Response to Reviewer Comments

Journal: The Cryosphere
Title: **Sensitivity of ice flow to uncertainty in flow law parameters in an idealized one-dimensional geometry**

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First of all, we would like to thank the editor Alexander Robinson and the two anonymous reviewers for their helpful and excellent comments and their efforts to create the detailed reviews! In our revision of the manuscript we addressed three main issues:

1. We extended the introduction and the discussion sections, based on the reviewers' suggestions and additional literature to give a more comprehensive overview of the state of the art and to allow for an in-depth discussion of our findings.
2. We improved the figures to make them generally more comprehensible and added further results: In particular, we included the discussion of the equilibrium state of the ice sheet before warming in the main part of the paper (see Figs. 2 - 5) and added comparisons to analytically approximated equilibrium states.
3. We further included a new section to investigate how robust our results are when including the effects of surface mass balance changes and sliding (see Figs. 11 and 12).

We provide detailed answers to all comments below. The reviewers' comments are given in black and the authors' in blue.
The changes made to the main document can be found at the end of this document (created with latexdiff).

Anonymous Referee #1

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This paper presents the results of a very creative and important inquiry into the question of how uncertainty in ice flow-law parameters may impact the predictions numerical ice-sheet models make for future sea level rise. The results of the work are very convincing and the case is well made to attend to flow-law uncertainty with greater effort in the future. The work is conducted with a simple numerical model under simple idealized experiments. Thus, the work reaches substantial conclusions that are not impacted by other, extraneous details.I have put most of my minor editorial comments and questions in the marked-up pdf manuscript that I provide as an attachment to the review.I think that panel B of figure 1 could be re-drafted either using a log scale (not sure if that would work) or just a focus on the top of the ice sheet, so that all the curves don't simply plot one on top of another (as is the case now).
We would like to thank Anonymous Referee #1 very much for their positive evaluation of our manuscript and appreciate their helpful comments.

Please also note the supplement to this comment:
https://www.the-cryosphere-discuss.net/tc-2020-79/tc-2020-79-RC1-supplement.pdf

- Line 1: editorial comment
  done
- Line 7: editorial comment
  done
- Line 9: editorial comment
  done
- Line 20: editorial comment
  done
- Line 24-28: suggestion to cut out the paragraph
  done
- Line 29: editorial comment
  done
- Line 31: "you should probably also cite a paper by Duval and the French group at LGGE, also there are papers by Jacka and Budd"
  Thanks for suggesting this important additional literature! We have now included the studies by Duva et al. (2010) and Budd and Jacka (1989).
- Line 38: two editorial comments
- Line 40: "I believe lab studies in Grenoble have also found this... The Duval group...."
  Thank you for pointing us to this literature, which we have included in the revised manuscript (in particular, Schulson and Duval (2009) and Duval et al. (2010)). However, we did not find a study by the Duval group which supports the higher value of the flow exponent (n=4). On the contrary, we found literature supporting that the flow exponent might be smaller than the standard value (n<3) (Schulson and Duval (2009) and Duval et al. (2010)).
- Line 53: editorial comment
  done
- Line 72-73: editorial comment
  done
- Line 89-90: editorial comment
  done
- Line 93: editorial comment
  done
- Figure 1: "would a logarithm scale be more useful in separating the profiles here? alternatively, just show the profile at the top of the ice sheet to highlight differences..."
  This is indeed a good point. We added an additional panel in the figure (Fig. 4 in the revised manuscript) to make it clearer how large the differences in initial geometries are.
- Line 148 and 155: missing reference
  done

Anonymous Referee #2

Received and published: 25 May 2020

Using idealised 2D ice sheet simulations this paper assess the influence of 2 parameters of the ice constitutive relation that is used in most ice flow models, the Glen flow law. The parameters are the activation energy, Q, and the Glen exponent, n, the pre-exponential factor is adapted to insure consistency between different values of Q and n.
We thank Anonymous Referee #2 very much for their very helpful comments and suggestions for additional literature which will greatly improve the manuscript.

I have several major comments.

First the experimental design is really poorly described and I found very difficult to understand what is really done; e.g.:

- For the equilibrium state we don't know how the accumulation rate is « adapted » to keep the volume of the ice sheet close to the reference. How close?
  Thank you for pointing this out! We have clarified our approach in the revised document.
  All simulations were started with an ice slab of 3 km thickness as an initial state. A first set of equilibrium runs (5 kyrs) was performed, with different parameters n and Q but fixed accumulation rate *a* of 0.5 m/yr. The ice volume of the steady state depends on the flow parameters, with relative differences between +10% and -15%, compared to the case with standard parameters (see the cross section and the relative differences in volume in Figure 2 of the revised manuscript).
  In order to keep the initial volume similar for all parameter combinations, we therefore increased the accumulation rate for those parameter combinations, which had a lower equilibrium volume and decreased the accumulation rates for the parameter combinations with a higher equilibrium volume. We aimed for less than 1% deviation in total ice volume and repeated simulations with different accumulation rates, until the volumes matched (similar to the bisection method), since an analytical solution cannot be derived for a polythermal ice sheet. The resulting equilibrium states with adapted accumulation rates used for the further analyses deviate by less than 0.8% in ice volume (Supplemental Figure S2).
  We restructured and expanded section 3.1.2, which now describes this process more clearly.
- We understand only in the results section that there is no melting only accumulation.
  Thank you for bringing this to our attention. We hope this will be clearer in the revised manuscript where we adapted the abstract and added two sentences to section 2.5, which state that the climatic mass balance remains unchanged in the warming simulations.
- Results are presented for several durations (100 years, 2000 years and 10 000 years)
  We apologize for the confusion around the timescales: We performed one set of warming simulations for 15.000 years. We here show a close up of the time series of the warming simulations for the first 2.000 and first 100 years as well as timeslices of the relative differences in ice volume and velocities after 100, 1.000 and 10.000 years, as we found that all simulations have reached equilibrium after 10 ka. We added clarifications in section 2 as well as the full time series in Supplemental Figure 3.

It would be very beneficial to have a clear description of the set-up and experimental design.

Second, as already noted by the first reviewer, the introduction ignores the contributions of several groups to the understanding of the ice rheology. We understand that a review paper by the same first author has been submitted; however, as this paper is not yet published, the authors should give a better review of previous works to motivate their contribution.

We are very grateful for the additional literature suggestions and expanded the introduction to include this previous work (see also answers below). In addition, we included the literature basis on which our assessment of the range of the flow parameters n and Q is based.

Finally, the flow law that is used here, is certainly the relationship that is used in most ice flow models but I don't think that we can say that it « has so far been assumed certain ». Many models have tested the sensitivity of the models, in different context, to some aspects of the flow law, the enhancement factor (e.g. Ritz et al. 1996, Quiquet, et al. 2018), the initial thermal regime (e.g. Seroussi et al. 2013), the initial viscosity (e.g. Nias et al. 2016, Humbert et al. 2005), the flow law (e.g. Peltier et al., 2000, Pettit and Waddington, 2003, Ma et al., 2010).

Thank you for bringing this to our closer attention. We agree that the phrase "has so far been assumed certain" is too strong and have revised this in the abstract as well as the main text of the manuscript.

Further, we now discuss the literature concerning the enhancement factor, the initial thermal regime and the mathematical form of the flow law itself in a more detailed way in the manuscript. However, as far as we know there has been no contribution discussing the impact of activation energies Q in the Arrhenius factor in the ice-sheet modeling community, nor has the impact of the flow exponent n been discussed in such a systematic manner.

Generally, changes in the enhancement factor and changes in activation energy Q or flow exponent n are not equivalent, as we now point out more clearly in the introduction and in the discussion: The response to a change in the enhancement factor can be expected to be independent of temperature or stress within the ice, while a change in activation energies Q impacts how the ice sheet responds to changes in the temperature distribution within the ice, and a change in the flow exponent n impacts how the ice sheet responds to changes in stress. Both enhancement factors as well as Q and n affect the ice viscosity, which in PISM is not a result of an optimization procedure but is inferred directly from the system variables, notably the stress and the temperature, and from the water content within the ice.

And there is also a many applications and papers describing anisotropic ice flow models. It is true that this aspect has not been too much explored or discussed in the last community efforts to assess the contribution of the ice sheet to sea level rise.

Indeed, several approaches on how to account for anisotropic ice flow in large ice-sheet models have been put forward by the community. We now discuss those approaches (e.g. Ma et al. 2010) in the introduction of the manuscript. However, as the reviewer also points out, to our knowledge the anisotropic ice-sheet models have not been included in the latest ice-sheet model intercomparison efforts.

It would be an interesting idea for future work to study if the uncertainty in ice flow parameters affects anisotropic ice flow in a similar way as isotropic ice flow. We intentionally limited this study to the isotropic Glen's flow law since it is the most widely used in the ice-sheet modeling community and depends on effectively only four parameters (the flow exponent n, the activation energies for warm and cold ice Q, and the prefactor A, and possibly an enhancement factor which reflects the effect of the anisotropy of the ice). As all of those parameters are less certain than often assumed, we first wanted to scrutinize how these uncertainties play out in the idealized flow-line case.

However, the idealised experiment presented here is very simple and it is not clear on which relevant time scales this uncertainty should be taken into account.

We chose the idealized experiment on purpose, to be able to disentangle the effect of the flow law from other effects like changes in the surface mass balance, but also geothermal heat flux, the topography of the bedrock and the model resolution.
The results indicate that the relative spread in flow-driven ice loss after warming is largest in the first centuries and relatively moderate when approaching equilibrium. A change in flow parameters seems not only to affect the equilibrium shape of the ice sheet itself, but also how fast the ice-sheet reaches the new equilibrium after a change in temperature. We rephrased the results section to clarify the relevance of the different time scales.

The results presented in this paper show ice losses of 300 Gt in 1000 years ; this value is the order of magnitude of what is lost by the Greenland ice sheet in one year ; So should we really take into account this uncertainty in sea level rise projections ?
While this is true, the initial volume of our idealized experiment is approximately 300 times smaller than the volume of the Greenland ice sheet. If the ice losses in our experiment would be scaled up to match the initial volume of the Greenland Ice Sheet, we would expect between 6000 Gt and 30000 Gt of ice loss after 100 years for 6 K of temperature increase, or between 60 and 300 Gt /yr. Further, in contrast to more realistic ice sheet simulations, here, the increased ice flow is the single driver of mass loss.
Relatively, the rates of mass loss in the idealized setup are of a similar order of magnitude compared to mass losses from e.g. the Greenland Ice Sheet (see also section 3.2).

To avoid confusion, we changed the absolute numbers of ice loss to relative numbers compared to the initial volume of the flow-line ice sheet in the revised manuscript. Even though the numbers here might seem small (up to 1% of the total ice volume is lost after 100 years), they compare to observations from the Greenland Ice Sheet, which has lost approximately 0.18% of its total mass in the period between 1972 and 2018 through melting, sliding and flow (assuming a mass loss of 4900Gt (see Mouginot et al., 2019) and a total present day mass of 268,500,000Gt).

Overall, while large uncertainties with respect to the Greenland Ice Sheet mass loss are of course connected to uncertainties in the climate and the melting parameterization (see, e.g., Aschwanden et al. (2019) and ISMIP6), we believe that our study provides an argument that uncertainties in the flow law need to be equally taken into account. We pick this point up in the discussion section of the revised manuscript.

Seroussi et al. (2013) have shown that 100 years simulations of the Greenland ice sheet are weekly sensitive to small changes in the initial thermal regime compared to other sources of uncertainty. On the other hand, for longer time scales, the sensitivity of the model results to the ice flow law as already been explored by several authors (see above) for real ice-sheet simulations. In conclusion I found that this simple experiment add very little to existing literature.
Many thanks for pointing us to these papers, which have indeed provided major contributions to the general understanding of ice-sheet dynamics and future projections. However, while they explore short term and long term effects of variations in rheology on ice loss from realistic ice sheets, the influence of the flow-law parameters themselves on ice velocities and mass balance cannot be inferred from these simulations.
What is new in our manuscript in comparison to previous studies? First, most other studies focus on the influence of enhancement factors, which are linear enhancements of the deformation rate, independent of temperature or effective stress. In contrast, our study specifically focuses on the activation energies Q and the flow exponent n. The activation energies determine how the ice softness changes with ice temperature,

which adds a new perspective in particular in the context of a changing climate. The influence of those parameters has not yet been systematically studied as far as we know.
Moreover, the idealized experimental design allows us to disentangle the effect from the flow law parameters from other influences, as e.g. bedrock topography, conditions at the ground, variations in the climatic mass balance etc. It also allows for comparing the simulation results with analytical approximations and thus to understand more deeply the effects of these parameters (which we have now done in sections 3.1 and 3.2).

In conclusion, I encourage the authors to improve their discussion on previous works to motivate their contribution and run more realistic experiments to better assess the magnitude and time scales relevant for this source of uncertainty.
Thank you very much for these suggestions. We extended the discussion on previous work and added a more detailed discussion on the magnitude of the flow parameter induced uncertainty, and the relevant time scales.
We took advantage of the idealized experimental design and complemented our work by analytical considerations (see new results in Figures 3, 5). In addition we estimated how relevant the influence of the flow parameters is when taking other drivers (such as surface mass balance changes or changes in sliding) into account. Including the effects of a lowering surface mass balance leads to similar results with respect to the influence of the flow law parameters (for a reduction of the SMB by 50%, we find a change in relative mass loss from 118% to 190% after 100 years, see Figure 11 in the revised manuscript). We find that with sliding, the effect of the flow uncertainty is even larger (up to +450% after 100 years, see Figure 12).
Encouraged by the referees comments, we improved the discussion about the implications and limitations of our study in section 4.

Sensitivity of ice ~~flow~~ loss to uncertainty in flow law parameters in an idealized one-dimensional geometry

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Abstract. ~~The~~ Acceleration of the flow of ice drives mass losses in both ~~,~~ the Antarctic and the Greenland Ice Sheet. The projections of possible future sea-level rise rely on numerical ice-sheet models, which solve the physics of ice flow~~and melt. While a number of important uncertainties have been addressed~~ , melt, and calving. While major advancements have been made by the ice-sheet modeling community in addressing several of the related uncertainties, the flow law, which is at the center of most process-based ice-sheet models, ~~has so far been assumed certain. Unfortunately~~is not in the focus of the current scientific debate. However, recent studies show that the ~~parameters in the flow law might be uncertain and~~ flow law parameters are highly uncertain and might be different from the widely accepted standard values. Here, we use an idealized flowline setup to investigate how these uncertainties in the flow law translate into uncertainties in flow-driven mass loss~~given a step-wise increase of surface temperatures~~. In order to disentangle the effect of future warming on the ice flow from other effects, we perform a suite of experiments with the Parallel Ice Sheet Model (PISM), deliberately excluding changes in the surface mass balance. We find that changes in the flow parameters within the observed range can lead up to a doubling of the ~~measured range of flow parameters can double the~~ flow-driven mass loss within the first centuries of warming, compared to ~~a setting with~~ standard parameters. The spread of ice loss due to ~~an~~ the uncertainty in flow parameters is of the same order of magnitude as the increase in mass loss due to ~~increasing surface temperatures~~surface warming. While this study focuses on an idealized ~~setting in order to disentangle the effect of the flow law from other effects~~flowline geometry, it is likely that this uncertainty carries over to realistic three-dimensional simulations of Greenland and Antarctica.

1 Introduction

Current and future sea-level rise is one of the most iconic impacts of a warming climate and affects shorelines worldwide (Hinkel et al., 2014; Strauss et al., 2015). The contribution of the large ice sheets in Greenland and Antarctica to sea-level rise sums up to 13.7 + 14.0 mm ~~during~~ over the last four decades (Mouginot et al., 2019; Rignot et al., 2019). It ~~is~~

~~accelerating quickly~~ has been accelerating in recent years and is expected to further increase with sustained warming (Levermann et al., 2014, 2020; Mengel et al., 2016; Seroussi et al., 2020; Goelzer et al., 2020; Aschwanden et al., 2019; Bamber et al., 2019). ~~Projections in the~~ Although some convergence can be observed in the projections of the median contribution of ~~the ice flow~~ ice loss from Antarctica and Greenland~~show some uncertainty but some convergence can be observed in these numbers. Unfortunately~~ , large uncertainties remain, and coastal protection cannot rely on the median estimate since ~~it has a likelihood of~~ there is a 50% ~~to~~ likelihood that it will be exceeded. ~~For societal purposes~~ Rather, an estimate of the upper uncertainty range is ~~important~~crucial. The most recent IPCC ~~special report on the ocean and the cryosphere~~ Special Report on the Ocean and Cryosphere in a Changing Climate provides projections of sea-level rise ~~until~~ for the year 2100 of 0.43 m ($0.29-0.59$ m) and 0.84 m ($0.61-1.10$ m) for RCP2.6 and RCP8.5 scenarios, respectively (Pörtner et al., 2019). Other studies find ~~wider (Levermann et al., 2020) or less wide ranges (Goelzer et al., 2011, 2016; Huybrechts et al., 2011)~~slightly different (Goelzer et al., 2011, 2016; Huybrechts et al., 2011) and partly wider ranges (Levermann et al., 2020). Such projections are typically performed with process-based ice-sheet models which represent the physics in the interior ~~of the ice~~ and the processes at the boundaries ~~on the ice.~~

~~Most of these representations are associated with uncertainties and many of those uncertainties are already taken into account in state-of-the-art sea-level rise projections, in particular within the framework of current~~ of the ice-sheet~~model intercomparison projects (Seroussi et al., 2020; Goelzer et al., 2020)~~.

~~In contrast, the flow of ice, one of the key processes driving mass loss in Greenland and Antarctica, is~~

In contrast to these processes at the boundaries of the ice sheet, many rheological parameters of the ice are typically not represented as an uncertainty in sea-level projections. The theoretical basis of ice flow, as implemented in ice-sheet models, has been studied in the lab and by field observations for more than half a century and is perceived as well established ~~(Glen, 1958; Paterson and Budd, 1982; Greve and Blatter, 2009; Cuffey and Paterson, 2010). However,~~ (Glen, 1958; Paterson and Budd, 1 . *Glen's flow law*, which relates stress and strain rate in a power law, is most widely used in ice-flow models. It is described in more detail in section 2.1. Some alternatives to the mathematical form of the flow law have been proposed: multi-term power laws like the *Goldsby-Kohlstedt* law or similar (Peltier et al., 2000; Pettit and Waddington, 2003; Ma et al., 2010; Quiquet et al., 2018) and anisotropic flow laws (Ma et al., 2010; Gagliardini et al., 2013) might be better suited to describe ice flow over a wide range of stress regimes. However, they have not been picked up by the ice-modeling community widely, possibly because this would require introducing another set of parameters which are not very well constrained.

Of all flow parameters, the enhancement factor is varied most routinely and its influence on ice dynamics is well understood (Quiquet et al., 2018; Ritz et al., 1997; Aschwanden et al., 2016). However, recent developments suggest that ~~the~~ also the other parameters of the flow law are less certain then typically acknowledged in modelling approaches: A review of the original literature on experiments and field observations ~~reveals~~ shows a large spread in the flow exponent n (which describes the nonlinear response in deformation rate to a given stress)~~and~~ , which can be between 2 and 4, and the activation energies Q in the Arrhenius law (which describe the dependence of the deformation rate on temperature), which also can vary by a factor of two (Jellinek and Brill, 1956; Butkovich and Landauer, 1960; Mellor and Smith, 1967; Weertman, 1968; Mellor and Testa, 1969; Muguruma, (Zeitz et al. submitted). New experimental approaches suggest a flow exponent larger than $n = 3$, which has been the most accepted value so far (Qi et al., 2017). ~~Thorough~~ Further, via an analysis of the thickness, surface slope and velocities of the Greenland Ice Sheet ~~, which are available through remote sensing , allow to~~ from remote sensing data, Bons et al. (2018) relate the driving stress to the ~~velocities and thus to infer the flow exponent n under~~ ice velocities in regions where sliding is negligible, and can thus infer a flow exponent $n = 4$ under more realistic conditions. ~~Bons et al. (2018) find a flow exponent $n = 4$ in regions where sliding is negligible.~~

Here we assess the implications of this uncertainty in simulations with the thermomechanically coupled Parallel Ice Sheet Model (the PISM authors, 2018; Bueler and Brown, 2009; Winkelmann et al., 2011), showing that variations in flow parameters have an important influence on flow-driven ice loss in an idealized flowline scenario.

This paper is structured as follows: in Section 2 we recapitulate the theoretical background of ice flow physics and describe the simulation methods used~~in this paper~~. The results of the equilibrium and warming experiments in a ~~flowline~~ flow-line setup with different flow parameters are presented in Section 3. Section 4 discusses the results and the limitations of the experimental approach and ~~section 5~~ draws conclusions and suggests possible implications of these results.

2 Methods

2.1 Theoretical background of ice flow physics

The flow of ice cannot be described by the equations of fluid dynamics alone, but needs to be complemented by a material-dependent constitutive equation which relates the internal forces (stress) to the deformation rate (strain rate). Numerous laboratory experiments and field measurements show that the ice deformation rate responds to stress in a nonlinear way. Under the assumptions of isotropy, incompressibility and uni-axial stress this observation is reflected in ~~the *Glen's flow law*~~Glen's flow law, which gives the constitutive equation for ice,

$$\dot{\epsilon} = A\tau^n, \tag{1}$$

where $\dot{\epsilon}$ is the strain rate, τ the dominant shear stress, n the flow exponent and A the softness of ice (Glen, 1958).

Both, the flow exponent and the softness are important parameters which determine the flow of ice. Usually, the exponent n is assumed to be constant through space and time. Until today, there is no comprehensive understanding of all the physical processes determining the softness A. It may depend, among others, on water content, impurities, grain size and anisotropy as well as temperature of the ice. Within the scope of ice-sheet modeling A is typically expressed as a function of temperature alone

$$A = A_0 \exp\left(-\frac{Q}{RT'}\right), \tag{2}$$

where A_0 is a constant factor, Q is an activation energy, R is the universal gas constant and T' the temperature difference to the pressure melting point (Greve and Blatter, 2009; Cuffey and Paterson, 2010).

Due to pre-melt processes the softness responds more strongly to warming at temperatures close to the pressure melting point, which is often described by a piece-wise adaption of the activation energy Q (Barnes et al., 1971; Paterson, 1991), with a larger value of Q at temperatures $T^{'} > -10°\mathrm{C}$. When using these piece-wise defined values for Q for warm and for cold ice in the functional form of the flow law, the respective factors A_0 ensure that the function is continuous at $T^{'} = -10°\mathrm{C}$. A_0 is therefore dependent on the values of the flow exponent n and both values of Q for cold and for warm ice.

The scalar form of ~~*Glen's flow law*~~ Glen's flow law (Equation (1)) is only valid for uni-axial stresses, acting in only one direction. For a complete picture the stress is described as a tensor of order two. The generalized flow law reads

$$\dot{\epsilon}_{\cancel{j,k}jk} = A(T')\tau_e^{n-1}\tau_{\cancel{j,k}jk}, \tag{3}$$

where ~~$\dot{\epsilon}_{j,k}$~~ $\dot{\epsilon}_{jk}$ are the components of the strain rate tensor and ~~$\tau_{j,k}$~~ τ_{jk} are the components of the stress deviator, τ_e is the effective stress, which is closely related to the second invariant of the deviatoric stress tensor:

$$\tau_e^2 = \frac{1}{2}\left[\tau_{xx}^2 + \tau_{yy}^2 + \tau_{zz}^2\right] + \tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2. \tag{4}$$

Each component of the strain rate tensor depends on all the components of the deviatoric stress tensor through the effective stress τ_e.

Glen's flow law (3) and the softness parametrization (2) are at the center of most numerical ice-sheet and glacier models, independently of the other approximations they might use (the PISM authors, 2018; Winkelmann et al., 2011; Greve, 1997; Pattyn, 2017; Larour et al., 2012; de Boer et al., 2013; Fürst et al., 2011; Lipscomb et al., 2018).

2.2 Ice flow model PISM

The simulations in this study were performed with the Parallel Ice Sheet Model (PISM) release stable v1.1. PISM uses shallow approximations for the discretized, physical equations: The shallow-ice approximation (SIA) (Hutter, 1983) and the shallow-shelf approximation (SSA) (Weis et al., 1999) are solved in parallel within the entire simulation domain. The shallow ice approximation is typically dominant in regions with high bottom friction, such that the vertical shear stresses dominate over horizontal shear stresses and longitudinal stresses. The shallow shelf approximation is typically dominant for ice shelves, with zero traction at the base of the ice, and for the fast flow regime in ice streams (Winkelmann et al., 2011). PISM assumes a non-sliding SIA flow and uses the results of the SSA approximations for fast flowing ~~ice.~~

and sliding ice. In PISM, the flow law enters both the SIA and the SSA part of the velocities, as detailed in Winkelmann et al. (2011). It is possible to choose different flow exponents n for the SSA and the SIA, but the softness is the same for both approximations.

The simulations performed here use mostly the SIA mode: The geometry of a two dimensional ice sheet sitting on a flat bed and the SIA mode serve to study the effects of changes in flow parameters onto internal deformation and to separate those effects from changes in sliding, etc. Including the shallow shelf approximation reproduces and even enhances the effect of changes in the activation energies Q (~~Supplemental Figure S3~~see section 3.1).

2.3 ~~Range of~~ Uncertainty in flow exponent and activation ~~energy values~~energies

The flow exponent n and the activation energies for warm and for cold ice, Q_w and Q_c, determine the deformation of the ice as a response to stress or temperature. A recent review (Zeitz et al. submitted~~) revealed that~~, see also literature in the introduction above) reveals a broad range of potential flow parameters n, Q_w and Q_c~~cannot be ruled out by reported studies~~. The activation energy for cold ice Q_c is varied between $42\,\mathrm{kJ/mol}$ to $85\,\mathrm{kJ/mol}$ (the typical standard value is $Q_\mathrm{c} = 60\,\mathrm{kJ/mol}$). The activation energy for warm ice Q_w is varied between $120\,\mathrm{kJ/mol}$ to $200\,\mathrm{kJ/mol}$ (the standard value is $Q_\mathrm{w} = 139\,\mathrm{kJ/mol}$). For the flow exponent n, values as low as 1 have been reported, but since many experiments and observations confirm a nonlinear flow of ice, n has been varied between 2 and 4, with the standard value of $n = 3$. The standard values serve as a reference point and correspond to the default values in ~~most~~ many ice-sheet models (the PISM authors, 2018; Greve, 1997; Pattyn, 2017; Larour et al., 2012; de Boer et al., 2013; Fürst et al., 2011; Lipscomb et al., 2018).

2.4 Adaption of the flow factor A_0

The flow factor A_0 in the flow law must be adapted to fulfill the following conditions: First, the continuity of the piece-wise defined softness $A(T^{'})$ must be ensured for all combinations of Q_w, Q_c and n. Secondly, a reference deformation rate $\dot{\epsilon}$ at a reference driving stress τ_0 and a reference temperature $T_0^{'}$ (the PISM authors, 2018) should be maintained regardless of the parameters. This is because the coefficient and the power are non-trivially linked when a power law is fitted to experimental data. These conditions give:

$$A_{0,\mathrm{old}} \cdot \exp\left(-\frac{Q_\mathrm{old}}{RT_0^{'}}\right) \cdot \tau_0^{n_\mathrm{old}} = A_{0,\mathrm{new}} \cdot \exp\left(-\frac{Q_\mathrm{new}}{RT_0^{'}}\right) \cdot \tau_0^{n_\mathrm{new}}, \tag{5}$$

$$A_{0,\mathrm{new}} = A_{0,\mathrm{old}} \cdot \exp\left(-\frac{Q_\mathrm{old} - Q_\mathrm{new}}{RT_0^{'}}\right) \cdot \tau_0^{n_\mathrm{old} - n_\mathrm{new}}. \tag{6}$$

If the reference temperature is $T_0^{'} < -10°\mathrm{C}$ the values for cold ice $A_{0,c}$ and Q_c are used in the equation above, or else $A_{0,w}$ and Q_w are used. The corresponding $A_{0,\mathrm{new}}$ for warm or cold ice respectively is calculated from the continuity condition at $T^{'} = -10°\mathrm{C}$. For e.g. $T_0^{'} < -10°\mathrm{C}$ it follows

$$A_{0,c,\mathrm{new}} = A_{0,c,\mathrm{old}} \cdot \exp\left(-\frac{Q_{c,\mathrm{old}} - Q_{c,\mathrm{new}}}{RT_0^{'}}\right) \cdot \tau_0^{n_\mathrm{old} - n_\mathrm{new}} \quad \text{and} \tag{7}$$

$$A_{0,w,\mathrm{new}} = A_{0,c,\mathrm{new}} \cdot \exp\left(-\frac{(Q_{c,\mathrm{new}} - Q_{w,\mathrm{new}})}{R \cdot 263.15\mathrm{K}}\right). \tag{8}$$

Here we choose $\tau_0 = 80\mathrm{kPa}$ as a typical stress in a glacier and $T_0^{'} = -20°\mathrm{C}$. Choosing another τ_0 in the same order of magnitude has only little effect on the differences in dynamic ice loss. Choosing another $T_0^{'}$ on the other hand influences how the softness changes with the activation energy Q, see Supplemental Figure S1. With $T_0^{'}$ closer to the melting temperature, the difference in softness at the pressure melting point decreases thus the ice loss is less sensitive to changes in the activation energy Q.

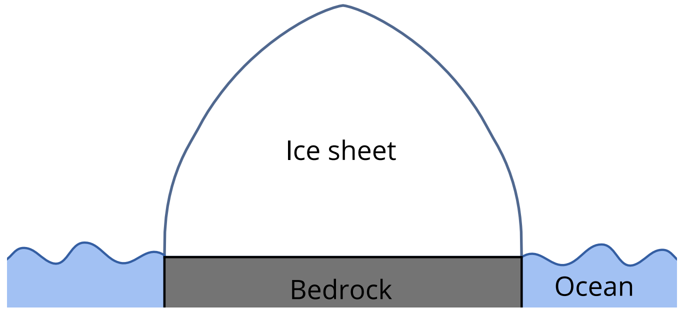

Figure 1. **Sketch of the flow-line setup** The ice is sitting on a flat bed, the fixed calving front does not allow ice-shelves. The accumulation rate is constant throughout the simulation domain and the temperature is altitude dependent.

2.5 ~~Simulation setup~~Experimental design

The study is performed in a ~~flowline~~ flow-line setup, similar to ~~(Pattyn et al., 2012), with~~ Pattyn et al. (2012), where the computational domain has an extent of 1000km in x-direction and 3km in y-direction (with a periodic boundary condition). The spatial horizontal resolution is 1 km. The ice rests on a flat bed of length L = 900 km with a fixed calving front at the edge of the bed, such that no ice shelves can form (Figure 1). In contrast to ~~(Pattyn et al., 2012)~~Pattyn et al. (2012), the temperature and the enthalpy ~~in the~~ of ice sheet are allowed to evolve freely.

The model is initialized with a spatially constant ice thickness and is run into equilibrium for ~~each of the ensemble members. The thickness profile of the equilibrium state is similar to the *Vialov profile* (see e.g. Cuffey and Paterson (2010)). The~~ different combinations of flow parameters Q_c, Q_w and n. The ice surface temperature is altitude dependent, $T_s = -6°\mathrm{C}/\mathrm{km} \cdot z - 2°\mathrm{C}$, where z is the surface elevation in km. The accumulation rate is constant in space and time for each simulation. In ~~warming scenarios~~ the warming experiments, for each ensemble member an instantaneous temperature increase of $\Delta T \in [1,2,3,4,5,6]°\mathrm{C}$ is applied ~~.~~ to the ice surface for the duration of 15,000 years (until a new equilibrium is reached), while the climatic mass balance remains unchanged. That means, the temperature increase can lead to an acceleration of ice flow, but is prohibited from inducing additional melt. This idealized forcing allows us to disentangle the effect of warming on the ice flow from climatic drivers of ice loss.

2.6 ~~Equilibrium states~~

~~**Effect of flow parameters on equilibrium state without warming** with the flow exponent $n = 3$ **a)** Sketch of the setup. **b)** Cross sections of equilibrium states for parameter combinations of Q_w and Q_c and **c)** Relative difference of average surface velocities and accumulation rates needed to keep the ice sheet volume in equilibrium close to the reference simulations with standard flow parameters.~~ ~~While the~~ The thickness profile of the equilibrium state is similar to the *Vialov profile* (see e.g. Cuffey and Paterson (2010); Greve and Blatter (2009)). However, in contrast to the isothermal Vialov profile, here the temperature of the ice is allowed to evolve freely, leading to a non-uniform softness of the ice (the PISM authors, 2018). The extent in x-direction is given by the geometry of the setup, a flat bed with a calving boundary condition at the margin, and the height and ~~the~~ shape of the ice sheet depend on the flow parameters n, Q_w and Q_c and the accumulation rate a. ~~Adapting the accumulation rate for each parameter combination allows to keep the volume of the ice sheet close to the reference value with standard flow parameters (with variation of only afew percent). Simulations with~~

3 Results

3.1 Effect of activation energies in model simulations compared to analytical solution

In order to gain a deeper understanding of the influences of Q_c and Q_w on the equilibrium shape of ice-sheets, we here compare the simulated results to analytical considerations based on the Vialov profile.

At a fixed accumulation rate of $a = 0.5\mathrm{m/yr}$, each flow parameter combination leads to an equilibrium state with a thickness profile similar to the Vialov profile but differences in maximal thickness and volume (Figure 2 a). Overall, high activation energies increase ice-flow velocities and reduce the ice-sheet volume. The activation energy for warm ice, Q_w~~need increased accumulation rates a in order to maintain an equilibrium volume~~, affects the volume and the velocities more strongly than the activation energy for cold ice, Q_c. A high Q_w leads to softer ice close to the pressure melting point (supplemental Figure 1) and at the base of the ice sheet, which leads to higher velocities and a lower equilibrium volume of the ice sheet while a low Q_w leads to stiffer ice close to the ~~reference value. For the highest combination of activation energies~~ pressure melting point and at the base of the ice sheet and in consequence the velocities decrease and the volume increases (Figure 2, b and c). For a fixed Q_w~~and~~, the volume appears to decrease linearly with increasing Q_c and the velocity appears to increase linearly with increasing Q_c~~, the relative differences $d_x = (x - x_0)/x_0$ of both, accumulation rates a and mean surface velocities v, increase by more than 300% (Fig. 4 a)~~.

The maximal thickness of an isothermal ice sheet can be estimated with the Vialov profile

$$h_m = 2^{n/(2n+2)} L^{1/2} \left(\frac{(n+2)a}{2A(T')(\rho g)^n} \right) \tag{9}$$

with the Glen exponent n, the ice sheet extent $2L$, the pressure adjusted temperature T', the gravity g, and the ice density ρ (Greve and Blatter, 2009). The Vialov thickness of a temperate ice sheet (isothermal at the pressure melting point), where the softness is evaluated at the pressure melting point depending on the activation energies Q_c and Q_w (see Equation (2)) gives a lower bound to the thickness, given the same geometry and flow parameters. The simulated maximal thickness is larger than the lower bound for all parameter combinations (Figure 3 a, lower bound indicated by grey line) and the ratio between the maximal thickness h_m from the PISM simulation to the lower bound from the Vialov profile depends on both, Q_w and ~~b)).~~ Q_c. The ratio increases with higher Q_w and decreases with higher Q_c (Figure 3 b). The ice-sheet thickness of the polythermal ice sheet, as simulated with PISM, matches well the Vialov thickness calculated with Equation (9), if an effective temperature $T'_\mathrm{eff} < 0°\mathrm{C}$ is assumed. The effective temperature T'_eff, which matches simulations best, varies for different Q_w. For $Q_\mathrm{w} = 120\,\mathrm{kJ/mol}$, an effective temperature of $T'_\mathrm{eff} = -5°\mathrm{C}$ matches well the equilibrium thickness of the

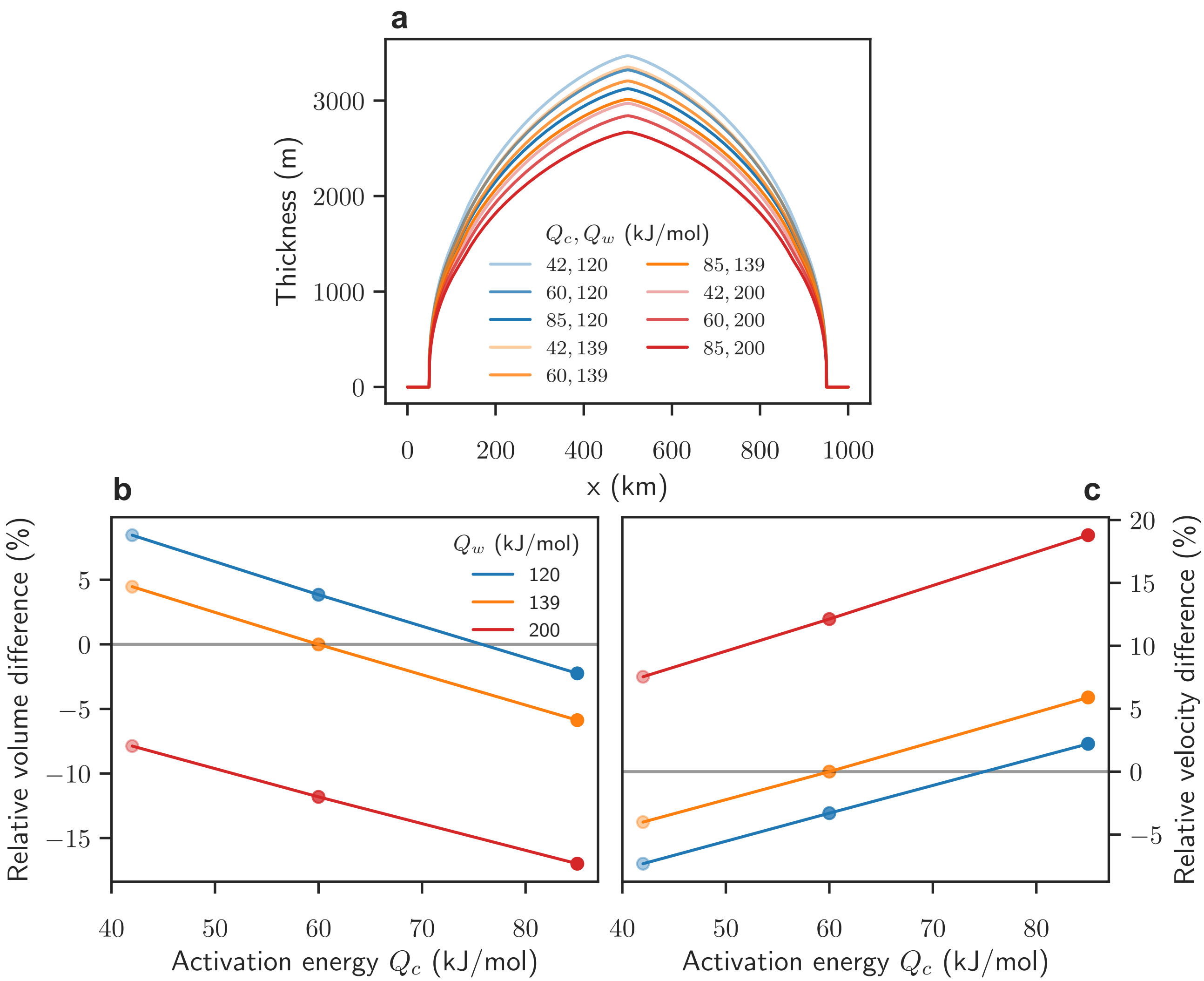

Figure 2. **Effect of activation energies on equilibrium volume and velocities** with fixed accumulation rate $a = 0.5$m/yr and flow exponent $n = 3$. Thickness profiles of equilibrium states for different combinations of activation energies Q_w and Q_c **(a)**. Relative difference of average equilibrium volumes **(b)** and velocities **(c)** compared to the reference state with standard parameters for parameter combinations of Q_w and Q_c. Q_c is shown on the x-axis and Q_w is given through the color of the the markers (Blue: $Q_\mathrm{w} = 120$ kJ/mol, orange: $Q_\mathrm{w} = 139$ kJ/mol, red: Blue: $Q_\mathrm{w} = 200$ kJ/mol)

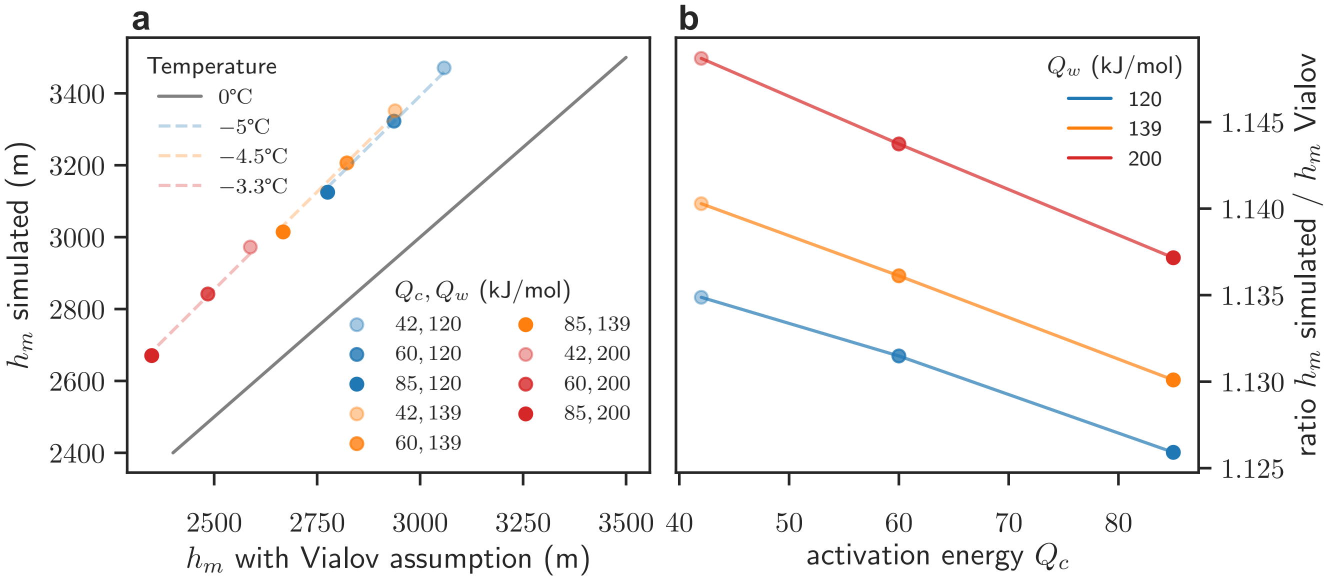

Figure 3. **Comparison of simulated equilibrium thickness with analytical results: (a)** Dots: Maximal thickness h_m of the simulated polythermal ice sheet versus the analytical solution for the maximal thickness h_m of a temperate Vialov profile with the same flow parameters and accumulation rate. Colors indicate the flow-parameter combination. The grey line indicates identity. Short, dashed lines indicate the analytical h_m with a temperature lower than the pressure melting point versus h_m at the pressure melting point with the same flow parameters and accumulation rate. The temperature, which fits the simulated results best, is indicated in the legend. **(b)** Ratio of the simulated h_m to the analytic h_m (assuming a temperate ice sheet) versus Q_c for different parameter combinations Q_c, Q_w. The value of Q_w is indicated by the color.

polythermal ice sheets. For $Q_w = 200\,$kJ/mol, an effective temperature of $T'_{\mathrm{eff}} = -3.3\,°\mathrm{C}$ matches well the equilibrium thickness of the polythermal ice sheets. These differences can be partly explained by the altitude-dependent surface temperature: The maximal thickness of the ice sheets varies by approximately 800 m, which leads to a difference in ice surface temperature of approximately 4.8 °C between the thickest and the thinnest ice and thus influences the temperature within the ice sheet.

~~If the accumulation rate is fixed at $a = 0.5$m/yr, the~~ The relative difference of average velocities $d_v = (\bar{v} - \bar{v}_0)/\bar{v}_0$ spans from $d_v = -7\%$ (with a corresponding relative difference in ice sheet volume of $d_{vol} = +10\%$) for the lowest combination of activation energies to $d_v = +18\%$ with a difference in volume of $d_{vol} = -15\%$ for the highest combination of values for Q_c and Q_w (~~Supplemental Figure S2 a) and b))~~Figure 2 b).

~~Overall,~~

3.2 Ice-sheet initial states

In order to keep the initial ice volume largely fixed (with variations of less than one percent) in the warming experiments, we adapt the accumulation rate for each parameter combination of Q_c and Q_w.

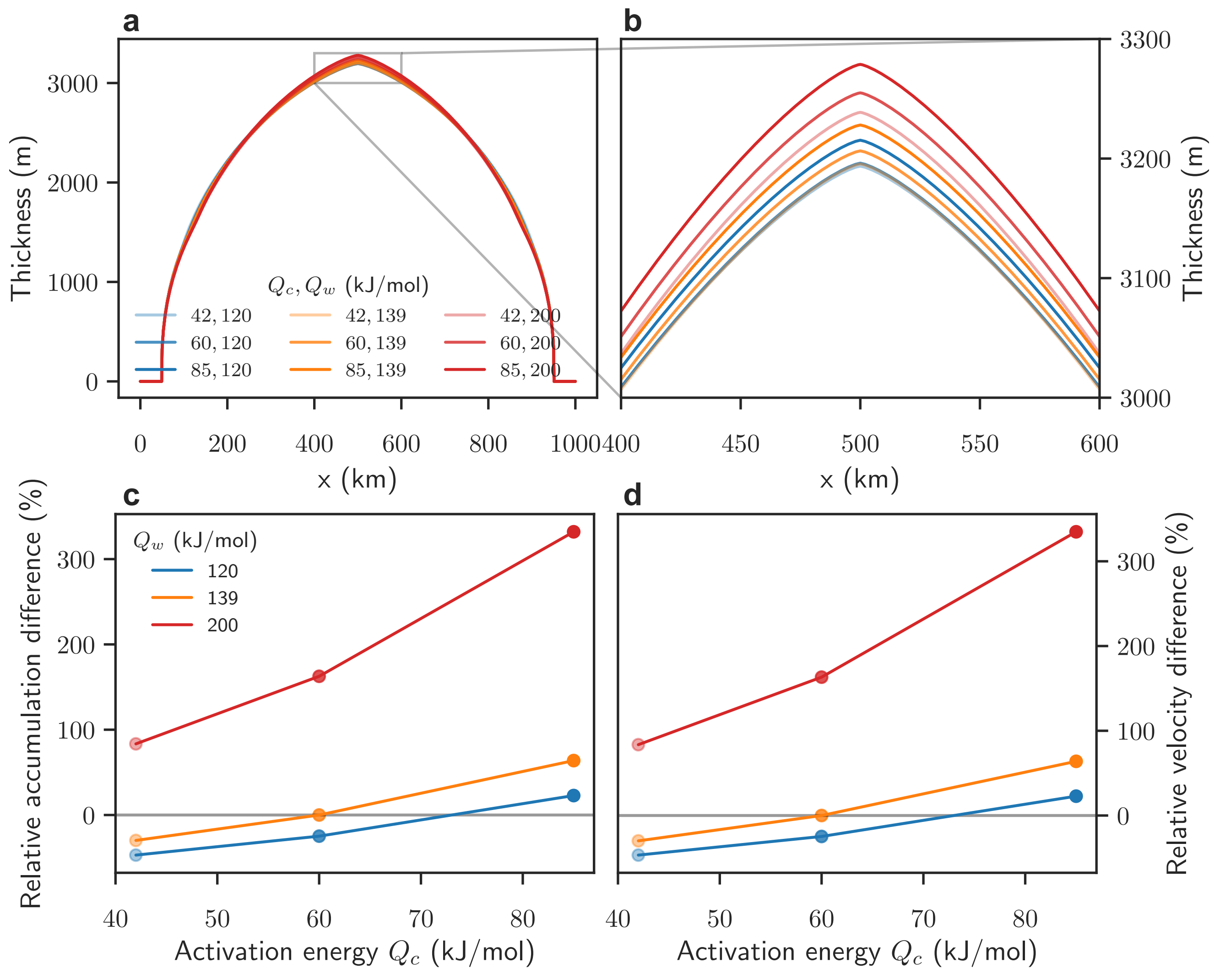

Figure 4. **Effect of flow parameters on equilibrium state without warming with adapted accumulation rates** and flow exponent $n = 3$. **(a)** Thickness profile of equilibrium states for parameter combinations of Q_w and Q_c with a zoom on the ice divide **(b)**. Relative difference of accumulation rates **(c)** needed to keep the ice sheet volume in equilibrium close to the reference simulations with standard flow parameters and relative difference in average surface velocities **(d)** versus Q_c. The value of Q_w is given by the color.

Since simulations with high activation energies ~~increase ice-flow velocities and reduce the ice-sheet volume under fixed accumulation rates. The activation energy for warm ice ,~~ Q_w ~~, affects the volume and the velocities more strongly than~~ have a smaller equilibrium volume at the same accumulation rate than simulations with standard activation energies, the accumulation rate a is increased to maintain an equilibrium volume close to the reference value. Simulations with low activation energies Q_c have a higher volume at the same accumulation rate, so the accumulation rate a is decreased. In the case of an isothermal ice sheet the maximal thickness and the ~~activation energy for cold ice,~~ volume can be computed analytically as shown above in Equation (9). In our model simulations, however, the temperature distribution within the ice can evolve freely, thus the softness is not uniform and an analytical solution cannot be found.

In order to find the right adaptation for the accumulation rates, we start from the volume calculated from the isothermal approximation as a first guess and run the model into equilibrium. If the relative difference between the new equilibrium volume and the standard equilibrium volume exceeds 1%, we further change the accumulation rate and repeat the equilibrium simulation, always starting from the same initial state. The final equilibrium states found via this iterative approach differ by max. 0.8% in ice volume (supplemental Figure S2) and the difference in maximal thickness is less than 100m (Figure 4, a and b).

For the combination of high activation energies Q_w and Q_c, the relative differences $d_x = (x - x_0)/x_0$ of both, adapted accumulation rates a and mean surface velocities v, increase by more than 300% (Fig. 4, c and d) and for the combination of low activation energies Q_c and Q_w both, adapted accumulation rates a and surface velocities v are approximately 50% lower compared to the case with standard parameters. Both, the accumulation rate and the velocities, change in the same way since they balance each other in equilibrium.

The maximal thickness of the polythermal simulated ice sheet is approximately 13-16% larger than the lower bound estimated with a temperate ice sheet (Figure 5, a and b) with the same flow parameters and accumulation rates. Similar to the case with fixed accumulation rates, the simulated thickness matches the Vialov thickness well, if an effective temperature $T'_\mathrm{eff} < 0°\mathrm{C}$ is assumed. The effective temperature, which mathces simulations best, varies for different Q_w, from $-5°\mathrm{C}$ for $Q_\mathrm{w} = 120\,\mathrm{kJ/mol}$ to $-3.6°\mathrm{C}$ for $Q_\mathrm{w} = 200\,\mathrm{kJ/mol}$. This difference can not be sufficiently explained by variations in surface temperature due to the difference in ice-sheet thickness. Rather the higher effective temperatures are linked to increased flow velocities of the ice, which in turn might lead to strain heating. In simulations with a high Q_w the simulated thickness has a higher discrepancy to the estimated lower bound (assuming a temperate ice sheet) than simulations with a low Q_w. In contrast to the case with fixed accumulation rate (Figure 3) the ratio between the estimated and the simulated thickness depends only very little on Q_c. ~~The softness of the ice at the bottom of the ice sheet , where temperatures are close to the pressure melting point, is more sensitive to variations in Q_w, which might explain the greater impact (Figure ??) .~~

4 ~~Results~~

~~Disentangling~~

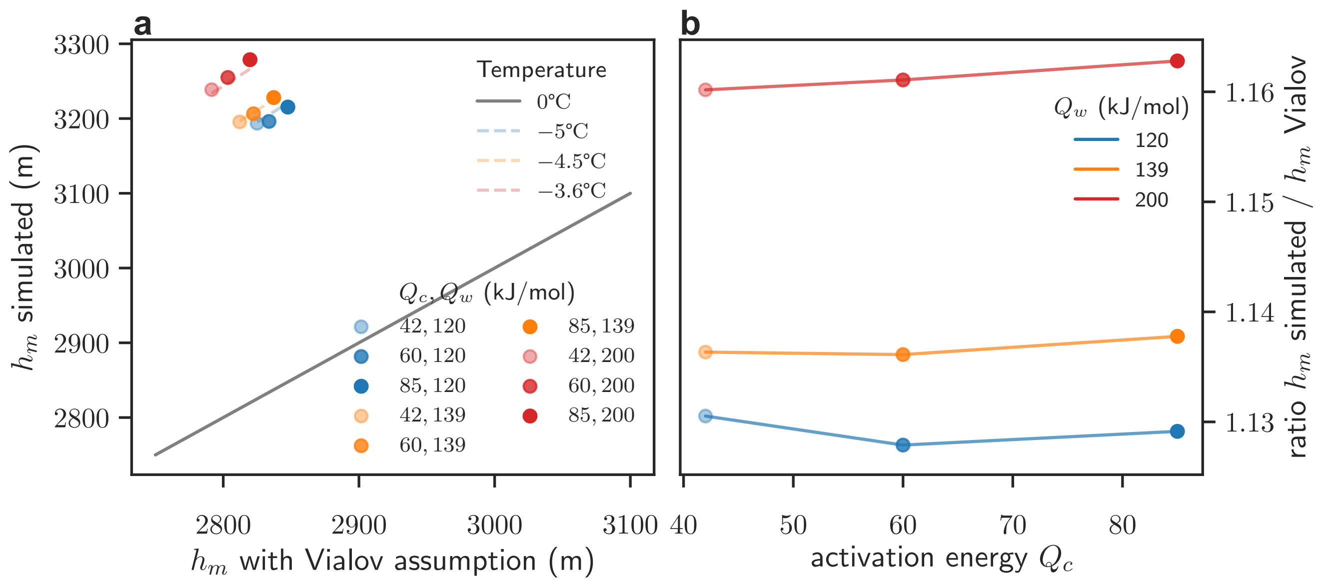

Figure 5. **Comparison of simulated equilibrium thickness with analytical results: (a)** Dots: Maximal thickness h_m of the simulated polythermal ice sheet versus the analytical solution for the maximal thickness h_m of a temperate Vialov profile with the same flow parameters and accumulation rate. Colors indicate the parameter combination. The grey line indicates identity. Short, dashed lines indicate the analytical h_m with a temperature lower than the pressure melting point versus h_m at the pressure melting point with the same flow parameters and accumulation rate. The temperature, which fits the simulated results best, is indicated in the legend. **(b)** Ratio of the simulated h_m to the analytic h_m (assuming a temperate ice sheet) versus Q_c for different parameter combinations Q_c, Q_w. The value of Q_w is indicated by the color.

3.1 Flow-driven ice loss under warming

Disentangling the purely flow-driven ice losses from the influences of melting, different initial temperature profiles and variations in sliding requires several ~~adjustments~~conditions: 1) The initial volume is fixed, which is here attained through adjustment of the accumulation rate for ~~all parameter combinations (Fig. 4, a) and b)).~~ the different flow parameter combinations as explained in section 3.2. 2) The surface mass balance is fixed, i.e., we do not allow for additional melt, and the accumulation rate does not change with ~~temperature, i. e. no melting occurs.~~ warming. 3) Sliding is effectively inhibited ~~by the SIA condition~~(which is here ensured by applying an SIA-only condition).

The effect of the temperature increase is ~~thus~~ limited to warming at the ice surface which can propagate ~~to~~ into the interior of the ice sheet though diffusion and advection. ~~Increasing temperatures increase the softness of the ice (Fig. ??) and thus increase~~ Warming makes the ice softer thus accelerates the flow and ice discharge. ~~Warming simulations show an ice loss for all temperature increases~~ Since temperature diffusion in an ice sheet is a very slow process, we apply the temperature anomaly for a total duration of 15,000 years. The total mass balance is evaluated and compared to the standard parameter simulation after

100, 1000 and 10,000 years of warming. A new equilibrium state is reached after 10,000 years for all parameter combinations (see longer time-series in Supplemental Figure S3).

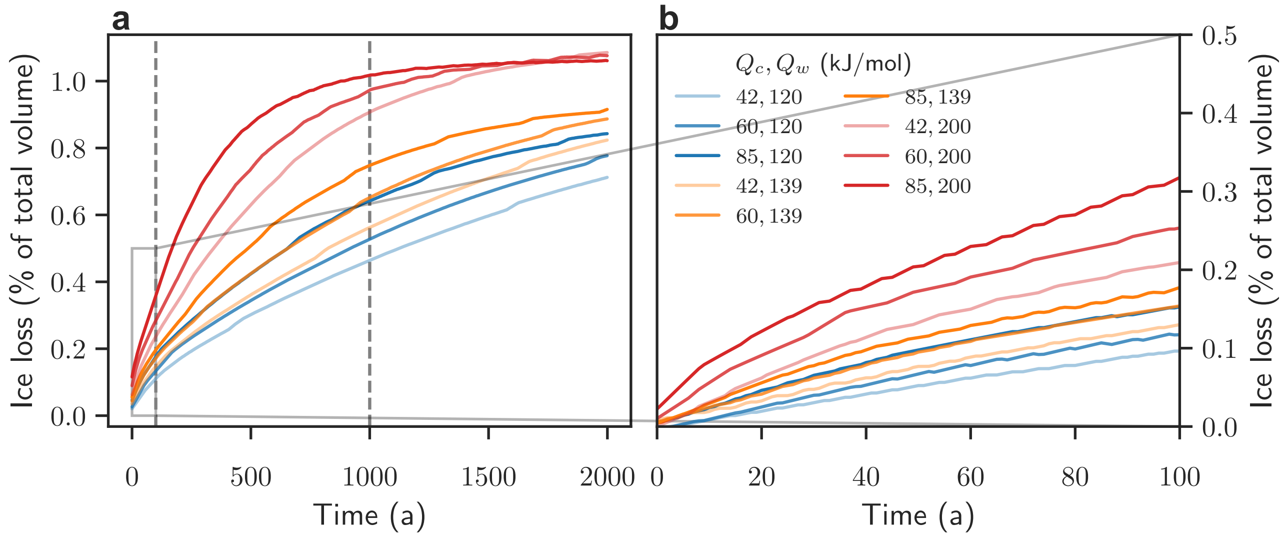

Figure 6. Time series for flow driven ice discharge under 2°C warming: (a) Time evolution of ice loss with different activation energies Q_c and Q_w and the flow exponent $n = 3$, subject to a temperature anomaly forcing of $\Delta T = 2$°C. **(b)** Zoom on the first 100 years.

In the experiments, the ice sheet loses mass for all warming levels and all parameter combinations~~, however,~~ . However, the amount and rate of the ice loss is dependent on the flow parameters.

~~**Effect of activation energy on flow driven ice discharge under warming: a)** Time evolution of ice loss with different activation energies Q_c and Q_w and the flow exponent $n = 3$, subject to a temperature anomaly forcing of $\Delta T = 2$°C. **b)** Relative difference of ice loss $d_m = (\Delta m - \Delta m_0)/\Delta m_0$ after 100, 1000 and 10000 years of warming. Ice loss with standard parameters Δm_0 serves as a reference.~~

~~A step forcing with a temperature increase of 2°C leads to immediate ice losses compared to the unforced state (Figure 6 a). The~~ Figure 6 shows the ice-sheet response to a warming of 2°C. For a fixed flow exponent of $n = 3$ the fastest ice loss is observed for the flow parameter combination of ~~$n = 3, Q_c = 85\,\mathrm{kJ/mol}$~~ $Q_c = 85\,\mathrm{kJ/mol}$ and $Q_w = 200\mathrm{kJ/mol}$ and the slowest ice loss for ~~$n = 3, Q_c = 42\,\mathrm{kJ/mol}$~~ $Q_c = 42\,\mathrm{kJ/mol}$ and $Q_w = 120\mathrm{kJ/mol}$. Simulations with $Q_w = 200\mathrm{kJ/mol}$ reach a new, temperature adapted equilibrium already after ~~2.000~~ 2,000 yrs, while simulations with lower Q_w continue to lose mass.

The sensitivity to variations in flow parameters is measured via the relative differences for flow-driven ice loss $d_m = (\Delta m - \Delta m_0)/\Delta m_0$, where the reference Δm_0 is always given by the simulation with standard parameters under the same temperature increase (~~6). The relative differences are particularly high immediately after the warming perturbation starts and decrease after approximately 300 years.~~

Figure 7). While the long term response to warming, ~~measured after 10.000~~ after 10,000 years, is not very sensitive to the particular choice of flow parameters, the rate of flow-driven ice loss is. The largest relative differences in ice loss is ~~seen~~

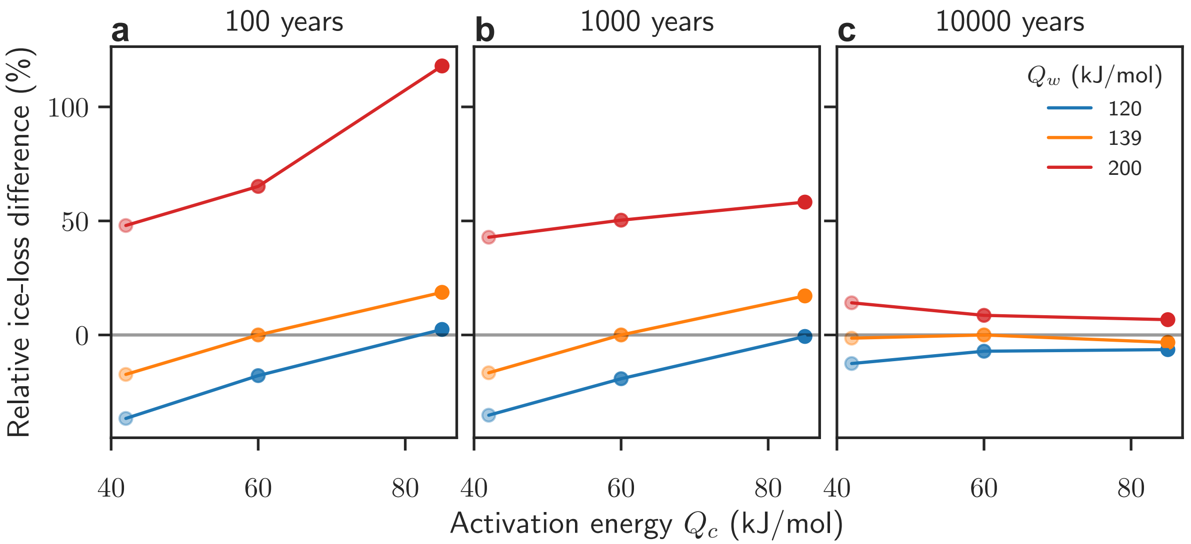

Figure 7. **Effect of activation energy on flow driven ice discharge under 2°C warming:** Relative difference of flow-driven ice loss after 100 **(a)**, 1000 **(b)** and 10,000 **(c)** years versus Q_c. The value of Q_w is given by the color. The simulations have reached a new equilibrium after 10,000 years.

found in the first ~~100–300 years after the start of the temperature forcing (solid bars in Figure **??**~~ century after the temperature increase (Figure 7, a), indicating that high Q_w ~~speeds~~ speed up the flow-driven ice loss. Under 2°C of warming, ice loss after 100 years is enhanced more than two-fold (i.e. increased by up to 118%) in simulations with ~~high activation energy for warm ice Q_w~~ $Q_w = 200$ kJ/mol, while low Q_w ~~may reduce~~ reduces the relative ice loss by ~~30~~up to 37%.

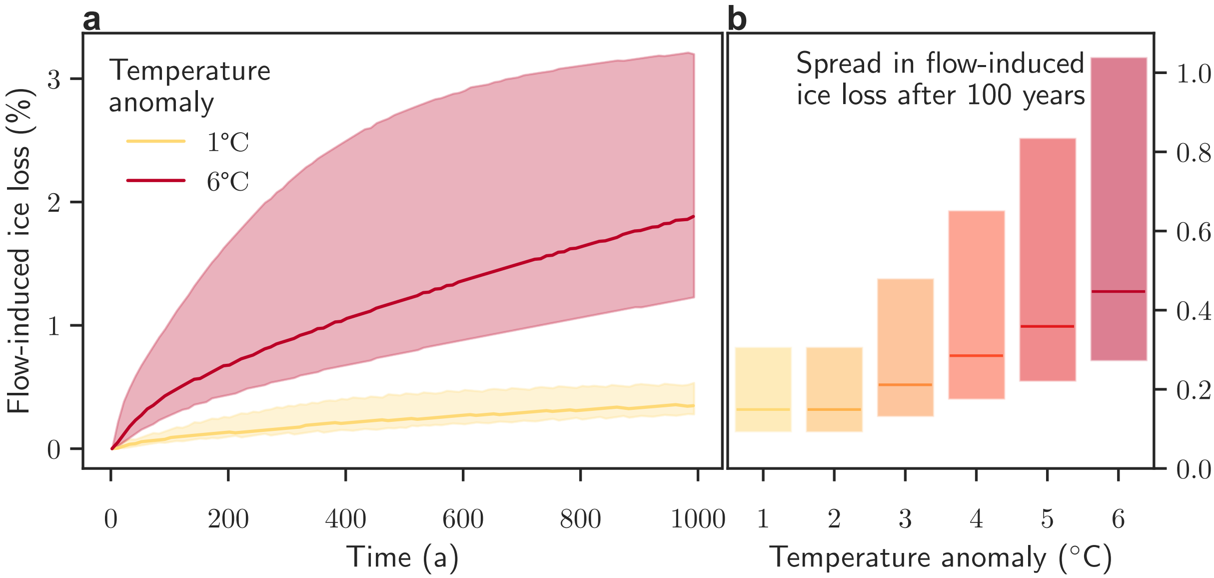

Figure 8. Effect of temperature forcing and activation energy on flow-driven ice ~~a)~~ **(a)** Time evolution of ice loss under warming of $1\,^{\circ}$C and $6\,^{\circ}$C. For warming of $\Delta T = 1\,^{\circ}$C the conceptual ice sheet ~~looses 30 Gt~~ loses 0.35% of ice after 1000 yr for standard parameters (solid, yellow line). Variations in the activation energies Q lead to variations in ice loss on the same order of magnitude (shaded area). For a warming of ~~ΔT=6 °C~~ $\Delta T = 6\,^{\circ}$C the conceptual ice sheet ~~looses 165 Gt~~ loses 1.89% of ice after 1000 yr for standard parameters (solid, red line). The variations in ice loss due to different parameters for the activation energy Q (shaded area) are strongly asymmetrical and, in particular during the first 300 years, high compared to the total amount of ice loss. ~~b)~~ **(b)** Uncertainty in flow induced ice loss after 100 years of simulation time over all combinations of Q_{w}, Q_{c} and temperature anomalies ΔT. The flow exponent $n = 3$ is kept fixed for all simulations.

The effect of the flow parameters on flow-driven ice loss upon warming is robust for different temperature increases. ~~Absolute ice~~ Ice losses as well as the ~~absolute~~ spread in flow-driven ice loss both increase ~~under higher warming scenarios~~ for higher warming levels (see Figure 8). ~~Variations in the flow parametersshape the ice-loss rates in particular during the first centuries of warming~~For a warming of $\Delta T = 1\,^{\circ}$C the idealized ice sheet loses 0.09% after 100 yr and 0.35% of ice after 1000 yr for standard parameters. For a warming of $\Delta T = 6\,^{\circ}$C the ice sheet loses 0.46% after 100 yr and 1.89% of ice after 1000 yr for standard parameters (solid, red line). For comparison, the Greenland Ice Sheet has lost approximately 0.18% of its mass in the period between 1972 and 2018 (Mouginot et al., 2019), which includes all processes: increase in flow, melting, and sliding.

The effect of flow parameter changes onto the purely flow-driven ice loss after 100 years is ~~on~~ of the same order of magnitude as ~~increasing the surface temperature~~ the effect of surface warming by several degrees. In particular ~~, at a warming of 2 °C the uncertainty~~ the uncertainty ranges of ice loss ~~overlaps with the uncertaintly of ice loss under a~~ for warming of $2\,^{\circ}$C and warming of $6\,^{\circ}$C overlap (Figure 8 b), when solely considering the ice loss is driven by changes in flow and excluding surface mass balance changes.

3.2 Influence of the flow exponent n

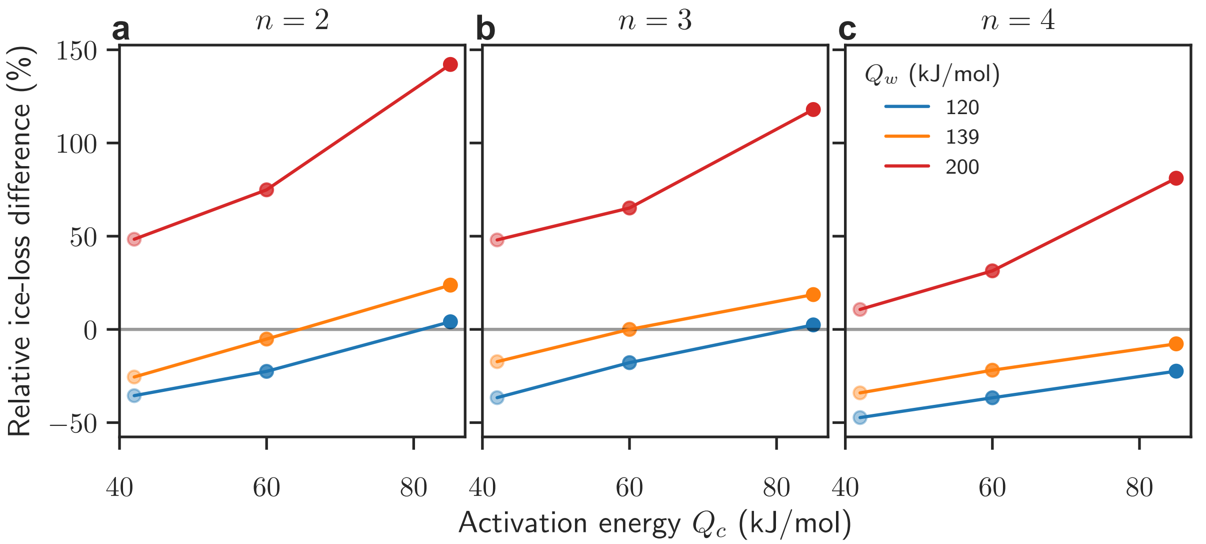

Figure 9. ~~**Effect of the flow exponent and activation energies on flow under** 2°C**C of warming:**~~ **Effect of the flow exponent and activation energies on flow-driven ice loss after 100 years under 2°C of warming:** Relative difference in flow-driven ice discharge for $n = 2$ **(a)**~~and average surface velocity~~ , $n = 3$ **(b)** and $n = 4$ **(c)** for different combinations of the flow exponent n and activation energies Q_c and Q_w. The reference is always a simulation performed with standard parameters $n = 3$, $Q_\mathrm{c} = 60$ kJ/mol and $Q_\mathrm{w} = 139$ kJ/mol. ~~Variations in the flow exponent n do not significantly influence the relative difference of mean velocities after 100 years.~~

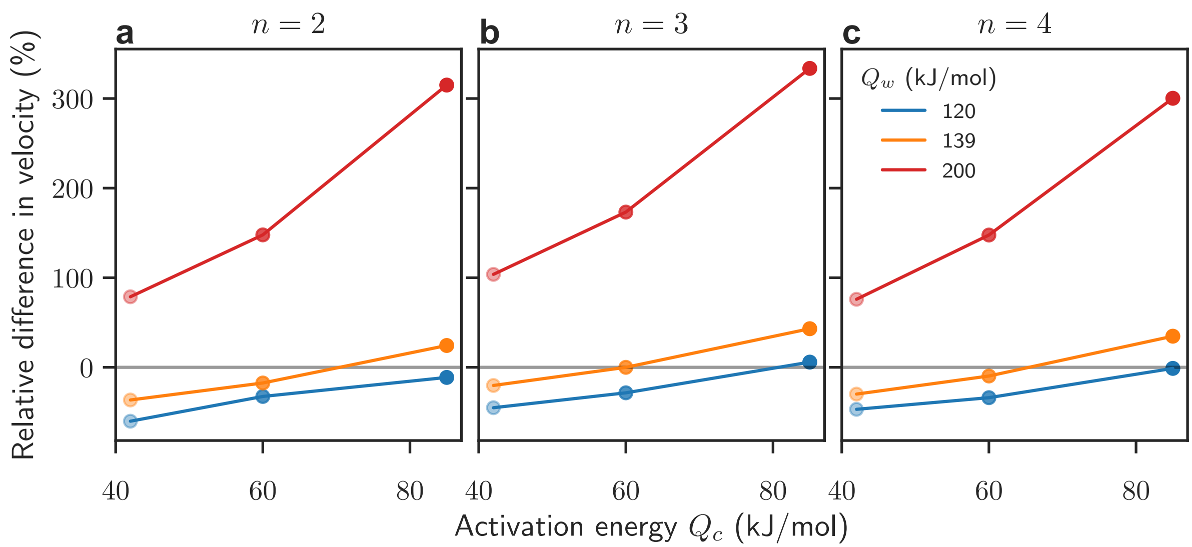

Figure 10. **Effect of the flow exponent and activation energies on mean velocity change after 100 years under 2°C of warming:** Relative difference in average surface velocity f or $n = 2$ **(a)**, $n = 3$ **(b)** and $n = 4$ **(c)** for different combinations of the flow exponent n and activation energies Q_c and Q_w. The reference is always a simulation performed with standard parameters $n = 3$, $Q_\mathrm{c} = 60$ kJ/mol and $Q_\mathrm{w} = 139$ kJ/mol. Variations in the flow exponent n do not significantly influence the relative difference of mean velocities after 100 years.

Variations in the flow exponent n do not change the qualitative effect of variations in activation ~~energy~~ energies Q on ~~difference in ice lossafter~~ the ice loss. After 100 years for a temperature anomaly of $\Delta T = 2°\mathrm{C}$ ~~(Figure ??a). In this setup a~~ a higher n seems to mitigate the effect of the activation energy on differences in ice loss~~after 100 years~~, while a lower n seems to enhance this effect (Figure 9). However, the effect of variations in activation energy on the ~~averaged~~ average surface velocity is almost independent of the choice for the flow exponent n (Figure ~~??b~~10).

The influence of the activation energies Q_c and Q_w on ice flow is similar even with different flow exponents n. This is robust for different warming scenarios from $+1$ to $+6\,°\mathrm{C}$. A higher flow exponent n, which leads to a more pronounced nonlinearity in ice flow, does not enhance but reduce variations in dynamic ice loss. This might be linked to the details of the parametrization in the flow law: As seen in both equations ~~and~~ (6) and (8), the factor A_0 decreases if the flow exponent n is reduced and $n_\mathrm{new} < n_\mathrm{old}$. Compared to the nonlinear stress dependency τ^n in the flow law the temperature dependent softness ~~$A(T') = A_0 \cdot \exp(-Q/RT)$~~ $A(T') = A_0 \cdot \exp(-Q/RT')$ becomes less important with increasing flow ~~exponents~~ exponent n.

4 ~~Discussion~~

~~In this study we present a first attempt to quantify the~~

3.1 Robustness of results to changes in accumulation and sliding

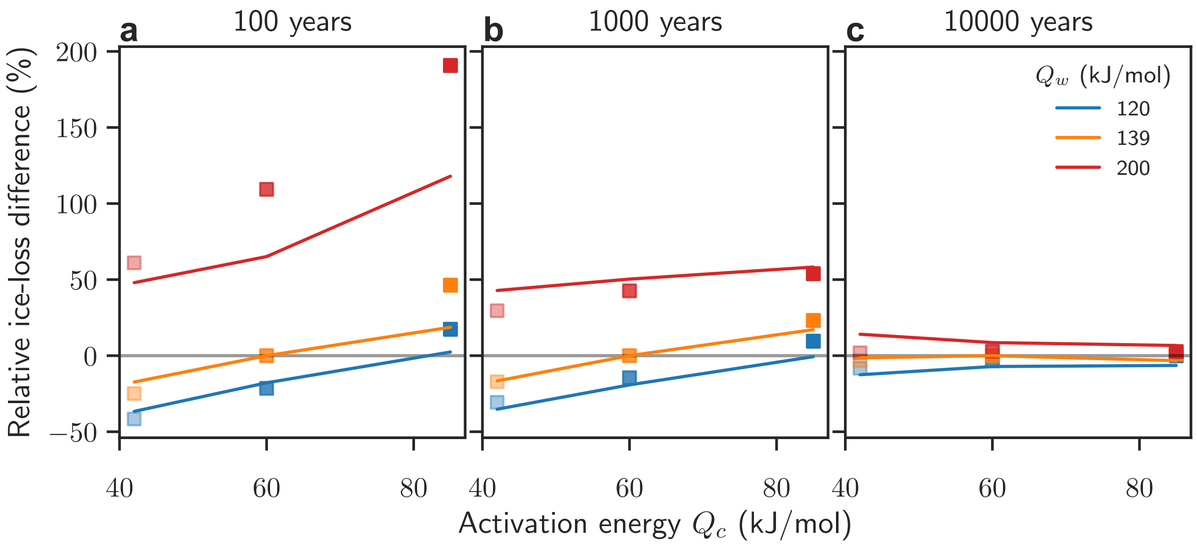

Figure 11. Effect of the flow exponent and activation energies on flow-driven ice loss under 2°C of ice warming in combination with a 50% reduction in accumulation rates: Relative difference in flow-driven ice discharge after 100 **(a)**, 1000 **(b)** and 10,000 **(c)** years. The ice sheet has reached a new equilibrium after 10,000 years. Relative difference for 2°C warming with an additional reduction of the accumulation rate of 50% (squares) are compared to the results without changes in the accumulation rate (lines, also see Figure 7).

The overall effect of uncertainties in the ~~flow law parameters onto ice dynamics, more specifically the flow-driven ice loss under warming. Uncertainties in the flow parameters alone account for a doubling in~~ activation energies Q remains robust, even if an additional driver of ice loss is taken into account. In a simulation where in addition to warming of 2°C we also reduce the accumulation rate by 50%, the ice losses remain dependent on the flow parameters Q_c and Q_w (Figure 11, lines indicate results without a change in accumulation rate, analogous to Figure 7 and squares indicate results with an additional 50% decrease in accumulation rate). After 100 years of forcing the relative spread of ice loss is slightly larger if accumulation changes are included. In particular, for $Q_w = 200\,$kJ/mol, the relative increase of mass loss mounts from 118% to 190%. On longer time scales, the spread in ice loss ~~during the first centuries of warming. The spread in flow-driven ice loss due to these uncertainties is on the same order of magnitude as the effects of increased temperature forcing~~is reduced (after 10,000 years of forcing, when the ice sheet has reached a new equilibrium, the relative spread is below ±10%).

~~The effect of the activation energies~~

When sliding is taken into account via the shallow ice approximation for sliding ice (see the PISM authors (2018)) the uncertainty in flow parameters leads to relative changes in ice loss from -30% to +470% after 100 years, which is a considerably larger spread than without sliding. The relative differences decrease with time, but remain ranger than without sliding. After 1000 years the ensemble member with low activation energies lost 40% less ice than the standard parametization and high

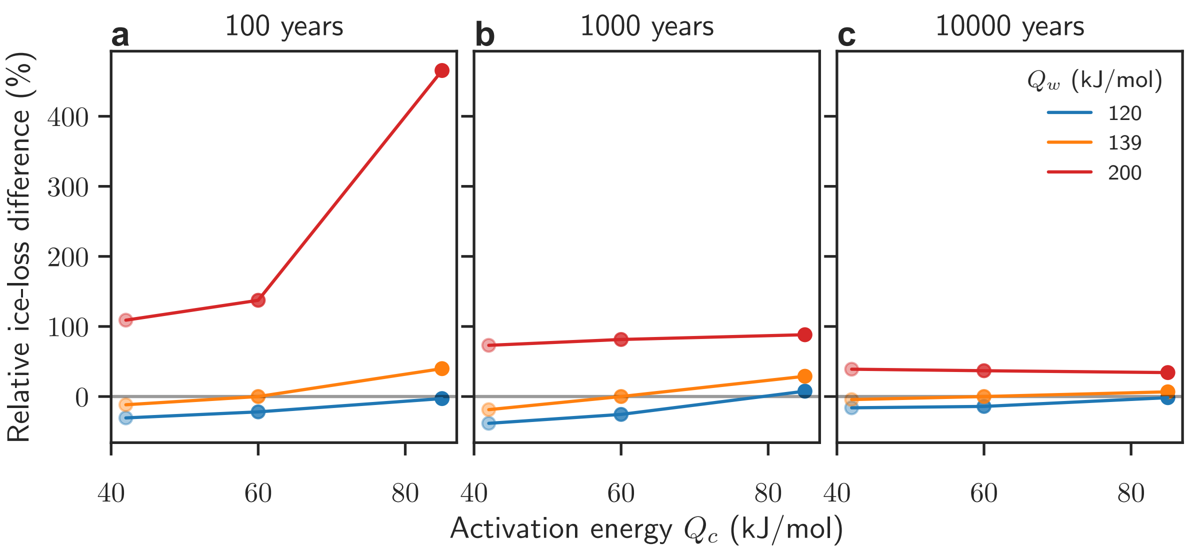

Figure 12. **Effect of activation energy on flow driven ice discharge under 2°C warming, including sliding:** Relative difference of flow and sliding driven ice loss after 100 **(a)**, 1000 **(b)** and 10,000 **(c)** years. The simulations have reached a new equilibrium after 10,000 years.

activation energies alsmost double the ice loss (+90%). After 10,000 years, when the ice sheets have reached a new equilibrium, the relative differences still range from -16% to +40% (see Figure 12).

4 Discussion and Conclusion

In this study we present a first attempt to disentangle and quantify the effect of uncertainties in the flow law parameters, in particular the activation energies Q ~~on ice dynamics (mean velocities and flow-driven ice loss) is stronger than the effect of the flow~~ and the flow exponent n~~. The activation energies determine how sensitive the softness of the ice is with respect to ice temperature. Parameters which increase the~~ , onto ice dynamics.

The effect of ice ~~softness close to the pressure melting point show to have the largest effect on ice dynamics.~~ rheology in ice-sheet models has been adressed in several studies with different experimental setups and different time frames. In particular the effect of the enhancement factors, which are often used to approximate the change in ice flow due to anisotropy, has been explored (Ritz et al., 1997; Ma et al., 2010; Humbert et al., 2005; Quiquet et al., 2018). In addition, the effect of the initial conditions (Seroussi et al., 2013; Nias et al., 2016; Humbert et al., 2005) and the effect of the mathematical form of the flow law itself (Quiquet et al., 2018; Peltier et al., 2000; Pettit and Waddington, 2003) have been studied. These studies have been crucial for the understanding of different enhancement factors in the shallow ice and the shallow shelf approximation (Ma et al., 2010), for the reconciliation of the aspect ratios of the Greenland Ice Sheet and the Laurentide Ice Sheet during the

last glacial maximum (Peltier et al., 2000) and the ice flow in Antarctica and the Greenland Ice Sheet (Ritz et al., 1997; Seroussi et al., 2013
.

~~The softness of~~ However, the ~~ice is not only dependent on the activation energies Q, but on the critical temperature where the ice changes from a cold to a warm regime. Uncertainties in the critical temperature are not easy to asses from literature. It still needs to be explored, how exactly the critical temperature influences the ice dynamics.~~

~~In this studywe chose an idealistic flowline geometry, which~~ approach presented in this manuscript is different in two important aspects: Firstly, the systematic study of not only the flow exponent n but also the activation energies Q has not been performed so far. Secondly, the idealized experimental setup, as presented in this study, allows to disentangle the ~~internal flow dynamics from other effects.~~ effects of the flow itself from other drivers and other sources of uncertainty. Several conditions need to hold to this end: The ice sheet is sitting on a flat bed and its maximal extent is determined by a calving front at the borders of the bed, thus no ice-ocean interactions or impacts of the bed geography influence the ice flow. Sliding is ~~inhibited,~~ generally inhibited (the ice dynamics is described by the shallow ice approximation, with zero basal velocity), no changes in sliding velocity influence the ice flow. The accumulation rate is fixed and independent of the temperature change, so that the ice loss is only driven by changes in flow and not by melting~~or sliding. While all those additional processes might reduce the importance~~ . These idealizations allow a clear understanding of the impact of the flow ~~parameters on computed ice loss, there are also possible positive feedbacks, if e. g. increased ice flow drives ice masses into regions with higher temperatures or into the ocean~~exponent and the activation energies on ice flow. In addition, they allow to compare the simulations of the polythermal ice sheet to the analytically solvable limit of an isothermal ice sheet by using the Vialov approximation.

~~Our analysis explores the effects of a long-overlooked uncertainty in the ice flow parameters . New experimental and data-analysis studies need to be conducted to further determine~~ In this setup the largest effect of the uncertainties in the flow parameters is observed in the first century after warming, while the effect of the uncertainties on ice loss becomes less important as the ice approaches a new equilibrium. Uncertainties in the activation energies ~~Q and the critical temperature for~~ alone account for up to a doubling in ice loss during the first 100 years of warming and are on the same order of magnitude as the effects of increased temperature forcing, under fixed surface mass balance. This effect remains robust, even if changes in the surface mass balance are taken into account. Reducing the surface mass balance by 50%, which is comparable to the ~~transition between warm and cold ice and further assess their influence on ice dynamics.~~

~~In this study, we try to disentangle the effect of variations in~~ changes in total surface mass balance of the Greenland Ice Sheet from 1972 to 2012 (Mouginot et al., 2019), increases the effect of the flow parameters on ~~the internal deformation of ice and find that the flow parameters are one of the determining factors for how strongly the Greenland and the Antarctic Ice Sheet are affected by anthropogenic warming, and thus need to be taken into account in future sea-level rise projections~~a timescale of 100 years and remains comparable on a timescale of 1000 years. Only as the ice sheet approaches its new equilibrium, the effect of the flow parameters becomes negligible. Allowing for not only flow but sliding while keeping all other conditions equal increases the effect of flow parameters substantially, leading to up to a five-fold increase in ice loss after 100 years compared with standard parameters.

~~Our idealized experiments in a flow-line setup reveal that~~ Acknowledging the uncertainty in flow parameters ~~can speed up flow driven ice loss up to two-fold on the timescale of centuries. For flow-driven ice loss in warming scenarios, the spread in the amount of ice loss increases with warming. The spread due to flow law parameter uncertainties can be as large or larger than the differences between different forcing temperatures.~~

~~In sea-level projections, naturally other processes such as sliding and melting are crucial in determining the overall mass balance of an ice sheet. This has strong implications for the Greenland and Antarctic ice sheets and respective sea-level projections:~~

~~First, incorporating the~~ might slightly shift the interpretation of previous studies. For instance, the effect of the initial thermal regime, as studied by Seroussi et al. (2013) could be enhanced if the the activation energies were higher than assumed, by making the ice softness more sensitive to changes in temperature. The crossover stress in the multi-term flow law presented by Pettit and Waddington (2003), at which the linear and the cubic term are of the same importance, is highly sensitive to the values of the activation energies. The positive feedback through shear heating, as studied for example by Minchew et al. (2018), could also be enhanced if activation energies were higher than usually assumed. The uncertainty in the flow-law parameters may further provoke a re-evaluation of other parameters, e.g. concerning melting and basal conditions. In particular, Bons et al. (2018) thorough analysis of observational data of the Greenland Ice Sheet supports a flow exponent of $n = 4$, not the standard value of $n = 3$~~. This is~~, which is is in line with recent ~~experiments which allow for large ice deformations in the lab, a regime similar to ice flow in glaciers, which also finds~~ laboratory experiments which also find $n > 3$ (Qi et al., 2017). ~~In turn, assuming~~ Assuming a higher flow exponent $n = 4$ ~~leads to a re-assessment of the basal velocity of the Greenland Ice Sheet, reducing the~~ has shown to significantly reduce the previously assumed area where sliding is possible ~~significantly (MacGregor et al., 2016).~~

~~Second~~(Bons et al., 2018; MacGregor et al., 2016). Moreover, both the flow exponent n and the activation energies Q feed into the grounding line flux formula (Schoof, 2007). In several ice-sheet models, this formula is used to determine the ~~movement~~ position and the flux over the grounding ~~lines~~ line in transient simulations (Reese et al., 2018). A change in the flow parameters n and Q ~~thus has additional~~ has thus implications for the advance and retreat of grounding lines ~~. This is especially true for~~ in simulations of the Antarctic Ice Sheet ~~, where the majority of the current ice loss is caused by ice flux across its grounding lines.~~

~~Third, this further affects the~~ and possibly the onset of the marine ice sheet instability, ~~which is particularly relevant~~ a particularly relevant process for the long-term stability of the Antarctic Ice Sheet.

~~Fourth, an uncertainty in flow might also mean that the ice is transported more quickly to the ice-sheet margins under global warming. In Greenland, where significant ablation happens at the margins increased ice transport might thus increase melting-induced ice loss.~~

~~Our results therefore imply that in~~ On the Greenland Ice Sheet increased ice flow might drives ice masses into ablation regions, where the ~~interplay of these processes, a potentially stronger flow response of the Antarctic and the Greenland Ice Sheet might increase their vulnerability to temperature changes~~ice melts. A possible effect of uncertainty in flow parameters on this particular feedback remains to be explored. Aschwanden et al. (2019) have found that uncertainty in ice dynamics plays a major role for mass loss uncertainty during the first 100 years of warming. While their study attributes the uncertainty mostly to large uncertainties in basal motion and only to a lesser extent to the flow via the enhancement factor, the uncertainties of the flow law and of the basal motion are not independent, as suggested by e.g Bons et al. (2018).

While the conclusions from the idealized experiments presented here cannot be transferred directly to assess uncertainty in sea-level rise projections, they are an important first step which allows informed choices about parameter variations in more realistic simulations of the Greenland or Antarctic Ice Sheet.

Code and data availability. Data and code are available from the authors upon request.

Author contributions. R.W. and A.L conceived the study. M.Z., A.L. and R.W. designed the research and contributed to the analysis. M.Z. carried out the literature review and the analysis. M.Z., R.W. and A.L. wrote the manuscript.

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Sensitivity of ice flow to uncertainty in flow law parameters in an idealized one-dimensional geometry
Supplementary material

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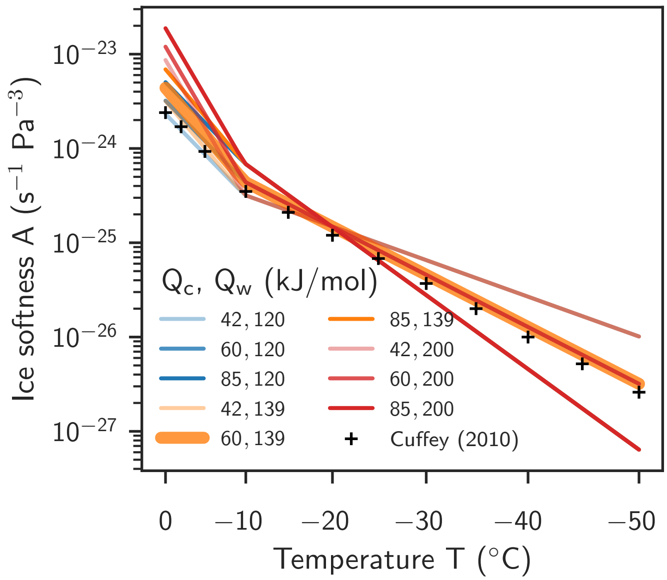

Figure S1. Effect of activation energy parameters on the temperature dependence of the softness A. The temperature dependence of the ice softness A is usually shown in an *Arrhenius plot*, where the softness is shown on a semi-log scale over the inverse temperature. Two parameters for the activation energy Q_c and Q_w for $T \leq -10\,^{\circ}\mathrm{C}$ and $T > -10\,^{\circ}\mathrm{C}$ parametrize the relationship of ice softness to pressure adjusted temperature. Here the softness is fixed at a reference temperature of $T = -20\,^{\circ}\mathrm{C}$. The softness at cold temperatures depends only on the choice of Q_c. Softness at pressure melting point is most sensitive for variations in the Q_w, but varies slightly with Q_c. At pressure melting point the softness increases 8-fold between the limits of parameter combinations, from -47% to + 335% compared to standard parameters. For comparison we show the textbook values of the softness for different temperatures (black crosses) (Cuffey and Paterson, 2010).

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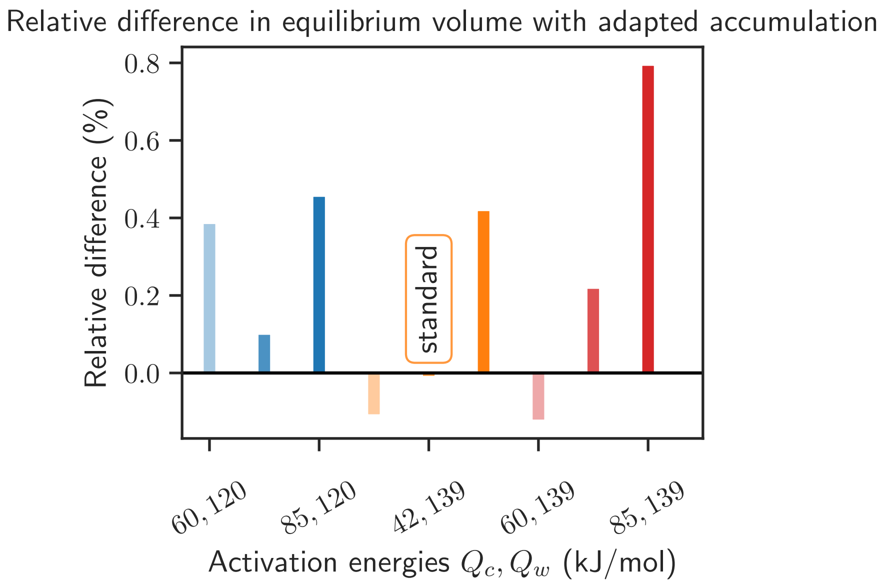

Figure S2. Relative difference in equilibirium volume with adapted accumulation rates

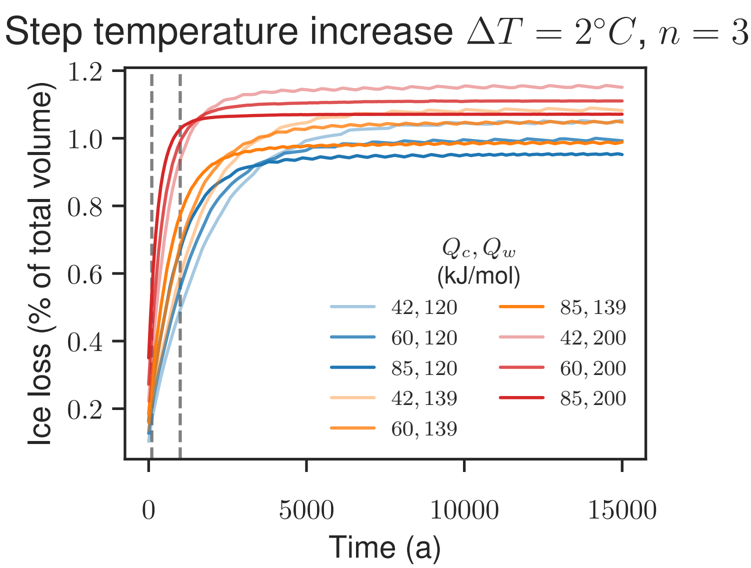

Figure S3. Effect of activation energy and flow exponent on flow-driven ice discharge : Ice loss in a conceptual flowline setup subject to a temperature anomaly forcing of $\Delta T = 2°\mathrm{C}$ and $n = 3$ in percent of the initial volume.