A mass conserving formalism for ice sheet, solid Earth and sea level interaction

Surendra Adhikari[1], Erik R. Ivins[1], Eric Larour[1], Lambert Caron[1], and Helene Seroussi[1]

[1]Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA 91109, USA.

Correspondence: Surendra Adhikari (surendra.adhikari@jpl.nasa.gov)

Abstract. Polar ice sheets are important components of any Earth System model. As the domains of land, ocean, and ice sheet change, they must be consistently defined within the lexicon of geodesy. Understanding the interplay between the processes such as ice sheet dynamics, solid Earth deformation, and sea level adjustment requires both consistent and mass conserving descriptions of evolving land and ocean domains, grounded and floating ice masks, coastlines and grounding lines, and bedrock and geoid height as viewed from space. Here we present a geometric description of an evolving ice sheet margin and its relations to sea level change, the position and loading of the solid Earth and include the ice shelves and adjacent ocean mass. We generalize the formulation so that it is applied to arbitrarily distributed ice, bedrock and adjacent ocean, and their interactive evolution. The formalism simplifies computational strategies that seek to conserve mass in Earth System models.

1 Introduction

Recently there has been intense interest in defining the physics involved in determining multidecadal change in the location and the migration rate of the grounding line, a boundary separating a grounded ice sheet from its floating extension, usually a floating ice shelf (e.g., *Nowicki and Wingham*, 2008; *Schoof*, 2012; *Sergienko and Wingham*, 2019). Indeed, how well a numerical ice sheet model projects its future sea level contribution largely depends on its ability to capture the subtle migration of grounding line. The non-equilibrium thermodynamics, fluid dynamics, plastic failure criteria, and conditions governing nonlinear stability of ice sheets are, quite generally, up for lively debate. In order to better tackle the difficult nonlinear physics involved and to better address the associated numerical challenges (e.g., *Schoof*, 2007; *Durand et al.*, 2009; *Sayag and Grae Worster*, 2013; *Favier et al.*, 2016; *Seroussi and Morlighem*, 2018) as well as to define proper observational criteria for locating the grounding lines and their migrations (e.g., *Hogg et al.*, 2018; *Milillo et al.*, 2019), it is important to agree on some of the baseline variables and boundary conditions. Direct interactions with the ocean (e.g., *Nakayama et al.*, 2018) and solid Earth (e.g., *Gomez et al.*, 2010; *Larour et al.*, 2019) are now seen as critical elements that must be incorporated into projections based on physical models, or retrospective paleoclimate simulations, of the rate of grounding line retreat in a warming climate (e.g., *Jones et al.*, 2015; *Whitehouse et al.*, 2017). Given the computational complexity of this problem, however, it is essential to

properly define the simple geometrical parameters, primarily moving boundaries at the ice-bedrock-ocean interfaces, for there to be rationally organized intercomparison among various research teams and their results. Defining geometry is a first order step for proper construction of models for tracking grounding line migration (e.g., *Gudmundsson et al.*, 2012) and in this paper we propose a simple set of definitions that the field may find useful for future debate and reconciliation.

5 Much of the basic configuration setup for mechanical analysis of ice sheet evolution at the ice-bedrock-ocean interfaces has been treated with great rigor, for example, in Chapter 3 of *Hutter* (1983). Similar setups are also familiar in the development of glacial isostatic adjustment (GIA) theory for ice, sea level and bedrock evolution following the Last Glacial Maximum (LGM) with migrating coast lines (e.g., *Milne*, 1998; *Lambeck et al.*, 2003; *Mitrovica and Milne*, 2003). We refer to these as traditional configurations. Modern satellite techniques have allowed us to gain knowledge of both the present-day locations and migration

10 rates of the grounding line (e.g., *Rignot et al.*, 2011; *Milillo et al.*, 2019). However, both observation and numerical simulation of subtle change in grounding line positions are complicated by the presence of complex geometric features including floating ice shelves, ice rises and rumples, and retrograde bedrock slopes. These features complicate the required geometrical simplifications used in traditional theory for ice-bedrock-ocean interface changes, especially if the interactions are two-way (e.g., *Lingle and Clark*, 1985; *Gomez et al.*, 2013; *de Boer et al.*, 2014). For example, estimating sea level contribution of an

15 evolving ice sheet in such cases becomes non-trivial (*Goelzer et al.*, 2019).

In the following, we begin by presenting a generalized definition of land, ocean and ice sheet domains and that of coastline and grounding line positions (Section 2). We consider a distributed system of ice domains comprised of glaciers and ice sheets of arbitrary geometric configurations that can be straightforwardly employed in any Earth System model in order to track the global mass transport and assess the evolution of a dynamic system of ice sheets, solid Earth and sea level. We then explore

20 the implication of this new geometrical setup for sea level and solid Earth loading studies (Section 3). Finally, in Section 4, we summarize the key conclusions of the study.

2 Land, ocean and ice sheet domains and their boundaries

To begin our discussion of a proposed geometrical setup, we consider a spherical planet whose surface is divided into land and ocean. Distributed ice domains including glaciers and ice sheets exist on both land and ocean (Figure 1). We define $B(\omega, t)$

25 and $S(\omega, t)$ to be the land/bedrock/seafloor and sea surface elevations, respectively, measured relative to the same datum, preferably consistent with the International Terrestrial Reference Frame (e.g., *Altamimi et al.*, 2016). Here ω denotes 2-D spatial coordinates on the planetary surface at time t. Depending upon the spatial scale, we interchangeably use ω to represent for geographic coordinates (θ, ϕ) or Cartesian coordinates (x, y), assuming that an appropriate coordinate transformation is applied. We define $H(\omega, t)$ as the thickness of ice bounded between its surface and the base. When the base of ice is in contact

30 with the underlying bedrock, ice is grounded. Ice may float on subglacial or proglacial lakes or on the ocean, with its base above the bedrock. Our focus here is on marine portions of the ice sheet, with $B(\omega, t) < S(\omega, t)$.

It is important to note that $S(\omega, t)$ may be highly variable both in space and time due to relatively short-term dynamic processes such as tides, wind stress, atmospheric pressure variability, and associated ocean circulation. At timescales on the
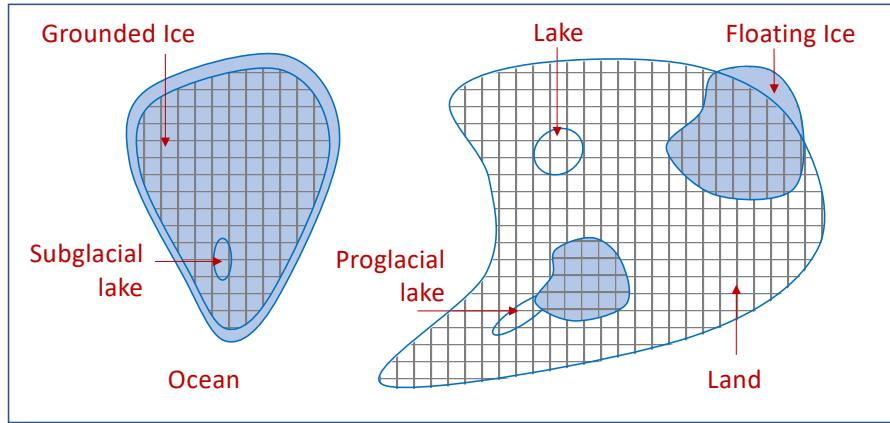
Figure 1. Conceptual depiction of land, ocean and ice domains in the Earth System. Gridded areas represent land and the rest ocean. Lakes are considered as part of the land. Ice can have multiple domains, shown here with blue sheds. The land-ocean boundary is generally defined as the coastline, which is called grounding line when it is part of the ice domain. Because our focus is on grounding line migration in marine portions of the ice sheet, we assume that all of ice on land (gridded portions of blue sheds) is grounded. Consequently, flotation of ice on subglacial and proglacial environments are not considered in this study.

order of 20 years or longer, changing interactions between sea level and ice sheet may become important (e.g., *Hillenbrand et al.*, 2017; *Larour et al.*, 2019). Hence, any long-term change in $S(\omega,t)$ caused, for example, by a sustained water and heat exchange between the land ice and the ocean is of central interest to our formulation. Here, $S(\omega,t)$ strictly refers to this quasi-static component of sea surface that is free from ocean dynamic signal and high-frequency signals of waves or meteorologically driven fluctuations. We hereafter refer $S(\omega,t)$ to as "sea level" for brevity. Sea level as defined above represents an equipotential surface whose spatial pattern mimics the geoid (*Tamisiea*, 2011; *Gregory et al.*, 2019). Under this definition, we may include areas that are non-oceanic, including the interior of marine ice sheets.

Figure 2a shows a cross-sectional view of land, ocean, and the grounded and floating portions of ice sheet domains. We introduce a function $F(\omega,t)$ that is used to define these domains in terms of the principles of hydrostatic equilibrium:

$$F(\omega,t) = H(\omega,t) + \frac{\rho_o}{\rho_i}\Big[B(\omega,t) - S(\omega,t)\Big], \tag{1}$$

where ρ_o and ρ_i are the densities of ocean water and ice, respectively. Note that, by definition, $R(\omega,t) = S(\omega,t) - B(\omega,t)$ where $R(\omega,t)$ is sea surface relative to the seafloor, termed the "relative sea level" (*Gregory et al.*, 2019). By design, the position with $F(\omega,t) = 0$ represents a boundary between the land and the ocean, provided that ω is in direct contact with the open ocean. This land-ocean boundary is generally known as a coastline. It follows from equation (1) that ice thickness at the coastline, $H_C(\omega,t)$, is given by:

$$H_C(\omega,t) = -\frac{\rho_o}{\rho_i}\Big[B(\omega,t) - S(\omega,t)\Big], \tag{2}$$

and that the coastline is free of ice where $B(\omega,t) = S(\omega,t)$. Since ice thickness cannot be negative, no coastline exists with $B(\omega,t) > S(\omega,t)$. Only in the marine sectors where $B(\omega,t) < S(\omega,t)$, does a coastline have finite ice thickness and then is replaced by the term grounding line.

Given the definition of coastline and grounding line, we may define the ocean domain, $\mathcal{O}(\omega,t)$, as follows:

$$\mathcal{O}(\omega,t) = 1 \quad \text{if} \ \ F(\omega,t) < 0 \ \text{ and } \omega \text{ is connected to the open ocean;}$$
$$\qquad\qquad = 0 \quad \text{otherwise, except when } \omega \text{ is precisely a coastline or a grounding line.} \tag{3}$$

The complementary land domain is simply given by $\mathcal{L}(\omega,t) = 1 - \mathcal{O}(\omega,t)$. Note that neither $\mathcal{O}(\omega,t)$ nor $\mathcal{L}(\omega,t)$ is defined at the coastline or grounding line. The concept of "connectivity" is introduced because the criterion $F(\omega,t) < 0$ in and of itself is not always sufficient to define the ocean domain. One obvious example where ω may not belong to ocean in spite of having $F(\omega,t) < 0$ is a deep continental trough with bathymetry well below sea level. Unless this trough or any such locations is physically connected to the open ocean via oceanic water, we consider this to be part of land rather than the ocean.

This assumption ensures an interconnected system of Earth's oceanic waters, termed global ocean, as has been considered traditionally in physical oceanography and sea level studies.

We define $\mathcal{I}(\omega,t)$ to be a globally distributed system of ice domains, such that

$$\mathcal{I}(\omega,t) = 1 \quad \text{if} \ \ H(\omega,t) > 0;$$
$$\qquad\qquad = 0 \quad \text{otherwise.} \tag{4}$$

15

For many applications, it may be useful to decompose $\mathcal{I}(\omega,t)$ into a number of sub-domains: $\mathcal{I}(\omega,t) = \{\mathcal{I}_1, \mathcal{I}_2, ..., \mathcal{I}_i, ...\}$, where $\mathcal{I}_i(\omega,t)$ represents the i-th ice domain. Individual ice sheets and glaciers can be thought of individual ice domains. As defined above, the grounding line within a given ice domain is where $F(\omega,t) = 0$, provided again that ω is in direct contact with the open ocean. Using equations (3) and (4), we may define the grounded ice mask simply as $\mathcal{G}(\omega,t) = \mathcal{I}(\omega,t)\mathcal{L}(\omega,t)$ and the floating ice mask as $\mathcal{I}(\omega,t)\mathcal{O}(\omega,t)$. Note that $F(\omega,t) \leq 0$ can exist even within the grounded ice mask. A condition where this may occur is when the interior of a marine ice sheet resides on a deep trough (see Figure 2a). If a column of ice is shielded by surrounding bathymetric highs from the open ocean, we expect it to remain grounded. The ice sheet domain as defined by equation (4) can accommodate complexities such as a pinning point that can modulate marine ice sheet instability on retrograde slopes (e.g., *Matsuoka et al.*, 2015; *Whitehouse et al.*, 2017). This is where the new setup diverges from traditional theory. The employed assumption in our description, however, limits us from capturing the floating ice on subglacial and proglacial lakes

25

that are not part of the global ocean (see Figure 1). We believe that the localized processes of ice-lake interactions are of secondary importance, at least, for the purpose of capturing large-scale interplay between the continental ice sheets, solid Earth and sea level in the first generation of Earth System models.

Our definition of coastline and grounding line, and hence that of land, ocean and ice domains, facilitates direct evaluation of the interaction between a dynamic system of ice sheets, solid Earth and sea level, as well as the estimation and interpretation

30

of ice sheet driven global and regional sea level change. Although a distributed system of ice domains is an integral part of the Earth System, in the following we consider, for brevity, a single domain as an ice sheet, while other ice domains are collectively referred to as farfield ice.

3   Sea level contribution and loading of the solid Earth

In order to estimate sea level contribution of an ice sheet, we define a flotation height of ice

$$H_0(\omega,t) = -\frac{\rho_o}{\rho_i} \min\Big[\big\{B(\omega,t) - S(\omega,t)\big\},0\Big], \tag{5}$$

so that the ice thickness in excess of $H_0(\omega,t)$ represents the so-called "height above flotation", $H_F(\omega,t)$. Mathematically,

5   $$H_F(\omega,t) = \mathcal{G}(\omega,t)\Big[H(\omega,t) - H_0(\omega,t)\Big]. \tag{6}$$

Here we invoke a grounded ice mask $\mathcal{G}(\omega,t)$ to ensure no net sea level contribution from the floating ice shelf. Clearly, $H_F(\omega,t) = H(\omega,t)$ for the grounded ice sheet that rests on the bedrock whose elevation is at or above sea level. For grounded portions of the marine ice sheet, $H_F(\omega,t) < H(\omega,t)$ and it can be negative (see regime 4 in Figure 2a), implying that this portion can take up ocean water when it is connected to the open ocean and hence contribute to sea level inversely.

10   The evolving ice sheet geometry is usually described in terms of evolving ice thickness and ice sheet domain. Indeed, prognostic simulations of ice sheet models track the transport of mass in terms of equivalent ice thickness distribution. This redistribution of ice mass and associated relative sea level change induce a solid Earth response in terms of its gravitation, rotation and viscoelastic deformation. This, in turn, modulates the bedrock topography as well as the geoid. We may therefore describe the evolving ice sheet geometry in terms of ice thickness, bedrock elevation, and sea level (Figure 2b-e). We denote

15   $\Delta H(\omega,\Delta t)$, $\Delta B(\omega,\Delta t)$, and $\Delta S(\omega,\Delta t)$ to be the change in respective fields over the time interval Δt. For the new ice sheet geometry at time $t+\Delta t$, equation (6) gives $H_F(w,t+\Delta t) = \mathcal{G}(\omega,t+\Delta t)\Big[H(\omega,t) + \Delta H(\omega,\Delta t) - H_0(\omega,t+\Delta t)\Big]$, where $H_0(\omega,t+\Delta t) = -\rho_o/\rho_i \ \min\Big[\big\{B(\omega,t) + \Delta B(\omega,\Delta t) - S(\omega,t) - \Delta S(\omega,\Delta t)\big\},0\Big]$ is given by equation (5). Similarly, we may rewrite equation (1) for $F(\omega,t+\Delta t)$ and define the new ocean domain $\mathcal{O}(\omega,t+\Delta t)$, land domain $\mathcal{L}(\omega,t+\Delta t)$, ice sheet domain $\mathcal{I}(\omega,t+\Delta t)$, and grounded ice mask $\mathcal{G}(\omega,t+\Delta t)$ as described in Section 2. Note that $\Delta B(\omega,\Delta t)$ and $\Delta S(\omega,\Delta t)$

20   may have components that are forced by external processes such as farfield ice melting or tectonics.

In what follows, we assume that the net change in ice sheet mass directly affects the ocean mass. Quantifying the fraction of ice mass change that contributes to sea level is a non-trivial problem. It cannot be approached analytically and is beyond the scope of this study. Despite the assumption, not all of $\Delta H(\omega,\Delta t)$ contributes to sea level change all the time. On the other hand, in response to the externally forced bedrock and sea level, the ice sheet may still contribute to sea level change even when

25   $\Delta H(\omega,\Delta t) = 0$. Traditionally, change in ice thickness above flotation, i.e. $\Delta H_F(\omega,\Delta t) = H_F(\omega,t+\Delta t) - H_F(\omega,t)$, is used to calculate sea level contribution of an ice sheet (e.g., *Bindschadler et al.*, 2013). As we show below this simplistic approach yields some error, particularly when evolving bedrock and sea level are considered (*Larour et al.*, 2019; *Goelzer et al.*, 2019).

We define $\Delta H_S(\omega,\Delta t)$ be a portion of ice thickness that contributes to sea level over the period Δt. The following relationship holds for generalized ice geometries and bedrock and/or sea level forcings:

30   $$\Delta H_S(\omega,\Delta t) = \Delta H(\omega,\Delta t)\,\mathcal{L}(\omega,t)\,\mathcal{L}(\omega,t+\Delta t) + \Delta H_F(\omega,\Delta t)\,|\Delta\mathcal{L}(\omega,\Delta t)| + \Big(1 - \frac{\rho_w}{\rho_o}\Big)H_0(\omega,t+\Delta t)\,\Delta\mathcal{L}(\omega,\Delta t), \tag{7}$$

where ρ_w is the fresh water density. The change in land domain is simply given by $\Delta\mathcal{L}(\omega,\Delta t) = \mathcal{L}(\omega,t+\Delta t) - \mathcal{L}(\omega,t)$. Since $\Delta H_F(\omega,\Delta t)$, unlike $H_0(\omega,t+\Delta t)$, can track the direction of margin migration (i.e., advance or retreat), absolute value of
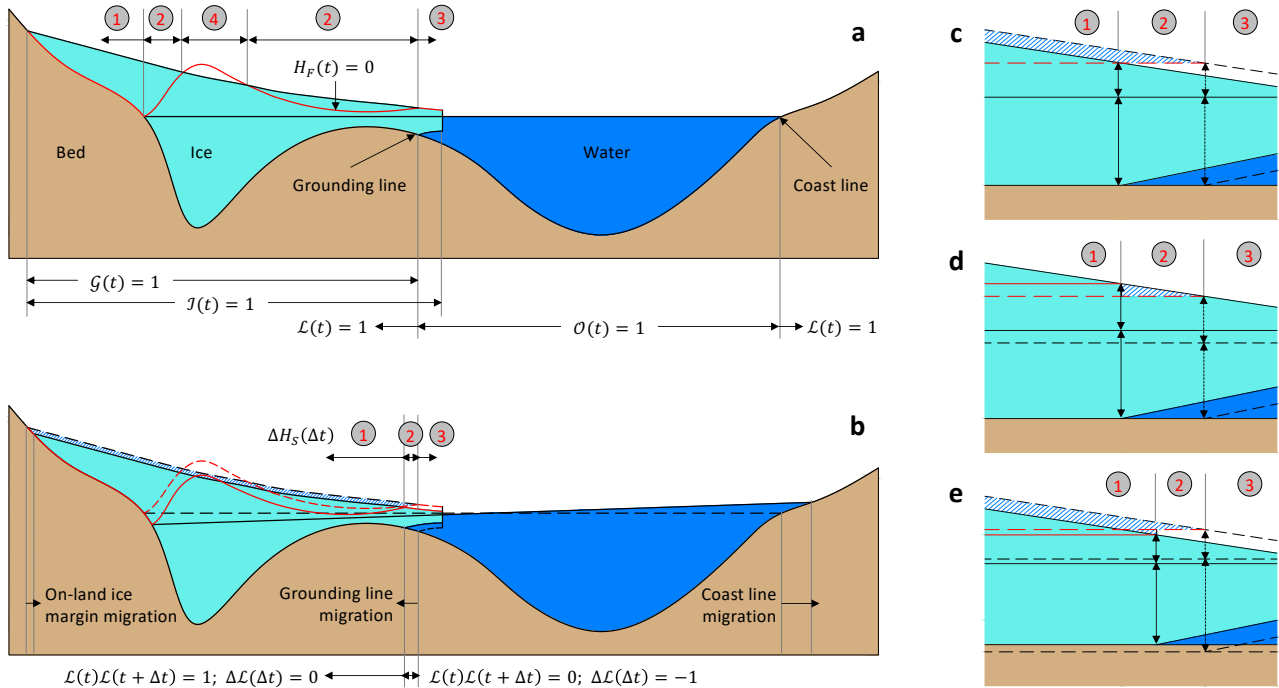
Figure 2. Conceptual depiction of evolving ice sheet geometry. (a) Domains of ocean $\mathcal{O}(\omega,t)$, land $\mathcal{L}(\omega,t)$, ice sheet $\mathcal{I}(\omega,t)$, and grounded ice $\mathcal{G}(\omega,t)$ at time t. Flotation line, having $H_F(\omega,t) = 0$, is represented by a red line. Ice thickness above flotation is given by $H_F(\omega,t) = H(\omega,t)$ in regime 1, $0 < H_F(\omega,t) < H(\omega,t)$ in regime 2, $H_F(\omega,t) = 0$ in regime 3, and $H_F(\omega,t) < 0$ in regime 4. **(b)** Ice sheet geometry at time $t + \Delta t$ after changes in ice thickness and associated sea level. Old geometry and field variables are shown with dashed lines. Ice thickness that contributes to sea level change (hashed area), $\Delta H_S(\omega,t)$, is given by $\Delta H(\omega,\Delta t)$ in regime 1. It is a function of $\Delta H_F(\omega,\Delta t)$ and $H_0(\omega,t + \Delta t)$ in regime 2 (see equation 7), and is zero in regime 3. We zoom in around the grounding line to assess $\Delta H_S(\omega,\Delta t)$ in different scenarios: **(c)** when ice thickness changes but relative sea level does not, typically assumed in stand-alone ice sheet models; **(d)** when (externally forced) sea level changes but ice thickness does not; and **(e)** when both ice thickness and (ice driven or externally forced) relative sea level change. In **b** and **d**, only sea level, not the bedrock, is changed for simplicity. Sketches are not to scale.

$\Delta\mathcal{L}(\omega,\Delta t)$ is used in the second term on the right-hand side. This method requires bookkeeping the global land domain. This is crucial for considering distributed ice-ocean domains with complex geometries in Earth System models. For the treatment of an individual ice sheet, however, it is sufficient to track a continental or regional land domain, provided that there is an understanding that ocean water may recede from, or reinundate, continental land (e.g., *Johnston*, 1993; *Milne*, 1998). In fact,
5 often it may be possible to replace all $\mathcal{L}$'s appearing in equation (7) by corresponding $\mathcal{G}$'s, the grounded ice masks. However, in an exceptional case with $H_0(\omega,t + \Delta t) > 0$ holding in the areas of on-land ice margin migration, using $\mathcal{G}$'s rather than $\mathcal{L}$'s incorrectly predict a non-zero contribution from the last term in equation (7).

Figure 2 shows a few schematics of evolving ice sheet geometries and their sea level contributions. We consider all plausible cases by combining scenarios of ice thickness and relative sea level changes. In reference to these sketches and equation (7), we outline three distinct regimes of an evolving ice sheet and their sea level contribution over the period Δt:

1. Where ice remains grounded on the bedrock at the elevations above, at, or below corresponding sea level at both times t and $t + \Delta t$, all of $\Delta H(\omega, \Delta t)$ contributes to sea level (first term on the right-hand side). It turns out that $\Delta H(\omega, \Delta t) \neq \Delta H_F(\omega, \Delta t)$ only in marine portions of the grounded ice and only when evolving bedrock and sea level are considered. *Goelzer et al.* (2019) present a method to backtrack $\Delta H(\omega, \Delta t)$ from $\Delta H_F(\omega, \Delta t)$ in such situations, assuming that $\Delta B(\omega, \Delta t)$ and $\Delta S(\omega, \Delta t)$ are known. Over the timescale of a few decades or shorter, it may be that relative sea level does not evolve significantly, and in these cases $\Delta H_F(\omega, \Delta t) \approx \Delta H(\omega, \Delta t)$. Stand-alone ice sheet models typically inherit this assumption even though simulation timescales can be on the order of centuries (e.g., *Bindschadler et al.*, 2013).

   This regime also includes land areas covered by the evolving ice sheet margins. When ice margin advances over the period Δt, newly glaciated areas must satisfy $H(\omega, t) = 0$ and $\Delta H(\omega, \Delta t) > 0$. When it retreats, $\Delta H(\omega, \Delta t) = -H(\omega, t)$ must hold in recently deglaciated areas. In both cases, all of $\Delta H(\omega, \Delta t)$ contributes to sea level change.

   Externally forced $\Delta B(\omega, \Delta t)$ or $\Delta S(\omega, \Delta t)$ does not affect the estimate of $\Delta H_S(\omega, \Delta t)$ in this regime although it may yield nonzero $\Delta H_F(\omega, \Delta t)$ and possibly modulate the ice flow dynamics.

2. Where ice has transitioned from the grounded to floating state or vice versa over the period Δt, sea level contribution from this regime depends on both $\Delta H_F(\omega, \Delta t)$ and $H_0(\omega, t + \Delta t)$ (the last two terms on the right-hand side). In the absence of externally forced bedrock and sea level, it follows that $|\Delta H_F(\omega, \Delta t)| \leq |\Delta H(\omega, \Delta t)|$ in this regime.

   The last term in the equation accounts for the fact that fresh water density evolves during the accretion and ablation of ice, whereas the average ocean water density in the vicinity of the grounding line acts to determine the flotation height. For typical values of ρ_w and ρ_o, we find that about 2.5% of $H_0(\omega, t + \Delta t)$ contributes to sea level change. This particular contribution has been considered by *Goelzer et al.* (2019).

   This is the only regime in which responses to the externally forced $\Delta B(\omega, \Delta t)$ and/or $\Delta S(\omega, \Delta t)$ contribute to sea level change, even when $\Delta H(\omega, \Delta t) = 0$ (see Figure 2d).

3. Where ice is floating at both times t and $t + \Delta t$, there is no net direct contribution from this regime of the ice sheet to sea level change. However, change in ice shelf thickness can greatly affect the interior ice sheet dynamics via modulation of buttressing force (e.g., *Gudmundsson et al.*, 2019).

In Figure 3, we present a quantitative assessment of $\Delta H_S(\omega, \Delta t)$ and $\Delta H_F(\omega, \Delta t)$ over the next 350 years for Pine Island and Thwaites glaciers. Notice the systematic error associated with a customary approach of using $\Delta H_F(\omega, \Delta t)$ to quantify sea level contribution from an ice sheet (Figure 3c). The discrepancy in regime 1 is due to the evolving bedrock and sea level, and that in regime 2 is due to the missing fraction of newly grounded or floating ice. These results are based on the recent work

Change in ice thickness, ΔH [m] ΔH that contributes to sea level, ΔH_S [m] $\Delta H_F - \Delta H_S$ [m]
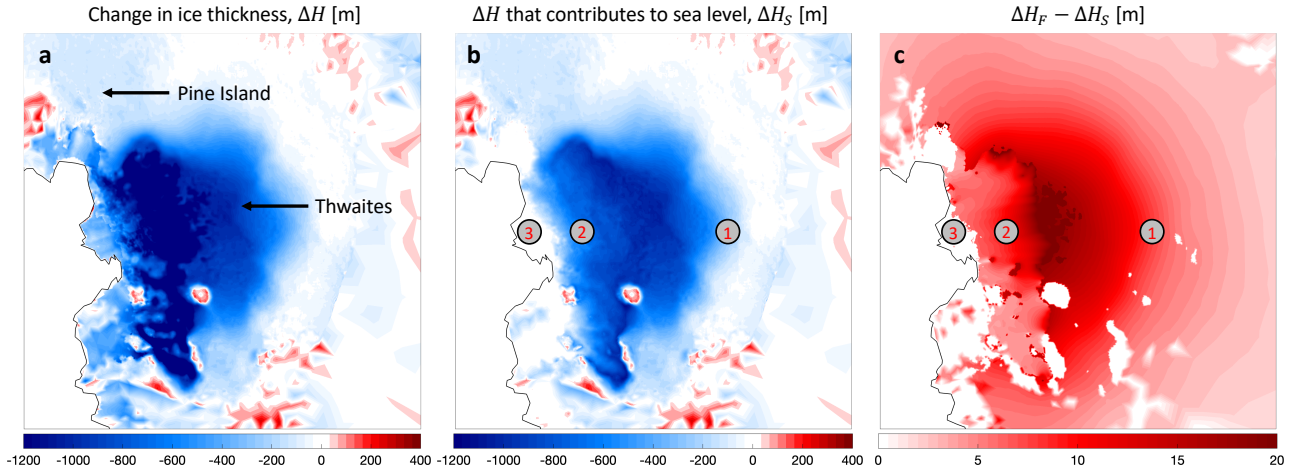
Figure 3. Example results of ice thickness change and its contribution to sea level. (a) Modeled change in ice thickness over the next 350 years, shown for portions of Pine Island and Thwaites glaciers adjacent to the Amundsen Sea. **(b)** Estimated ice thickness change that directly contributes to sea level, given by equation (7). **(c)** Error associated with a conventional method that assumes that sea level contribution of an ice sheet is given by $\Delta H_F(\omega, \Delta t)$. In the latter two panels, we point out three regimes of the ice sheet as defined in Section 3 (also see Figures 2b-e) in which the following holds true: $\Delta H_F(\omega, \Delta t) \neq \Delta H_S(\omega, \Delta t) = \Delta H(\omega, \Delta t)$ in marine portions of regime 1; $\Delta H_F(\omega, \Delta t) \neq \Delta H_S(\omega, \Delta t) \neq \Delta H(\omega, \Delta t)$ in regime 2; and $\Delta H_F(\omega, \Delta t) = \Delta H_S(\omega, \Delta t) = 0$ in regime 3. In this particular example, the conventional method underpredicts sea level contribution from the ice sheet by the amount that corresponds to panel **c**.

of *Larour et al.* (2019), who provide consistent solutions of evolving $H(\omega, t)$, $B(\omega, t)$ and $S(\omega, t)$ for Antarctica over the next 500 years. They simulate a high-resolution dynamical ice-flow model (*Larour et al.*, 2012) that is fully coupled with a global solid-Earth deformation and sea-level adjustment model (*Adhikari et al.*, 2016) under the present-day surface climatology and a realistic sub-ice shelf melting scenario. The effect of farfield ice mass change, extrapolated into the next 500 years based on

5   the contemporary trend, on the evolution of bedrock and sea level in Antarctica is also accounted for.

The barystatic sea level change, defined as the ocean mass related global mean sea level change (*Gregory et al.*, 2019), due to ice mass change over the period Δt is given by

$$\overline{\Delta R_C^I}(\Delta t) = \left[-\rho_i \int \Delta H_S(\omega, \Delta t) \, \mathrm{d}\omega \right] \bigg/ \left[\rho_w \int \mathcal{O}(\omega, t + \Delta t) \, \mathrm{d}\omega \right]. \tag{8}$$

The numerator in the equation owes to the net change in ice mass over Δt, and the integral in denominator represents the ocean

10   surface area at time $t + \Delta t$. Assume that an ice sheet collapses instantaneously and that all of melt water contributes to sea level. Resulting $\overline{\Delta R_C^I}(\Delta t)$ represents the "potential sea level" of the ice sheet at time t, and it can be readily derived by setting $\Delta H(\omega, \Delta t) = -H(\omega, t)$ and $\mathcal{G}(\omega, t + \Delta t) = 0$ in the limit of $\Delta t \to 0$ in equation (7).

Ocean surface area changes as grounding line and coastline positions migrate. Such a migration over Δt is controlled by a number of processes that contribute to $\Delta R(\omega, \Delta t)$. A comprehensive synopsis of these processes can be stated as follows:

15    $$\Delta R(\omega, \Delta t) = \Delta R_C^I(\omega, \Delta t) + \Delta R_C^L(\omega, \Delta t) + \Delta R_P^I(\omega, \Delta t) + \Delta R_P^L(\omega, \Delta t) + \Delta R_D(\omega, \Delta t) + \Delta R_V(\omega, \Delta t). \tag{9}$$

The first four terms on the right-side of the equation represent the processes that contribute to sea level by changing the ocean mass (termed the barystatic components), while $\Delta R_D(\omega, \Delta t)$ represents the density related steric sea level change and $\Delta R_V(\omega, \Delta t)$ represents a component due to vertical land motion that is not captured by the barystatic processes. We use the superscript I to refer to the *ice sheet* under consideration and superscript L to the other parts of the *land*, including farfield

5 ice sheets, glaciers and hydrological basins, that contribute to barystatic sea level change. When these sources of freshwater contribute to ocean mass change over a *contemporaneous* period $[t, t + \Delta t]$, corresponding changes in relative sea level fields are denoted with the subscript C. The ice sheet and other parts of the land may have contributed to the evolution of barystatic sea level in the *past*, i.e. over the period $(-\infty, t]$, and the induced viscous response of the solid Earth may still contribute to the change in relative sea level over the period $[t, t + \Delta t]$. These components of sea level are denoted with the subscript P.

10 Assuming that the contemporaneous period Δt is on the order of 10 years, we may interpret $\Delta R(\omega, \Delta t)$ as ongoing change in relative sea level monitored by satellite gravimetry and altimetry (e.g., *WCRP Global Sea Level Budget Group*, 2018). Along the same lines, we may interpret $\Delta R_C^I(\omega, \Delta t) + \Delta R_C^L(\omega, \Delta t)$ and $\Delta R_P^I(\omega, \Delta t) + \Delta R_P^L(\omega, \Delta t)$ as ongoing sea level change driven by contemporary global mass redistribution (e.g., *Adhikari et al.*, 2019) and by global GIA processes (e.g., *Peltier et al.*, 2015; *Caron et al.*, 2018), respectively.

15 As ice sheet evolves, it not only contributes to sea level but also loads the underlying solid Earth. For the period $[t, t + \Delta t]$, we may define a mass conserving field $\Delta M(\omega, \Delta t)$ that describes the net change in mass per unit area on the solid Earth surface as follows:

$$\Delta M(\omega, \Delta t) = \rho_i \, \Delta H_S(\omega, \Delta t) + \rho_w \, \Delta R_C^I(\omega, \Delta t) \, \mathcal{O}(\omega, t + \Delta t). \tag{10}$$

Here $\Delta H_S(\omega, \Delta t)$ is given by equation (7) and $\Delta R_C^I(\omega, \Delta t)$ represents the associated relative sea level change, whose spatial

20 pattern is dictated by the perturbation in Earth's gravitational and rotational potentials and associated viscoelastic deformation of the solid Earth (*Farrell and Clark*, 1976; *Milne and Mitrovica*, 1998). This component of sea level is precisely the same as the first term on the right-side of equation (9), whose ocean averaged value is given by equation (8). Because sea level is defined globally, including on land, we ensure mass conservation by use of an ocean mask in equation (10).

4 Conclusions

25 We have divided the Earth's surface into complementary domains of land and ocean, which are separated by coast lines. While there may be multiple land domains, we maintain a single global ocean as in traditional studies of physical oceanography and sea level. Distributed bodies of ice intersect land and ocean to form glaciers, ice sheets and ice shelves. Grounding lines are defined as the coastlines that belong to the ice domains. The set of generic, and quite simple, descriptions given in this paper handle the complex geometries of both the coastlines and evolving grounding lines, complementary to those of farfield land,

30 ocean and ice domains and their respective overall evolutionary history.

The main importance of this paper is that a unified method is outlined to determine the exact fraction of ice thickness change that contributes to sea level change, $\Delta H_S(\omega, \Delta t)$, over the period Δt. Along with its obvious application to estimate global

mean sea level, this field is absolutely critical to track the global mass transport and assess the response of a dynamic system of ice sheets, solid Earth and sea level, while accounting for fine-scale complexities in geometry. In the most simplified case when bedrock and sea level do not evolve, our method reduces to the traditional approach that assumes that change in ice height above flotation, $\Delta H_F(\omega, \Delta t)$, directly contributes to sea level. We recommend that the ice sheet modeling community

5 considers $\Delta H_S(\omega, \Delta t)$ rather than $\Delta H_F(\omega, \Delta t)$ as a metric to quantify the sea level contribution of evolving ice sheets. This is especially appropriate for model analyses that is informed by ice and ocean mass monitoring from space assets, such as ocean and ice altimetry, radar interferometry and space gravimetry (e.g., *Bentley and Wahr*, 1998).

Notation

B	Bedrock elevation
ΔB	Change in bedrock elevation over the period Δt
F	A function defined such that $F = 0$ represents a grounding line or coastline
$\mathcal{G}$	Mask of grounded portions of the ice sheet
H	Ice thickness
ΔH	Change in ice thickness over the period Δt
H_0	Flotation height of grounded ice
H_C	Ice thickness at a coastline or a grounding line
H_F	Ice height above flotation
ΔH_F	Change in (ice) height above flotation over the period Δt
ΔH_S	Change in ice thickness that directly contributes to sea level over the period Δt
$\mathcal{I}$	Ice sheet domain
$\mathcal{L}$	Land domain
$\Delta \mathcal{L}$	Change in land domain over the period Δt
ΔM	Change in mass per unit area on the solid Earth surface over the period Δt
$\mathcal{O}$	Ocean domain
R	Sea level relative to the bedrock, termed "relative sea level"
ΔR	Change in relative sea level over the period Δt
ΔR_C^I	Change in relative sea level over the period Δt due to contemporary ice sheet mass change
$\overline{\Delta R_C^I}$	Change in barystatic sea level over the period Δt due to contemporary ice sheet mass change
ΔR_C^L	Change in relative sea level over the period Δt due to contemporary land water/ice mass change
ΔR_P^I	Change in relative sea level over the period Δt due to past ice sheet mass change
ΔR_P^L	Change in relative sea level over the period Δt due to past land water/ice mass change
ΔR_D	Change in relative sea level over the period Δt due to density related steric change
ΔR_V	Change in relative sea level over the period Δt due to vertical land motion

ρ_i	Density of ice
ρ_o	Mean density of the ocean water
ρ_w	Density of fresh water
S	Sea surface elevation
ΔS	Change in sea surface elevation over the period Δt
t	time
Δt	time interval
ω	2-D spatial coordinates: geographic (θ, ϕ) or Cartesian coordinates (x, y)

Author contributions. SA conceived and conducted the research, and wrote the first draft of the manuscript with help of ERI. EL helped produce Figure 3. All authors contributed to the analysis of the results and to the writing and editing of the manuscript.

Competing interests. The authors declare that they have no competing interests.

Acknowledgements. This research was carried out at the Jet Propulsion Laboratory (JPL), California Institute of Technology, under a contract
5 with National Aeronautics and Space Administration (NASA), and was funded through the JPL Research, Technology & Development programs (grant no. 01STCR-R.17.235.118; 2017-2019, and 01STCR-R.19.021.241; 2019).

References

Adhikari, S., E.R. Ivins and E. Larour: ISSM-SESAW v1.0: mesh-based computation of gravitationally consistent sea level and geodetic signatures caused by cryosphere and climate driven mass change, *Geosci. Model Dev., 9*, 9769–9816, 2016.

Adhikari, S., E.R. Ivins, T. Frederikse, F.W. Landerer and L. Caron: Sea-level fingerprints emergent from GRACE mission data, *Earth Syst. sci. data, 11*, 629–646, 2019.

Altamimi, Z., P. Rebischung, L. Metivier and X. Collilieux: ITRF2014: A new release of the International Terrestrial Reference Frame modeling nonlinear station motions, *J. Geophys. Res. Solid Earth, 121*, 6109–6131, 2016.

Bentley, C.R. and J.M. Wahr: Satellite gravity and the mass balance of the Antarctic ice sheet, *J. Glaciology, 44,* 207–203, 1998.

Bindschadler, R.A., S.M.J. Nowicki, A. Abe-Ouchi, A. Aschwanden, H. Choi, J. Fastook, G. Granzow, R. Greve, G. Gutowski, U. Herzfeld, C. Jackson, J. Johnson, C. Khroulev, A. Levermann, W.H. Lipscomb, M.A. Martin, M. Morlighem, B.R. Parizek, D. Pollard, S.F. Price, D. Ren, F. Saito, T. Sato, H. Seddik, H. Seroussi, K. Takahashi, R.T. Walker and W.L. Wang: Ice-sheet model sensitivities to environmental forcing and their use in projecting future sea-level (The SeaRISE Project), *J. Glaciol., 59*, 195–224, 2013.

Caron, L., E.R. Ivins, E. Larour, S. Adhikari, J. Nilsson and G. Blewitt: GIA model statistics for GRACE hydrology, cryosphere and ocean science, *Geophys. Res. Lett., 45*, 2203–2212, 2018.

de Boer, B., Stocchi, P. and R.S.W. van de Wal: A fully coupled 3-D ice-sheet–sea-level model: algorithm and applications, *Geosci. Model Dev., 7*, 2141–2156, 2014.

Durand, G., O. Gagliardini, B. de Fleurian, T. Zwinger and E. Le Meur: Marine ice sheet dynamics: Hysteresis and neutral equilibrium, *Geophys. Res. Lett., 114*, F03009, 2009.

Farrell, W.E. and J.A. Clark: On postglacial sea level, *Geophys. J. Roy. Astr. S., 46*, 647–667, 1976.

Favier, L., F. Pattyn, S. Berger and R. Drews: Dynamic influence of pinning points on marine ice-sheet stability: a numerical study in Dronning Maud Land, East Antarctica, *The Cryosphere, 10*, 2623–2635, 2016.

Goelzer, H., V. Coulon, F. Pattyn, B. de Boer and R. van de Wal: Brief communication: On calculating the sea-level contribution in marine ice-sheet models, *The Cryosphere Discuss.*, doi: 10.5194/tc-2019-185, 2019.

Gomez, N., J.X. Mitrovica, P. Huybers and P.U. Clark: Sea level change as a stabilizing influence on marine ice sheets, *Nature Geosci. 3*, 850–853, 2010.

Gomez, N., D. Pollard and J.X. Mitrovica: A 3-D coupled ice sheet – sea level model applied to Antarctica through the last 40 ky, *Earth Planet. Sc. Lett., 384*, 88–99, 2013.

Gregory, J.M., S.M. Griffies, C.W. Hughes and others: Concepts and Terminology for Sea Level: Mean, Variability and Change, Both Local and Global, *Surv. Geophys., 40*, 1251–1289, 2019.

Gudmundsson, H., J. Krug, G. Durand, L Favier and O. Gagliardini: The stability of grounding lines on retrograde slopes. *The Cryosphere, 6*, 1497–1505, 2012.

Gudmundsson, G.H., F.S. Paolo, S. Adusumilli and H.A. Fricker: Instantaneous Antarctic ice- sheet mass loss driven by thinning ice shelves. *Geophys. Res. Lett., 46*, doi: 10.1029/2019GL085027, 2019.

Hillenbrand, C.-D., J.A. Smith, D.A. Hodell, M. Greaves, C.R. Poole, S. Kender, M. Williams, T. Joest Andersen, P.E. Jernas, H.Elderfield, J.P. Klages, S.J. Roberts, K. Gohl, R.D. Larter and G. Kuhn: West Antarctic Ice Sheet retreat driven by Holocene warm water incursions, *Nature, 217*, 43–48, 2017.

Hogg, A.E., A. Shepherd, L. Gilbert, A. Muir and M. R. Drinkwater: Mapping ice sheet grounding lines with CryoSat-2, *Adv. Space Res., 62*, 1191–1202, 2018.

Hutter, K.: *Theoretical Glaciology*, Reidel Publishing Co., Dordrecht, Netherlands, pp. 510, 1983.

Johnston, P.: The effect of spatially non-uniform water loads on predictions of sea level change, *Geophys. J. Int., 114*, 615–634, 1993

5   Jones, R.S., A.N. Mackintosh, K.P. Norton, N.R. Golledge, C.J. Fogwill, P.W. Kubik, M. Christl and S.L. Greenwood: Rapid Holocene thinning of an East Antarctic outlet glacier driven by marine ice sheet instability. *Nat. Comm, 6*, 8910, 2015.

Lambeck, K., A. Purcell, P.J. Johnston, M. Nakada and Y. Yokoyama: Water-load definition in the glacio-hydro-isostatic sea-level equation, *Quatern. Sci. Rev. 22*, 309–318, 2003.

Larour, E., H. Seroussi, M. Morlighem and E. Rignot: Continental scale, high order, high spatial resolution, ice sheet modeling using the Ice

10   Sheet System Model (ISSM), *J. Geophys. Res., 117*, F01022, 2012.

Larour, E., H. Seroussi, S. Adhikari, E.R. Ivins, L. Caron, M. Morlighem and N. Schlegel: Slowdown in Antarctic mass loss from solid Earth and sea-level feedbacks, *Science, 364*, 6444, eaav7908, 2019.

Lingle, C. and J. Clark: A numerical model of interactions between a marine ice sheet and the solid earth: application to a west Antarctic ice stream, *J. Geophys. Res., 90*, 1100–1114, 1985.

15   Matsuoka, K., R.C.A. Hindmarsh, G. Moholdt, M.J. Bentley, H.D. Pritchard, J. Brown, H. Conway, R. Drews, G. Durand, D. Goldberg, T. Hattermann, J. Kingslake, J.T.M. Lenaerts, C. Martin, R. Mulvaney, K. Nicholls, F. Pattyn, N. Ross, T. Scambos and P.L. Whitehouse: Antarctic ice rises and rumples: Their properties and significance for ice-sheet dynamics and evolution, *Earth-Science Reviews 150*, 724–745, 2015.

Milne, G.A.: Refining models of the glacial isostatic adjustment process, *PhD Thesis*, Physics Dept., University of Toronto, Canada, 1998.

20   Milne, G.A. and J. Mitrovica: Postglacial sea-level change on a rotating Earth, *Geophys. J. Int., 133*, 1–19, 1998.

Mitrovica, J.X. and G.A. Milne: On post-glacial sea level: I. General theory, *Geophys. J. Int., 154*, 253–267, 2003.

Milillo, P., E. Rignot, P. Rizzoli, B. Scheuchl, J. Mouginot, J. Bueso-Bello and P. Prats-Iraola: Heterogeneous retreat and ice melt of Thwaites Glacier, West Antarctica, *Science Adv., 10*, eaau3433, 2019.

Nakayama, Y., D. Menemenlis, H. Zhang, M. Schodlok and E. Rignot: Origin of Circumpolar Deep Water intruding onto the Amundsen and

25   Bellingshausen Sea continental shelves, *Nat. Commun., 9*, 3403, 2018.

Nowicki, S.M. and D.J. Wingham: Conditions for a steady ice sheet-ice shelf junction, *Earth Planet. Sci. Lett., 265*, 246–255, 2008.

Peltier, W.R., D.F. Argus and R. Drummond: Space geodesy constrains ice age terminal deglaciation: The global ICE-6G_C (VM5a) model, *Geophys. Res. Lett., 122*, 450–487, 2015.

Rignot, E., J. Mouginot and B. Scheuchl: Antarctic grounding line mapping from differential satellite radar interferometry, *Geophys. Res.*

30   *Lett., 38*, L10504, 2011.

Sayag, R. and M. Grae Worster: Elastic dynamics and tidal migration of grounding lines modify subglacial lubrication and melting, *Geophys. Res. Lett., 40*, 5877–5881, 2013.

Schoof, C.: Ice sheet grounding line dynamics: Steady states, stability, and hysteresis, *J. Geophys. Res., 112*, F03S28, 2007.

Schoof, C.: Marine ice sheet stability. *J. Fluid Mech., 698*, 62–72, 2012.

35   Sergienko, O.V. and D.J. Wingham: Grounding line stability in a regime of low driving and basal stresses, *J. Glaciology, 65*, 833–849, 2019.

Seroussi, H. and M. Morlighem: Representation of basal melting at the grounding line in ice flow models, *Cryosphere, 12*, 3085–096, 2018.

Tamisiea, M.E.: Ongoing glacial isostatic contributions to observations of sea level change, *Geophys. J. Int., 186*, 1036–1044, 2011.

WCRP Global Sea Level Budget Group: Global sea-level budget 1993–present, *Earth Syst. Sci. Data, 10*, 1551–1590, 2018.

Whitehouse, P.L., M.J. Bentley, A. Vieli, S.S.R. Jamieson, A.S. Hein and D.E. Sugden: Controls on Last Glacial Maximum ice extent in the
   Weddell Sea embayment, Antarctica, *J. Geophys. Res., 122*, 371–397, 2017.