Flexural strength of freeze-bonds

General comments

This is a very interesting experimental study. It described what seems to be well conducted experiments and it is well written. However, to major things should be improved:

1. More in-depth explanation of experimental procedures.
2. The discussion should be improved. Below I suggest several interesting perspectives that could be used to analyse your nice results. But I understand if you do not have the time to include all these.

I am not sure if it is **major** or **minor** modifications, but I encourage you to improve the paper. I will be happy to review the revised manuscript.

1. Introduction

Be aware that also Szabo and Schneebeli (2007) did experimental studies of tensile strength of freeze-bonds. Their work was different than your, but it may be nice to read.

2. Experimental procedure

1. Lines 65-67. If I understand correctly you are making the freeze-bonds so that they simulate the strength of refrozen vertical cracks in the ice cover, and not the bond between layers of rafted ice? A simple sketch may help to clarify, it may also help to explain which natural physical mechanisms you are trying to mimic or address.
2. Lines 69-75. I don't understand how you treated the ice. How cold was the ice when it was sprayed with mist? Please explain carefully the procedure of storing and handling the ice. The reader may use this to try to understand the thermal regime of the ice. It was stored in -10°C, but the spraying of mist was done at +2°C. How long was the ice at +2°C before the mist was applied?
3. Lines 108-11. Which strain-rate does a loading rate of 0.1 mm/s in the outer-fiber (according to Linear-elastic first order beam theory I assume?) correspond with?
4. You indicate an interesting and important distinction between simplified beam theory and ordinary continuum mechanics. Would it be possible to give the analytical expressions for the continuum 3D stresses in a linear-elastic beam? You may use appendix to explain the derivation and possible correction for perfect plastic material.

   If you want you could bring this further by discussing different failure theories (max tensile stress, max difference between major and minor tensile stress, Coulomb-Mohr etc.) can explain the observations. This may be too much for this experimentally based paper, but I encourage you to study theoretical models and expand your work.

5. The statement about real major and minor stresses is interesting, but should be expanded (as suggested above) or deleted. You should at least let the reader know if all principle stresses are tensile. If we compare the continuum stress states in the same material in a tensile and beam test it may be possible to explain the experimental 1.7 factor. A very simple suggestion follows here. Let us assume Tresca failure criterion (c as the material property) then the following applies at failure:

   $$c = 1/2(\sigma_{major} - \sigma_{minor}) \qquad \textbf{[1]}$$

   For a tensile test there is only one non-zero principal stress:

   $$c = 1/2(\sigma_{tensile} - 0) = 1/2\sigma_{tensile} \qquad \textbf{[2]}$$

   For a beam test there are both non-zero major and minor stresses and one gets:

   $$c = 1/2(\sigma_{major} - \sigma_{minor}) = 1/2(\sigma_f - 1/3\sigma_f) = 1/3\sigma_f \qquad \textbf{[3]}$$

   Since the material (c) is the same in both tests this gives

   $$\sigma_f/\sigma_{tensile} = 3/2 \qquad \textbf{[4]}$$

   I am not sure how good this suggestion is, but you may see if you think it makes sense. In reality the stress state in a tensile test is also affected by different stresses, but the match with the experimental data you refer to is veey good.

3. Results and observations

1. Lines 133. I suggest you remove Appendix 1 including all references. See below for why I think so.
2. Lines 133 continued. I am not sure if it is surprising that freeze-bonds formed under cold conditions become stronger than parent ice. It has been observed earlier by Høyland and Møllegaard (2014) and I believe also Shafrova and Høyland (2008). If the grains in the freeze-bond are smaller it may help to understand this phenomenon.
3. Lines 148-149. This is not an observation, so I suggest you move it to **Discussion**

4. Discussion

1. Lines 185-193. The observation around the 1.7 factor for σ_f vs. σ_t is interesting. It would be great if you cold argue a bit, at least suggest possible mechanisms. Here you could explain that there are several reasons for flexural strength to be different than tensile strength. Even if we forget the discussion above on the existence of a minor principle stress, the tensiel and flexural strength are equal **only** if linear-elastic first-order beam theory (LEFB) applies, and in reality it rarely does. We may distinguish between two things:

   - Three-dimensional continuum (real?) stresses are different than LEFB stresses. Earlier in the paper you hint about this, and if you could argue that LEFB theory systematically predicts higher strength it would be excellent. Anyway, please mention this possibility.

- If the vertical stress distribution is non-linear it will also give different strengths compared to LEFB. The flexural strength can be both higher and lower, see Ervik et al. (2014) for a discussion on this. This probably does not apply to your experiments as the beam were isothermal, but it applies since you compare with Timco and O'Brien (1994).

2. Lines 195-206. It is good that you summarize earlier literature, you could add Høyland and Møllegaard (2014), Marchenko and Chenot (2009), Szabo and Schneebeli (2007) and Bueide and Høyland (2015) to complete the list. The fact that you used freshwater ice and that your experiments in many ways were more carefully conducted when it comes to ice quality (same ice all the time) and the testing rig (compared to the ones I was involved in), it is not surprising that you find high values of strength. However, a short discussion around the word *strength* would be good. In the (rare) case of one-parameter models such as von-Mises the word strength may be appropriate, but even then this material parameter is generally not equal to the highest force with dimension of stress (such as equation 1 in this paper). In the literature the words strength sometimes means the highest force with unit of stress obtained in the experiments, and sometimes it means some kind of material parameter. This is very important in the shear experiments where the average shear force often do not represent the strength, see Bueide and Høyland (2015) for a discussion here.

3. Lines 221-231. This is not the first study that reports higher FB strength than parent material strength. There are several explanations of this, but in your case it could be that the FB would be more granular than the columnar ice and that the *strength* of tearing apart the columnar ice with larger grains would be less than the smaller more granular texture of the FB?

4. FB and presence of water. It seems that the presence of water is essential in the formation of freeze-bond. Szabo and Schneebeli (2007) tested tensile strength of fresh-water freeze-bonds formed in air, and found stronger bonds for warmer ice. You add only a little bit of water, probably enough to create bonds, but not enough to heat the ice and this should produce some kind of maximal strength.

5. Appendix 1

I don't understand this appendix and I suspect that it is wrong. It starts with the Stefan condition and then claims to integrate it without explain neither which domain the integration yields nor the boundary conditions. I suggest you skip this appendix. Alternatively you can expand it and explain properly, but I suspect you will end of with another independent article. A major challenge with applying continuum thermodynamics is the quantification of the thickness of the bond.

References

Bueide, I. M., Høyland, K. V., 2015. Confined comprssion tests on saline ice and freeze-bonds. In: Proc. of the 23th Int. Conf. on Port and Ocean Eng. under Arctic Conditions (POAC), NTNU, Trondheim, Norway. paper #186.

Ervik, A., Høyland, K. V., Marchenko, A., Karulina, M., Karulin, E., 2014. In-situ experimental investigations of the vertical stress distribution in sea ice covers; a comparison of tensile and flexural strength. In: Proc. of the 22 Int. Symp. on Ice (IAHR), Singapore. Paper #1113.

Høyland, K. V., Møllegaard, A., 2014. Mechanical behaviour of laboratory made freeze-bonds as a function of submersion time, initial ice temperature and sample size. In: Proc. of the 22 Int. Symp. on Ice (IAHR), Singapore. Paper #1202.

Marchenko, A., Chenot, C., 2009. Relegation of ice blocks in the water and on the air. In: Proc. of the 20th Int. Conf. on Port and Ocean Eng. under Arctic Conditions (POAC), Luleå, Sweden. Vol. POAC09-67.

Shafrova, S., Høyland, K. V., 2008. Morphology and 2D spatial strength distribution in two Arctic first-year sea ice ridges. Cold Regions Science and Technology (51), 38-55.

Szabo, D., Schneebeli, M., 2007. Subsecond sintering of ice. Applied Physics letter (90:151916), 3 p.

Timco, G. W., O'Brien, S., 1994. Flexural strength equations for sea ice. Cold Regions Science and Technology (22), 285-298.