E., Aspinall, W. P., and Cooke, R. M.: Ice sheet contributions to future sea-level rise from structured expert judgment, Proceedings of the National Academy of Sciences, 116, 11 195–11 200, https://doi.org/10.1073/pnas.1817205116, https://www.pnas.org/content/116/23/11195, 2019.

Barnes, P., Tabor, D., and Walker, J. C. F.: The friction and creep of polycrystalline ice, Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences, 324, 127–155, 1971.

Bons, P. D., Kleiner, T., Llorens, M.-G., Prior, D. J., Sachau, T., Weikusat, I., and Jansen, D.: Greenland Ice Sheet: Higher Nonlinearity of Ice Flow Significantly Reduces Estimated Basal Motion, Geophysical Research Letters, 45, 6542–6548, https://doi.org/10.1029/2018GL078356, http://doi.wiley.com/10.1029/2018GL078356, 2018.

Bueler, E. and Brown, J.: Shallow shelf approximation as a "sliding law" in a thermomechanically coupled ice sheet model, Journal of Geophysical Research: Solid Earth, 114, 1–21, https://doi.org/10.1029/2008JF001179, 2009.

Cuffey, K. M. and Paterson, W. S. B.: The Physics of glaciers, Elsevier Inc., 4 edn., http://linkinghub.elsevier.com/retrieve/pii/0016718571900868, 2010.

de Boer, B., van de Wal, R. S. W., Lourens, L. J., Bintanja, R., and Reerink, T. J.: A continuous simulation of global ice volume over the past 1 million years with 3-D ice-sheet models, Climate Dynamics, 41, 1365–1384, https://doi.org/10.1007/s00382-012-1562-2, http://link.springer.com/10.1007/s00382-012-1562-2, 2013.

Fürst, J. J., Rybak, O., Goelzer, H., De Smedt, B., De Groen, P., and Huybrechts, P.: Improved convergence and stability properties in a three-dimensional higher-order ice sheet model, Geoscientific Model Development, 4, 1133–1149, https://doi.org/10.5194/gmd-4-1133-2011, 2011.

Glen, J. W.: The mechanical properties of ice I. The plastic properties of ice, Advances in Physics, 7, 254–265, https://doi.org/10.1080/00018735800101257, 1958.

Goelzer, H., Huybrechts, P., Loutre, M.-F., Goosse, H., Fichefet, T., and Mouchet, A.: Impact of Greenland and Antarctic ice sheet interactions on climate sensitivity, Climate dynamics, 37, 1005–1018, 2011.

Goelzer, H., Huybrechts, P., Loutre, M.-F., and Fichefet, T.: Last Interglacial climate and sea-level evolution from a coupled ice sheet–climate model, Climate of the Past, 12, 2195–2213, https://doi.org/10.5194/cp-12-2195-2016, https://www.clim-past.net/12/2195/2016/, 2016.

Goelzer, H., Nowicki, S., Payne, A., Larour, E., Seroussi, H., William, H., Gregory, J., Abe-ouchi, A., Shepherd, A., Simon, E., and Agosta, C.: The future sea-level contribution of the Greenland ice sheet : a multi- model ensemble study of ISMIP6, The Cryosphere Discussions, pp. 1–43, https://doi.org/10.5194/tc-2019-319, 2020.

Greve, R.: Application of a Polythermal Three-Dimensional Ice Sheet Model to the Greenland Ice Sheet: Response to Steady-State and Transient Climate Scenarios, Journal of Climate, 10, 901–918, https://doi.org/10.1175/1520-0442(1997)010<0901:AOAPTD>2.0.CO;2, http://journals.ametsoc.org/doi/abs/10.1175/1520-0442{%}281997{%}29010{%}3C0901{%}3AAOAPTD{%}3E2.0.CO{%}3B2, 1997.

Greve, R. and Blatter, H.: Dynamics of Ice Sheets and Glaciers, Advances in Geophysical and Environmental Mechanics and Mathematics, Springer Berlin Heidelberg, Berlin, Heidelberg, https://doi.org/10.1007/978-3-642-03415-2, http://link.springer.com/10.1007/978-3-642-03415-2, 2009.

https://doi.org/10.5194/tc-2020-79
Preprint. Discussion started: 7 April 2020

Hinkel, J., Lincke, D., Vafeidis, A. T., Perrette, M., Nicholls, R. J., Tol, R. S., Marzeion, B., Fettweis, X., Ionescu, C., and Levermann, A.: Coastal flood damage and adaptation costs under 21st century sea-level rise, Proceedings of the National Academy of Sciences, 111, 3292–3297, 2014.

Hutter, K.: Theoretical glaciology material science of ice and the mechanics of glaciers and ice sheets, 1983.

Huybrechts, P., Goelzer, H., Janssens, I., Driesschaert, E., Fichefet, T., Goosse, H., and Loutre, M.-F.: Response of the Greenland and Antarctic ice sheets to multi-millennial greenhouse warming in the Earth system model of intermediate complexity LOVECLIM, Surveys in Geophysics, 32, 397–416, 2011.

Larour, E. Y., Seroussi, H., Morlighem, M., and Rignot, E.: Continental scale, high order, high spatial resolution, ice sheet modeling using the Ice Sheet System Model (ISSM), Journal of Geophysical Research: Earth Surface, 117, https://doi.org/10.1029/2011JF002140, http://doi.wiley.com/10.1029/2011JF002140, 2012.

Levermann, A., Winkelmann, R., Nowicki, S., Fastook, J. L., Frieler, K., Greve, R., Hellmer, H. H., Martin, M. A., Meinshausen, M., Mengel, M., et al.: Projecting Antarctic ice discharge using response functions from SeaRISE ice-sheet models, 2014.

Levermann, A., Winkelmann, R., Albrecht, T., Goelzer, H., Golledge, N. R., Greve, R., Huybrechts, P., Jordan, J., Leguy, G., Martin, D., et al.: Projecting Antarctica's contribution to future sea level rise from basal ice shelf melt using linear response functions of 16 ice sheet models (LARMIP-2), Earth System Dynamics, 11, 35–76, 2020.

Lipscomb, W. H., Price, S. F., Hoffman, M. J., Leguy, G. R., Bennett, A. R., Bradley, S. L., Evans, K. J., Fyke, J. G., Kennedy, J. H., Perego, M., Ranken, D. M., Sacks, W. J., Salinger, A. G., Vargo, L. J., and Worley, P. H.: Description and Evaluation of the Community Ice Sheet Model (CISM) v2.1, Geoscientific Model Development Discussions, pp. 1–65, https://doi.org/10.5194/gmd-2018-151, https://www.geosci-model-dev-discuss.net/gmd-2018-151/, 2018.

MacGregor, J. A., Colgan, W. T., Fahnestock, M. A., Morlighem, M., Catania, G. A., Paden, J. D., and Gogineni, S. P.: Holocene deceleration of the Greenland Ice Sheet, Science, 351, 590–593, https://doi.org/10.1126/science.aab1702, http://doi.wiley.com/10.1002/2015JF003803http://www.sciencemag.org/cgi/doi/10.1126/science.aab1702, 2016.

Mengel, M., Levermann, A., Frieler, K., Robinson, A., Marzeion, B., and Winkelmann, R.: Future sea level rise constrained by observations and long-term commitment, Proceedings of the National Academy of Sciences of the United States of America, 113, 2597–2602, https://doi.org/10.1073/pnas.1500515113, 2016.

Mouginot, J., Rignot, E., Bjørk, A. A., van den Broeke, M., Millan, R., Morlighem, M., Noël, B., Scheuchl, B., Wood, M., and Morlighem, M.: Forty-six years of Greenland Ice Sheet mass balance from 1972 to 2018, Proceedings of the National Academy of Sciences, 116, 201904 242, https://doi.org/10.1073/pnas.1904242116, http://www.pnas.org/lookup/doi/10.1073/pnas.1904242116, 2019.

Paterson, W. S. B.: Why ice-age ice is sometimes "soft", Cold Regions Science and Technology, 20, 75–98, https://doi.org/10.1016/0165-232X(91)90058-O, 1991.

Paterson, W. S. B. and Budd, W. F.: Flow parameters for ice sheet modeling, Cold Regions Science and Technology, 6, 175–177, https://doi.org/10.1016/0165-232X(82)90010-6, 1982.

Pattyn, F.: Sea-level response to melting of Antarctic ice shelves on multi-centennial timescales with the fast Elementary Thermomechanical Ice Sheet model (f.ETISh v1.0), The Cryosphere, 11, 1851–1878, https://doi.org/10.5194/tc-11-1851-2017, https://www.the-cryosphere.net/11/1851/2017/, 2017.

Pattyn, F., Schoof, C., Perichon, L., Hindmarsh, R. C. A., Bueler, E., De Fleurian, B., Durand, G., Gagliardini, O., Gladstone, R., Goldberg, D., Gudmundsson, G. H., Huybrechts, P., Lee, V., Nick, F. M., Payne, A. J., Pollard, D., Rybak, O., Saito, F., and Vieli, A.: Results of the marine ice sheet model intercomparison project, MISMIP, Cryosphere, 6, 573–588, https://doi.org/10.5194/tc-6-573-2012, 2012.

Pörtner, H.-O., Roberts, D., Masson-Delmotte, V., Zhai, P., Tignor, M., Poloczanska, E., Mintenbeck, K., Nicolai, M., Okem, A., Petzold, J., Rama, B., and Weyer, N.: The IPCC Special Report on the Ocean and Cryosphere, Tech. rep., 2019.

Qi, C., Goldsby, D. L., and Prior, D. J.: The down-stress transition from cluster to cone fabrics in experimentally deformed ice, Earth and Planetary Science Letters, 471, 136–147, https://doi.org/10.1016/j.epsl.2017.05.008, http://dx.doi.org/10.1016/j.epsl.2017.05.008http://linkinghub.elsevier.com/retrieve/pii/S0012821X17302625, 2017.

Reese, R., Winkelmann, R., and Hilmar Gudmundsson, G.: Grounding-line flux formula applied as a flux condition in numerical simulations fails for buttressed Antarctic ice streams, Cryosphere, 12, 3229–3242, https://doi.org/10.5194/tc-12-3229-2018, 2018.

Rignot, E., Mouginot, J., Scheuchl, B., van den Broeke, M., van Wessem, M. J., and Morlighem, M.: Four decades of Antarctic Ice Sheet mass balance from 1979–2017, Proceedings of the National Academy of Sciences, 116, 1095–1103, https://doi.org/10.1073/pnas.1812883116, http://www.pnas.org/lookup/doi/10.1073/pnas.1812883116, 2019.

Schoof, C.: Ice sheet grounding line dynamics: Steady states, stability, and hysteresis, Journal of Geophysical Research: Earth Surface, 112, 1–19, https://doi.org/10.1029/2006JF000664, 2007.

Seroussi, H., Nowicki, S. M. J., Payne, A. J., Goelzer, H., Lipscomb, W. H., Abe-Ouchi, A., Agosta, C., Albrecht, T., Asay-Davis, X., Barthel, A., Calov, R., Cullather, R. I., Dumas, C., Gladstone, R., Golledge, N. R., Gregory, J. M., Greve, R., Hattermann, T., Hoffman, M. J., Humbert, A., Huybrechts, P., Jourdain, N. C., Kleiner, T., Larour, E. Y., Leguy, G. R., Lowry, D. P., Little, C. M., Morlighem, M., Pattyn, F., Pelle, T., Price, S. F., Quiquet, A., Reese, R., Schlegel, N. J., Shepherd, A., Simon, E., Smith, R. S., Straneo, F., Sun, S., Trusel, L. D., Van Breedam, J., van de Wal, R. S. W., Winkelmann, R., Zhao, C., Zhang, T., and Zwinger, T.: ISMIP6 Antarctica: a multi-model ensemble of the Antarctic ice sheet evolution over the 21st century, The Cryosphere Discussions, pp. 1–54, https://doi.org/10.5194/tc-2019-324, https://www.the-cryosphere-discuss.net/tc-2019-324/, 2020.

Strauss, B., Tebaldi, C., Kulp, S., Cutter, S., Emrich, C., Rizza, D., and Yawitz, D.: Maryland and the surging sea: a vulnerability assessment with projections for sea level rise and coastal flood risk., 2015.

the PISM authors: PISM, a Parallel Ice Sheet Model, http://www.pism-docs.org, 2018.

Weis, M., Greve, R., and Hutter, K.: Theory of shallow ice shelves, Continuum Mechanics and Thermodynamics, 11, 15–50, https://doi.org/10.1007/s001610050102, 1999.

Winkelmann, R., Martin, M. A., Haseloff, M., Albrecht, T., Bueler, E., Khroulev, C., and Levermann, A.: The Potsdam Parallel Ice Sheet Model (PISM-PIK) – Part 1: Model description, The Cryosphere, 5, 715–726, https://doi.org/10.5194/tc-5-715-2011, https://www.the-cryosphere.net/5/715/2011/, 2011.