Author's response to Referee Comment 3

I sincerely appreciate the detailed comments and suggestions provided by the reviewer as Referee Comment 3. This input helped me in rewriting the manuscript to make it more accessible for readers of The Cryosphere and others. In my personal view, thanks to the efforts of the reviewer, the manuscript has now improved significantly from the initial submission. My detailed responses to the list of review items are provided below, with citations from the present referee comment in italics. In response to some of the concerns raised by the reviewer, the revised manuscript will include two appendices: Appendix A and B are attached at the end of this document.

Major revision items

1. ***Section 2.1 needs major rewriting.*** *Since the author aims to maximize generalization, the description is too generic and therefore it is not clear (until you read section 3). A not exhaustive list of suggestions (for improvement) are:*

   - *State more explicitly and since the beginning (e.g. at lines 33, in the paragraph at lines 39-45, and then at line 47) that you aim to verify the displacement over time of the sea-ice edge, and compare the forecast versus observed displacements. (e.g. at line 47 the phrasing "the difference in maximum edge displacement between two products" can be mis-interpreted as the "distance between observed and forecasted ice-edges").*

     The reviewer is correcting when pointing out that this aspect, which is central to the present work, was not given proper attention in the original manuscript. In the present revision this aspect is now described explicitly near the end of the Introduction section, and in the first paragraph of the following section.

   - *Do not use "product". You can use "dataset", "ice-edge" or other terms. You can as well refer to model (or forecast) and observation, explicitly.*

     I agree that the reviewer's suggestion is a better choice of words, and I have replaced 'product(s)' nearly everywhere in the manuscript, mostly by substitutions with 'data set(s)'.

   - *The titles of Sections 2.1 and 2.2 should include "displacement" somewhere, e.g. "2.1 verification measures of the displacement of a single ice-edge" and "2.2. comparison of the displacements of forecast and observed ice-edges"*

     I have rephrased the titles in question along the lines suggested by the reviewer. I have however used slightly different formulations.

   - *Lines 58-59: "$L^{(1)}$,$L^{(2)}$ denote the sea ice edges for two representations..." is not clear at all: here* ***you need to state explicitly here that (1) indicate the time t_0 and (2) indicate the time t_0+Δt, and that you measure the ice-edge displacement that occurred between the times t_0 and t_0+Δt*** *(for either the forecast or the observation ice-edge). In fact, I suggest to change all the notation here (otherwise the reader will naturally associate 1 for forecast and 2 to observations), replacing (1) with (t0) and (2) with (t0+Δt), and then $d_n^{2:1}$ can become $d_n^{\Delta t}$.*

     The description on lines 58-70 was meant as a general approach for computing distances between any two lines, e.g., edge lines in two different data sets or edge lines at different times within the same dataset. Nevertheless, I realize that this can potentially give rise to some confusion. Consequently, I have decided to follow the reviewer's suggestion, and later add a comment that the approach is not only valid for distances between edge lines at two different times, but also generally, for any set of two edge lines.

   - *The mathematical equations (3) and (4) need to be defined in a more mathematical rigorous way. Especially equation (4), the sign is not accounted for in the mathematical formula and there is a* *missing absolute value* $|.|$.

Regarding Eq. (3), I agree that there was a room for improvement. Consequently, I have rewritten the contents on l. 55-63 ("Denoting [...] distance of z.") to

Denoting the N grid cells that satisfy this condition for time t by $e_1^{(t)}, e_2^{(t)}, \ldots, e_N^{(t)}$ the ice edge for time t is then the line

$$L(t) = \{e_1^{(t)}, e_2^{(t)}, \ldots, e_N^{(t)}\}$$

This follows the algorithm presented in Melsom et al. (2019). Let $L(t_0), L(t_0 + \Delta t)$ denote the sea ice edges at times t_0 and $t_0 + \Delta t$, respectively. Furthermore, let $d_n^{\Delta t}$ be the displacement distance between a grid cell $e_n^{(t_0+\Delta t)}$ in $L(t_0 + \Delta t)$ and line $L(t_0)$, *i.e.*,

$$d_n^{\Delta t} = s_n \min ||e_n^{(t_0+\Delta t)} - L(t_0)||$$

where s_n is +1 or -1 when $e_n^{(t_0+\Delta t)}$ is on the no ice or ice side of $L(t_0)$, respectively, explicitly defined by Eq. (4) below. $||z||$ is the Euclidean distance of z, and the length of dashed lines in Fig. 1 correspond to the displacements $\min ||e_n^{(t_0+\Delta t)} - L(t_0)||$ for selected cells $e_n^{(t_0+\Delta t)}$.

If we denote the sea ice concentration at the time t_0 for a grid cell $e_n^{(t_0+\Delta t)}$ (belonging to the ice edge at $t_0 + \Delta t$) by $c[e_n^{(t_0+\Delta t)}](t_0)$, s_n is given as follows:

$$s_n = \begin{cases} -1 & \text{if } c[e_n^{(t_0+\Delta t)}](t_0) \geq c_{edge} \\ +1 & \text{if } c[e_n^{(t_0+\Delta t)}](t_0) < c_{edge} \end{cases}$$

Regarding Eq. (4), I realize that the description of the quantity could lead to confusion, and I have rewritten (l. 63) "maximum distance" as "maximum expansion displacement". Eq. (4) is correct, as it returns the intended value. If $d_n = 1, 3, -5, d_{max} = 3$, and if $d_n = -1, -3, -5, d_{max} = -1$. This was by design, and was detailed on l. 65-67. The rationale for this definition was stated on l. 66-67 ("The definition of s was designed so that $d_{max}^{2:1}$ will represent the displacement of the largest sea ice advance from $L^{(1)}$ to $L^{(2)}$.") The paragraph has been slightly rephrased in the revision, to make this even more clear.

- *Lines 62-63: the text is not precise; you need to state that in Eq. (3) you consider the minimum of all distances (between the single point in the edge at time $t_0 + \Delta t$ and all points of the edge at time t_0). Similarly, for Eq(4) you need to state that you consider the max (over all n) of the $d_n^{\Delta t}$*

  In the revision I have rewritten these items as suggested by the reviewer.

- *Lines 65: I believe this should state " ... will return the largest absolute positive value ..."*
  AND

- *Line 66: what happens if the $d_n^{2:1}$ are partially positive and partially negatives? Mention the canceling errors.*

  As explained for the item "*The mathematical equations* ... **Regarding Eq. (4)**" above, the statement the reviewer's assumption regarding l. 65 is wrong. This should be clear from the revisions introduced in response to that item. Regarding the question with reference to l. 66, this was in fact described on l. 66 ["Eq. (4) will return the largest positive value among $d_n^{2:1}$", i.e., if there is a mix, the largest positive value is returned, which is simply spelling out the definition provided in Eq. (4)]. I fail to see any relevance of canceling errors in this context.

2. *Section 2.1, Lines 70-75: **Table 1 (as well as Table 2** in Section 3) lists the frequencies of the histograms of the set of distances $d_n^{2:1}$ defned by Eq. (3), for the idealized model and observation ice-edges of figure 1. This is not clear from the text at line 75, nor from the caption (which I believe has a mistake, since it should refer to Eq. 3, and not Eq. 4) and the heading of the table is wrong (it should state "Frequencies" rather than "fraction of grid cells"). Here are my suggestions:*

- ***Representing a distribution with a table is quite unconventional: I strongly recommend plotting the histograms of the distances $d_n^{2:1}$ instead.*** *(You can choose if to leave the table or eliminate it, but please add the histogram). Please show the histograms also for Table 2.*

  These results are now shown as two histograms, which are displayed as Fig.s 2 and 5 in the revised manuscript.
- *Please rephrase the caption ("category distribution of displacement distances" is not clear; this can simply be "distribution of ice-edge displacements").*

  The caption has been rewritten as suggested by the reviewer.
- *Please rephrase lines 71-74: you can shrink it all as a couple of sentences such "The maximum distance in Eq. (4) provides a single measure to examine the ice-edge displacement. However, it can be more informative to analyse the whole distribution of the displacements $d_n^{2:1}$ defined by Eq. (3), rather than their maximum only. This can be done by analysing the histogram of the displacements (Figure HIST) and its corresponding frequencies (Table 1); the distribution of the displacements $d_n^{2:1}$ can as well be represented by their cumulative probability distribution (Figure 2)."*

  I agree that the reviewer's suggestion is a better choice of words, and have chosen to adopt it (with very slight modifications) in the revised manuscript.
- *Please rephrase line 75: "the distribution of selected distance categories" is not clear.*

  I have rewritten the entire paragraph in question (i.e., l. 71-77), and this phrase is no longer included in the manuscript.

3. *The* ***decorrelation length****, used to sub-sample the ice-edge, is lightly mentioned at lines 78-80. It is thereafter used (e.g. caption of Figure 2, and then more heavily at lines 110-115), however a more thorough definition and how the author calculate Δn is missing in the article. Please add some text about it (maybe this can be done in an appendix).*

   An appendix (A) with the mathematical formulation for computing the decorrelation length is included in the revised manuscript, as suggested by the reviewer.

4. *Section 2.3 is too long, needs rewording, and need to be accompanied by a visual example. I strongly suggest to:*

   a *summarize it in few sentences, such as in Melsom 2019, page 617, left column third paragraph ("A variant ..."). In the re-wording it is important that you still retain the explanation at lines 138-139, where you correctly state that you expect the distances to be smaller when adding ocean open boundaries and coastal lines (because when adding these artificial "fixed" edges you automatically include in your verification some perfectly matched edges, aka trivial skill).*

   b ***Include subsection 2.3 in Section 3****, after the data description and prior the results (aka after line 173 and before line 174). You can actually split section 3 in three subsections: 3a Sea-ice data description, 3b open boundaries and coasts, 3c verification results. In this fashion the reader has an immediate visual example (figure 3) on your need of adding these fictitious boundaries (especially when considering an Arctic sub-region).*

   c *The example presented at lines 180-187 and illustrated in figure 4 is excellent!*

   These suggestions for changing the structure of the manuscript are well explained. Consequently, the revised manuscript is modified in response to items a and b, albeit somewhat differently from the reviewer's recommendations. I have decided to move the original subsection 2.3 to a new Appendix (B), where I also include a conceptual figure in order to illustrate the modifications to the algorithm in Section 2. I have also split Section 3 into two parts (not three, since the modifications due to open boundaries and coastlines are now extensively detailed in Appendix B).

5. *Give a new title to Section 3: Application of the new verification method on sea-ice forecast (this is not a “case study” ... also at line 211).*

   The title has been rewritten, using a slightly shorter phrase than proposed by the reviewer: “Application of the new validation method”

6. *Lines 115-119:* ***rather than considering the ranks****, and a variable numbers of bins, I suggest using the* ***quantiles****, which have a fix range in [0,1] (or which range between 0 and 100, if you consider centiles): in this fashion it is more natural to aggregate and compare (you avoid any issue related to the variable binning).*

   The use of quantiles makes sense when the degrees of freedom allows a large number of statistically independent values to be included in the analysis. In the present analysis, this is related to the decorrelation length Δn along the ice edges. For the results in Section 3, the number of independent displacement distances is variable, and not large. This was why a rank histogram analysis using 10 bins was applied. An explanation along these lines have been added in the revised manuscript. The issue is further highlighted in a new analysis, when the domain is split in two (in response to a suggestion from another reviewer). In that case, the number of bins needed to be limited to eight in order to keep the majority of the dates in the analysis.

   Consequently, I have kept the approach of using rank histograms in the manuscript.

7. *Lines 120-126: you need to state here that the larger is the quantile (or rank), the better the geographical correspondence between maximum observed displacement and max (or at least large) forecast displacement.*

   I agree, in the revised manuscript I have included two sentences where these general properties are explained.

8. *Lines 199-206: the expected histogram for a random process would be a flat histogram (each bin is equiprobable), and you could compare your histogram to a flat one as described in Wilks (2019). I do however suggest to simply describe visually the intuitive result: your histogram in figure 5 shows a mode for the highest rank, which shows some skill in detecting the location of the maximum displacement. Rephrase (and join) these paragraphs. The reference is:*

   - *Wilks, D, 2019: Indices of Rank Histogram Flatness and Their Sampling Properties. Mon Wea Rev, 147: 763-769. https://doi.org/10.1175/MWR-D-18-0369.1*

   I have rewritten, and joined, the two paragraphs in question, trying to follow the reviewer’s advice. The reference to equiprobable bins is included in Sect. 2.2. in the revised manuscript. A reference to Wilks (2019) has been included.

Minor (technical) revisions

1. *Lines 3 (and line 5): I suggest replacing “expansion” with “decline” and replacing “advancing” with “retreat”. This is because in the first few sentences the authors relate with climate change, so it sounds a bit counter-intuitive to talk about sea-ice expansion (after reading the article it is clear that the methodology applies to both spring melting and autumn freeze-up, but in the abstract and introduction -if you maintain the climate flavour- you might prefer focussing on “decline”)*

   This suggestion definitely warrant attention. The reference to climate change is made in response to the Arctic being expected to become a region with more activity over the coming decades, i.e. the phrase is a motivation for the topic at hand, but not to the present theory per se. The present work relates to evolutions over much shorter time scales, and the abstract of the present revision reflect this. The focus here is on the relevance for forecasted expansion of sea ice, which is a hazard for open ocean operations in a polar environment. This was already reflected in the original manuscript, by the mathematical definitions in section 2.1, and in the text by e.g. pointing out that this leads to a *one-sided Hausdorff distance* variation, and also indicated by the choice of words elsewhere such as in the Abstract. Nevertheless, this reviewer comment, along with the question regarding Eq. (4) in Major item 1, made me realize that this aspect needs to be spelled out explicitly in Sect. 2. Consequently, I also rewrote the start of Sect. 2.1 to further highlight the focus on expansion.

2. *Lines 28-32: citing literature from the verification community is welcome! You could add to Ebert and McBride (2000) also Davis et al (2006a,b), since MODE is now the most commonly used object-oriented verification method in the weather community. You could as well add that both these methods were designed for (several) precipitation-like (convex-shaped) features, and that there is no equivalent for a single linear feature such as ice-edge.*

    a *Davis, C. A., B. G. Brown, and R. G. Bullock, 2006: Object-based verification of precipitation forecasts, Part I: Methodology and application to mesoscale rain areas.* Mon. Wea. Rev. *textbf134, 1772 - 1784, doi: 10.1175/MWR3145.1.*

    b *Davis, C. A., B. G. Brown, and R. G. Bullock, 2006: Object-based verification of precipitation forecasts, Part II: Application to convective rain systems.* Mon. Wea. Rev. *textbf134, 1785 - 1795, doi: 10.1175/MWR3146.1*

    Thanks! In the revised manuscript a reference to Davis et al. (2006a) is added.

3. *End of line 33: please be specific in stating that "We begin this study by presenting a new algorithm for assessing the quality of representation of the displacement over time of the sea-ice edge ... "*

    This issue was attended to in relation to the reply to the first bullet point under Major item 1.

4. *Line 34-36: please rephrase these two sentences to align with the suggestion of major revision 4b.*

    The final paragraph in the Introduction is rewritten in the revised manuscript in order to reflect the changes in the manuscript's composition.

5. *Line 39: replace "some idealized distributions are ..." with "an idealized ice-edge is"*

    The phrase in question is rewritten in the revised manuscript, to "a set of idealized ice edges is".

6. *Line 49 (and 70): I am sure the Hausdorff distance was introduced earlier than Dukhovskoy et al (2015), can you please provide the original reference?*

    The original reference was not provided in any article where I've seen the Hausdorff distance applied (e.g. not in Dukhovskoy et al (2015), and not in either of the four articles that are cited on p. 5914 in that paper). According to Wikipedia (`https://en.wikipedia.org/wiki/Grundz%C3%BCge_der_Mengenlehre`), the original reference is
    Hausdorff, F.: Grundzüge der Mengenlehre, First edition, Verlag von Veit & Comp., Leipzig, Germany, 476pp, 1914
    This book is available from `https://archive.org/details/grundzgedermen00hausuoft/page/n5/mode/2up`
    This is a book of nearly 500 pages, and in German which is not my strongest side. After spending half an hour browsing the book half-blinded due to my limited skill in German, I was not able to find the definition. Consequently, I choose to refrain from listing this citation (but would be happy to include it, should the editor encourage me to include it). In any event, I find the description of the Hausdorff distance in Dukhovskoy et al (2015) useful, accessible, and easy to comprehend.

7. *Lines 78-83: you can join these two sentences in a single paragraph. You should replace "when time serie results are examined" with "when results aggregated over multiple cases over an extended time period", or something similar (the point is that you aggregate multiple cases, and not that you consider a time serie)*

The sentences have been rewritten, and put in a single paragraph, along the lines suggested by the reviewer.

8. *Lines 85-87: rephrase these two sentences to twin them with the beginning of Section 2.1, something like "In the previous section we focussed on measures which describe the displacement of a single (forecast or observed) ice-edge. In this section we extend these to assess the differences in the displacements of the forecast versus observed ice-edge."*

   I have rewritten the first paragraph of Sect. 2.2 along the lines suggested by the reviewer. However, the second sentence is kept (slightly rephrased), in order to link this paragraph explicitly to the set of ice edges displayed in Fig. 1.

9. *Rephrase lines 88-89.*

   These lines have been rephrased.

10. *In the caption of Figure 2, lines 3-4, replace "the mean separation distance difference" with "A measure of the overall displacement difference" (be careful, that grey-shaded integral is not the mean displacement difference). The idea of using the area between the two curves (the grey shaded area) is excellent!*

    For clarity, I have added "for the present subsample of ice edge grid cells" to make clear that I refer to the subsample, and not the full set. Aside from that, the integral indicated by the grey-shaded area is the mean displacement difference (when using the axis units in integration), as it is displayed for fractions (from 0 to 1): Say e.g. that the difference in displacements are 10 grids everywhere. Then, the area (the gray shaded integral) and the mean separation distance difference will both be 10 grid cells.

11. *Line 92: replace "property" with "attribute" (verification term).*

    Rewritten as suggested.

12. *Line 93: I suggest writing "a simple measure which provide this type of information is ..."*

    Rewritten along the lines suggested.

13. *Line 99: replace "site-specific" with "local".*

    Rewritten as suggested.

14. *Lines 99-100: write "... of the model ice edge in proximity of the maximum displacement found in the observations".*

    Rewritten as suggested.

15. *Line 110: "In order* to *examine ..."*

    Added 'to' as pointed out.

16. *The original reference for the rank (Talagrand) histogram is Talagrand et al (1999) and Anderson (1996)*

    a *Talagrand, O., R. Vautard, and B. Strauss, 1999: Evaluation of probabilistic prediction systems.* Proc. Workshop on Predictability, *Reading, United Kingdom, ECMWF, 125.*

    b *Anderson, J. L., 1996: A method for producing and evaluating probabilistic forecasts from ensemble model integrations.* J. Climate, ***9**, 15181530*

These references are added. Thanks!

17. *Line 171: change "integrated" to "interpolated" or "upscaled" (I imagine it is a mass-conservative upscaling from the ice charts at 1km resolution to the SVIM 4km resolution domain). Is the smoothing (the second order checkboard suppression mentioned at line 161) performed on the observations too?*

    "Interpolation" is an estimation of an intermediate value based on a set of discrete existing values. I find this to be an improper description here, as grid cell values do not represent a point, but the average for a grid cell. I suppose "upscaled" is correct, but somewhat lacking in precision. At any rate, I find "integral" to be the proper designation here. For clarity, I rewrote the relevant passage to "results are integrated onto the SVIM domain using a mass conserving Riemann integral approach". The smoothing is not performed on the observations, since the data algorithm that produces the two-dimensional representation of sea ice concentration is not subject to the type of noise which arises from numeric dispersion from the model configuration. This is stated in the revised manuscript.

18. *Line 172: what is meant here with "dry"?*

    "Dry" has been rewritten to reflect that these are cells that lack proper values (due to the presence of land in the present context).

19. *Line 180: Eliminate "Category" and write "The distribution frequencies in Table 2 change only moderately when including open ocean boundaries and coastal lines."*

    Rewritten along the lines suggested; "Table 2" becomes "Fig. 5" and I have chosen to retain the reference to the algorithm, which now is detailed in Appendix B.

20. *Line 185: replace "unreasonable results" with "mis-matches ice-edge points"*

    "gives unreasonable results" has been rewritten as "mis-matches ice edge grid cells".

21. *Rephrase lines 188-189: I suppose you consider a fix number of nine ranks (and not nine randomly chosen points of the ice-edge).*

    The paragraph with these lines have been rephrased. This is where I set the rank size for the application example, as the reviewer points out (albeit to a rank size of ten, not nine). In the following text, the random draw required for the analysis is described, and an explanation for this particular choice of rank size is given.

22. *Lines 193-198: please consider using quantiles, rather than ranks, as suggested in major comment 6.*

    I disagree that the use of quantiles is a better approach, as explained in my response to major comment 6.

23. *The first paragraph of the conclusion need polishing / rephrasing.*

    The first paragraph has been rephrased. Note that in the revision the paragraph has also been rewritten in response to comments from another reviewer.

Appendix A. Decorrelation length of displacements

Assume that a we have a set of N edge grid cells e_n (i.e. satisfying Eq. (1)) that form a line

$$L = \{e_1, e_2, \dots, e_N\} \tag{A1}$$

where L is continuous in the sense that grid cells e_n and e_{n+1} are neighbors. Furthermore, associate displacement distances d_n to each e_n as defined in Sect. 2.1. Then, the spatial autocorrelation of displacements can be estimated using a sample Pearson correlation coefficient approach:

$$r(\eta) = \sum_{n=1}^{N-\eta} [(d_n - \overline{d_n})(d_{n+\eta} - \overline{d_{n+\eta}})] \Bigg/ \left[\sum_{n=1}^{N-\eta} (d_n - \overline{d_n})^2 \sum_{n=1}^{N-\eta} (d_{n+\eta} - \overline{d_{n+\eta}})^2 \right]^{1/2} \tag{A2}$$

We have $r(0) = 1$ and we define the decorrelation length of the displacements, Δn, as

$$\Delta n = \min_{\forall \eta} (\eta \mid r(\eta) < 1/e) \tag{A3}$$

where e is Euler's number. If, for a given time, the ice edge is discontinuous, each continuous line segment is treated separately, and the weighted mean value of the results for Δn from each segment is used. In that case, weights are applied according to the number of edge grid cells in each line segment.

Appendix B. Open boundaries and coasts

As discussed by Melsom et al. (2019), open boundaries and coastlines can potentially have significant impacts on the results for the metrics for sea ice displacements. Here, we introduce a method which will give more physical meaningful results if ice either freezes near a coastline, or enters into a domain across an open boundary. Moreover, the method affects the results modestly or not at all for an edge that is displaced inside of the domain.

First, set the open boundary grid lines to

$$L^{OB}(t_0) = \{e_{1_{OB}}, e_{2_{OB}}, \ldots, e_{N_{OB}}\} \quad , \quad c[e_{n_{OB}}](t_0) < c_{edge} \qquad \text{(B1)}$$

where $e_{n_{OB}}$ is any ocean grid cell along the boundary of the domain which was on the open ocean side of the ice edge at $t = t_0$. Then $L(t_0)$ in Eq. (3) can be replaced by

$$\widetilde{L}(t_0) = L(t_0) \cup L^{OB}(t_0) \qquad \text{(B2)}$$

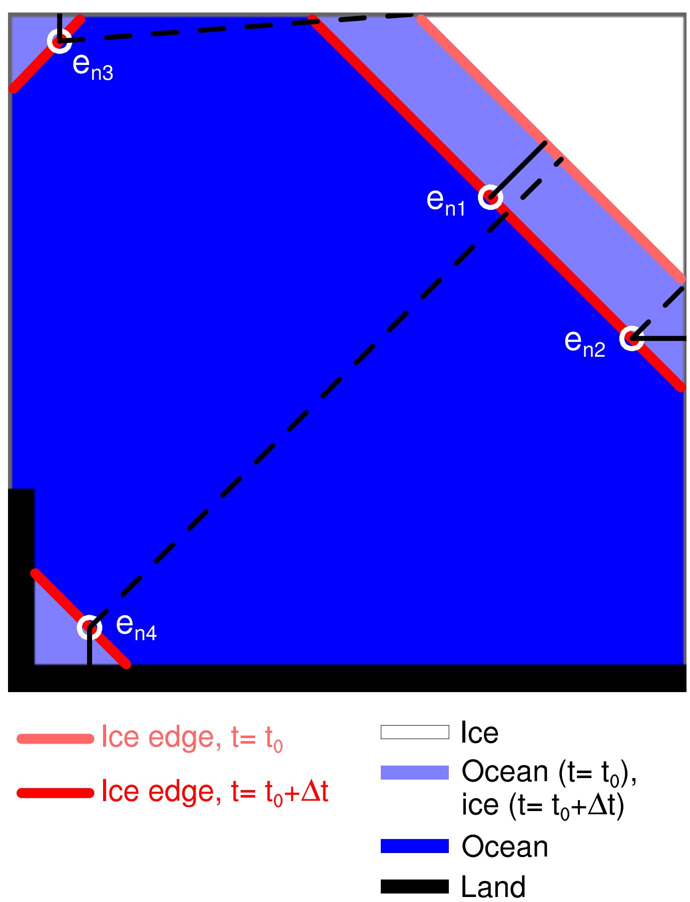

Figure B1: Binary fields with values of 1 (ice) and 0 (no ice/ocean) are displayed by white and blue color shading, respectively. Land is indicated as a black region. Light shades of blue indicate regions with a non-overlapping ice cover, as indicated by the color legend. Open boundary grid cells are depicted as gray lines. Ice edges for t_0 and $t_0 + \Delta t$ are drawn as light red and red and lines, respectively. Dashed black lines show the edge displacements as defined in Sect. 2.1, for a selection of labeled ice edge grid cells from $t_0 + \Delta t$, marked by white circles. Full black lines display the displacements $\widetilde{\widetilde{d}}$ that result from the modifications described in Appendix B, see Eq. (B8). For the set of grid cells that are highlighted here, only e_{n1} is unaffected by the modified definitions.

and for the corresponding distances we introduce the notation $\widetilde{d}$, so Eq. (3) becomes

$$\widetilde{d_n}^{\Delta t} = \min ||e_n^{(t_0+\Delta t)} - \widetilde{L}(t_0)|| \tag{B3}$$

where $e_n^{(t_0+\Delta t)}$ is a grid cell on $L(t_0 + \Delta t)$, as before. Note that the set of grid cells $e_n^{(t_0+\Delta t)}$ is not affected, so the number of displacement distances considered in Eq. (4), $N(t_0 + \Delta t)$, is unchanged.

A sample grid cell to which the displacement distance is significantly affected by this modification, is displayed as e_{n3} in Fig. B1. It must be noted that if the ice is imported into the domain, the distances $\widetilde{d}$ associated with such a displacement will be underestimated, since the real position of the ice edge outside of the analysis domain at t_0 is unknown. Moreover, for regular displacement of ice inside the domain, results will be affected slightly when occurring in the vicinity of the open boundary (e.g. e_{n2} in Fig. B1).

Similarly, there can be cases where freezing of ice occurs along the coastline, e.g. as an effect of colder air in the vicinity of continents, or less salty water masses close to the coastline. This is another case where the algorithm above can yield grossly exaggerated displacement distances. Again, the problem can be alleviated by including additional grid lines.

Set the coastal grid lines as

$$L^C(t_0) = \{e_{1_C}, e_{2_C}, \ldots, e_{N_C}\} \quad , \quad c[e_{n_C}](t_0) < c_{edge} \tag{B4}$$

where e_{n_C} is any ocean grid cell along the coastline which which was ice free or had a sea ice concentration below c_{edge} at $t = t_0$. Then $L(t_0)$ can be replaced by

$$\overline{L}(t_0) = L(t_0) \cup L^C(t_0) \tag{B5}$$

Here, Eq. (3) will be replaced by

$$\overline{d_n}^{\Delta t} = \min ||e_n^{(t_0+\Delta t)} - \overline{L}(t_0)|| \tag{B6}$$

and the computation of the displacement distance to grid cell e_{n4} in Fig. B1 is severely affected.

For a regional model, the typical situation is that there are both open boundaries and coastlines. In that case, we may combine the above modifications of the algorithm by adopting

$$\overline{\widetilde{L}}(t_0) = L(t_0) \cup L^{OB}(t_0) \cup L^C(t_0) \tag{B7}$$

and Eq. (3) can be written

$$\overline{\widetilde{d_n}}^{\Delta t} = \min ||e_n^{(t_0+\Delta t)} - \overline{\widetilde{L}}(t_0)|| \tag{B8}$$