Review of: Estimating the snow depth, the snow–ice interface temperature, and the effective temperature of Arctic sea ice using Advanced Microwave Scanning Radiometer 2 and ice mass balance buoy data

1 Introduction

1.1 Summary of Review

This manuscript addresses the issue of converting satellite radar ranging data to sea ice thickness estimates, while simultaneously retrieving the depth of the overlying snow. This is done through a novel linear mapping of the temperature gradients in the snow and ice to the thickness values of the materials.

However the manuscript does contain a technical issue which affects its application to CryoSat-2 data (described in Sect. 2.2), which must be remedied before publication. In short, the freeboard data used in the study have been calculated using the modified Warren climatology, so any thickness estimates resulting from them cannot be independent of that climatology.

Otherwise, this manuscript contains a novel approach to estimating the depth of snow on sea ice, and the authors show that the related thickness values match OIB thickness values more closely than those resulting from use of the modified Warren climatology.

I believe that if the issues presented in this review can be resolved, subsequent publication of this manuscript would be of significant interest to the sea ice community.

1.2 Summary of the Technique Presented

Sea ice thickness is conventionally estimated from ice freeboard as such (using the notation of this paper):

$$H = \frac{h_f \rho_w + h \rho_s}{\rho_w - \rho_i} \tag{1}$$

Where H is sea ice thickness, h_f the height of the snow ice interface above the waterline, and h the depth of the overlying snow. ρ values represent the densities of overlying snow, water and ice. From the above, it can be seen that for h_f to be converted to H, h must be known (as well as all the density values). Unfortunately despite a major research effort to map the spatial distribution of h, it remains highly uncertain.

One of the novelties of this paper is the authors' framing of the problem thus:

$$H = h_f \frac{\rho_w}{\rho_w - \rho_i - (h/H)\rho_s} \tag{2}$$

This framing allows h_f to be converted to H without direct knowledge of h, instead it is sufficient just to know the ratio h/H - this is referred to as α. The authors then show that this formulation is useful, by estimating (h/H) in a Pan-Arctic way using satellite-measured temperatures of the air-snow and snow-ice interfaces.

A byproduct of this approach is that snow depth (h) is also trivially extracted:

$$h = h_f \frac{\rho_w}{\rho_w - \rho_i - (h/H)\rho_s} \times (h/H) \tag{3}$$

$$H = \frac{h_f \rho_w + h \rho_s}{\rho_w - \rho_i}$$

$$h = h_0$$

Sea Ice Thickness, H (m)

Snow Depth, h (m)

h_0

$$H = \frac{h_f \rho_w + h \rho_s}{\rho_w - \rho_i}$$

$$H = \frac{1}{\alpha} h$$

Sea Ice Thickness, H (m)
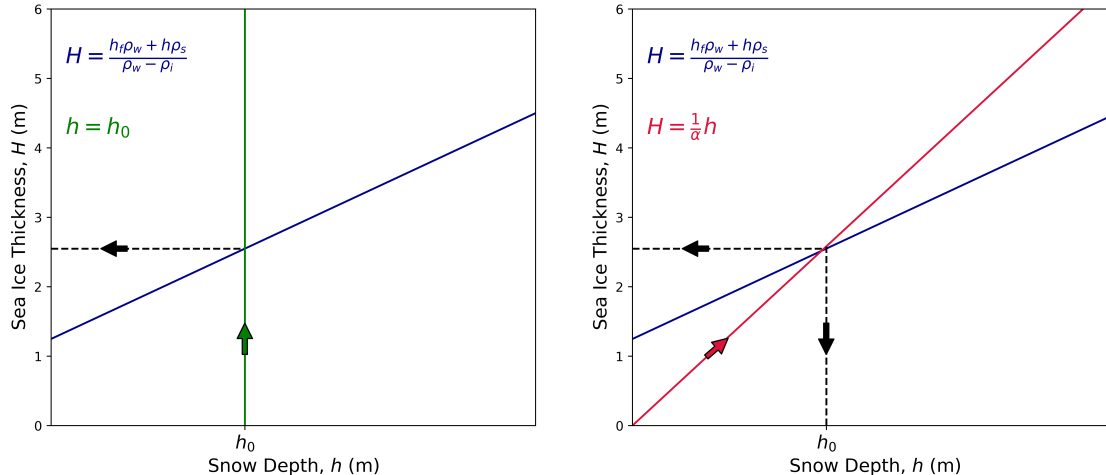
Snow Depth, h (m)

h_0

Figure 1: Graph of H, h space for a case with typical values of densities, snow depth and ice thickness

It should be noted that this approach of 'simultaneously' working out h and H is mathematically equivalent to first working out h using Eq. (3) and substituting the result into Eq. (1).

The method can be further understood graphically in h, H space, as in Figure 1. When the ice freeboard is known, the corresponding snow depth (h) and ice thickness (H) lie unconstrained in h, H space along the blue line described by Eq. (1).

Thickness is then traditionally determined by a priori knowledge of the snow depth (h_0) - this is depicted in the left hand plot by the intersection of the green line (known snow depth) with the blue line.

This paper introduces a different relationship to constrain sea ice thickness, depicted on the right, revolving around the empirical parameter α. By finding the intersection with the red (α) line and the blue line, the authors are able to simultaneously derive h and H.

2 Major Points

2.1 Difference between radar freeboard and scattering horizon height

L22: The statement "altimeters ... measure sea ice freeboard" is only approximately correct in the case of radar altimeters.

The instrument on board CS2 measures a time of flight, which can be related to the height of some radar scattering horizon only where no snow lies in between the scattering horizon and the instrument. When snow is in between and fully penetrated by the radar, the radar range to the scattering surface is overestimated due to slower pulse propagation in the overlying snow. Correcting for this and estimating the height of the ice-snow interface requires knowledge of the overlying snow (Mallett et al., 2020).

This issue surfaces again when the authors identify the radar freeboard as the height from the sea surface to the radar scattering horizon in L28. This is only the case for bare ice. Where overlying snow is present and fully penetrated, the radar freeboard is a finite distance below the ice freeboard (the assumed scattering surface). In the freeboard product used in this manuscript (Kurtz et al., 2014) this displacement is $h_s(1 - c_s/c)$.

This is relevant to Fig. 1, where h_{rf} is depicted as being above the ice freeboard. While it may be true that the radar scattering horizon is above the snow-ice interface, in products that assume full radar penetration of the snowpack the radar freeboard is lower than the ice freeboard. Theoretically for total radar penetration and a freeboard depressed to near the water by snow, the radar freeboard can be below the waterline (while the ice freeboard and scattering horizon are above).

2.2   The freeboard product used by the authors has been created with mW99

In the final sentence of the abstract, the authors state:

> "In conclusion, the developed α-based method has the capacity to derive ice thickness and snow depth, without relying on the snow depth information as input to the buoyancy equation for converting freeboard to ice thickness."

However, the method presented here works directly from ice freeboard data which can only be derived by relying on snow depth information (Sect. 5.1 & Eq. 15 of Kurtz et al., 2014).

I feel that what the authors would like to present is a way to convert *radar freeboards* to ice thickness without relying on snow depth data, and this should be done before publication. I think it is possible for the authors to adapt their processing chain to deal with this, although it may complicate things.

2.3   Uncertainty Analysis

The authors state in their Discussion and Conclusions section:

> Overall, the developed α-based method yields ice thickness and snow depth, without relying on a priori 'uncertain' snow depth information, which results in uncertainty in the ice thickness retrieval.

They are of course correct to identify that uncertainty in snow depth leads to uncertainty in the ice thickness retrieval. To avoid having to quantify snow depth, they instead rely on a parameter equal to h/H, which they empirically derive from the temperature of the air-snow and ice-snow interfaces.

Clearly there is significant uncertainty in the value of α, and the authors should try to quantify how this propagates through into uncertainty in ice thickness. It's possible that their α parameter is more uncertain than other published data for h, and if so this method will deliver lower quality estimates of H than the traditional method.

It seems (looking at Fig. 1 of this review) that a given error in α would have a more serious impact on H than the same error in h, because the gradients of the lines are much more similar in on the left panel of Fig. 1 than on the right. This issue scales with the alpha parameter (i.e. as the freeboard goes down), and at high α very small uncertainties in alpha will lead to large uncertainties in H.

As alpha becomes so large that the freeboard tends to zero (not that uncommon in the Atlantic sector of the Arctic), the method seems to lose its usefulness, whereas the traditional method continues to function. That is to say in the case of near-zero freeboard, the traditional method still provides an estimate of H, but that proposed by the authors does not (see Eq. 7 as $h_f \to 0$).

This is addressed in L 161/162, where a critical value is given for alpha, and it is explained that for alpha above this value data are not produced. How often does this occur? And what is the effect on H of small errors in alpha just below this critical threshold?

3   Minor Points

3.1   General

- L21: The authors should consider directing the reader to Laxon et al. (2003) when illustrating that thickness has been estimated for nearly two decades.

- When discussing studies indicating the height difference between the scattering horizon and the snow-ice interface, the authors should consider directing the reader to Nandan et al. (2017) and Willatt et al. (2010, 2011).

- L62 & 64: Define RTM before using the acronym.

- The font sizes of some annotations to Figure 3 should be increased so as to be legible and comparable to the (a), (b), (c) lettering.

- In Fig. 2, the box that reads 'Find temperature discontinuity point'. It is my understanding that the temperature is continuous (but not a smooth function), and therefore it has no discontinuities (but its gradient does). Should this box then read 'Find temperature gradient discontinuity point'?

- I think the notation of H and h in combination with h_f, h_{rf} and h_{tf} is confusing to the casual reader. For instance, the fact that h and h_f look so similar but are in fact unrelated confused me initially. Even changing $H \to H_{ice}$ and $h \to h_{snow}$ would clarify this.

3.2 Validation of H against OIB Data

The authors are able to create two products from freeboard data obtained by OIB and CryoSat-2, one for snow depth (h), and one for ice thickness (H). They then rightly try to assess the quality of these data products against other datasets, namely the OIB snow depth and ice thickness data.

There are at least five algorithms published to process the raw OIB radar returns into along-track snow depth data, and they produce a spread in the mean snow depth (Kwok et al., 2017). 'Validation' of a model or data implies comparison to true or certain values, and it is unclear which OIB snow depth product (if any) represents *the truth*. This limits the strength of the validation exercise. Nonetheless, I understand that OIB snow depth values have historically been taken as *the truth* in published work so this is a perhaps not a big issue. It might also be argued that the spread of different OIB data is sufficiently small relative to other methods of snow depth estimation to allow OIB to approximate the truth for validation purposes.

I feel that there is however a more significant issue with the authors' claims to have 'validated' their ice thickness data against OIB ice thickness data (H_{OIB}). OIB aircraft instruments do not measure thickness (H_{true}) directly, but instead estimate it based on freeboard, snow depth, snow density and ice density values. As such, OIB thickness data (while likely to be the most accurate data on H_{true} outside of in-situ measurement), undoubtedly suffer from biases involving snow depth, snow density and ice density, and therefore should not be mistaken for H_{true}.

The technique for determining H_{OIB} is very similar to that presented in this manuscript: the authors use identical freeboard, snow density and ice density values to estimate thickness with the hydrostatic equilibrium assumption. Given these similarities, comparing the thickness estimates in this paper with OIB thickness estimates doesn't really qualify as independent validation.

It seems more like the exercise of comparing H estimates is in fact comparing the novel snow depth estimates with OIB snow depths (Fig 7 top row; a valuable analysis), and then investigating how that singular difference propagates into sea ice thickness estimates.

I suspect that the strong agreement between the two datasets presented in the middle row of Fig. 7. is largely a result of the identical radar freeboards and geophysical parameters used in each processing chain. After all, much of sea ice thickness is determined by radar freeboard information, independent of snow data. The fact that the 'simultaneous' method matches H_{OIB} data more closely than MW99 is therefore evidence that the snow depth product produced by the 'simultaneous' method is closer to OIB snow depths than MW99 (because everything else is equal).

I think it is perfectly reasonable (and in fact expected) to compare H estimates from the new method with H_{OIB}. However, I think this should be presented as a 'comparison with' or 'evaluation against' OIB data, rather that implying that the new data are being validated against some true value. It is also an understandable bit of reasoning to say that values which are closer to H_{OIB} are likely to be closer to H_{true}, but if this assumption is made it should be stated explicitly.

3.3 Limitations of Other Data

L66 - 69:

> Other approaches worth mentioning are snow depth retrieval using dual-frequency altimetry (Guerreiro et al., 2016; Lawrence et al., 2018, Kwok and Markus, 2018), snow on sea ice model accumulating snowfall from reanalysis (Petty et al., 2018), multilinear regression (Kilic et al., 2019), and the neural network approach (Braakmann-Folgmann and Donlon, 2019). However, these methods do not satisfactorily meet the criteria required for freeboard to ice conversion over the entire Arctic Ocean basin scale or multi-year time scale.

The approach of Guerreiro et al. (2016) and Lawrence et al. (2018) are limited latitudinally by the AltiKa orbital inclination and Lawrence et al. (2018) additionally through calibration with OIB which only operates in Spring. As the authors identify, they are limited in spatial or temporal extent.

While there are limitations to the data products of Petty et al. (2018), Kilic et al. (2019) and Braakmann-Folgmann and Donlon (2019), it's not obvious that these can be characterised by failure to cover the entire basin on a multiyear timescale. As such, the statement on L69 that they do not satisfactorily meet these criteria should be clarified.

3.4 Rainbow Color Schemes

Where possible, authors should avoid presenting continuous data with 'rainbow' color schemes as in Figures 6 & 8. This is because (among other reasons) the scheme tends to imply sharp transitions in the data where they do not exist (Borland and Taylor, 2007). Alternatives for geoscientists are given by Light and Bartlein (2004), Stauffer et al. (2015) and Thyng et al. (2016).

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Review of 'Simultaneous estimation of wintertime sea ice thickness and snow depth from space-borne freeboard measurements', Shi Hoyeon et al.
By Isobel R Lawrence.

Paper synopsis

A methodology is presented for estimating sea ice thickness and snow depth (on sea ice) using ice or snow freeboard, and snow surface and snow/ice interface temperatures (available from satellite data). The method is based on the assumption that conductive heat flux through the snow/ice interface is continuous, therefore the bulk temperature difference across the ice layer, divided by the ice thickness, is proportional to the temperature difference across the snow layer divided by its depth. The proportionality is a function of the conductivities of ice and snow, but can be derived empirically using temperature profiles from drifting buoys. The authors find a piecewise (2-part) linear relationship between the ratio of the temperature differential across the snow layer to the temperature change through the ice, and the ratio of snow to ice thickness. They present a thorough analysis of the possible physical mechanisms behind the two-part function, in the appendix.

Using the derived relationship, the ratio of snow depth to ice thickness can be estimated purely from satellite measurements of snow surface and snow/ice interface temperature. Then, combined with either snow or ice freeboard, this ratio can be used to simultaneously estimate snow depth and ice thickness. The appeal of the methodology lies in the fact that independent estimates of snow depth are not required to calculate ice thickness. The authors compare their ice thickness and snow depth estimates to those from OIB, and also against 'CryoSat-like' and 'ICESat-like' ice thicknesses, where snow depth from the Warren climatology is implemented in the traditional freeboard-to-thickness buoyancy equations. Finally the method is extended to the basin scale, using satellite-derived snow surface and snow/ice interface temperatures and sea ice freeboard data from CryoSat-2.

The manuscript is very well written, coherent and extremely thorough in its explanations, however something major has been overlooked which I believe means that this method cannot be used to retrieve snow depth and ice thickness from *radar* altimetry. That said, the method remains applicable to laser altimetry, e.g. IceSat-2, and is therefore still very valuable. I therefore recommend the paper for publication following this major revision:

Major comments

In order to use this methodology with satellite data from CryoSat-2, ice freeboard (the elevation of the snow/ice interface above the ocean surface) is required. However it is impossible to retrieve ice freeboard from CryoSat-2 without a-priori knowledge of the snow layer. Since the radar pulse slows down as it travels through the snow, snow depth is required in order to correct for the slower speed of propagation and estimate sea ice freeboard.

To compound the issue, the equation to convert radar freeboard into ice freeboard is incorrectly reported in a number of studies, including that of Kurtz et al. (2014, eq. 16) which describes the CS2 ice freeboard dataset you use in your analysis. Please see Mallett et al. (2020) for the correct derivation of the equation and details of its misreporting in the literature.

The correct equation for sea ice thickness from *radar* altimetry (assuming full snow penetration) is:

$$H = \left(f_r + h \left(\frac{c}{c_s} - 1 \right) \right) \left(\frac{\rho_w}{\rho_w - \rho_i} \right) + h \left(\frac{\rho_s}{\rho_w - \rho_i} \right),$$

where f_r is radar freeboard, as estimated from radar altimeters like CryoSat-2. If this equation cannot be solved by the proposed methodology (I do not believe it can be), then the paper should be restructured to focus on the laser case. The methodology remains valid for use with snow freeboards, and these are available from ICESat and now ICESat-2, so perhaps section 4.3 could be changed to an application to ICESat data. I appreciate that this will require a substantial amount of work, which is why I consider this revision major. However I find this methodology novel and valuable and the results in section 4.2 are encouraging; I would like to reiterate therefore that I think the paper deserves publication subject to this alteration and the following minor revisions:

Minor comments

I think you need to include an uncertainty budget for your sea ice thickness and snow depth estimates.

L30: "However, the radar scattering horizon is often treated as the snow–ice interface". Include Hendricks et al. (2016), Guerreiro et al. (2017), Tilling et al. (2018) as refs here since AWI, LEGOS and CPOM CryoSat-2 ice thickness products all make the same assumption.

L72: …"for given densities and freeboard" – (and assuming no snow penetration for laser and full snow penetration for radar)

L138: Could you say how many are discarded based on this criterion, and out of how many total.

L159: Can you provide a reference for the OIB data processing document where the densities are given?

L160: I understand that you keep ice density constant in order to compare with OIB data, but later when comparing with satellite-derived ice thickness should you not then use the densities used in those products for a fair comparison?

L198: "It was reported…" – By who?

L200: Where does T_{si} for March come from if the Lee et al. (2018) dataset is only December-February ?

L201: "…if data frequency is over 20". Do you mean if 20 days out of the month contain data? Or are you referring to a number of points per grid cell?

L205: Please could you provide the details and a reference for which OIB dataset you used and where it is available from? i.e. L2, L4, Quicklook?

Figure 4: Why do you choose to show us the 7-day averaged plot in Fig.4 when Figure 3 was showing 15-day averaged temperature profiles?

L235: At the end of this sentence you could refer the reader to the appendix.

L244: bias is not near-zero in Fig 4b, it *is* zero.

L269: Did you calculate a different alpha for each year, and apply the different alpha to each year of OIB data? Or did you just average all the years together? Please clarify this in the text.

L295: Do you get the MW99 for input into Eqs. (4) and (5) from the CS2 data? If so is it monthly grid-averaged? How do you assign each OIB point a snow depth?

L301: "Therefore, if there are decreasing trends in both ice thickness and snow depth, the decreasing trend of ice thickness estimated from the constant snow depth will be diminished in radar, while being amplified in lidar" – This sentence seems overcomplicated. To me, all that the bottom two plots of Figure 7 demonstrate is that MW99 snow depth is larger than OIB snow depth. For the laser case, this means that using W99 causes ice thickness to be *underestimated* compared to H(OIB), and for the radar case using W99 results in ice thicknesses that are *too thick* compared to OIB. Perhaps you could plot MW99 against h(OIB) to clarify this? The retrieval of sea ice thickness from ICESat has not traditionally used the Warren climatology- see Kwok and Cunningham (2008) and Petty et al. (2020). Therefore I don't think it's justified to call this 'ICESat-like thickness' since you are not using the same snow depth product that they do.

Typos / Grammar

L128: "Therefore, the interface searching algorithm…" -> "Therefore, an interface searching algorithm…"

L165: "Sea ice thicknesses converted from MW99 using Eqs. (4) and (5) are also compared to examine how simultaneous retrievals…" **->** "Sea ice thicknesses are also calculated from Eqs. (4) and (5), using MW99 as snow depth, to examine how simultaneous retrievals…"

L194: "This reformatted AASTI-v2 data are called…" -> "This reformatted AASTI-v2 dataset is called…"

L293: "Examining how the current practices of retrieving the sea ice thickness through ICESat and CS2 measurements are compared with the simultaneous retrievals is of interest" **->** "We now examine how the current practises of retrieving sea ice thickness from ICESat and CS2 measurements compare with our method."

L294: "In doing so, OIB-measured…" -> "To do so, OIB-measured…"

L297: "Apparently, ICESat-like thickness tends…." -> "According to our analysis, ICESat-like thickness tends…."

L416: "…which are hard to be quantified explicitly." -> "…which are hard to quantify explicitly."