

1 The flexural strength of bonded ice

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8 **Abstract.** The flexural strength of ice surfaces bonded by freezing, termed freeze-bond, was studied by performing
9 four-point-bending tests of bonded freshwater S2 columnar-grained ice samples in the laboratory. The samples were
10 prepared by milling the surfaces of two ice pieces, wetting two of the surfaces with water of varying salinity, bringing
11 these surfaces together, and then letting them freeze under a compressive stress of about 4 kPa. The salinity of the
12 water used for wetting the surfaces to generate the bond varied from 0 to 35 ppt. Freezing occurred in air under
13 temperatures varying from -25 to -3 °C over periods that varied from 0.5 h to ~100 hours. Results show that an increase
14 in bond salinity or temperature leads to a decrease in bond strength. The trend for the bond strength as a function of
15 salinity is similar to that presented in Timco and O'Brien (1994) for saline ice. No freezing occurs at -3 °C once the
16 salinity of the water used to generate the bond exceeds ~25 ppt. The strength of the saline ice bonds levels off (i.e.,
17 saturates) within 6-12 hours of freezing; bonds formed from fresh water reach strengths that are comparable or higher
18 than that of the parent material in less than 0.5 hours.

19 1. Introduction

20 Freeze bonds form when distinct ice features, such as floating ice floes or ice blocks of a rubble pile, become
21 and remain in contact over a period of time at low enough temperature. Insight into the strength of the bonds is
22 important when, for example, the strength of an ice cover formed of refrozen floes or the strength of an ice rubble pile
23 is estimated. There are several factors that affect the failure of a cover of sea ice, surface waves being a major one
24 that has gained an increasing amount of interest recently (Shen, 2017; Squire, 2020). Under the action of waves, ice
25 covers bend and may undergo flexural failure (Ardhuin et al., 2020; Aspin et al., 2012; Collins et al., 2015; Hwang
26 et al., 2017; Kohout et al., 2014, 2016; Shackleton, 1982). It is relevant to ask if the freeze bonds forming into vertical
27 cracks within a broken and refrozen ice cover form the weakest link at which wave-induced cracks initiate and
28 propagate. During the wave-ice interaction, the freeze bonds deform and failure occurs under a tensile state of stress
29 arising from flexural deformation. Szabo and Schneebeli (2007) performed tensile tests on sintered ice grains on scale
30 ~ 10^{-3} m, but to our knowledge, no data on freeze-bond strength under tensile loading at time and length scales relevant
31 to geophysical or ice engineering problems have been published.

32

33 The strength of freeze bonds has been tested only under combined compressive and shear loading. Such tests
34 have been related to continuum modeling of ice rubble using material models having yield surfaces resembling that
35 of a Mohr-Coulomb material model (Ettema and Urroz, 1989; Heinonen, 2004; Liferov et al., 2002, 2003; Serré,
36 2011b, 2011a). The critical shear stress of a Mohr-Coulomb material is given by $\tau = c + \sigma \tan \varphi$, where c is the
37 cohesion, σ the compressive stress, and φ the internal friction angle of the material. The underlying assumption in
38 testing has been that the failure of the individual freeze bonds within the rubble occurs through the same mode as the
39 failure of the rubble itself. No evidence of this type of similarity between the two scales exists. Instead, the numerical
40 simulations (Polojärvi and Tuhkuri, 2013) suggest that the individual freeze bonds within deforming rubble do not
41 fail due to shear, but rather under tensile stresses as the bonded ice blocks move relative to each other. This implies
42 that data on the shear strength of the freeze bonds may not lead to reliable estimates of the shear strength of ice rubble.
43

44 In this paper, the strength of freeze bonds under tensile loading is studied. For this purpose, we conducted
45 four-point-bending tests using the apparatus described and used by Murzda et al. (2020). All procedures for testing
46 were designed with the aim of reducing the number of variables for reliable analysis: bonds were formed between
47 milled surfaces of freshwater ice specimens (termed the parent material) and bond freezing and testing were performed
48 in air under a small compressive stress of about 4 kPa. The experimental variables were the freezing time (0.5 h...~100
49 h), the sample temperature (-3°C...-25°C), and the salinity of the water used to form the bond (0...35 ppt). Bond
50 strength initially increases with freezing time, but then appears to level off and to reach a plateau (i.e., to saturate)
51 over several hours. Depending on the salinity of the water from which the bond is formed, the saturation time for bond
52 strength ranges from 0.5 h to 12 h. The “saturated strength” of freshwater bonds with finer microstructure appears to
53 reach levels higher than the strength of the parent material with a larger grain size. The results from these experiments,
54 presented below, represent the first set of results on the failure of freeze bonds under tension.

55 2. Experimental procedure

56 Freshwater ice, used here as the parent material for the freeze-bonded samples, was produced in the
57 laboratory as described in Smith and Schulson (1993) and Golding and others (2010). Tap-water was frozen
58 unidirectionally from top to bottom in a cylindrical 800 L polycarbonate tank, forming pucks of ~1 m in diameter and
59 ~25 cm in thickness. The ice was generally bubble-free and columnar-grained. Thin-section analysis showed that the
60 average column diameter, as measured in the horizontal plane normal to the direction of ice growth using the linear
61 intercept method, was 5.5 ± 1.3 mm. The c-axes were randomly oriented within, and confined to, the horizontal plane,
62 suggesting that the ice had an S2 growth texture (in the terminology of Michel and Ramseier, 1971). The ice density
63 was 914.1 ± 1.6 kg·m⁻³ (Golding and others, 2010); Young’s modulus in the horizontal plane was 9.52 GPa (Snyder
64 and others, 2016). Once grown, the ice was cut into blocks and stored in plastic cooler boxes in a cold room at -10° C.
65 Specimen preparation is described in detail elsewhere (Iliescu et al., 2017; Murzda et al., 2018, 2019, 2020b, n.d.).
66

67 Samples to be freeze-bonded were prepared from the ice blocks by milling them into thin plates. The plates
68 had dimensions of $h \sim 15$ mm in thickness (parallel to the long axis of the grains), $b \sim 85$ mm in width, and $l \sim 300$ mm
69 in length. Specimens were allowed to equilibrate to the test temperature for at least 24 hours prior to testing.

70
71 The plates were then cut perpendicular to their long axis into two parts. In most samples the sawn surfaces
72 were milled after cutting (more below). The two parts of the specimen were then placed in a cold room with a
73 temperature of $+2^\circ\text{C}$ for a few minutes. To initiate freeze-bond growth, the sawn and milled surfaces were sprayed
74 with a fine mist of water at a temperature of $+2^\circ\text{C}$ and quickly brought into contact by setting the two pieces into a
75 freeze-bonding rig (Figure 1). The surfaces were wet when brought into contact, but in addition, a syringe was used
76 to inject about 0.1 ml of water to the bond to ensure uniform wetting of the surfaces. Excess water, if any was observed
77 around the bond, was wiped with a tissue. All of the above steps were performed at $+2^\circ\text{C}$ to prevent freezing from
78 occurring before setting the sample into the rig. Afterwards, the freeze-bonding rig was moved to another cold room
79 with a desired test temperature.

80
81 To investigate whether the roughness of the faces in contact affects the bond strength, a few samples had
82 their faces produced by cutting the parent plate either with a coarse (1/2 inch in width, 1/40 inch in thickness and
83 6 teeth per inch) or a fine (13/64 inch in width, 1/64 inch in thickness and 24 teeth per inch) band saw. Although few
84 in number, results from these initial experiments suggested that surface roughness of the kind we explored had no
85 significant effect on flexural strength. Thus, for all further tests (that led to the results reported below) sawn surfaces
86 were milled for consistency and reproducibility (more in Discussion).

87
88 Figure 1 shows a sketch (a) and photograph (b) of the freeze-bonding rig. The rig had a system consisting of
89 two plastic bars and two springs for applying a desired pressure (i.e., compressive stress) to the bond during freezing.
90 In the present experiments, a confining pressure of ~ 4 kPa was chosen which is in accordance with the maximum
91 hydrostatic pressure within submerged 10-meter-thick ice rubble mass (Ettema and Schaefer, 1986). The rig was kept
92 in a cold room of the desired temperature (i.e. from -25°C to -3°C) during freezing. The base of the rig was made from
93 an acrylic plate having low heat conductivity, ensuring the heat flux in the bond area was mainly along the long axis
94 of the sample. Wax paper was placed between the ice and the acrylic to prevent freezing of ice onto the rig. All
95 materials of the rig were such that the frictional resistance between them and ice was low. This enabled good control
96 of the confining pressure and sample alignment.

97
98 To investigate the effect of the salinity on the bond strength, fresh water and saline water of salinity ranging
99 from 2 to 35 ppt (parts per thousand), was used in spraying. Saline water was prepared in the manner described by
100 Golding et al. (2010, 2014) by adding the commercially available salt mixture “Instant Ocean” to tap water. Salinity
101 was measured using a calibrated YSI Pro30 conductivity salinity meter.

103 After a desired time of freezing, varying from 0.5 to ~100 h, the freeze-bonded sample was removed from
104 the rig and its flexural strength under four-point bending was measured. For this purpose, a servo-hydraulic loading
105 system (MTS model 810.14) with a custom-built four-point loading frame was utilized. The sketch of the apparatus
106 is shown in Figure 2 of Murdza et al. (2020), the photograph of the apparatus is shown in Figure 5a and the apparatus
107 is described in detail elsewhere (Ilieșcu et al., 2017; Murdza et al., 2018, 2019, 2020b). The outer loading rollers are
108 immobile during testing while the inner loading rollers are attached to the actuator. The hydraulic actuator was driven
109 under displacement control and loading was controlled using a FlexTest-40 controller. A calibrated load cell was used
110 to measure the load.

111
112 The experiments were performed at an outer-fiber center-point displacement rate of 0.1 mm s⁻¹ (or outer-
113 fiber strain rate of about 1.4 x 10⁻⁴ s⁻¹ according to linear-elastic first order beam theory). This displacement rate
114 resulted in an outer-fiber stress rate of about 1 MPa s⁻¹. As was indicated earlier (Murdza and others, 2020), the 0.1
115 mm s⁻¹ displacement rate in cycling results in a period of ~20 s which is approximately the frequency of ocean swells
116 (Collins and others, 2015). The major outer-fiber stress σ_f was calculated as:

$$\sigma_f = \frac{3PL}{4bh^2}, \quad (1)$$

117 where P is the applied load and L is the distance between the outer pair of loading cylinders and is set by the geometry
118 of the apparatus to be $L = 254$ mm. The flexural strength that we refer to throughout this paper is the maximum major
119 outer-fiber stress that the ice plate can withstand before breaking. It is important to note that in all experiments
120 described in this paper the bond formation and breaking of bonded ice occurred at the same temperature. Owing to
121 the confining impact of the loading cylinders of the 4-point flexing apparatus (see Figure 5a and Figure 2 of Murdza
122 et al. (2020)) and to the Poisson effect, a biaxial state of tension developed in the ice. Based on isotropic elasticity and
123 plasticity theories, the minor stress was approximately between one-third to one-half of the major stress (Appendix
124 A).

125 3. Results and Observations

126 3.1. Flexural strength of parent material

127 Two measurements on the flexural strength of pristine ice plates, that is, plate-like samples of parent material
128 without a freeze bond, were conducted at -10 °C. The strength values obtained were 1.51 and 1.63 MPa. Only two
129 experiments were performed as these values compare favorably with the earlier measurements by Murdza et al. (2020)
130 on the same kind of ice using the same loading system. Murdza et al. (2020) reported that the average and the standard
131 deviation of the flexural strength at -3, -10 and -25 °C were 1.42±0.16, 1.67±0.22 and 1.89±0.01 MPa, respectively.
132 Further, the measured values are in agreement with the data that are reviewed in Timco and O'Brien (1994), where
133 the average and standard deviation of 1.73±0.25 MPa is reported for the flexural strength of freshwater ice at
134 temperatures below -4.5 °C.

135 **3.2. Flexural strength of bonded ice**

136 **3.2.1. Freshwater bond**

137 The experiments with a freshwater bond were conducted at -3 and -10 °C. The results are listed in Table 1.
138 The time for the bond formation (0.5 hours was the shortest freezing period used here, implying that the bond formed
139 in less time) is reasonably consistent with analytical estimates, Appendix B. Surprisingly, in all of these experiments,
140 the failure occurred outside of the bond. This suggests that even after only a relatively short period of freezing, the
141 strength of the freshwater bond reaches and exceeds that of the parent material. Even though the results listed in Table
142 1 show scatter, at -10°C comparison of the measured flexural strengths to those described in Section 3.1 showed that
143 they are not statistically different from the flexural strength of pristine freshwater ice samples (*p-value* = 0.21 and 0.08
144 for tests at -3 and -10 °C, respectively). This is important because it indicates that the above-described bond generation
145 procedure did not hamper the samples by, for example, leading to geometrical misalignments in them.

146 **3.2.2. Saline bond**

147 Figures 2 and 3 show the results from the experiments performed to investigate the effect on bond strength
148 of the salinity of the water used to create the freeze bond. The data are given in Tables 2-4. The tables indicate the
149 experiments where no freezing occurred (“No”) and the experiments where bonding occurred, but the bond was too
150 weak to be tested (“Low”). These data are excluded in the figures below.

151

152 Figure 2 shows that the strength of the saline bonds increases over time and levels off, or saturates, after
153 about 6-12 h. Thus, the strength of the saline bonds increases at a considerably lower rate than that of the freshwater
154 bonds. A comparison of these results to those in Section 3.1 shows that the strength of the saline bonds is well below
155 the strength of the freshwater ice used as the parent material. A comparison of the two data sets in Figure 2 shows
156 that the saturated strength of the bonds made from water of higher salinity but at lower temperature (35 ppt and -
157 10 °C) is about twice of that of the bonds with lower salinity but higher temperature (12 ppt and -3 °C).

158 Figure 3 illustrates how the salinity of the water used to generate the freeze-bond at -3 °C affects its saturated
159 strength at -3 °C. While the measured strength values for low salinities are close to those measured for freshwater ice,
160 the bond strength decreases rapidly with an increase in salinity and no freezing occurs once the salinity of the salt
161 water used to generate the bond reaches ~25 ppt; even at 17 ppt some bonds were too weak to be tested. This agrees
162 reasonably well with analytical estimates, Appendix C, where formulas that relate strength to volume fraction of solid
163 phase suggest that at salinities of 30 ppt and above at -3 °C no freezing occurs. Figure 3 additionally shows two
164 exponential fits, one directly fitted to our data and one by Timco and O’Brien (1994) for the flexural strength of saline
165 ice (equations for these fits provided in Appendix D, where σ_b is flexural strength in MPa and ν_b is liquid brine content
166 in parts per thousand). It is important to notice that the fit by Timco and O’Brien (1994) yields lower values than the
167 measured bond strength in the present study for the whole range of salinities used. Likewise, the actual strength values
168 for the freshwater bonds are greater than the ones suggested by Figure 3, since the failure in these cases occurred
169 outside the bond, indicating that the bond is stronger than the parent material. Both saline and freshwater bonds that

170 develop through freezing appear to reach strengths higher than that of S2 type parent material of the same salinity
171 (strength of saline parent material is assumed to be the same as in Timco and O'Brien (1994)).

172 Temperature has a strong effect on the saturated strength of the freeze bonds. Figure 4 and Table 5 summarize
173 the data from experiments on specimens having bonds made from water of salinity 20 ppt at temperatures from -3 °C
174 to -25 °C. Three out of the four specimens at -25 °C failed outside of the bond with a measured strength of
175 1.61 ± 0.12 MPa, which is close to 1.89 MPa measured earlier at -25 °C on the same type of freshwater ice (Murdza
176 and others, 2020). Figure 4 also suggests that no freezing occurs at temperatures above about -3 °C, which is in fair
177 agreement with analytical estimates of no strength at $T = -2$ °C in Appendix C. Though the analytical equation from
178 Appendix C predicts well when no freezing occurs, it does not yield a trend that describes most of the data in Figure
179 4. The reason may be that for the microstructure of bonds in the present study, strength may not be directly proportional
180 to volume fraction of the solid phase as the model in Appendix C assumes, but rather a non-linear function of the
181 volume fraction of solid.

182 Figure 5a-c show an example of the typical samples after failure. Figure 5a shows a case where the crack had
183 initiated at the bond and started to propagate along it, but then deviated from it and continued to grow through the
184 parent material. Figure 5b shows a close up of a bond face-on after the most common type of failure, which occurred
185 along the bond. In this case, both surfaces of the failed freeze-bond had a fairly uniform “blurry” appearance which
186 indicates that failure occurred through the ice of the bond. It was also fairly usual for the samples having low salinities,
187 low temperatures and long freezing times, that the crack initiated and started to propagate along the bond and then
188 slightly deviated and moved parallel to the bond but inside the parent material, as shown by Figure 5c.

189 **4. Discussion**

190 The above results are the first measurements to be reported for the strength of freeze bonds under tensile
191 loading. Although the experiments were performed under flexural loading, they provide unique data on the tensile
192 strength of the freeze bonds. Under the loading conditions, the flexural strength of ice is governed by tensile strength,
193 although measured strengths are greater by a factor of about 1.7 than strengths measured under pure tensile loading
194 (Ashby and Jones, 2012). The reason is that in bending only a thin layer close to one surface of the sample (and thus
195 a relatively small volume) carries the peak tensile stress and it is less likely that this volume contains larger flaws, while
196 in tension the entire sample carries the tensile stress and it is more likely that it will contain larger flaws. Murdza et
197 al. (2020b) showed that the flexural strength of freshwater S2 ice tested on the same loading system as used here
198 compares well with direct measurements of the tensile strength of the same type of ice at the same conditions (Carter,
199 1971) when divided by 1.7. Murdza et al. (2020a) showed that the flexural strength of lake Arctic ice tested under
200 three-point bending is also similar to the one obtained in Murdza et al. (2020b). By using this 1.7 factor to scale
201 the values for saturated bond strengths shown in Figure 2 leads to tensile strength values of about 0.3 MPa and 0.18
202 MPa for bonds at -10 °C and -3 °C, respectively.

203

204 While there are no other data on the tensile strength of freeze bonds, the results can be compared to the
205 relatively large amount of earlier work on the shear strength of freeze bonds (Bailey et al., 2012; Boroojerdi et al.,
206 2020a, 2020b; Bueide and Høyland, 2015; Ettema and Schaefer, 1986; Helgøy et al., 2013a, 2013b; Høyland and
207 Møllegaard, 2014; Marchenko and Chenot, 2009; Repetto-Llamazares et al., 2011b, 2011a; Shafrova and Høyland,
208 2008; Szabo and Schneebeli, 2007). Common values for the shear strength in those studies ranged from 0.01 to 0.1
209 MPa, which are considerably lower than the flexural strength values measured here. Usually, these strengths have
210 been measured for bonds grown under water over periods that have not been long enough to reach saturated bond
211 strengths. On the other hand, the highest reported shear strength values ~0.3...0.7 MPa (Bailey et al., 2012; Boroojerdi
212 et al., 2020b; Shafrova and Høyland, 2008) are within the same range as the flexural strength values measured here.
213 Given that the tensile strength of ice is, on average, lower than the shear strength (Timco and Weeks, 2010), the
214 strength values measured here are perhaps surprisingly high. The high strength values here likely relate to the well-
215 controlled bond growing procedure and possibly to a finer microstructure of the material that comprises the bond.
216

217 Work on the shear strength of freeze bonds has led to a conclusion that the evolution of the bond strength has
218 three phases (Boroojerdi et al., 2020b; Repetto-Llamazares et al., 2011b, 2011a): (1) an initial period of a few minutes
219 of increasing strength due to heat flux from the bond to the parent material; (2) a period of some hours of weakening
220 as the temperature of the bond increases due to water surrounding it; and, (3) a period of several days of strengthening
221 due to sintering. The evolution of the flexural strength of the bonds in the present experiments is likely similar to that
222 of phases (1) and (3). The initial bond strengthening can be related to the transfer of heat along the long axis of the
223 specimen and the accompanying advance of the ice/water interface. Given that the water layer after wetting the contact
224 surfaces is very thin, the bond strength would be expected to saturate quickly; Appendix B describes a simple model
225 and suggests that the process, similar to above described phase (1), takes fewer than 10 minutes at -10 °C and a greater
226 amount of time of about 1.5 h at -3 °C, aligning with earlier studies and the result here. This means that for saline
227 bonds, phase (3) has a duration of about 6...12 hours, whereas earlier experiments have occasionally had relatively
228 long freezing times, varying from 60 h to 12 days (Bailey et al., 2012; Shafrova and Høyland, 2008).
229

230 As part of the studies on the evolution of bond strength, it has been fairly common to investigate the ratio of
231 the bond strength values to that of the parent material (Bailey et al., 2012; Boroojerdi et al., 2020b; Shafrova and
232 Høyland, 2008). Shafrova and Høyland (2008) found that specimens with bonds grown in the field had the strength
233 ratio varying from 0.008 to 0.082 (with a mean of 0.03 after 48 hours of bonding). For laboratory-grown bonds, they
234 measured ratios in the range 0.06 to 0.69 (0.21 ± 0.12). The latter values are in line with values reported by Bailey et
235 al. (2011) and Boroojerdi et al. (2020b), who reported ratios up to about 0.70 and 0.85, respectively. Boroojerdi et al.
236 (2020b) suggested an empirical formula to describe the strengthening of a freeze bond during the above-described
237 phase (3). The formula was based on curve fitting and an assumption that the shear strength of the bond approaches
238 asymptotically that of the parent material with increasing sintering time. The experiments here indicate that such an
239 assumption may not be always justified, as at least the flexural strength of the freeze bonds can reach values that are
240 above that of the parent material. Aligning with our observations, the results by Høyland and Møllegaard (2014)

241 provide an indication of the shear strength of freeze bonds reaching strengths comparable to that of the parent material.
242 In their uniaxial compression tests on bonded cylindrical samples having an inclined freeze-bond, the failure changed
243 from shearing (along the plane of the bond) to axial splitting of the sample in some of the cases.

244

245 Ettema and Schaefer (1986) and Repetto-Llamazares et al. (2011b) studied whether freezing in the presence
246 of water has an effect on the shear strength of the freeze-bond. The results indicate that shear strength was higher
247 when bonds froze under water. While Ettema and Schaefer (1986) let the bonding occur with samples submerged in
248 fresh water, Repetto-Llamazares et al. (2011b) used 7 ppt saline water for submerging. Earlier studies on the effect of
249 freezing conditions have not had the opposing surfaces wetted before bringing them together when generating bonds
250 in air. This effectively removes the above-described phase (1) from the bond strength evolution, if the result of the
251 heat transfer during the initial period of bond strengthening is assumed to simply be freezing of the liquid at the bond
252 interface. In addition, in these earlier studies, the maximum freezing times for the bonds grown in air varied only
253 from 0.5 min to 3 min. As phase (3) takes at least several hours, it seems likely that the mentioned studies have not
254 yielded data on saturated bond strengths for bonds grown in air.

255

256 While the new data from the present tests yielded clear trends for the strength of the freeze-bonds, they also
257 showed significant scatter. This scatter, even when bond generation was performed in a simplified manner using
258 carefully prepared milled samples (Section 2), is an indication that the strength of freeze bonds is a parameter that
259 inherently shows wide scatter. One reason for this, amongst others perhaps, is the detailed microstructure/phase
260 distribution of the bond. The microstructure probably varies somewhat from specimen to specimen, thereby leading
261 to variations in bond strength. The variation is actually of similar magnitude to that observed in experiments on the
262 flexural strength of pristine ice samples made with the same apparatus (Murdza et al., 2020b). The other reason,
263 perhaps, can be attributed to the fact that a small volume of material is tested in bending and, hence, the variation in
264 tensile strength if measured using a traditional methodology would likely be smaller, given the correlation coefficient
265 between tensile and flexural strength is 1.7.

266

267 The fact that in samples with freshwater bonds failure initiated and propagated outside of the bond suggests
268 that the strength of the freshwater bond is greater than the strength of pristine freshwater S2 ice. This may indicate a
269 difference in microstructure between the ice in the freeze bond and the ice of the parent plates. A finer grain size
270 within the bonds may be due to the initial water layer, which was produced by spraying a very fine mist, creating small
271 water droplets working as nucleation sites for the ice grains in the bond. Our argument is supported by the work of
272 Schulson and others (1984) who showed that tensile strength strongly depends on the grain size, increasing as grain
273 size decreases. A difference in grain size could also explain the fact that the strength versus salinity curve from Timco
274 and O'Brien (1994) is below the trend obtained in the present study (Figure 3). Concerning of the microstructure of
275 the bonds, one may think that because one phase is dominant it should form the matrix; however, there is at least one
276 class of materials, namely high-temperature nickel-based superalloys (Sims, 1984) where the minor component forms

277 the matrix. Since we do not know the structure of the bonds in the present study, we cannot be conclusive in this
278 regard.

279

280 As expected, both temperature and salinity affect the flexural strength of bonded ice samples. The strength
281 of both freshwater and saline ice increases over time and saturates, although saturation occurs significantly slower in
282 the case of saline bonds. The reason is likely related to the rejected salts and entrapped air at the ice-water interface
283 that slows the rate of the interface advance. The trend of saturated strength versus salinity (Figure 3) has an exponential
284 functionality similar to what has been suggested by Timco and O'Brien (1994), while the trend of saturated strength
285 versus temperature for saline bonds (Figure 4) appears, to a first approximation, to be roughly linear. It is important
286 to mention here that the salinities provided in this paper are salinities of the spray and not of melt-water from the bond
287 itself, and this begs the question: Is the bond salinity the same or lower than the salinity of spray solution? In the
288 formation of a natural floating sea ice cover, of course, the rejection of salts from ice results in melt-water salinities
289 lower than bulk water salinity (Weeks and Ackley, 1986). Given that in our experiments the bond thickness is very
290 small (<1 mm) and freezing time is relatively short, it is unlikely that all the salt is expelled from the freeze bond,
291 resulting in the bond salinity similar to the spray salinity. Therefore, while the resulting bonds might have salinities
292 slightly lower than the sprayed water, the results yield a reasonably reliable trend of strength as a function of salinity
293 which is very similar to the relationship proposed by Timco and O'Brien (1994).

294

295 The effect of surface roughness at the freezing interface was also briefly addressed when performing the
296 experiments. In addition to the milled surfaces with a roughness of $0.43\pm0.24 \times 10^{-6}$ m in the direction of milling and
297 $2.01\pm0.47 \times 10^{-6}$ m in the orthogonal direction (Schulson and Fortt, 2012), experiments were performed on samples
298 with surfaces produced by using a fine and coarse band saw blade, which resulted in ice surface roughness of up to
299 ~ 1 mm. The results from the experiments with differently produced surfaces showed no significant difference on the
300 strength of freshwater and saline bonds (1.39 MPa vs 1.43 ± 0.15 MPa for freshwater bonds and 0.39 ± 0.13 MPa vs
301 0.34 ± 0.16 MPa for saline bonds of 12 ppt salinity). As milling could be performed with the highest accuracy from the
302 aspect of sample dimensions and alignment, it was chosen as the technique we used here. Unlike what was observed
303 in the present study, Helgøy et al. (2013a) observed that the surface roughness does affect freeze bond shear strengths,
304 with rougher surface leading to bonds having higher strength. The discrepancy between their results and results in the
305 present study suggests that there may exist a threshold value for the surface roughness, after which it affects the bond
306 strength; it is likely that both milled and sawn surfaces used in the present study are too smooth for the effect of surface
307 roughness to be observed. On the other hand, experiments on shear strength usually involve sliding motion between
308 the blocks of the parent material. This motion may become restricted by rough surfaces, which could lead to higher
309 shear loads interpreted to be due to an increase in freeze-bond strength. In tests under tensile loading, such kinematic
310 restrictions do not exist.

311

312 Finally, it is worth noting that while this is the first study on the flexural strength of freeze bonds, it is not
313 the complete story. Further work is needed to investigate the effects of other factors such as bond pressure, the
314 character of parent ice plate, bond microstructure, the width of the opening to be bonded, etc.

315 **5. Conclusions**

316 Systematic experiments on the flexural strength of freeze bonds were conducted for the first time. The bonds
317 were grown in the air under 4 kPa confining pressure. The parent material was S2 columnar-grained freshwater ice.
318 The salinity of the bond varied from 0 to 35 ppt and freezing temperatures from -3 to -25 °C. It is concluded that:

319 (i) Freshwater bond strength exceeds the strength of parent ice in less than 0.5 h upon freezing.
320 (ii) The saline bonds reach their saturated strength within about 6-12 h of freezing.
321 (iii) An increase in bond salinity and in freezing temperature leads to a decrease in bond strength.
322 (iv) The relationship between bond strength and its salinity is similar to the one suggested by Timco and O'Brien
323 (1994).
324 (v) No freezing occurs once the salinity of the water used to generate the bond reaches values of about ~25 ppt
325 at -3 °C.

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443

444 **Appendix A. The relation between major and minor stresses in a sample in four-point bending**

445 To find a relationship between major and minor principal stresses in the ice plate we consider two cases: the
446 ice behaves as an isotropic linear elastic material and ice behaves as plastically isotropic material.

447 Stress tensor for the ice plate bent in a four-point manner (x_1 direction is along the long axis of the ice plate;
448 x_2 direction is along the width b of the specimen; x_3 direction is along the thickness of the ice plate):

449

450
$$\sigma_{ij} = \begin{pmatrix} \sigma_{11} & 0 & 0 \\ 0 & \sigma_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

451
452 Strain tensor for the ice plate bent in a 4-point manner:
453

454
$$\varepsilon_{ij} = \begin{pmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \varepsilon_{33} \end{pmatrix}.$$

455
456 Incremental plastic strain-tensor:
457

458
$$d\varepsilon_{ij}^p = \begin{pmatrix} d\varepsilon_{11}^p & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & d\varepsilon_{33}^p \end{pmatrix}.$$

459
460 A constitutive relationship for isotropic linear elastic material (Hooke's law):
461

462
$$\varepsilon_{22} = \frac{1}{E} [\sigma_{22} - \nu(\sigma_{11} + \sigma_{33})];$$

463
464
$$\sigma_{22} = \nu\sigma_{11} = \frac{1}{3}\sigma_{11}.$$

465
466 A constitutive relationship for plastically isotropic material beyond yielding (Levy-Mises Flow Rule):
467

468
$$d\varepsilon_{22}^p = \frac{d\bar{\varepsilon}^p}{\bar{\sigma}} \left[\sigma_{22} - \frac{1}{2}(\sigma_{11} + \sigma_{33}) \right];$$

469
470
$$\sigma_{22} = \frac{1}{2}\sigma_{11},$$

471
472 where $\bar{\sigma}$ is effective stress (also significant or equivalent stress); $d\bar{\varepsilon}^p$ is an effective plastic strain increment.
473
474 Hence, the minor stress σ_{22} is approximately between one-third to one-half of the major stress $\sigma_{11}.$

475 **Appendix B: Time for the freeze-bond formation**

476 To estimate the time to form a freeze-bond we assume that heat fluxes are along the long axis of the sample,
477 i.e. horizontal temperature gradients are much larger than vertical gradients at the freezing interface. The other

478 assumptions are that the heat flow through a material is equal to the energy for the solidification of the water along
479 the bond and that no heat losses occur, i.e.

480

$$k \frac{dT}{dx} = \rho \lambda \frac{dx}{dt} \quad (A1)$$

481

482 where t is the time, T is the ice temperature, k is the thermal conductivity of ice, λ is the latent heat of fusion of ice
483 per unit mass, ρ is the ice density and Δx is the characteristic conduction length or, in our case, the thickness of the
484 bond.

485 Taking into account that thermal diffusivity $\alpha = k/\rho c_p$, where c_p is specific heat capacity, and that $x = \sqrt{\alpha t}$,
486 after integration of Equation A1, and further assuming time interval to be $[0, t]$ and interval for the characteristic
487 conduction length to be $[0, x]$, we obtain the relationship:

488

$$t = \frac{1}{4} \left(\frac{x \lambda}{\Delta T} \right)^2 \left(\frac{\rho}{k c_p} \right) \quad (A2)$$

489

490 A note of caution is necessary here. As ice-water interface advances during freezing in saline ice, both air
491 and salt are rejected and build up at the interface. Unlike freezing in nature, there is not enough space for rejection
492 and, as a result, this slows the rate of advance of the interface.

493

494 According to Equation A2, and using parametric values of $c_p = 2100 \text{ J/kg}^\circ\text{C}$, $\lambda = 330 \text{ kJ/kg}$, $k =$
495 $2.2 \text{ W/m}^\circ\text{C}$, $\rho = 914 \text{ kg/m}^3$ for freshwater ice at -10°C and bond thickness of 1 mm we need only 1 min for the
496 bond formation, while for freshwater ice at -3°C a similar bond forms in about 10 minutes. While this estimate is
497 consistent with observations, it is also in accord with earlier experimental results by Repetto-Llamazares et al. (2011a,
498 2011b) and Borojeerdi et al. (2020a) for phase (1) increase during freeze bond shear strength evolution (Section 4).

499

500 **Appendix C: The strength of freeze-bonds as a function of salinity and temperature**

501 Principle:

502

503 The freeze bond is comprised of essentially two phases, solid (ice) plus liquid (water), barring entrapped air.
504 To a first approximation, we assume that its strength, σ_{fb} , is proportional to the volume fraction, f_s , of the solid phase.
505 The constant of proportionality, σ_{f0} , is the strength of freshwater ice. The relationship:

$$\sigma_{fb} = \sigma_{f0} f_s . \quad (A3)$$

506 The volume fraction of the solid phase is obtained from the lever rule:

$$f_s = \frac{X_l - X_0}{X_l - X_s}, \quad (A4)$$

507 where X_l and X_s denote the limit of solubility of salt in the liquid (water) and in the solid (ice) phases, respectively,
508 and X_0 is the concentration of salt in the water before freezing is initiated. Over the temperature range of interest, the
509 phase diagram for the H₂O-NaCl system (i.e., thermodynamics) dictates that both X_l and X_s increases with decreasing
510 temperature, T, according to the relationships:

$$X_l = \frac{T - T_0}{m_l}, \quad (A5)$$

$$X_s = \frac{T - T_0}{m_s}, \quad (A6)$$

511 where T_0 denotes the melting point of "pure" ice (273 K) and m_l and m_s , respectively, denote the slope of the liquidus
512 and the solidus on the phase diagram; both slopes are negative. The solubility of salt in ice is very low and so for
513 practical purposes $X_s \sim 0$. Writing the temperature difference as $T - T_0 = \Delta T$, the volume fraction of ice within the
514 freeze bond from Eqn (A4) is given by:

$$f_s = \left(1 - \frac{m_l X_0}{\Delta T}\right). \quad (A7)$$

515 Thus, upon equating X_0 to salinity, S , the strength of the freeze bond is given by:

$$\sigma_{fb} = \sigma_{f0} \left(1 - \frac{m_l S}{\Delta T}\right). \quad (A8)$$

516 Taking m_l to be independent of concentration, its value is $m_l = -0.1 \text{ Kpsu}^{-1}$, giving:

$$\sigma_{fb} = \sigma_{f0} \left(1 + \frac{0.1 S}{\Delta T}\right), \quad (A9)$$

517 where $\Delta T < 0$.

518

519 The model thus dictates that once freezing is complete the strength of the freeze bond decreases linearly with
520 increasing salinity, reaching the limit of zero strength when $S = \Delta T / -m_l$.

521

522 Both dictates are in reasonable agreement with observation.

523

524 **Appendix D: Trends in Figure 3**

525

526 The red trend in Figure 3 is taken from (Timco and O'Brien, 1994) where the authors report values for
527 flexural strength of saline ice over the range of salinities used in the present study and for temperatures above -4.5°C
528 (σ_f in MPa), i.e.

529

530
531
532 To calculate salinity S (in ppt) based on the liquid brine content v_b (brine volume fraction) in Timco and
533 O'Brien (1994) we used the following relationship suggested by (Frankenstein and Garner, 1967):
534

$$\sigma_f = 1.76e^{-5.88\sqrt{v_b}}. \quad (\text{A10})$$

535
536 where T is the ice temperature in degrees Celsius between -0.5 °C and -22.9 °C. The fit to our data in Figure 3 (black
537 curve) was made according to the least square method which resulted in the following equation (σ_f in MPa):
538

$$\sigma_f = 1.12e^{-5.88v_b} \quad (\text{A12})$$

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562 **Table 1. Results from testing freshwater bond experiments. The time here is the bond formation time, the strength is the**
563 **flexural strength (temperature during flexural testing and bond formation was the same). The reader should notice that in**
564 **all of these experiments the failure occurred outside of the bond and within the parent material.**

Sample #	Temperature [°C]	Time [h]	Strength [MPa]
1	-10	24	1.43
2	-10	25	1.39
3	-10	24	1.28
4	-10	3	1.58
5	-3	1.5	1.02
6	-3	1.5	1.28
7	-3	0.5	1.4

565

566 **Table 2. Results from testing saline bond experiments at -10 °C and 35 ppt.**

Sample #	Time [h]	Strength [MPa]
8	1.5	0.15
9	3	0.1
10	26	0.34
11	34	0.54
12	25	0.64
13	82	0.61
14	6	0.38
15	12	0.54

567

568 **Table 3. Results from testing saline bond experiments at -3 °C and 12 ppt.**

Sample #	Time [h]	Strength [MPa]
16	1.5	Low
17	1.5	0.31
18	3	0.17
19	3	0.18
20	6	0.25
21	6	0.22
22	14	0.48
23	24	0.14
24	24	0.35
25	72	0.29
26	97	0.32

569

570

571 **Table 4. Results from testing saline bond experiments at -3 °C.**

Sample #	Salinity [ppt]	Time [h]	Strength [MPa]
27	35	1.5	No*
28	35	24	No*
29	25	24	No*
30	20	28	0.12
31	17	3	Low*
32	17	13	Low*
33	17	25	0.3
34	17	113	0.28
35	10	24	0.34
36	10	24	0.34
37	10	24	0.41
38	10	26	0.77
39	10	73	0.54
40	5	21	0.37
41	5	24	0.46
42	5	24	0.75
43	2	25	0.62
44	2	24	0.91

572 * "No" and "Low" correspond to "No freezing occurred" and "Strength was too small to be measured", respectively.

573

574 **Table 5. Results from testing of ice with bond salinity of 20 ppt after ~24 h of freezing.**

Sample #	Temperature [°C]	Strength [MPa]
45	-25	1.69*
46	-25	1.67*
47	-25	1.47*
48	-25	1.13
49	-20	1.25
50	-20	0.71
51	-15	0.87
52	-15	0.76
53	-15	0.63
54	-15	0.55
55	-15	0.35
56	-15	0.2
57	-10	0.66
58	-10	0.64

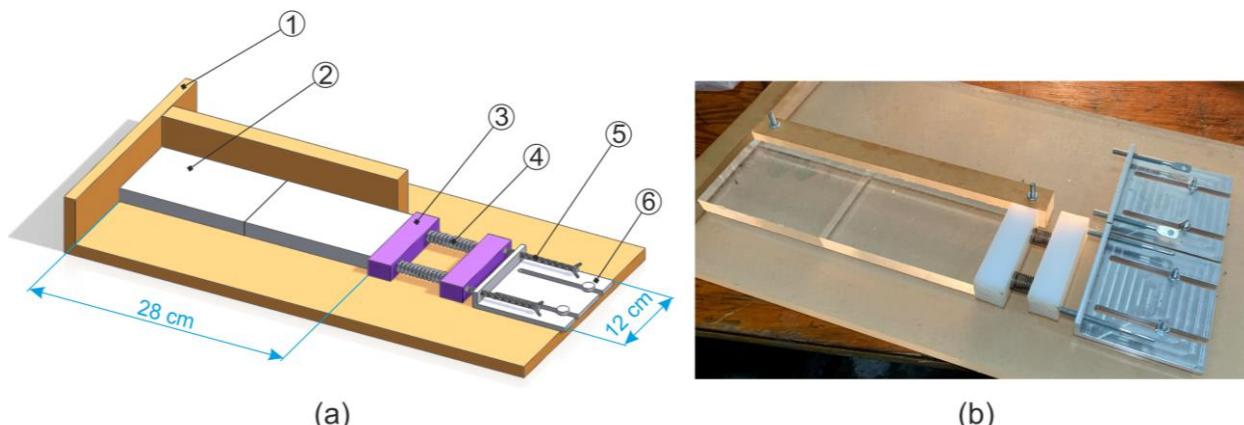
Sample #	Temperature [°C]	Strength [MPa]
59	-10	0.46
60	-10	0.4
61	-5	0.2
62	-5	0.1
63	-3	0.12

575 *failure occurred outside the bond.

576

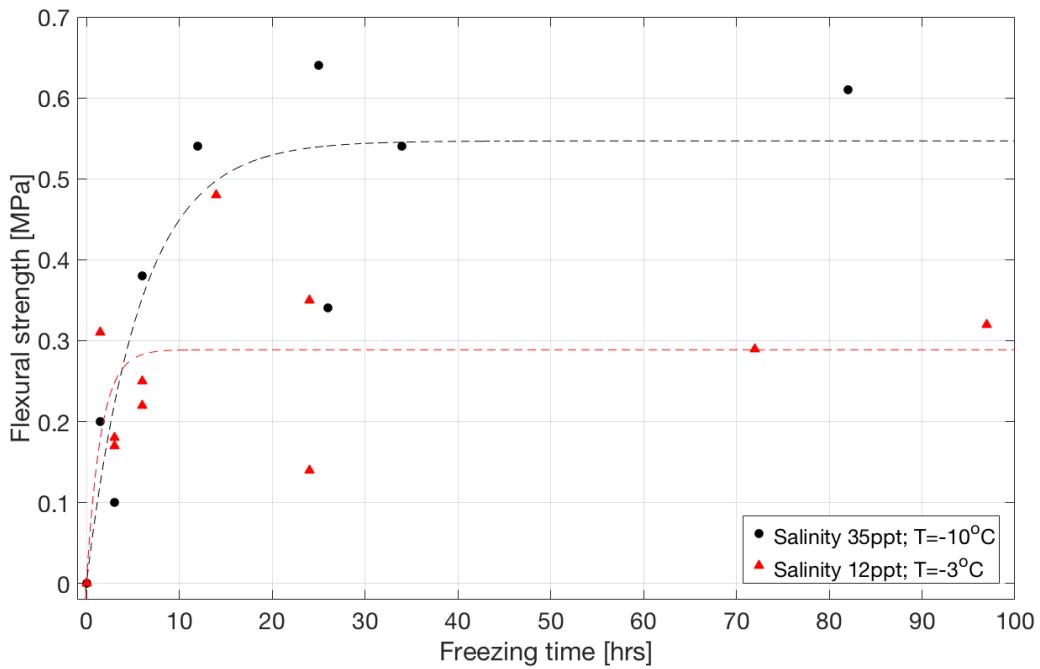
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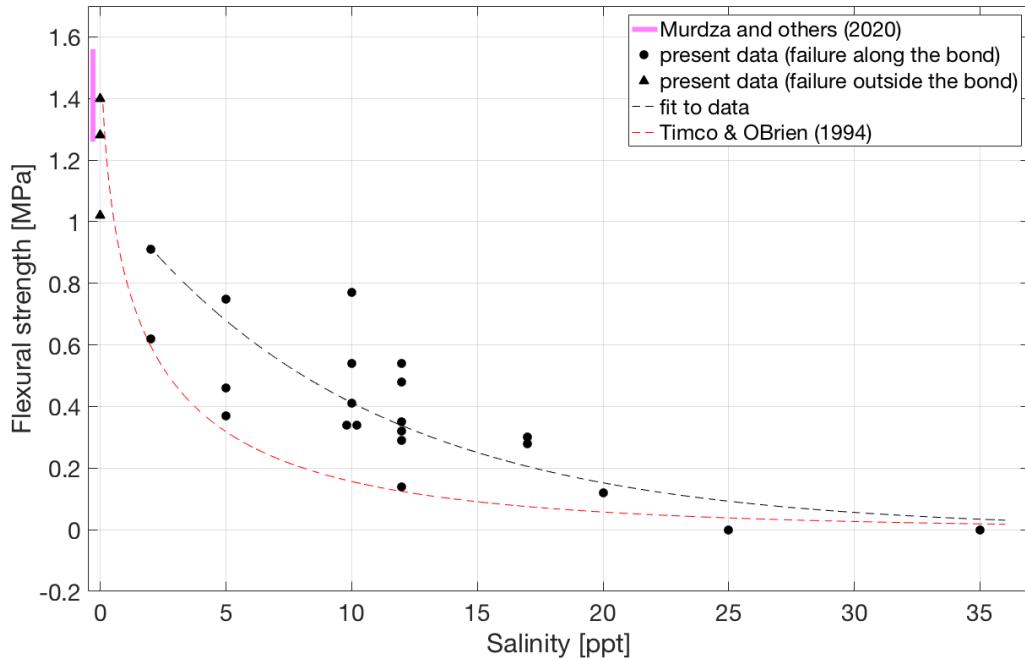
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580 **Figure 1.** Sketch (a) and photograph (b) of the freeze-bonding rig with an ice sample having the shape of a thin plate: 1 –
581 acrylic plate; 2 – ice specimen; 3 – plastic bar; 4 – spring; 5 – bolt; 6 – fixation plate.



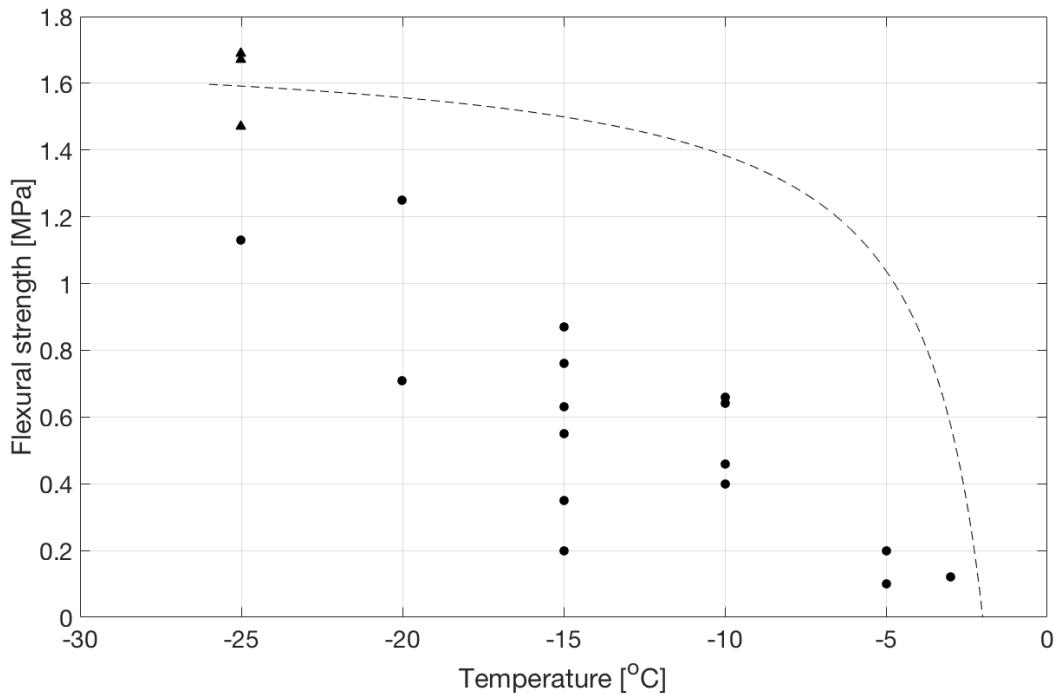
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583 **Figure 2. Flexural strength as a function of freezing time for bonded ice prepared from salt water of 35 ppt salinity at -10°C**
584 (in black) and from salt water of 12 ppt salinity at -3°C (in red).



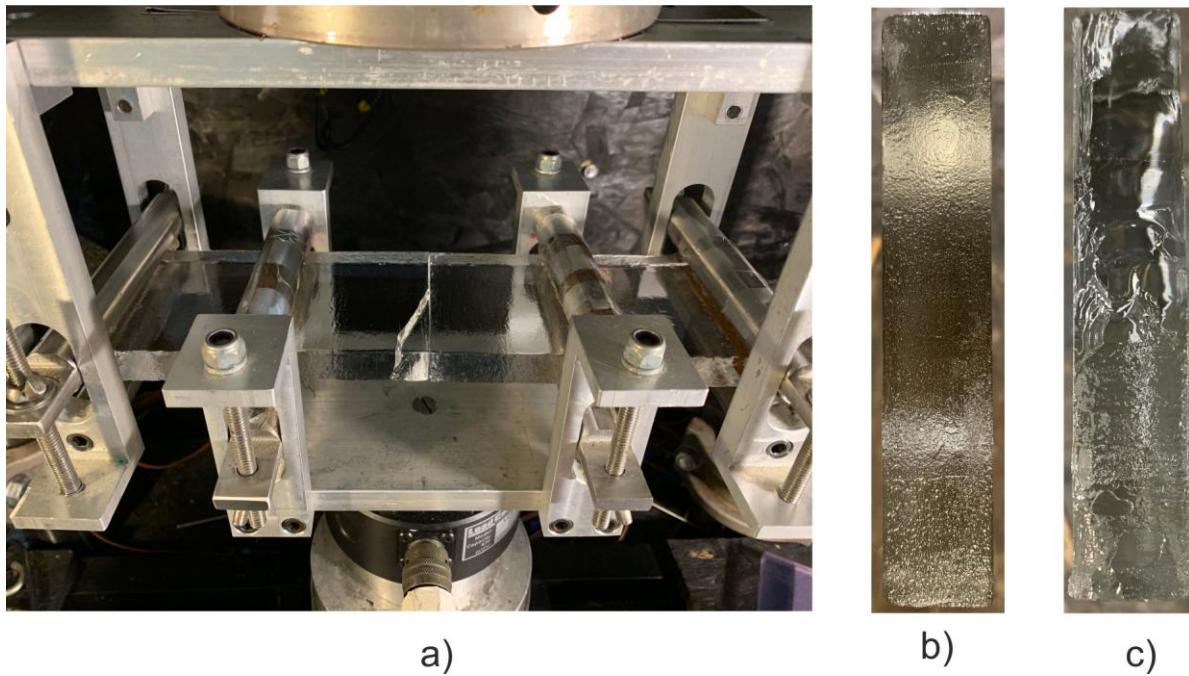
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586 **Figure 3. Flexural strength at -3°C of bonded ice as a function of the salinity of the salt water from which the bond was**
587 **formed. The solid pink line indicates the flexural strength 1.42 ± 0.16 MPa of parent freshwater ice at -3°C (Murdza and**
588 **others, 2020). A red dashed line is taken from Timco and O'Brien (1994) for the ice at -3°C . A black dashed line is a fit to**
589 **the present data.**



590

591 **Figure 4.** Flexural strength of bonded ice as a function of temperature for bonds formed from water of salinity of 20 ppt.
 592 Triangular-shaped points at -25 °C indicate that actual bond strength is greater than that of the parent material as the
 593 failure occurred outside the bond. The dotted line is drawn according to the model in Appendix C.



594

595 **Figure 5.** Photographs of an ice sample #38 right after forced failure (a); the saline bond surface of 10 ppt after a crack
 596 propagated fully through the bond, sample #19 (b) and partially through the parent material, sample #47 (c).

597