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# *Interactive comment on* "An X-ray micro-tomographic study of the pore space, permeability and percolation threshold of young sea ice" *by* Sönke Maus et al.

**Anonymous Referee #2**

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This paper presents an interesting and detailed study of the connectivity properties of the porous brine microstructure of young natural sea ice via X-ray tomography of centrifuged samples, and the associated fluid transport properties of imaged reconstructions of the brine phase via numerical simulation. The results on the connectivity and fluid permeability at very low brine volume fractions and very small length scales are particularly significant, given the improved imaging resolution over previous studies of similar sea ice properties. This is a valuable study, a carefully written manuscript, and an important contribution to sea ice physics. However, I think the significance of this work as described in the abstract is somewhat misplaced, and implications for the so-called "rule of fives" that they draw from their results at very low brine volume



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Discussion paper

fractions and small scales are similarly off base and should be stated more carefully. Nevertheless, with a re-focus of some of the writing, results and conclusions, as well as careful consideration and addressing of the substantive specific issues raised below, I would recommend publication in The Cryosphere - again, after thoroughly revising the manuscript to take care of these concerns.

1. First, a general remark. Consider the two dimensional square bond lattice where bonds are open with probability p and closed with probability 1-p. In general, percolation thresholds are rigorously defined for infinite systems, with the threshold for the infinite square lattice of exactly 1/2. For a 10x10 sample of the lattice, there will be many realizations of the bond configurations where there exist paths of open bonds that connect one side to the other, even for p much less than 0.5. However, it can be proven for the infinite lattice that for any $p<0.5$, there does not exist a percolating (or infinite) cluster of open bonds, but that for p larger than or equal to 0.5 such a cluster does exist, which defines the threshold. Obtaining percolation thresholds or other critical points or even critical exponents from finite samples is a pervasive problem in statistical physics, and involves consideration of the correlation length and its relation to sample size, as discussed in detail for sea ice X-ray tomography in [Pringle et al., 2009]. One of my concerns about this paper is that there does not appear to be any consideration at all of the relationship between the one sample size they look at and their conclusions. Perhaps samples with vertical extent of 8 cm (if they could have scanned those) might typically require a brine volume fraction of 3.5% for there to be connections from top to bottom which include the micro-scale features that have been resolved with their instrument and analysis. Figure 3 in [Pringle et al., 2009] shows a transition around brine volume fraction of about 5% in the behavior of the fractional connectivity (fraction of brine voxels on one face connected to the opposite face, regardless of path characteristics) for sea ice single crystals, and its dependence on sample size, and a corresponding divergence of the correlation length as 5% is approached from below. Do you have data below 2.4% that shows a similar transition or correlation length divergence as you approach the threshold from below, which would

support the notion of a small scale threshold at 2.4%? Or if one could accurately image even smaller features, would one find an even smaller threshold, or a series of smaller thresholds?

2. For binary lattice percolation models, such as the 2D square lattice with bonds open or closed, given a finite sample and a bond configuration, there either is or is not a path of open bonds connecting one side of the sample to the other. However, if the bonds are pipes with arbitrarily small radii, that is, the radii are chosen from a probability distribution with support down to 0, then the question of whether a configuration or cluster percolates or not is now determined by how "thick" one requires the spanning pathways in the cluster to be. In other words, given a cut-off radius, one can then ask if connected clusters of pipes whose radii exceed the cut-off percolate or not. If they do, then fluid flowing through a percolating cluster of large enough pipes will generally be forced to travel through some of the smallest pipes whose radii are near or at the cut-off. Moreover, these "bottlenecks" or throats determine the leading order behavior of the fluid permeability, or the effective electrical conductivity if the bonds are conductors. There are rigorous theorems (and analogous techniques in theoretical solid state physics) to this effect that form the basis of critical path analysis [Golden and Kozlov, in Homogenization: Serguei Kozlov Memorial Volume, V. Berdichevsky et al. (Eds.), 1999; Golden, in Homogenization and Porous Media, U. Hornung (Ed.), 1997; Golden et al., GRL, 2007; Ambegaokar, Halperin and Langer, Phys. Rev. B, 1971]. In the context of sea ice, which is of course a continuum material, the percolation characteristics of the porous brine microstructure can be thought of in terms of the pipe network described above. One way of putting a principal result of this paper is that in sea ice samples of vertical dimension 2 cm to 3 cm with brine volume fractions exceeding 2.4%, there are fluid pathways through the brine phase connecting the top to the bottom whose minimal "diameter" exceeds 0.07 mm (or in terms of the pipe network, configurations of pipes whose diameters exceed a cut-off of 0.07 mm span the sample vertically, or percolate).

Now, let us discuss how this result is related to the so-called “rule of fives” and the generally accepted value of 5% brine volume fraction for the “percolation threshold” of sea ice. The concise statement of this “rule of thumb” in the first paragraphs of the papers [Golden et al., Science, 1998; Golden et al., GRL, 2007, Pringle et al., JGR, 2009] is that columnar sea ice is “effectively impermeable” to bulk fluid flow for brine volume fractions below about 5%. This is not stated as a mathematical theorem, and there is an understanding that by the very nature of percolation theory for finite samples, and the complex multiscale structure of the brine phase, one would expect the possibility of some fluid flow over relatively small scales through relatively small pore spaces, even for brine volume fractions below 5%. (Figure 3b in [Golden et al., 2007] shows that the fractional connectivity for samples of 8 mm in vertical extent remains non-zero down to below 4% brine volume fraction.) However, I have personally made hundreds of in situ measurements of the vertical fluid permeability of sea ice in the Arctic and Antarctic, by removing partial cores and then measuring the rate at which water fills the hole through the ice at the bottom of the hole by various techniques. Even with the uncertainties in this “sack hole” method, I can unequivocally state that if the sea ice at the bottom of the hole is columnar and has brine volume fraction below about 4% or 5% (and horizontal flow is blocked with “packers”), there will most likely be very little or no measurable fluid in the hole even after a few hours. As the brine volume fraction of columnar sea ice decreases from high values associated with quite permeable ice, there is a noticeable, clear transition to bulk fluid flow over the scale of tens of centimeters relevant to the experiment, being essentially shut down, or the ice becoming effectively impermeable, once the brine volume fraction gets below about 5%. Roughly speaking, permeability values then generally lie below about 10ˆ{-12}.

The spirit in which this rule was developed was in terms of whether or not the brine microstructure could enable various geophysical processes such as surface flooding and subsequent snow-ice formation, melt pond drainage, and changes in salinity. For example, suppose we consider upward percolation of sea water and brine due to snow loading of the ice surface, and the subsequent freezing of the flooded surface snow layer. If the upper layer of sea ice through which fluid must pass to reach the surface, say, has permeability around 10ˆ{-13} with brine volume fraction just below the threshold, we may wind up with very little water on the surface and essentially no new snow-ice production. On the other hand, if the permeability of this restrictive layer is around 10ˆ{-11} or larger with brine volume fraction a bit above the threshold, then after several hours there may be a few centimeters of water on the surface which could produce a significant amount of snow-ice that affects ice mass-balance accounting.

From the point of view of the pipe network, the 5% threshold for bulk flow in practice means that in sea ice samples of vertical dimension on the order of, say, 20 cm to 50 cm with brine volume fractions above about 5%, there are fluid pathways through the brine phase connecting the top to the bottom whose minimal "diameter" exceeds a certain cut-off value, which is much larger than 0.07 mm. As reported in [Weeks and Ackley, 1982], S. Martin and co-workers over many studies found that most vertically oriented "channels" through which the bulk of fluid is transported through sea ice over tens of centimeters have diameters that range from about 1 mm to 1 cm, with some channels much larger. In Equation (4) of [Golden et al., GRL, 2007] the cut-off, bottleneck, or minimal diameter was then chosen to be 1 mm, which leads via critical path analysis to the prediction of the scaling factor in front. This percolation formula in Equation (4) with a bulk transport threshold of 5% then agrees very closely with in situ data for brine volume fractions above the threshold. By way of comparison with the scales considered in the current paper, the amount of fluid that flows per unit time through a circular pipe of diameter 1 mm is 10ˆ{4} times the amount that flows per unit time through a pipe of diameter 0.1 mm, which is just a bit larger than the critical diameter of 0.07 mm in this paper.

Thus, it is not really appropriate to state without careful explanation and qualification that the 2.4% value found here is considerably lower than the 5% threshold which has been widely used for bulk flow over larger scales. What is being referred to is quite different for these two situations, in the setting of a multiscale porous medium like sea ice as described above, with the rule of fives and the 5% threshold describing fluid transport behavior on significantly larger sample and pore size scales than what is considered in this paper. (In fact, the vertical sample size of 5 mm for the permeability simulations calls into question the applicability of this work beyond the smallest of scales. See Figure 3 in [Pringle et al., 2009] on the dependence of the correlation length with brine volume fraction.) The results described in [Golden et al. 2007, Pringle et al., 2009] with a vertical threshold of 4.6% (and higher thresholds in the horizontal directions) were a first step in imaging the connectivity of the brine phase and building toward larger scales, with the analysis of the most basic building blocks - sea ice single crystals. As stated in [Golden et al. 2007], "These images provide insight into and constraints for more detailed modeling of micro-scale inclusion connectivity. A similar analysis of large-scale pore networks remains challenging, but is inherently reflected in the in situ permeability data." Indeed, extending such analyses to the scales relevant for the rule of fives remains a challenge today.

3. One major significance of the paper, in my opinion, is that they have explored a new level of fine scale structure and conducted a high resolution analysis of the habitats of microbial life in sea ice, and also carefully computed a key property of sea ice which is critical for local nutrient fluxes, namely, fluid permeability, on scales which may be particularly relevant for small scale biological processes.

4. I am concerned that while centrifuging may leave major inclusion structures intact, if this process modifies the brine microstructure by opening up new pathways that then appear to be connections in the X-ray tomographic images, it will be at these finest scales that are the focus of this paper. I think there should be some discussion of how the centrifuging process may or may not affect the 2.4% value thus obtained.

5. The results in [Perovich and Gow, JGR, 1996] should be referenced and briefly discussed in light of the results in this paper.

6. A few sentences, or synopsis, about the scales of features the authors can resolve

and how their resolution compares to previous works would be welcome, or if this were made a little clearer with a sentence like: “The inclusions we see that form connected fluid pathways at lower brine volume fractions than have been observed before have the following characteristic ranges of dimensions, etc.” The paragraph around line 135 needs to be expanded and clarified.

7. In line 375, it is stated, “So far sea ice permeability has been studied in terms of isotropic percolation (Petrich et al., 2006; Golden et al., 2007; Pringle et al., 2009)”. However, in [Pringle et al., JGR, 2009] the finding of different values of the percolation threshold in three perpendicular directions for a sea ice single crystal is certainly anisotropic percolation, and should be noted as such.

8. The idea of analyzing the effective centrifuged brine volume fraction compared to the total is excellent. However, by a certain point there seem to be so many different types of porosities running around that it is difficult to keep them straight. Perhaps a synopsis and overall explanation would be helpful, as well as clearer definitions with diagrams of all the parameters in Table 2.

9. Line 23 - should refer to [Polashenski et al., JGR, 2018] which deals explicitly with the issue of how initially permeable sea ice supports melt ponds.

10. Line 92 - missing a "c" in subscript of phi.

11. The exponent in Equation (8) is 2.6, which I assume is a best fit. The corresponding exponent in Equation (4) in [Golden et al., GRL, 2007] is 2. This is a theoretical prediction, based on an argument that even though sea ice is a continuum material that could exhibit so-called non-universal behavior different from lattices (with its exponent larger than 2, see [Golden, PRL, 1990] for rigorous results on lattices), it exhibited universal behavior due to the lognormal distribution of brine inclusion sizes. It is interesting to speculate if the exponent of 2.6 in this paper is a demonstration of non-universal behavior at these fine scales. Perhaps a sentence could be added addressing this?

12. Line 15 - I believe the authors meant to say “Sea ice is a porous medium that covers, on average, about 12 percent of the earth’s oceans.” (Or about 7 percent of earth’s surface).

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