Supporting Information for "Geophysical constraints on the properties of a subglacial lake in northwest Greenland"

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1. Seismic and radar velocity structure

Estimates of lake depth using traveltimes of seismic or radar reflections depend on knowing the velocity of the medium through which the waves travel. In order to reliably compare depth estimates from each technique, it is important to determine a self-consistent model of the structure and material properties of the ice sheet from top to bottom. In this study, the average seismic velocity structure of the ice sheet was determined using a normal moveout velocity analysis of the primary lake top reflection, which yielded a velocity of Vp = 3700 + 40 m/s. Since this value averages the entire ice column consisting of both firn and glacial ice, it is lower than most values reported for glacial ice, which typically range between 3750 - 4000 m/s (e.g, Gusmeroli et al., 2012). Because radar velocity varies significantly between ice and firn, accurate lake depth determination with GPR requires knowing the depth and properties of both the firn and underlying glacial ice. Here, we estimate the firn depth and material properties using a seismic refraction survey. For each shot gather, we measured the P wave travel time at all geophones (Figure S1) and inverted for a best fitting shallow velocity structure using the software REFRACT (Burger et al., 2006). Given the maximum source receiver offset of 230 m, the velocity model is limited in depth resolution to approximately 50 m. At the lower bound of the refraction profile Vp is approximately 2850 m/s, indicating that the firn layer extends deeper than the resolution of the survey (Figure S2 A). However, given an average velocity within the ice sheet of 3700 m/s, we can place constraints on the depth of the firm by assuming a velocity in both the lower firn layer (i.e., the firn that lies below the depth of resolution) and the underlying glacial ice. This is illustrated in Figure S2 B, which shows how the estimated firn thickness trades off with the velocity of the lower firn layer and underlying glacial ice. In our preferred model, we assume the lower firn layer and underlying glacial ice to have velocities of Vp = 3000 m/s and Vp = 3800 m/s, respectively, which yields a firn thickness of 80 m. A thicker firn layer is possible if the velocity of either the lower firn layer or the underlying glacial ice is faster than our preferred model, although a thinner firn layer is unlikely since the ice sheet velocity below the firn would need to be unrealistically slow for glacial ice.



Figure S1. Results of seismic refraction survey. Scatter points indicate the measured P wave arrival time, and the red line shows the predicted travel times for the preferred model.



Figure S2. (A) Preferred seismic velocity and density structure, which includes an 80 m thick layer of firn overlying glacial ice with a Vp of 3800 m/s. Vs is assumed to be half of Vp, and density is determined using Equations S1 and S2. The dashed black line shows the density profile of the DYE-3 ice core from southern Greenland. (B) Firn thickness analysis. Each line corresponds to an assumed firn velocity below 50 m. The velocity of the ice below the firn is calculated for total firn thicknesses ranging from 50 - 100 m, given the constraint of an average (RMS) velocity of 3700 m/s in the firn and ice column.

The density profile is scaled from V_P using a simple porosity model. First the porosity is determined using Equation S1, where φ is porosity, and $V_{P_{firn}}$, $V_{P_{ice}}$, and $V_{P_{air}}$, are the P velocities of firn, glacial ice, and air respectively. Next, the density profile $\rho(z)$ is determined with a two phase mixing model between ice and air, assuming that the density of ice ρ_{ice} is 950 kg/m³ and the density of air ρ_{air} is 1.22 kg/m³ (Equation S2).

$$\varphi = \frac{V_{P_{firn}} - V_{P_{ice}}}{V_{P_{air}} - V_{P_{ice}}}$$

Equation S1.

$$\rho(z) = (1 - \varphi)\rho_{ice} + \varphi\rho_{air}$$

Equation S2.

Lastly, to obtain radar velocity, use the scaling law of Kovacs et al., (1995):

$$V(z) = \frac{V_{air}}{[1 + 8.45x10^{-4}\,\rho(z)]}$$

Equation S3.

Where V(z) is the radar velocity in m/us, and V_{air} is the radar velocity in vacuum (300 m/µs). Figure S3 shows the scaled radar velocities in each layer of the seismic velocity model. The average radar velocity of the profile of 172 m/µs is used to convert radar two-way travel time to depth.



Figure S3. GPR velocities scaled from seismic velocities.

2. Hydraulic Head Estimates

Water will flow down the hydraulic head gradient according to hydraulic potential theory. We use the theory from Shreve (1972) and similarly applied in Badgeley *et al.* (2017):

$$H = S + \left(\frac{\rho_w}{\rho_i}\right)B$$

Where S is the surface elevation, B is the bed elevation (Figure S4 A), ρ_w is the density of water (1000 kg/m³) and ρ_i is the density of ice (we used 920 kg/m³). We assume that basal water pressure equals overburden pressure (i.e. zero effective pressure).

The hydraulic head (H) is shown in Figure S4 B. Because the head is dominated by surface topography, the gradient in hydraulic head is downslope from the ice divide despite bedrock lows where the lake is located. Higher resolution bed topography might result in stronger subglacial connections; however, at this point the surface topography dominates the general subglacial pathways and would prevent water from flowing into the subglacial lake from the ice margins.



Figure S4: A) Bed elevation from BedMachine v3 (Morlingham *et al.*, 2017). The red lines show the area outlined as lake detection from Palmer *et al.* (2013). The orange dots are the radar profile we use in this study. B) Hydraulic head calculated from the surface and bed elevations.