Answers to tc-2020-354 RC1

June 21st, 2020

Note:

- The referee comments are shown in black,
- The authors answers are shown in blue,
- *Quoted texts from the revised manuscript are shown in italic and in dark blue.*

- Note that the exact pages and line numbers in our responses are subjected to change as the revised manuscript is being prepared.

Review of "A generalized stress correction scheme for the MEB rheology: impact on the fracture angles and deformations" by Plante and Tremblay (tc-2020-354).

This manuscript is a generally well written and easy to follow description of an extension to the MEB model of Plante et al (2020), but also has implications for other MEB implementations. This extension addresses two problems of the original model: large (numerical) error growth, which may be model-code specific) and too large fracture angles (which is probably not model-code specific. The new scheme allows to specify a more general correction of stress states that exceed the Mohr-Coulomb failure criterion. Sensitivity experiments in uniaxial compression illustrate that the parameterization indeed reduces the error growth and also reduces the fracture angles towards more realistic values. From the sensitivity experiments a preferred parameter set is determined. This is a very useful addition to the development of MEB rheology (and code) and should be published subject to minor revisions.\

We thank the referee for his or her thorough review of the manuscript and constructive comments.

My main point of critique is that the manuscript is missing a bit of general introduction and a clear problem statement. To my mind, the manuscript can be improved by taking the reader more by the hand than is done. This only requires a few sentences here and there or maybe an additional paragraph, e.g.

(1) what do we expect from a "brittle" model in contrast to a "granular material". The concept of "granular material/flow" is used often in the text, but it is not clear (from the text) how the brittle part of the model relates to that. Do we expect that a brittle model represents a granular material properly?

Brittle and ductile are types of fractures: a brittle fracture occurs with little prior plastic (permanent) deformation, and ductile fracture occurs after significant plastic deformations. A granular material, on the other hand, is a type of material: i.e., a composite of aggregated granules, as opposed to metals or crystals). Granular theories describe the fractures and deformations of granular materials in terms of the distribution of contact normal between individual grains.

A brittle model is thus expected to represent the nucleation and propagation of cracks in a material, effectively producing a material discontinuity in the material properties. A granular model typically uses a Mohr-Coulomb yield curve and granular flow rules (sliding along fracture planes with more or less dilatation) to govern the material dynamics.

In the MEB model for instance, the damage parameterization corresponds to the “brittle” behaviour of sea ice, and the granular behaviour is reflected in the choice of a Mohr-Coulomb yield curve.

This is now clarified in a new paragraph at the beginning of the model section 2, L72-75 and in section 4.3.3.

(2) state the issues with sea models and MEB in particular that are addressed in this paper in separate paragraphs. Now the angle-issue is mentioned in the middle of a paragraph that is introduced by: “The damage parameterization is relatively new, …”

>> We re-wrote the introduction (as suggested) to better introduce these issues, as well as addressing the previous and following comments from the reviewer. We address the challenges in representing the fracture of sea ice in the context of large-scale sea-ice models that are based on the continuum assumption, and then address issues that are more specific about the MEB rheology.

(3) discuss if the new scheme can be also useful for other implementations of MEB (e.g. neXtSim)

>> Yes, the new scheme can be used in other implementation of the MEB model (e.g. neXtSIM). Specifically, our implementation represents a generalization of the damage parameterization that can be easily implemented numerically and used to improve the performance of MEB models. Our results also show that the new scheme can be used to tune the simulated fractures closer to observations. These statements are added in the discussion section, L315-318 and in conclusions.

There are some technical issues (figure referencing and captions) detailed below.

The points below sometimes repeat my main points.

page 1

Abstract: General background and (more importantly) a clear problem statement is missing from the abstract. E.g., the problem of too large angles is not stated and the error growth is also only mentioned as the target of the new parameterization. It would help to have more context here already (1-2 extra sentences).

>> A clearer statement for the motivation of this study in included in the revised manuscript. The abstract now reads:

“The Maxwell Elasto-Brittle (MEB) rheology differs from other sea ice models by relying on a damage parameterization that aims at representing the brittle character of sea ice, without involving granular or plastic laws to constrain the sea-ice deformations. This approach efficiently produces localized fractures and reproduce observed sea-ice deformation statistics (spatial and temporal scaling and

spatio-temporal coupling) but tends to overestimate the observed fracture angle between conjugate pairs of fractures. The damage parameterization relies on a correction scheme to relax super-critical stresses back onto the yield criterion. The standard correction scheme is found to cause large errors in the stress field, building up asymmetries in fully-symmetric problems. Here, a generalized damage parameterization is developed where the super-critical stresses can be relaxed back onto the yield curve at different angles in stress invariant space, using a decohesive stress tensor. The sensitivity of the simulated fracture and post-fracture deformations to the decohesive stress tensor is investigated in simple uniaxial compression simulations. Results show that the decohesive stress tensor influences the growth of residual errors associated with the damage parameterization, the orientation of the lines of fracture and the short-term deformation associated with the damage but does not influence the long-term post-fracture deformations. We show that when the ice fractures, divergence initially occurs as the elastic strains are released but convergence develops in the longer-term when the viscous response dominates and the post-fracture deformations become linearly viscous. The post-fracture viscous deformations in the MEB model are shown to be dissociated from the fracture process itself, an important difference with classical Viscous Plastic (VP) models where plastic deformations are uniquely defined from the state of stress on the fracture plane by a flow rule. Using the generalized damage parameterization together with a stress correction path normal to the yield curve brings the simulated fracture angles closer to observations (from $40-50^\circ$ to $35-45^\circ$, compared to $20-30^\circ$ in observations) and reduces the growth of errors, allowing for longer-term simulations."

l3: any correction path: unclear "any"

>> This was replaced "*at different angles in stress invariant space*" in the revised manuscript.

l18: significantly: repetition

>> Corrected as suggested by the reviewer.

l22: the presence of and deformations along LKFs

>> Corrected as suggested by the reviewer.

l30: Hunke, 2001: not sure if this is an appropriate reference (for what)?

>> We removed this reference in the revised manuscript.

l44: "The fracture angle simulated by the MEB and standard VP models" It will be easier to follow, if you dedicate a separate paragraph (or at least an introductory sentence to this paragraph) to the fracture angles as a problem statement before describing what VP and MEB models do wrong.

>> We agree with the reviewer. We revised the introduction and added a new paragraph that focuses on the representation of fracture angles in both the VP and MEB models.

l160 I think that the problem statement is not clear enough. Unless you are very familiar with the details of the implementation of MEB models, it's not clear where Plante et al (2020) had numerical difficulties and if this is specific to their implementation. It should be clear if this will also be of value for, e.g. neXtSIM, or Dansereau et al.

Also the fracture angle problem is somewhat buried in the introduction and should be more prominent, because the paper devotes a large part to this.

>> We now devote a paragraph on the MEB model behaviour where these points are clarified as suggested. We specify that the numerical difficulty is related to the integration of the residual errors in the damage parameter, and is associated with the damage equation used in all MEB models.

l62: (Sulsky and Peterson, 2011) fix parentheses

>> Corrected as suggested by the reviewer.

l81: (Plante et al., 2020) fix parentheses

>> Corrected as suggested by the reviewer.

l96: is -> in

>> Corrected as suggested by the reviewer.

l104: “resulting in dominant elastic component”? not clear, something missing?

>> This refers to the dominance of the elastic term vs. the negligible viscous term in the constitutive equation. We clarified these lines in the revised manuscript, which now read:

“[...] the elastic term dominates when the ice is undamaged while the viscous term dominates when the ice is heavily fractured.”

ll117: maybe put \mu, \phi, c into Fig1 for better illustration?

>> We added the parameters as suggested by the reviewer.

eq 16: where does the “some algebra” start from? Maybe add a little more explanation here to guide the reader.

>> We added more information in the revised manuscript. Eq. 16 is found by finding the intersection point between the yield curve (Eq. 10) and the line corresponding to the stress correction. We add the mathematical expression of the stress correction line, such that the algebra is more straightforward.

l148: “something that is not possible in the standard parameterization otherwise \Psi …” please rephrase.

>> This sentence is clarified in the revised manuscript, and now reads:

“[...] as opposed to the standard parameterization in which case any super-critical stress is instantaneously corrected to the origin.”

l166: on -> of

>> Corrected as suggested by the reviewer.

l208: asymmetry factor: not immediately clear why this measures error. I assume that you expect perfectly symmetric solutions about the center line, but I think that this needs to be explained.

>> This diagnostic is explained in more details in section 4.3.1, L212-L215 in the revised manuscript, including its definition about the center line. We specify that it measures the cumulated far-field response to all residual errors produced from the start of the simulation. As opposed to the amplification factor R, which only measures the maximum local amplification of the residual error by the damage parameterization, the asymmetry measures the cumulative and longer-term effect of the residual errors on the model solution.

The same is true for “damage activity”, what do you want to use this for and how does this diagnostic achieve that.

>> The damage activity is only used to indicate the onset of fracturing and the short time scale associated with the development of fracture. It serves to show that the onset of the growth of errors in the solution is associated with the fracture. This is specified in section 4.3.2, L215-217 in the revised manuscript.

eq.26/27. the notation is a bit unusual and looks a little like (pseudo-) code. Why not use standard indexing as one would expect in a maths text?, e.g. \left(\sigma_{II}\right)_{n_x-i,j}

>> We agree and the format is corrected in the revised manuscript.

l235: 0.29 N/m? units?

>> This is an error and is corrected to N m^{-2}.

l243: “mostly elastic with divergence along the fracture line” Where do we see that divergence? In Fig3 I mostly see negative divergence = convergence.

>> This is illustrated in Fig. 4b (the reference is added in the revised manuscript). Figure 3 shows the deformations after 2 hours of simulations, in which points the deformations are dominated by the post-fracture (viscous) convergence. This is also clarified at the beginning of this paragraph, at L243-245 in the revised manuscript.

l248/9 The references to figure 4 are not correct. There is no Fig 4i, then it’s not clear from the caption, what we are seeing in color (damage?). It would help to add the timing in the plot (maybe top right or bottom left of rhs column).

>> There were errors in the labelling. This is corrected in the revised manuscript. We also improved the labels and captions in this figure.

l251: here and everywhere else: Units should NOT be in italics).

>> Corrected as suggested by the reviewer.

l254: 10^{-6} Nm^{-2} (unit not in italics): in 4.1 it was 1e-8!! In Fig5 it seems to be 1e-8 as well.

>> It should indeed indicate 10-8, this is corrected in the revised manuscript.

l254:are -> is

>> Corrected as suggested by the reviewer.

l254: “damage error amplification ratio R” maybe refer to equation 23 here?

>> We agree and added the reference in the revised manuscript.

l258: indicate -> indicates

>> Corrected as suggested by the reviewer.

In Figure 6 the panels for eps_asym and R_max are exchanged wrt to Figure 5. Why confuse the reader?

>> We agree with the reviewer and interchanged the panels in the revised manuscript.

l263: “the production of” could be removed

>> Corrected as suggested by the reviewer.

l264: I would argue for \gamma \ge 0 the improvement is significant (including 0). But the asymmetry also grows for \gamma > 0 and only for values > 45 it seem to stay low. Why not discuss that here?

>> We added a few lines in the revised manuscript in section 5.2, L265-267 to address this comment, instead of only bringing this point in the discussion section. We note that the improvement by the generalized parametrization is limited by the fact that the damage remains an integrated parameter, and that the residual error remains very influential in heavily damaged ice due to by the non-linear relationship between the sea ice deformation and the damage. We specify that the main improvement here is the removal of the spikes in the amplification ratio. We also note that the slower growth of asymmetries in the case of large correction angles are partly attributed to the slower development of the discontinuity. Thus, as we increase gamma, the improvement comes increasingly at the cost of losing the brittle behaviour of sea ice.

l286: “Based on these results, we suggest the use of a correction path that is normal to the yield criterion (\gamma = arctan \mu, see black points in Fig. 9).” my say, \gamma = \phi in this case, (isn’t it)?

>> It is not the case. \gamma and \mu are defined in the stress invariant space, whereas the friction angle \phi is defined in the Mohr stress space. That is, the angle of friction \phi does not correspond to the angle spanned from the x axis to the yield curve in the stress invariant space. Rather, the slope of the yield curve in the stress invariant space is \mu = sin(\phi). Thus, writing \gamma as a function of \phi would yield: \gamma = arctan(sin(\phi)).

l331: “are robust to the exact” -> are not sensitive to the exact, are robust with respect to the exact …

>> Corrected as suggested by the reviewer.

l335: reach -> reaches

>> Corrected as suggested by the reviewer.

l335: elastic wave are however no-longer -> elastic waves, however, are no longer …

>> Corrected as suggested by the reviewer.

l344: in -> is

>> Corrected as suggested by the reviewer.

l349: the uniaxial -> a uniaxial

>> Corrected as suggested by the reviewer.

l351: not sure if "post-fracture" (or pre-fracture) is grammatically correct. I would use "after (and before) fracture" in most places in this manuscript

>> We prefer to keep "post-fracture" as it is concise and often used in the field to describe material behaviour.

l352: "contrary to laboratory experiments of granular materials and satellite observations of sea ice." A short discussion about to what extent we expect granular behavior in an MEB model seems in place (not here in the conclusions but somewhere in the introduction?), in order to understand if this is an encouraging or a discouraging result

>> We agree with the reviewer and provide more background on the fracture angles and dilatancy in section 4.3.3. This section (4.3.3) and section 4.2 are re-written in the revised manuscript to clarify the granular character of sea ice and how it is related to fracture angles in uniaxial compression tests.

l361: "the production of" remove, see above

>> Corrected as suggested by the reviewer.

Figure3: miximum -> maximum/minimum?

>> This is corrected to "maximum" in the revised manuscript.

Figure 4. What are the meaning and the units of the color scale? Is this for the control simulation only?

>> This figure is for the control run and the color indicates the local damage (unitless) of each scatter points. These precisions are added in the revised manuscript.

Fig9: "The theoretical fracture angle from the Mohr-Coulomb and Roscoe theories are indicated by dashed and dash-dotted lines for reference." something like this, could also be useful in Fig7.

>> We agree and added these precisions in the captions of both Fig 7 and 9.