Authors' reply

T. Schlemm, A. Levermann

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We thank all referees for their helpful comments that stipulated further investigations into the gradient of the mélange thickness and mélange buttressing beyond the steady state. We improve the model presented in the manuscript by showing that it can also be solved without the assumption of a fixed mélange geometry. For application in glacier retreat modelling, an adaptive approach can be used in which the upper bound on calving rates is updated when the mélange geometry changes.
We first reply to the referees individually and then present the new work inspired by the referees' comments in the appendix.

1 Anonymous Referee 1

Main comments

The authors also assume that the ice mélange volume is in steady- state, and then apply their model to non-steady-state situations. That seems dangerous, especially without further justification. I don't understand the consequences of that assumption, which the authors also don't address. In particular, the parameter a is treated as a constant, but it depends on the width of the calving face, the width of the end of the ice mélange, the length of the ice mélange, the velocity of icebergs at the end of the ice mélange, and some unknown flow parameterization b. Most or all of these could change with time as the glacier terminus advances/retreats through a fjord and the ice mélange geometry evolves.
Response: The mélange buttressing model can be modified to allow free evolution of mélange geometry and this justifies using an adaptive approach in glacier retreat simulations, in which the upper bound C_{max} is calculated after each time step to account for the change in mélange geometry (see sections B and C).

Essentially, the authors started off with an assumption that there is a negative feedback loop between calving and ice mélange buttressing, and then demonstrated that their model produces a negative feedback loop. This also makes the title feel misleading. I think a more effective approach would be to ask "If ice mélange produces a negative feedback loop with calving, what properties must it have in order to appreciably affect tidewater glacier retreat?"
Response: We argue that there are good reasons to assume a negative feedback loop between mélange thickness and calving rates. We do not claim to prove the existence of this feedback loop, rather we show that this negative feedback loop causes an upper bound on the calving rate which depends on the embayment gemoetry (width and length) and on mélange properties (internal friction and exit velocity). Thus we do not prove that mélange buttressing exists (other papers show good evidence for this assumption) but rather show how it may effect calving rates. This means we essentially present a model for mélange buttressing, and that's why we find the title of the manuscript appropriate and ask the referee to allow it.

Minor comments

P1, L22: Most studies also neglect the impact of iceberg meltwater on ocean heat transport.
Response: Yes, that's true. We approach this from an ice-sheet-modelling approach rather than an ocean-modelling view-point, so we do not consider iceberg meltwater, either.

P3, L6: Amundson and Burton (2018) arrive at a similar result using a very different (continuum mechanics) approach to modeling ice mélange.
Response: Thank you for pointing this out.

P4, L25: This equation is ad hoc and, as written, not entirely consist with observations. Why does the ice mélange thickness have to equal the terminus thickness to prevent calving from occurring? In general, ice mélange thickness is considerably less than the terminus thickness. Note also that here d is used to refer to the effective ice mélange thickness, but later d cf is used to refer to the thickness at the calving front and substituted into this equation, which is confusing.
Response: As stated in our previous authors' reply, we assume now that calving is inhibited when mélange thickness has reached some fraction $h = \gamma H$, $0 < \gamma < 1$ of the ice thickness. This introduces the factor γ into equation (4) without changing the result qualitatively.

P6, L9-13: This is unnecessarily wordy. You could just write that conservation of mass dictates that $dV/dt = \ldots$, and then explain each of the three terms.
Response: We corrected this.

P6, L11-12: The overall rate of mélange volume "change"?
Response: Yes, this was corrected..

P6, L13: This equation shouldn't be set to 0, because its not until the next equation that you assume steady-state.
Response: Thank you for noticing this, we corrected it.

P6, L18: How does b parameterize the flow? Are you just suggesting that this is something that could be taken from observations? Please elaborate.
Response: The parameter b giving the mélange gradient along the embayment is now determined by linearizing the implicit exponential equation given in Amundson, Burton (2018). It then depends on the coefficient of internal friction of the mélange μ_0 (see section A).

P7, L8-9: "as also suggested by previous studies."
Response: This was included.

P13, L6-7: Please elaborate on what sort of observations could be made. How do you move forward from using steady-state assumptions?
Response: As section B shows, it is justified to use the steady-state model for glacier retreat if an adaptive upper bound on calving rates is used. Observations could further constrain the internal friction of mélange and the velocity of mélange exiting the embayment.

2 Douglas Benn

We thank Doug Benn for his thoughtful review and the positive feedback. The minor comments have been taken into account and corrections made.

3 Anonymous Referee 3

Major comments

Numerical experiment: Because the boundary condition on the sides of the channel are periodic, in this setup any potentially formed ice shelf would be unconstrained and therefore incapable of providing ice shelf buttressing. To some extent ice melange can be though of as a weak ice shelf with different rheology, and therefore melange buttressing will also be absent in a setup that does not allow ice shelf buttressing. I find the fact that a melange buttressing parameterization is tested in a setup that does

not allow for ice shelf/melange buttressing to begin with inconsistent. Using no slip boundary conditions on the side walls would solve this inconsistency. For the no slip wall case then, the effective melange buttressing can be diagnosed from the model and compared with observed and modeled values of melange strength.
Response: This is a misunderstanding. The setup has rocky fjord walls and where the bedrock wall is below sea level, there is grounded ice resting on it. The Spinup has an ice shelf constrained by these grounded ice walls which is exerting a buttressing force on the glacier. This is why the removal of the ice shelf leads to a rapid glacier retreat already without any calving parametrisation applied (MISI only in fig. 5). This will be clarified in the manuscript.

Simplified calving relations: Section 3.4 doesn't make much sense. Calving relations are simplified by a fitting a function to a region generated by considering different water depths and freeboards. This simplified relation is then used in the numerical simulation. Because the water depth is known exactly in a given setup (the numerical experiment) an exact calving relation should be used directly, rather than a fit to the range of values generated from multiple water thicknesses. If this were not computationally feasible, linearization locally using Taylor expansion should be used, not an arbitrary global line fit.
Response: The purpose of the simplifications is not to replace the full calving parametrisations in numerical simulations, but rather to be illustrative. The combination of a nonlinear calving relation and nonlinear buttressing makes it difficult to isolate the effect of mélange buttressing. The simplifications make the relation a bit clearer. We ask the reviewer to allow this.

Melange properties: The authors ignore the granular character of the melange. Because melange is a sea ice/ice berg mixture, it is its concentration that has bigger impact on its strength than its thickness. Thickness becomes relevant only when the concentration is close to 1. Yet, in this paper it is thickness that is the key variable in deriving the bound on calving rate. It should be either stated that concentration is assumed to be 1, which is unrealistic, or the melange concentration should be taken into account, perhaps by elaborating on the relationship between melange thickness and melange effective thickness.
The authors use the terms melange thickness and melange effective thickness interchangeably, however these are not the same. This has an effect on the mass conservation in equation 2. Because melange thickness is not melange effective thickness, the calving rate does not equal the rate of melange formation at the calving front. This needs to be addressed/corrected.
Melange flow and material properties are all lumped into one parameter b, it should be justified what the reasonable range of b is. There should also be a way to translate this parameter b to melange strength (under some assumptions) so that there is a clear way to evaluate the parameterization in the future when more observations become available. Also, as b is likely to be bounded because realistic melange has a finite maximum strength; this has implications for constraining the value of Cmax for a given embayment geometry.
Response: We have chosen to stick with the average mélange thickness rather than using an effective mélange thickness. The parameter b giving the mélange gradient along the embayment is now determined by linearizing the implicit exponential equation given in Amundson, Burton (2018). It then depends on the coefficient of internal friction of the mélange (which ranges from 0.1 to above 1) (see section A).

Minor comments

Forcing in the numerical experiments is unclear - why is the ice shelf removed throughout the simulations, rather than just at the initial time of each experiment?
Response: PISM tends to regrow shelves very quickly. If floating ice was removed only in the first time step, at least one cell of floating ice would regrow within the first simulation year and form the new glacier terminus. Since the calving parametrisations are applied only to grounded termini, the shelf would not be calved off and continue to grow. In order to prevent this spurious regrowth of a floating tongue, floating ice is removed at every time step.

Figures not well referenced through the text - there is a lot of statements floating around and it is unclear if they are based on a figure or equation or some previous work.
Response: We have corrected this in a number of places.

Please also note the supplement to this comment
Response: Comments in the supplement were taken into account and corrections made.

Next, we present some work that was inspired by the referees' comments and will be included in the revised manuscript.

A Mélange thickness gradient

In sec. 2 in the manuscript, the mélange thickness was assumed to thin linearly along the embayment length with $d_{cf} = \beta d_{ex}$. This can be justified as follows:
[Amundson and Burton(2018)] give an implicit exponential relation for the mélange thickness:

$$d_{cf} = d_{ex} \exp\left(\mu_0 \frac{L_{em}}{W} + \frac{d_{cf} - d_{ex}}{2d_{cf}}\right) \tag{1}$$

where μ_0 is the coefficient of internal friction of the mélange and ranges from about 0.1 to larger than 1. The embayment width, W, is assumed to be constant along the embayment in [Amundson and Burton(2018)], here we can replace it with the average embayment width. In a linear approximation, eq. 1 becomes

$$d_{cf} = d_{ex}\left(1 + \mu_0 \frac{L_{em}}{W} + \frac{d_{cf} - d_{ex}}{2d_{cf}}\right) \tag{2}$$

This equation has one physical solution for d_{cf}:

$$d_{cf} = d_{ex} \cdot \frac{1}{4}\left(3 + 2\mu_0 \frac{L_{em}}{W} + \sqrt{1 + 12\mu_0 \frac{L_{em}}{W} + 4\left(\mu_0 \frac{L_{em}}{W}\right)^2}\right) \approx \beta d_{ex} \tag{3}$$

The parameter β can be linearized to take the form

$$\beta = b_0 + b_1 \mu_0 \frac{L_{em}}{W} \tag{4}$$

where the parameters b_0 and b_1 are determined by the way of obtaining the linear approximation: Completing the square under the squareroot gives the asymptotic upper limit with $b_0 = 1.5$, $b_1 = 1.0$. Taylor expansion can be used to get a more accurate approximation around a specific value of $\mu_0 L/W$: expansion around $\mu_0 L/W = 0.5$ gives $b_0 = 1.11$, $b_1 = 1.21$ while expansion around $\mu_0 L/W = 1.0$ gives $b_0 = 1.17$, $b_1 = 1.11$. The choice of linearisation parameters b_0 and b_1 should depend on the expected range of values for $\mu_0 L/W$. Fig 1 shows that each of the linear approximations given in the text overestimates β slightly but that it is possible to achieve a small error ($< 5\%$) over a rather large range of values for L/W.

B Beyond a steady-state solution

The mélange buttressing model derived in section 2 of the manuscript assumes mélange to be in a steady state with a fixed mélange geometry. This implies a fixed calving front position. This assumption is not fulfilled if glacier retreat is considered.
If the mélange geometry changes in time, the change in the volume can also be expressed as:

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t} \int_0^{L(t)} \mathrm{d}x \, W(x)\, d(x,t) \tag{5}$$

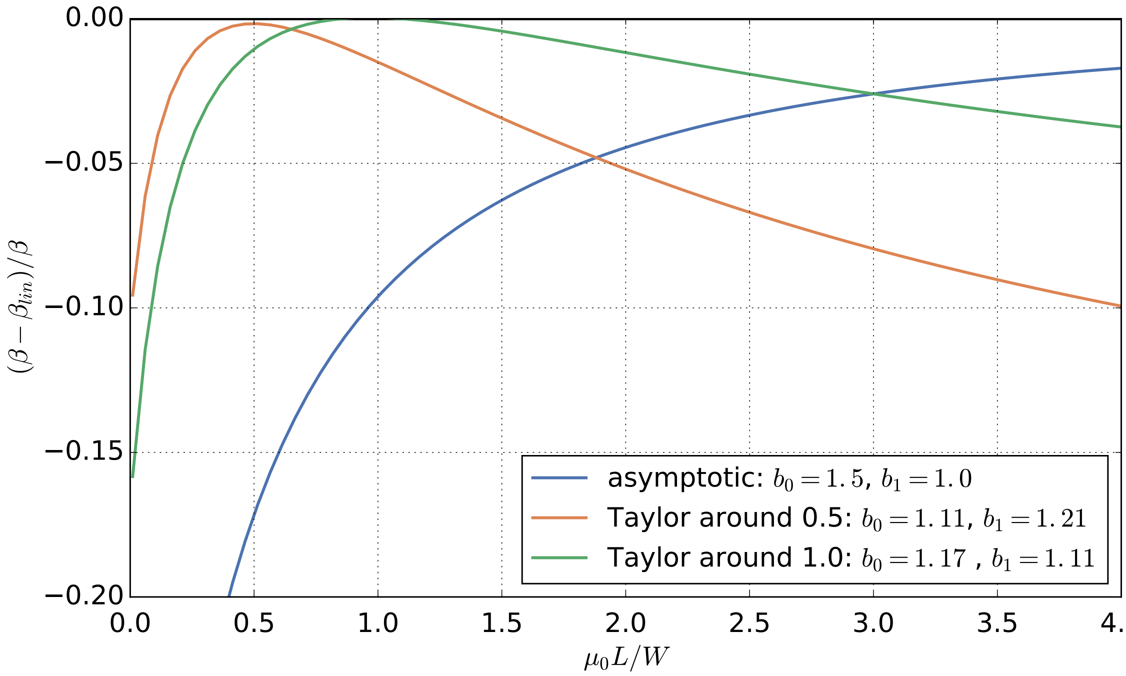

Figure 1: The relative difference between β given by eq. 3 and different linear approximations of β.

where $L(t)$ is the distance between the the embayment exit and the calving front, $W(x)$ the width of the embayment at a distance x from the embayment exit and $d(x,t)$ is the mélange thickness. This expression is equal to the sum of mélange production and loss terms given in eq. 2 in the manuscript. By applying the Leibniz integral rule to the volume integral of eq. 5 as well as rewriting the mélange production and loss terms as functions of time and calving front position, eq. 2 in the manuscript becomes

$$W(L(t))H(L(t))C(t) - W(0)d(0,t)u_{ex} - m\int_0^{L(t)} \mathrm{d}x\, W(x)$$

$$= \frac{\mathrm{d}}{\mathrm{d}t}(L(t)) \cdot W(L(t))\,\beta(L(t))d(0,t) + \frac{\mathrm{d}}{\mathrm{d}t}\left(\beta(L(t))d(0,t)\right) \cdot \int_0^{L(t)} \mathrm{d}x\, W(x) \qquad (6)$$

where the first three terms on the left hand side are the mélange production through calving, the mélange loss at the embayment exit and the mélange melting, respectively, and the left hand side is the rewritten volume integral. This differential equation for $d(0,t)$ can be solved if the embayment geometry $W(x)$ as well as ice thickness at the calving front $H(L(t))$ are known, the calving rate $C(t)$ is given by

$$C(t) = \left(1 - \frac{\beta(L(t))d(0,t)}{\gamma H(L(t))}\right) C^* \qquad (7)$$

and the change rate of the embayment length, $L(t)$, is given by

$$\frac{\mathrm{d}}{\mathrm{d}t} L(t) = C(t) - u_{cf}(t) \qquad (8)$$

where the ice flow velocity at the calving front, $u_{cf}(t)$, depends on the bed topography and the ice dynamics.

We will now consider an idealized setup with constant ice thickness, $H(x) = H$, as well as constant embayment width, $W(x) = W$, while neglecting ice flow by setting $u_{cf} = 0$. Eqs. 6 - 8 are solved numerically for the parameter values $H = 1000\,\mathrm{m}$, $W = 10\,\mathrm{km}$, $\mu = 0.3$, $\gamma = 0.2$, $C^* = 3\,\mathrm{km/a}$, $u_{ex} = 100\mathrm{km/a}$, $b_0 = 1.11$, $b_1 = 1.21$, and the initial conditions $L(0) = 10\,\mathrm{km}$ and $d(0) = 10\,\mathrm{m}$. We consider a scenario without mélange melting, $m = 0$, and a scenario with mélange melting, where the melt rate is set to $m = 10\,\mathrm{m/a}$ (see fig. 2). In the scenario without melting, mélange length and thickness at the calving front increase, while mélange thickness at the embayment exit and buttressed calving rate decrease. If melting of mélange is considered, the mélange thickness at the calving front increases initially, and then decreases until the embayment is mélange-free, since the volume of mélange melted increases with mélange area. A comparison between these solutions, where the mélange geometry is free to evolve, and the steady-state solution obtained by plugging the mélange length, $L(t)$, into eq. 4 and 6 in the manuscript, respectively, (see bottom panels of fig. 2) shows good aggreement. This is a justification for the adaptive approach discussed in next section.

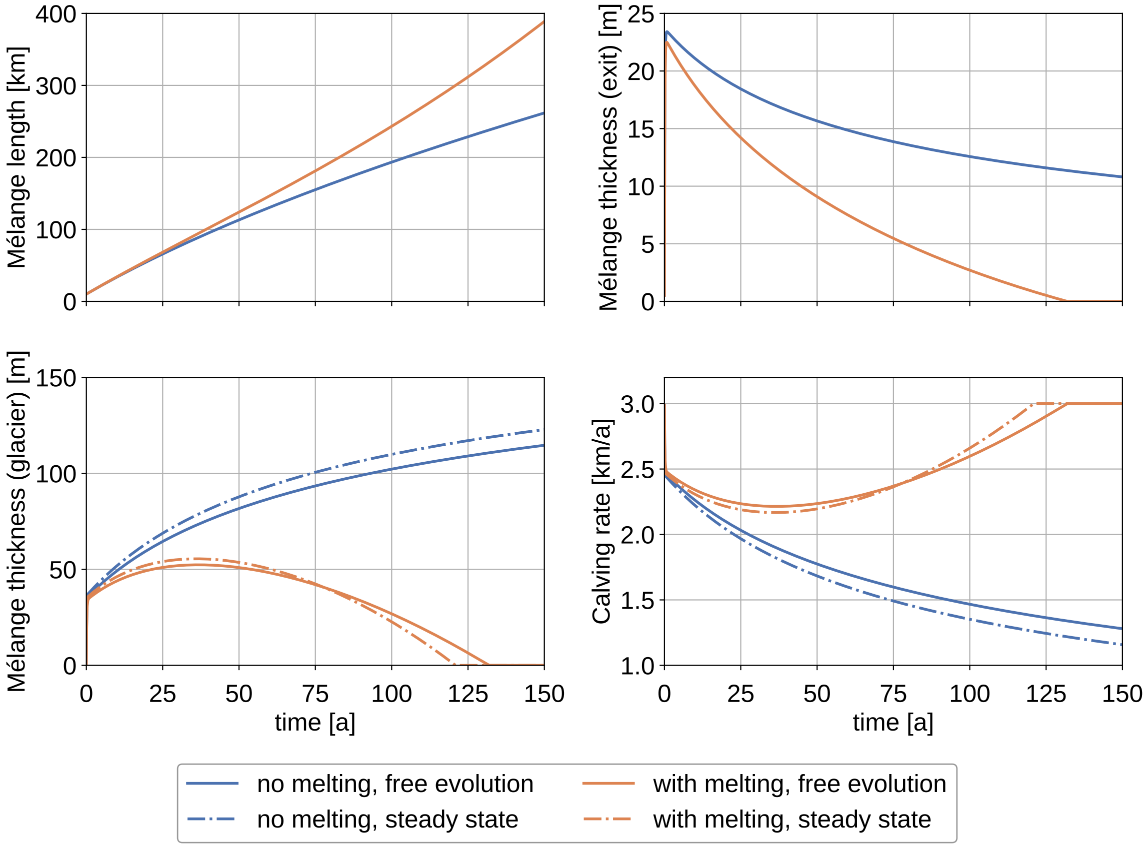

Figure 2: Top row panels show the numerical solutions of mélange length, $L(t)$, and mélange thickness at the embayment exit, $d(0, t)$, given by eqs. 6 - 8.. Two scenarios are considered: without melting (blue line) and with melting (orange). The bottom panels show the mélange thickness at the calving front, $d(L(t), t)$, and the resulting buttressed calving rate, $C(t)$. The solution with free evolution of the mélange geometry (continuous line) is contrasted with the steady-state solution (dashed line).

C An adaptive upper limit on calving rates

Assuming that mélange equilibration is faster than glacier retreat, the upper bound C_{max} can be calculated as a function of mélange length L_{em}.
Here we assume that the postion of the embayment exit remains fixed, so that the mélange length grows with the same rate with which the glacier retreats. We calculate a new C_{max} each year. and perform the same experiments as described in section 4 in the manuscript.
This slows down the glacier retreat significantly (compare fig. 3 to fig. 7 in the manuscript). In the case with C_{max0} equals $2.5\,\mathrm{km/a}$ prevents the complete loss of ice. The upper cound in calving rate is reduced to down to 30% of its original value (see fig. 4).

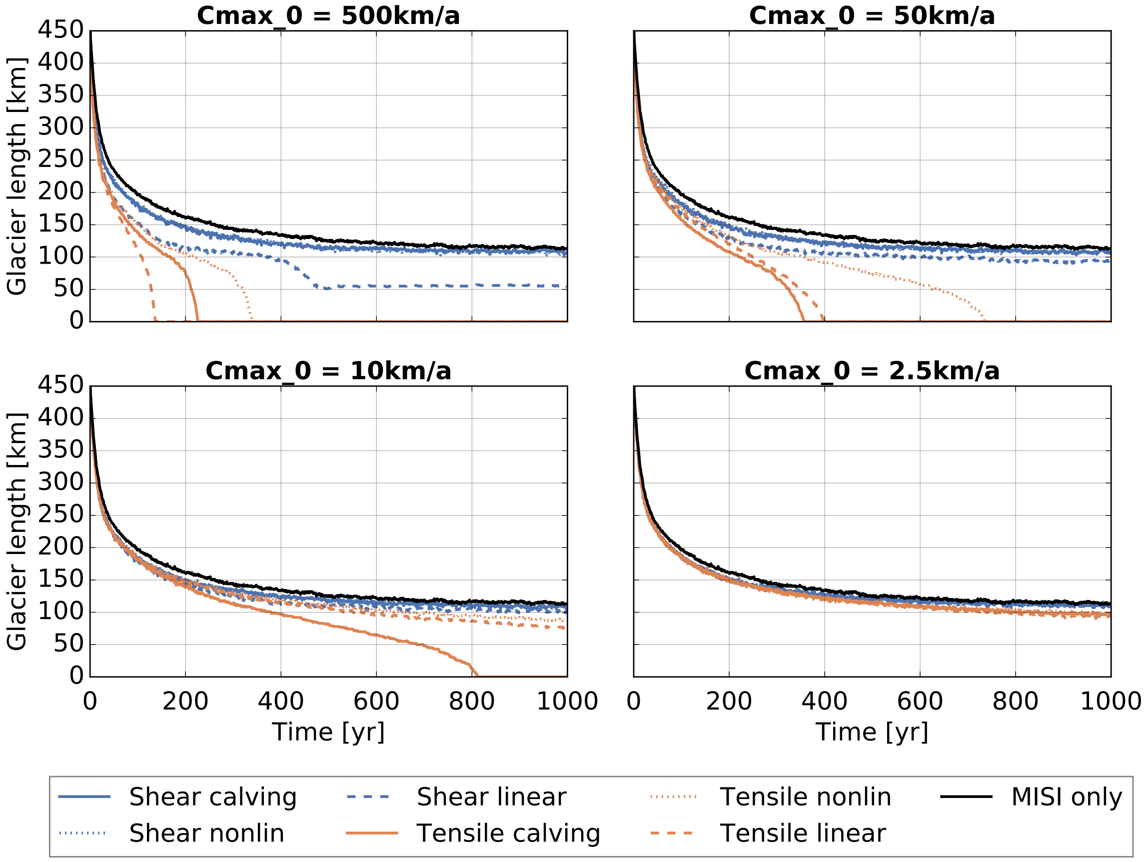

Figure 3: Glacier length timeseries with an adaptive calving limit.

References

[Amundson and Burton(2018)] Amundson, J. M. and Burton, J. C.: Quasi-Static Granular Flow of Ice Mélange, Journal of Geophysical Research: Earth Surface, 123, 2243–2257, https://doi.org/10.1029/2018JF004685, URL `https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2018JF004685`, 2018.

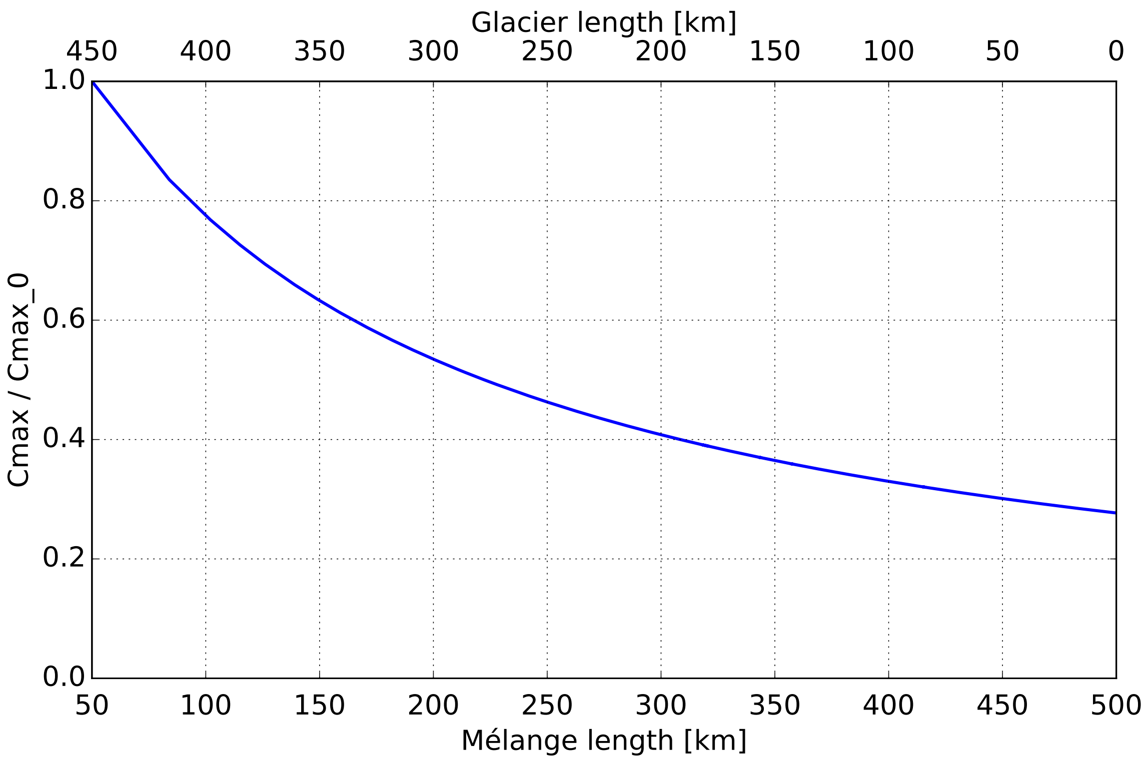

Figure 4: Reduction of the upper limit on calving rates as a function of mélange length and glacier length.