Review of ‘Simultaneous estimation of wintertime sea ice thickness and snow depth from space-borne freeboard measurements’, Shi Hoyeon et al.
By Isobel R Lawrence.

Paper synopsis

A methodology is presented for estimating sea ice thickness and snow depth (on sea ice) using ice or snow freeboard, and snow surface and snow/ice interface temperatures (available from satellite data). The method is based on the assumption that conductive heat flux through the snow/ice interface is continuous, therefore the bulk temperature difference across the ice layer, divided by the ice thickness, is proportional to the temperature difference across the snow layer divided by its depth. The proportionality is a function of the conductivities of ice and snow, but can be derived empirically using temperature profiles from drifting buoys. The authors find a piecewise (2-part) linear relationship between the ratio of the temperature differential across the snow layer to the temperature change through the ice, and the ratio of snow to ice thickness. They present a thorough analysis of the possible physical mechanisms behind the two-part function, in the appendix.

Using the derived relationship, the ratio of snow depth to ice thickness can be estimated purely from satellite measurements of snow surface and snow/ice interface temperature. Then, combined with either snow or ice freeboard, this ratio can be used to simultaneously estimate snow depth and ice thickness. The appeal of the methodology lies in the fact that independent estimates of snow depth are not required to calculate ice thickness. The authors compare their ice thickness and snow depth estimates to those from OIB, and also against ‘CryoSat-like’ and ‘ICESat-like’ ice thicknesses, where snow depth from the Warren climatology is implemented in the traditional freeboard-to-thickness buoyancy equations. Finally the method is extended to the basin scale, using satellite-derived snow surface and snow/ice interface temperatures and sea ice freeboard data from CryoSat-2.

The manuscript is very well written, coherent and extremely thorough in its explanations, however something major has been overlooked which I believe means that this method cannot be used to retrieve snow depth and ice thickness from *radar* altimetry. That said, the method remains applicable to laser altimetry, e.g. IceSat-2, and is therefore still very valuable. I therefore recommend the paper for publication following this major revision:

Major comments

In order to use this methodology with satellite data from CryoSat-2, ice freeboard (the elevation of the snow/ice interface above the ocean surface) is required. However it is impossible to retrieve ice freeboard from CryoSat-2 without a-priori knowledge of the snow layer. Since the radar pulse slows down as it travels through the snow, snow depth is required in order to correct for the slower speed of propagation and estimate sea ice freeboard.

To compound the issue, the equation to convert radar freeboard into ice freeboard is incorrectly reported in a number of studies, including that of Kurtz et al. (2014, eq. 16) which describes the CS2 ice freeboard dataset you use in your analysis. Please see Mallett et al. (2020) for the correct derivation of the equation and details of its misreporting in the literature.

The correct equation for sea ice thickness from *radar* altimetry (assuming full snow penetration) is:

$$H = \left(f_r + h\left(\frac{c}{c_s} - 1\right)\right)\left(\frac{\rho_w}{\rho_w - \rho_i}\right) + h\left(\frac{\rho_s}{\rho_w - \rho_i}\right),$$

where f_r is radar freeboard, as estimated from radar altimeters like CryoSat-2. If this equation cannot be solved by the proposed methodology (I do not believe it can be), then the paper should be restructured to focus on the laser case. The methodology remains valid for use with snow freeboards, and these are available from ICESat and now ICESat-2, so perhaps section 4.3 could be changed to an application to ICESat data. I appreciate that this will require a substantial amount of work, which is why I consider this revision major. However I find this methodology novel and valuable and the results in section 4.2 are encouraging; I would like to reiterate therefore that I think the paper deserves publication subject to this alteration and the following minor revisions:

Minor comments

I think you need to include an uncertainty budget for your sea ice thickness and snow depth estimates.

L30: “However, the radar scattering horizon is often treated as the snow–ice interface”. Include Hendricks et al. (2016), Guerreiro et al. (2017), Tilling et al. (2018) as refs here since AWI, LEGOS and CPOM CryoSat-2 ice thickness products all make the same assumption.

L72: …”for given densities and freeboard” – (and assuming no snow penetration for laser and full snow penetration for radar)

L138: Could you say how many are discarded based on this criterion, and out of how many total.

L159: Can you provide a reference for the OIB data processing document where the densities are given?

L160: I understand that you keep ice density constant in order to compare with OIB data, but later when comparing with satellite-derived ice thickness should you not then use the densities used in those products for a fair comparison?

L198: “It was reported…” – By who?

L200: Where does T_{si} for March come from if the Lee et al. (2018) dataset is only December-February ?

L201: “…if data frequency is over 20”. Do you mean if 20 days out of the month contain data? Or are you referring to a number of points per grid cell?

L205: Please could you provide the details and a reference for which OIB dataset you used and where it is available from? i.e. L2, L4, Quicklook?

Figure 4: Why do you choose to show us the 7-day averaged plot in Fig.4 when Figure 3 was showing 15-day averaged temperature profiles?

L235: At the end of this sentence you could refer the reader to the appendix.

L244: bias is not near-zero in Fig 4b, it *is* zero.

L269: Did you calculate a different alpha for each year, and apply the different alpha to each year of OIB data? Or did you just average all the years together? Please clarify this in the text.

L295: Do you get the MW99 for input into Eqs. (4) and (5) from the CS2 data? If so is it monthly grid-averaged? How do you assign each OIB point a snow depth?

L301: "Therefore, if there are decreasing trends in both ice thickness and snow depth, the decreasing trend of ice thickness estimated from the constant snow depth will be diminished in radar, while being amplified in lidar" – This sentence seems overcomplicated. To me, all that the bottom two plots of Figure 7 demonstrate is that MW99 snow depth is larger than OIB snow depth. For the laser case, this means that using W99 causes ice thickness to be *underestimated* compared to H(OIB), and for the radar case using W99 results in ice thicknesses that are *too thick* compared to OIB. Perhaps you could plot MW99 against h(OIB) to clarify this? The retrieval of sea ice thickness from ICESat has not traditionally used the Warren climatology- see Kwok and Cunningham (2008) and Petty et al. (2020). Therefore I don't think it's justified to call this 'ICESat-like thickness' since you are not using the same snow depth product that they do.

Typos / Grammar

L128: "Therefore, the interface searching algorithm…" -> "Therefore, an interface searching algorithm…"

L165: "Sea ice thicknesses converted from MW99 using Eqs. (4) and (5) are also compared to examine how simultaneous retrievals…" **->** "Sea ice thicknesses are also calculated from Eqs. (4) and (5), using MW99 as snow depth, to examine how simultaneous retrievals…"

L194: "This reformatted AASTI-v2 data are called…" -> "This reformatted AASTI-v2 dataset is called…"

L293: "Examining how the current practices of retrieving the sea ice thickness through ICESat and CS2 measurements are compared with the simultaneous retrievals is of interest" **->** "We now examine how the current practises of retrieving sea ice thickness from ICESat and CS2 measurements compare with our method."

L294: "In doing so, OIB-measured…" -> "To do so, OIB-measured…"

L297: “Apparently, ICESat-like thickness tends….” -> “According to our analysis, ICESat-like thickness tends….”

L416: “…which are hard to be quantified explicitly.” -> “…which are hard to quantify explicitly.”