June 30, 2020

Dear Editor,

We thank you for the opportunity to revise the manuscript and for generously granting a month-long extension of submission deadline. We thank all three reviewers for their positive and thoughtful comments that have led to a greatly improved manuscript. We are happy to report that we have addressed all of reviewers' comments and revised the manuscript suitably. Much of the reviewers' criticisms are on the clarity and focus of the manuscript, which we have considered sincerely and made some significant reshuffling of the text. Here is the list of key changes we have made in the revisions.

1. We have changed the title so that it better aligns with the key goals of the paper.
2. We have spelled out the key goals of the paper both in the Abstract and Introduction.
3. We have added in the Introduction more description of the existing methods and their limitations and have contrasted our goals against them.
4. We have added mathematical description of a generic level-set function that can track both the grounding lines and coastlines in a seamless manner (Section 2.1).
5. We have explained the key mathematical terms concisely, with their physical interpretations in light of space observations when possible.
6. We have added new materials (about 2.5 pages, with 1 figure) in the Appendix to facilitate the interpretation of some mathematical terms.
7. We have introduced a new section (Section 4) to place our formalism in a broader context of sea-level change and mass conservation in the Earth System.
8. We have added new figure panels to demonstrate the level of improvements possible by employing the new method of estimating ice sheets' contribution to sea level, which is significantly large (on the order of 10-15%).
9. We have inserted several subsections to make the paper more accessible to readers.

Let me know if you have any question or need further information.

On behalf of co-authors,
Surendra Adhikari
Jet Propulsion Lab, Caltech

Referee #1

Summary

The paper presents a formalism to geometrically interpret changes in ice sheets and underlying bedrock and their combined effect on the ocean and sea level. The approach defines two distinct domains (land and ocean) that can both intersect areas of ice cover. It then traces ice and bedrock changes and transitions between different domains to determine the sea-level contribution of the ice sheet.

General comments

The paper is well written, clearly structured and deals with the important question of how to calculate the sea-level contribution of a marine ice sheet, among others. I believe it would make an interesting contribution to The Cryosphere given that the points raised below are addressed adequately.

We thank the reviewer for a positive and constructive review. Please find our response to individual comments below. The changes made in the revised manuscript are pointed by the **bold text**.

One of the main conclusions of this paper reads very similar to the one in Goelzer et al. (2020), both papers proposing an alternative to conventionally calculating the sea-level contribution of marine ice sheets based on volume above flotation. It seems important to clarify what the similarities and differences are. The results presented by Goelzer et al. (2020) imply that ice and bedrock changes have to be considered together at least in any place where ice could ground over the course of the experiment. This is the direct consequence of the claim that the sea-level contribution calculated from one point in time to another should be independent of what happens in between. Since the example that is put forward (see their Fig 2a and related text) matches with regime 3 here, there seems to be a direct disagreement between the two approaches: bedrock changes are taken into account in their case, but not here. This may point to a flaw in the approach that should be clarified and discussed. If bedrock changes are not considered as part of the ice sheet change in regime 3, what other component is taking it into account (if any) and how does the domain separation between those components work? If both approaches are not compatible, why and under what circumstances do they differ? What would needs to be changed to make the two approaches compatible? Are the two approaches addressing a different modelling framework, which explains the differences?

We break this question into two parts. (1) What are the similarities and differences between our work and the recent work by Goelzer et al. (2020)? (2) Why are Figure 2a of the Goelzer paper and our Regime 3 (as defined in our Figure 2b) inconsistent?

The goal of the Goelzer paper was to provide a correction to a common approach of estimating sea-level contribution from ice sheets. The approach, based on the concept of ice height above flotation (HAF), predicts an incorrect sea-level contribution when the ice-sheet model accounts for evolving bedrock and sea level, especially in the marine portions of the ice sheet. In particular, Goelzer et al. (2020) provide a correction for the effects of bedrock elevation change and externally-forced sea level in stand-alone ice-sheet models **(page 2 lines 24-27)**. Our goal here is two-fold: (1) to formulate a *new* field – rather than a correction term – that can be utilized to accurately quantify ice sheets' contribution to sea level by accounting for the effects of evolving bedrock and sea level driven by any geophysical processes **(page 2 lines 28-29)**; and (2) to develop a generic level-set for accurate tracking of both the grounding lines and coastlines in a seamless manner **(page 2 lines 16-18)**. Our formalism can be applied to a whole suite of modeling architectures: from stand-alone ice-sheet models (e.g., many of ISMIP6 participating models), to models that account for isostatic adjustment of bedrock (e.g., Le Meur and Huybrechts, 1996; Pattyn et al., 2017) or a self-consistent GRD (gravitational, rotational, deformational) response of the solid Earth (e.g., Gomez et al., 2013; Larour et al., 2019). In the latter set of models, our formalism ensures mass conservation in the Earth System by exchanging mass between the land and the ocean, accounting for induced GRD response of solid Earth, and adjusting the ocean area through grounding lines and coastlines migration, simultaneously **(page 2 lines 29-35)**.

Regarding the second question, it appears that we were not clear enough in the manuscript in terms of explaining the physical significance of three regimes. To facilitate discussions, we compile relevant figures below (Figure 1). Based on the evolving ice thickness ΔH, bedrock elevation ΔB, and mean sea level ΔS, we classify 3 regimes of the ice sheet. In Regimes 1 and 3, grounded ice remains grounded and floating ice remains floating, respectively, over the considered time period Δt. Regime 2 only captures the portion of the ice sheet that experiences transition from grounded to floating state, or the reverse, over the course of Δt. Now let us interpret Figure 2a of Goelzer et al. (2020) from the lens of the three regimes we've classified. Over the period $\Delta t = t_4 - t_1$ (see top panel in the figure below), as correctly pointed out by the reviewer the floating ice remains floating (equivalently, our Regime 3) and hence the column of ice does not contribute to the barystatic sea-level change during this period. Even though ΔB and ΔS do not appear to play any role in this regime (as they do not appear explicitly in **equation 10**), the fact is that they determine the new grounding line position and hence the domain of Regime 3. The reviewer's remark that we do not consider the effect of the evolving bedrock elevation in Regime 3 is therefore not accurate. If we consider $\Delta t = t_2 - t_1$, on the other hand, the column of ice does contribute to the barystatic sea level because the then-floating ice is now grounded (equivalently, our Regime 2), and we will have to consider the effect of ΔB and ΔS explicitly while estimating the sea-level contribution of the ice column over this shorter period. This is consistent with the interpretation made by Goelzer et al. (2020).

While the first few words in the abstract seems to say that this paper is about representing ice sheets in models, the mention of geodesy later may suggest that observations of ice sheets are equally addressed. If the aim is indeed modelling, I think this should be made clear and a clearer distinction be made from observations. If both should be addressed at the same time, I suggest to make sure that the presented formulations make sense in both realms.

By design, the proposed formalism is very generic and it can be used for applications with modeling, observations or combination of models and observations. Many aspects of our

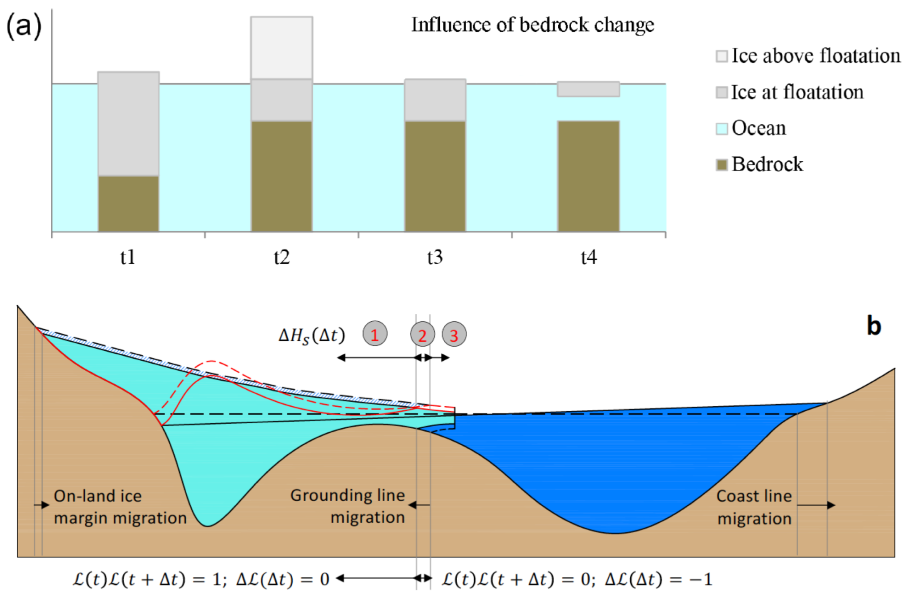

Figure 1: Top: Figure 2a of Goelzer et al. (2020). Bottom: Figure 2b of our paper.

formalism including a generic level set for coastlines and grounding lines **(Section 2.1)** can be used for both modeling and observational applications (**page 2 lines 18-19**). However, due to lack of observed data for some fields (e.g., change in bedrock elevation beneath the ice sheet), some other aspects of the formalism (e.g., **Section 3.2**) are more suited for model-based studies (**page 2 lines 30-35**), that too for Earth System models that seek to conserve mass **(page 1 lines 9-10)**.

While I appreciate the formal description of the case, I miss better guidance of the reader through what is a difficult problem to understand and visualise. Recurrent redefinitions of variables (see example S(w,t) below) should be avoided, individual terms in the equations should be better explained and examples should be given where possible. This particularly applies to cases where the formalism uses familiar concepts and applies them to something else (e.g. floatation condition for ice applied to define the ice free coastline).

Based on this and similar comments from two other reviewers, we have provided more precise definitions of the key terminologies and expanded descriptions of individual terms appearing in the key equations. You will find it in our responses below. We appreciate this comment, as it greatly helps to improve the focus and clarity of the manuscript.

For understanding and reproducing the results it would be useful to provide access to the data and tools used to produce the results and plots in Figure 3. Please consider making the geometry and scripts available.

Agreed. We have made the data and Matlab script publicly available at https://doi.org/10.7910/DVN/9LUJTD **(page 15 line 1)**.

Specific Comments

P1.l1 Not all ESMs include ice sheet components. Reformulate.

Agreed. We simply write "...components of the Earth System." **(page 1 line 1)**.

P1.l2 The connection between ESMs and geodesy is not clear to me. E.g. observation don't exist for ESM paleo simulation where the formalism should also hold.

ESMs are intimately tied to geodesy, even if these models are not constrained by paleo data. Some of the fields considered in our formalism such as mean sea level (MSL) or geoid and bedrock topography are defined relative to a common geodetic datum, such as the International Terrestrial Reference Frame. These fields essentially describe the "shape" of the Earth, and modeling and measuring of which is the very definition of geodesy. Also note that some geodetic observing systems relevant for ESMs have been in place for more than a century. Earth rotation and gravity fields were measured during the first half of the 20th century, for example.

P1.l5 "grounded and floating masks" suggests a modelling perspective, but "as viewed from space" relates to an observational dataset. What is the perspective of this paper?

As noted above, our formalism is generic and applied to both modeling and observational realms, although some aspects of it are more suited for model-based studies due to the apparent lack of observational data **(page 1 lines 9-10)**. We have revised the referred sentence **(page 1 lines 2-5)**.

P1.l5 "Here we present ...". The subject in this sentence is not clear. Reformulate.

Toward more focused and impactful abstract, we have completely rewritten its second half **(page 1 lines 5-10)**. As a result, this sentence does not appear in the revised manuscript.

P1.L13-15 This is clearly true for simulations of the Antarctic ice sheet, but not really for the Greenland ice sheet, which is dominated by surface mass balance changes. Similar cases may also exist during other climates with ice sheets that were mainly land based. Suggest to reformulate.

Agreed. We now write "...numerical model of marine ice sheets..." **(Page 1 line 16)**.

P2.l2 "Defining geometry". Do you mean "Defining the bedrock"? How could the geometry be defined upfront for an intercomparison when the models are supposed to produce an evolving geometry?

We agree that this statement is ambiguous. We have deleted it altogether. The removal of this sentence does not break the flow of the write-up.

P2.l5 What do you mean with "basic configuration setup" and "Similar setups"? Could you describe this in more detail?

We have rephrased the sentences in a clearer way **(page 2, lines 5-9)**. The quoted phrases do not appear in the revised sentences. See our response to Referee #2's comments #p.2,l.8 on page 17 and #p.4,l.23 on page 19 of this document.

P2.l12 Why are "floating ice shelves" and "retrograde bedrock slopes" complex features? They occur in the very simple flowline model setups you may be referring to above. Clarify.

Agreed. We have replaced the quoted phrased by "rugged fjord geometries" and "uneven bed topography". These kilometer-scale features **(page 2 line 11-15)** are "complex" relative to the simplified geometries considered in previous studies of mechanical analysis of grounding line migration (e.g., Hutter, 1983; Lambeck et al., 2003). We have avoided the usage of word "complex" in the revised manuscript.

P2.l13 What is the "traditional theory for ice-bedrock-ocean interface changes"? Clarification needed.

We meant to imply the previous studies that considered simplified geometries (e.g., Hutter, 1983; Lambeck et al., 2003; Mitrovica and Milne, 2003), lacking kilometer-scale features such as pinning points and rugged fjords perhaps due to the lack of constraining data or computational resources **(page 2 lines 5-9 and lines 13-15)**. See also our response to Referee #2's comments #p.2,l.8 on pages 17 of this document. We have excluded the usage of the phrase "traditional theory" in the revised manuscript.

p2.l23 "setup" → maybe "interpretation"?

The quoted word does not appear in the revised manuscript. We simply write "To begin our discussion, we consider a spherical planet..." **(page 3 line 10)**.

p2.l24 What is the difference between "glaciers" and "ice sheets" in your description? Clarify if the two terms are interchangeable or distinct. If the latter, what sets them apart in your formalism?

The formalism does not need to distinguish between glaciers and ice sheets. In fact, we prefer a generic term such as "distributed ice domains". We use glaciers and ice sheets to give a sense to the readers of what we meant by distributed ice domains **(page 3 line 14)**.

p2.l24 I found the upfront separation between land and ocean confusing for your context, because it is not intuitive where that separation is to be made for a marine-based ice sheets. The definition what is to be considered land and ocean comes too late. I suggest to make that clearer much earlier.

Agreed. We now make it explicit upfront in the Section 2 **(page 3 lines 11-15)**.

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P2.l25 Same for S(w,t), defined here first as the sea-surface elevation. How should we think of S for an ice shelf? Why not start with defining S as the geoid as you do later.

We agree that the definition of mean sea level (MSL) S is indeed confusing. We have clarified it in reference to Gregory et al. (2019) on **page 3 lines 22-33**.

P2.l26 I admit, I had to look up what the ITRF is. For other readers not familiar with it, you may want to add a sentence or two to say what the ITRF provides. In practice, if I use Bedmachine data, is it registered on the ITRF or are you suggesting this is something the user would have to take care of herself?

ITRF is the standard reference frame defined by the International Earth Rotation and Reference Systems Service (www.iers.org). It is being updated every few years. Glaciological datasets (e.g., surface and bedrock DEMs) are intimately tied to a reference frame. BedMachine v3 data, for example, are referenced to the WGS 84 ellipsoid. This and ITRF's recommended ellipsoid, GRS80, should not differ from each other by more than a few centimeters. This difference is much smaller than the uncertainties in BedMachine data. It is also possible to project data from one ellipsoid to another.

We have decided not to specifically mention ITRF in the revised manuscript (whose description would divert the flow of the paper), because the theoretical formalism presented here is valid as long as both the bedrock and MSL are measured with respect to the same reference frame **(page 3 lines 20-21)**.

P2.l31 So far $S(\omega, t)$ is defined as sea-level. As such, any case $B > S$ is not well defined. The interpretation of S as the geoid must come earlier for this to make sense.

The MSL, as we have defined in this study **(page 3 lines 22-30)**, has the same spatial pattern as the geoid and hence is a global field **(page 3 lines 32-33)**.

P2.l32 You say here that $S(\omega, t)$ includes high-frequency noise and variability, but on the next page you want $S(w, t)$ to refer only to the quasi-static component of the sea-surface. Why not introduce S directly as the quasi-static component of the geoid, rather than going through three redefinitions along the way (sea surface $\rightarrow$ quasi-static sea-surface $\rightarrow$ sea-level $\rightarrow$ geoid).

We appreciate the comment. We believe that this issue is fully addressed above. See **page 3 lines 22-33** for the definition of MSL.

P3.l1 Remove "changing" before interactions?

Agreed. See **page 3 line 24**.

p3.l7 What is the "interior of marine ice sheets"? Clarify

We meant to imply that MSL is a globally-defined field **(page 3 line 33, see also Figures 2a-b)**. This sentence does not appear in the revised manuscript.

P3.l9 Say and explain what $F(\omega, t)$ means. Traditionally it determines if the ice is floating. But you seem to extend it to locations with $H = 0$? Maybe it would be worth it to mention that.

$F(\omega, t)$ is indeed a generalization of the concept of flotation condition for ice based on the principle of hydrostatic equilibrium **(page 4 lines 1-6, see also line 9)**.

p3.l13 Please define what "open ocean" means and what "contact with the open ocean" means. This definition comes too late in the manuscript. Does w have to be part of $\mathcal{O}$? Maybe we need the definition of $\mathcal{O}$ already before this part not on page 4?

We simply write "the ocean" consistently throughout the manuscript. We have defined it upfront in the Section 2 **(page 3 lines 11-12)**. To avoid the ambiguity, we have provided a detailed mathematical description of a generic level set for seamless delineation of land-ocean-ice interfaces **(page 4 line 16 – page 5 line 14)**. Also see our response to your comment #p4.l6 on pages 8-9 of this document. The quoted words do not appear in the manuscript. The mathematical definition of $\mathcal{O}$ **(equation 5)** remains where it originally was, because it relies on the definition of $F(\omega, t)$.

p3.l14 A more obvious definition of a generalised coastline for me, that also exists in presence of a marine-based ice sheet would be the point where the bedrock and the geoid intersect (1/2 in Fig2). That doesn't help for your formalism, but it goes to show that it is not immediately obvious to think the coastline at the grounding line. Better guidance needed.

We would like to respectfully disagree with the reviewer on this point. We believe that our formalism, which treats grounding lines and coastlines as a seamless level set of $\mathcal{F}(\omega, t) = 0$, makes it appealing for Earth System models. See our response to comment #P4.l6 on pages 8-9 of this document. The reviewer is suggesting the boundary between Sector A and Sector B to be the coastline **(see Figure 2a)**. Unlike the grounding line, tracking the migration of this boundary neither improves our understanding of ice-sheet dynamics nor helps us determine an evolving ocean-surface area, for example.

P3.l8-15 I found this paragraph difficult to follow. You start with reference to Fig. 2a, where our focus is on the left hand side and with $F(\omega, t)$ which suggests it is about ice. But then you describe $R(w, t)$ and the coastline, which are difficult to visualise in a place with an ice shelf. It may help to guide the reader by being explicit about the two "generalisations" that take place here: floatation criterion (for ice) $\rightarrow$ definition of the coastline (everywhere). grounding line (for ice) $\rightarrow$ coast line (everywhere).

We have already addressed most of the issues noted here. Also see our responses to your comments #p3.l9, p3.l13 and p.4l1 on pages 7-8 of this document. We have moved the definition of relative sea level $R(\omega, t)$ to **Section 4 (page 12 line 8)**. With these changes, we believe that the paragraph should be clearer and easier to follow (See **Section 2.1**).

P4.l1 I don't see why there could not be a grid point in a model with $B = S$ and $H > 0$. A glacier terminating on land or on a sill exactly at sea-level? Please clarify.

We are glad that the reviewer raised this point. Indeed, there is a subtle assumption that goes into our interpretation of equation (1). This has now been clarified **(page 4 lines 10-15)**. Since the assumption is reasonably valid **(page 4 line 14)**, no "grid point" exits in a model with $B = S$ and $H > 0$ **(page 5, line 16)**.

P4.l6 The fact that neither O nor L are defined at the grounding line seems problematic. How can your formalism be mass conserving when grounding line grid points in an ice sheet model are not part of these masks? How do you track the grid cells that fulfil this condition, do they form a separate category? Why would it not matter to consider them?

O and L are complementary fields such that their surface areas make up the total area of

Earth's surface. The coastlines and grounding lines form their own level set, with no surface areas, which can be tracked straightforwardly. As such, there should not be any "cells" outside both O and L, and therefore there should be no problem regarding mass conservation. This has been now clarified in the manuscript **(page 5 lines 21-26)**. We agree that we did not provide a mathematical description of the coastlines and grounding lines in the original manuscript, which is now included in terms of a level-set function in the revised version **(page 4 line 16 – page 5 line 14)**.

P4.l9 Remove "deep" and "well". I suppose the condition could also be true for shallow troughs with bathymetry moderately below sea-level.

Agreed. See **page 4 lines 6-7**.

P4.l10 While I understand the use of this connectivity concept in your formalism, I find it problematic in practice. It means that small changes in ice or bedrock can lead to very large changes in O and L. In an unfavourable configuration, the short term grounding and ungrounding of a critical point could e.g. switch an entire system of connected fjords on and off.

With a newly-added mathematical description of a level-set function $\mathcal{F}(\omega, t)$ **(page 4 line 16 – page 5 line 14)**, the (relatively subjective) concept of "connectivity" does not need to be invoked anymore. However, our formalism still supports the idea of potentially having large changes in O and L due to relatively small changes in bedrock or mean sea level. And, this is precisely what happens in reality (at least from mass conservation perspective) and it should be an essential feature in paleo simulations of a system of ice sheets, solid Earth and sea level.

p4.l18 With the above, combining the grounded and floating ice masks leaves a hole at the grounding line. Is this desired?

Again we understand the confusion created by the lack of mathematical description of grounding line level set, which has now been included **(page 4 line 16 – page 5 line 14)**. Adding grounded and floating ice masks to grounding line level set would leave no hole in the ice domain. See our response to your comment #P4.l6 on pages 8-9 of this document.

P5.l13 The geoid typically changes first, then the bedrock. Maybe re-order in the sentence.

We would like to respectfully disagree with the reviewer. For a given relaxation frequency that is relevant for the present study (with periodicity of decades or longer), both geoid and bedrock elevation evolve simultaneously.

p5.l22-23 I am confused about this sentence. Isn't "quantifying the fraction of ice mass change that contributes to sea level" exactly what you are doing by defining ΔH_S below? Reformulate?

We meant to imply that not all of ice mass loss makes it to the ocean. A fraction of it, for example, may get impounded in proglacial lakes. However, the quoted statement is in direct conflict with the assumption that "the net change in grounded ice mass results in the equivalent change in ocean mass..." **(page 7 lines 12-13)**, so we have simply removed the sentence. We thank the reviewer for catching this apparent inconsistency. The reviewer is correct that ΔH_S (well, in fact, its one component ΔH_M) provides the accurate estimate of ice mass loss **(page 7 equation 10 and the text that follows)**.

p5.23 "the assumption" appears three sentence back, maybe refer to it more specifically.

The in-between sentences have been deleted (see our response to the previous comment). This should fix the issue noted by the reviewer.

p5.23 Remove "all the time".

Agreed. See **page 7 line 13**.

p5.26 "As we show below". This has been shown before by others (see references). Reformulate to avoid confusion.

Agreed. We have removed the quoted phrase **(page 7 lines 24-25)**.

p5.l30 Can you please explain what the three terms mean physically. E.g. the first term accounts for thickness changes of ice that is and remains grounded ...

To facilitate easier interpretation, we have restructured the definition of ΔH_S **(equation 10)** and split it into two parts: the component ΔH_M that modulates both the mass and volume of the oceanic water and induces the GRD response of the solid Earth **(equation 11)**, and the component that only modulates the ocean volume **(equation 12)**. The revised description of the equations **(page 7 line 26 – page 9 line 31)** along with the Appendix **(page 15 line 3 – page 18 line 5)** should provide enough information to interpret the individual terms appearing in these equations.

p5.l30 Could you explain why H0 appears as an *absolute* contribution in the third term compared to considering *changes* in H_S and H_F in term one and two?

We appreciate this comment very much. In fact, it was a mistake on our end. The correct term should be $(\Delta H - \Delta H_F)$ rather than H_O, which is now corrected **(equation 12)** and its physical justification is given in the Appendices A2 and A3 **(page 16 line 8 – page 18 line 5; see also Figure A1)**. This term, in essence, accounts for the difference in volume between the freshwater that would be produced when ice melts and the oceanic water that would be displaced when it floats **(page 9 lines 20-23 and 26-29)**.

p6.l6 Not clear what "holding in the areas of on-land ice margin migration" means. Reformulate.

We realize that the quoted statement is in direct conflict with the assumption that "the net change in grounded ice mass results in the equivalent change in ocean mass..." **(page 7 lines 12-13)**, so we have deleted the sentence and written "...it may often be possible to use grounded ice masks in place of the land domains..." **(page 14 lines 5-6)**. We thank the reviewer for helping us catch this minor inconsistency.

p7.l15 I suppose you mean that the nonzero ΔH_F is compensated by other terms in Eq. 7. Which ones? This is important to understand.

No, but thanks for the opportunity to clarify. What we mean is that externally-forced ΔB or ΔS may possibly modulate ice dynamics in this regime, even if they do not contribute to ΔH_F or ΔH_S. We have elaborate in the manuscript **(page 9 lines 13-16)**.

p7.l22-23 This sentence may be confusing because Goelzer et al. (2020) consider not only transitions between grounded and floating ice, but all three regimes. Reformulate.

This is correct. Now that we have added a paragraph in the Introduction to summarize the Goelzer paper and contrast it against our goals **(page 2 lines 26-35)**, we really do not need this citation here. We have simply deleted the sentence.

p7.l26 Important to note that in regime 3 bedrock changes can contribute to sea-level change, even if you are not considering them as part of the ice sheet change. Also important to realise that ice floating at t and $t + \Delta t$ may have been grounded at some point in between. I think this makes your solution dependent on the time stepping. See Fig 2a in Goelzer et al. (2020) for an example of such a case.

We acknowledge that the bedrock change in this regime may contribute to spatial pattern of sea level. By referring to **equation (12)**, all we are saying here is that there is no mass contribution to the ocean from this regime of the ice sheet **(page 9 lines 27-29)**.

Once ΔH_S is defined **(equation 10)**, which is what our focus is at this point, a GRD (gravity, rotation, deformation) model of solid Earth yields self-consistent solutions for bedrock elevation and MSL change. This is a whole different story, which has been briefly summarized in Section 4 **(page 13 lines 9-21)**.

Regarding the last question referring to Fig 2a of Goelzer et al. (2020): Given our explanation on pages 3-4 (see Fig. 1) on the topic, we believe that there is no confusion left in this regard. Nevertheless, here is our brief response to the questions listed above. Yes, ice could have been grounded in this region at some point in time between t and $t + \Delta t$. In this instance, the part of the region that is grounded belongs to Regime 2 (not 3; see Fig. 1b) and it should be interpreted accordingly. Yes, our solution is time dependent and this is what it should be. See Fig. 1a. There is no net mass contribution to sea level from the ice column during the period $[t_1, t_4]$, while there is clearly non-zero mass contribution from ice during the sub-period $[t_1, t_2]$.

p8.l7 The change in ΔH_S and $\Delta R^I_C(\omega, t)$ is not only due to ice mass changes, but also due to bedrock changes under the ice. It may be good to mention that here.

We think that the reviewer got confused between "ice mass change" and "ice thickness change". $\rho_i \Delta H_M$, the mass component of ΔH_S, essentially describes the change in ice mass per unit area (units of kg/m^2). And, this field is contributed by ice thickness change, bedrock elevation change and MSL change, which we have thoroughly discussed in the manuscript **(Section 3.2 and Appendix)**. We do not wish to list all of these contributors explicitly again.

p8.l8 For clarity, could you mention why potential changes in ocean area from $O(\omega, t)$ to $O(\omega, t + \Delta t)$ do not matter for $\Delta R^I_C(\omega, \Delta t)$ in Eq 8? I suppose the underlying assumption is that we should be interested in sea level at time $t + \Delta t$?

The change in ocean area from $O(\omega, t)$ to $O(\omega, t + \Delta t)$ does matter. The ocean area at time $t + \Delta t$ is the sum of the ocean area at time t and change in the ocean area over the period Δt. The original Eq. 8 does not appear in the revised manuscript. We rather provide explanation in the text (**page 9 lines 30-31**).

p8.l12 $\mathcal{G}(\omega, t)$ is not explicit in Eq. 7 only implicitly by evoking Eq. 6. This could be mentioned.

Agreed. See **page 10 line 4**.

p8.l14. Maybe remind us what R(w,t) is here, as it is only introduced inline and back at p3.

We have now introduced the relative sea level (RSL) here in Section 4 **(page 12 lines 8-9)**, not earlier in the manuscript.

p8.l15 To make this equation more digestible, maybe start with combined symbols. E.g. by combining all the barystatic components like you do in the text.

Other reviewers also have similar concerns. To address these, we have introduced a new section (**Section 4**) and restructured (and expanded) the original materials as follows.

1. We contextualize why we need to consider all contributors of relative sea-level (RSL) change in a mass conserving Earth System framework in order to compute ΔH_S and $\mathcal{T}$ **(page 12 lines 2-14)**;
2. We introduce non-steric components of RSL including the one induced by ΔH_S itself over the period Δt **(page 12 lines 15-27)**, and interpret them in light of space observations and existing GRD models **(page 12 lines 27-31)**;
3. We dissect ΔH_S into two parts: the component that modulate both the mass and volume of the oceanic water and drives the GRD response of solid Earth, and the (smaller) component that only modulates the ocean volume **(page 12 line 32 – page 13 line 6)**;
4. We summarize how the spatial patterns of the "mass" component is computed, which necessitates conservation of mass in the Earth system **(page 13 lines 9-18)**.

We hope that the reader finds it more accessible.

p9.l1-24 My experience with the paragraph including Eq. 9 is that a lot of new concepts are suddenly thrown in here without much preparation. Especially the idea to separate the effect of the past from the contemporary would profit from some more introduction. May be a new section with a few introductory sentences could be started p8.l6 to prepare the ground for this discussion.

We appreciate the feedback. We have restructured and expanded the original materials **(page 12 lines 15-31)**. See our response to the previous comment.

p9.l3 Since $\Delta R_V(\omega, \Delta t)$ may be the component that takes into account bedrock changes in regime 3 and is complementary to the bedrock changes happening under the ice, I would be interested to see how it is calculated and how the masking works in that case.

One of the strengths of our formalism is that ΔH_S as defined by **(equation 10)** considers the effects of ΔB and ΔS perturbed by all kind of processes (e.g., glacial/oceanic loads or tectonics). This is clearly stated in the manuscript **(page 7 line 5)**. Our partitioning of the ice domain into 3 distinct regimes is for a convenient interpretation of ΔH_S. Regime 3 & other 2 regimes for that matter are defined by considering the effects of ΔS and ΔB, including ΔB_V – the component of bedrock elevation change that corresponds to ΔR_V **(now denoted by R_O)**. **Equation (13)** summarizes the processes that contribute to ΔR **(page 12 lines 15-27)**. We have interpreted these components in light of space observations and existing GRD models **(page 12 lines 27-31)**. We have also provided summary of how to compute RSL induced by ΔH_S **(page 13 lines 9-18))**. Providing a detailed theoretical/numerical description of each term, including ΔR_O, is beyond the scope of this study. In terms of masking, both ΔB and ΔR are globally-defined fields and there is no need for masking.

p9.l28 By your definition, the grounding lines are neither part of the grounded ice nor part of the floating ice domain. What is this category called and how are you accounting for it?

Just like how the ocean O and the land L are complementary fields such that their surface areas make up the total area of the Earth's surface (**page 5 lines 21-22**), the grounded and floating masks are also complementary fields they together make up the ice domains. The grounding lines, like coastlines, form their own level set, with no surface areas (**page 6 line 3**), which can be tracked straightforwardly as discussed on **page 5 lines 25-26**. We agree that we did not provide a mathematical description of the coastlines and grounding lines in the original manuscript, which is now included in terms of a level-set function in the revised version **(page 4 line 16 – page 5 line 14)**.

p10.l4-5 How does your approach compare to/ differ from that proposed by Goelzer et al. (2020).

In a nutshell, they provide correction terms associated with an evolving bedrock and an externally-forced (spatially uniform) sea level for a common method of estimating ice sheets' contribution to the barystatic sea level change (BSLC). Their focus is the marine portions of stand-alone ice sheet models. In contrast, we provide a new field altogether, ΔH_S, that yields BSLC by accounting for spatially variable $\Delta B(\omega, t)$ and $\Delta S(\Delta t)$ caused by any geophysical processes. Our method is applied to all plausible settings of ice sheets (e.g., terrestrial- vs. marine-based and grounded vs. floating). In addition, our goal is also to track grounding lines and coastlines in a seamless manner in order to conserve mass in the Earth System and facilitate complex interactions between ice sheets, solid Earth and sea level. While we do not wish to be critical of the Goelzer paper, the new materials that summarizes their work and contrast it against our goals in the Introduction (see our response on pages 2-3 of this document) should provide enough information in this regard.

Figures

Figure 1. It seems confusing to introduce lakes, subglacial lakes and proglacial lakes and then not consider them at all. It would make the figure much clearer to remove them.

Our goal is to make our formalism as generic as possible. Given the complexities of Earth's surface features, it is an extremely challenging task. Figure 1 depicts some of the relevant features (and associated modeling challenges), and in the caption we have clearly stated what is considered and what is not considered in our analysis. We think this is important. We prefer to keep the figure as is.

Figure 2. The geoid (and S) is also defined over land. Please add in both panels.

Agreed. In the revised **Figure 2**, the MSL is drawn over land as well.

Figure 3. Please include a panel with ΔH_F. This is important as it is discussed as the conventional method and appears in the difference in panel c. Please also add contour lines to delineate the regions 1-3. Mention that regime 4 does not exist in this region if that is true. Otherwise, delineate regime 4. If different from results in Goelzer et al (2020) it would also be interesting to add a comparison as figure here.

Since the difference between ΔH_S and ΔH_F is smaller than the either field by a factor of 10 or so (compare **Figure 3b** vs **Figure 3c**), we are hesitant to include a new panel for the latter field (which looks almost same as Figure 3b). We have added contours to show 3 Regimes, as suggested. We find that 4 regimes defined in Fig. 2a to interpret H_F may be confused with 3 regimes defined in Fig. 2b to interpret ΔH_S. We have therefore labelled the formers as Sectors A, B, C, and D. These sectors are not relevant for Figure 3. In principle, the Goelzer method should yield similar, if not the same, result as ours. Because it was not clear to us how the effect of MSL is handled, we decided not to compare against their method. However, we have added two new panels **(Figure 3d-e)** to show the time series of the Antarctic ice-volume change that is attributable to the sea-level change, computed by ours and the more common HAF method. We find significant difference (on the order of 10-15%) between the two methods (see **Figure 3e**).

Referee #2

Summary

This paper presents a framework for describing the geometry of an evolving ice sheet margin in Earth system models, in which the geometry of the bedrock and mean sea level also can be dynamic. The authors define relevant terms and give a mathematical description of the way different quantities are related. In particular, they quantify the portion of ice thickness change that contributes to changes in ocean mass and global mean sea level. As Earth system modelers work to integrate dynamic ice sheet models with solid-Earth and sea-level models, this paper will be a useful reference.

In general, the paper is well written, and the figures are very helpful. Sometimes, however, technical terms related to sea level are used without precision, or are introduced without giving enough background information. Some of the equations contain terms that are not clearly defined or explained physically. In the comments below, I suggest where the exposition could be improved, especially for readers approaching these concepts for the first time.

We thank the reviewer for a positive and constructive review. We have clarified concepts and terminologies that are ambiguous, supplied additional background materials as required, and explained physical meanings of key mathematical terms that were missing. Find our response to individual comments below. The changes made in the revised manuscript are pointed by the **bold** text.

Also, the text refers to “traditional” or “customary” approaches of estimating sea-level contributions from ice sheets based on the change in height above flotation, in contrast to the approach described here. It would be helpful to see some specific examples of customary approaches from published papers, with estimates of the magnitude and sign of the associated errors. This would enable readers to better assess the value of the proposed formalism.

Most of the stand-alone ice-sheet models use ΔH_F to estimate ice sheet's contribution to barystatic sea-level change (BSLC), termed the HAF method **(page 2 lines 24-26, page 7 lines 15-20)**. Many of participating models of SeaRISE or ISMIP6 projects are some examples. Unless we compare these BSLC estimates with those derived from better/improved methods (e.g., our methods or Goelzer et al., 2020), which is not obviously in literature, we cannot quantify/report errors associated with these estimates. This is why the results presented in Figure 3, based on a particular model run by Larour et al. (2019), is useful. We have now included two additional panels in the figure **(Figures 3d-e)** that show the time series of Antarctic ice-volume change that is attributable to sea-level change. We find a significant difference between ours and the HAF method, which is on the order of 10-15% (see **Figure 3e**). This is now reported in the main text **(page 10 lines 24-31)** and highlighted in the Conclusion (**page 14 line 8**).

Major comments

p. 1, Title: The title includes the words “mass conserving”, but in the text I did not find an operational definition of what this means in an ESM with evolving ice sheets. Please provide such a definition, and perhaps an example of how mass conservation would be violated.

Although "mass conservation" is still an important element of the paper (see, for example, **page 1 line 10**), we also think that the original title was a bit too vague. We have now suitably modified the title, **A kinematic formalism for tracking ice-ocean mass exchange on the Earth's surface and estimating sea-level change**, which is better aligned with the key goals of the paper **(page 1 lines 5-8)**. As a compensation, we have now introduced **Section 4** wherein we place the presented formalism in a broader context of sea-level change and mass conservation in the Earth System. We have now defined what mass conservation means in the context of our formalism **(page 2 lines 32-35)** and also provided example cases where mass conservation is violated **(page 13 lines 18-21)**.

Also, the title implies that there will be a detailed analysis of ice-sheet interactions with solid Earth and sea level, but the actual scope seems narrower: to accurately compute the contribution of dynamic ice sheets to barystatic sea level rise (i.e., the sea-level component associated with a redistribution of mass between land-based ice and the ocean) in ESMs. I suggest a revised title that better reflects the scope.

We fully understand the reviewer's concerns, and we agree that the original title was a bit vague. We have revised it appropriately to better align with the the key goals of the paper (see our response to the previous comment). We appreciate the comment.

p. 1, Abstract: The abstract is very general and does not provide a clear sense of the scope of the paper. If a central goal is to describe how to accurately compute the ice sheet contribution to barystatic sea level rise, then this goal should be clearly stated.

We agree that our original abstract was also a bit too vague. We have rewritten the second half of it, aiming at more focused and impactful abstract (**page 1 lines 5-10**).

p. 1, l. 1: Although I am fully in favor of including dynamic ice sheets in ESMs, I would not go so far as to suggest that "any Earth System model" already includes them. For some ESMs, ice sheets are still on the back burner.

Agreed. We now write "...components of the Earth System..." **(page 1 line 1)**.

p. 2, l. 2: "Defining geometry". I am not sure what this means – something like defining geometric concepts that are relevant to models with dynamic ice sheets?

Referee #1 also had a concern about the quoted phrase. We agree that this sentence is ambiguous. We have decided to delete it altogether. The removal of this sentence does not break the flow of the write-up.

p. 2, l. 4: "future debate and reconciliation." Ideally, the concepts are set forth clearly in a way that leads to greater mutual understanding and less debate.

This phrase and "defining geometry" (see previous comment) both appear in the same sentence, which we have removed from the revised manuscript.

p. 2, l. 5: "basic configuration setup". I am not sure what this means. It seems an odd way to describe what seem to be theoretical or analytical frameworks.

Agreed. Referee #1 also found this phrase unclear. We have rephrased the sentences in a much clearer way **(page 2, lines 5-9)**. The quoted words do not appear in the manuscript. Also see our response to your next comment.

p. 2, l. 8: I am not clear why these previous analyses are referred to as "traditional configurations". The word "traditional" suggests a contrast with something novel and untraditional to be introduced here. However, I don't see this work as heading off in a different direction from the cited papers, but rather as clarifying concepts that are particularly relevant for ESMs. As above, "configuration" doesn't seem to be the right word.

We no longer label the previous analyses as "traditional", and have avoided its usage in the manuscript along with the word "configuration". We have deleted this sentence.

We labeled the previous studies that considered simplified geometries of ice/bedrock/ocean system (e.g., Hutter et al., 1983; Lambeck et al., 2003; Mitrovica and Milne, 2003) as "traditional" because these studies do not capture kilometer-scale geometric features, e.g. ice rises and rumples and rugged fjord geometries **(page 2 lines 11-15)**, that are critical to understand grounding line dynamics. One of our key goals is to present a method that can track the grounding lines and coastlines of arbitrary geometries within a system of ice sheet, solid Earth and sea level models **(page 2 lines 16-19)**.

p. 2, l. 13: Similarly, what is meant by "traditional theory for ice-bedrock-ocean interface changes"?

We believe that our response to the previous comment fully addresses this question. See the revised paragraph (**page 2 lines 5-19**) in the manuscript.

p. 2, l. 25: "sea surface elevation". This is an ambiguous term; it can refer either to the mean (on some appropriate time scale) or to a quantity that varies on short time and spatial scales. There is some explanation below, but it is better to be as clear as possible when introducing the quantity $S(\omega, t)$. I think that what is meant here is what Gregory et al. (2019) call "mean sea level", a term they recommend in place of the deprecated term "mean sea surface". If S is actually meant to represent the geoid, which is not quite the same as mean sea level, then this should be stated clearly.

In general, Gregory et al. (2019) is a comprehensive, carefully written reference. I suggest that the authors adopt similar terminology, paraphrasing and referring to that paper as appropriate.

We agree that Gregory et al. (2019) is the key reference here. In the revised manuscript, we have complied with their definitions and made any difference explicit (**page 3 lines 22-29**). The key difference is that the mean sea level (MSL) in the context of our formalism does not account for the steric and dynamic components. Specifically, the global-mean value of MSL is given by the barystatic sea level, and its spatial pattern is dictated by

GRD (gravitational, rotational, deformational) response of the solid Earth to land-ocean mass exchange. This definition of MSL is familiar in GIA modeling (**page 3 lines 30-33**).

p. 2, l. 26: What is the International Terrestrial Reference Frame, and how is it defined? Is it similar to what Gregory et al. (2019) call the reference ellipsoid?

The International Terrestrial Reference Frame (ITRF) is the standard reference frame defined by the International Earth Rotation and Reference Systems Service (www.iers.org). It is being updated every few years. Yes, the ITRF solutions (e.g., GRS80) are reference ellipsoids as also noted by Gregory et al. (2019). We have decided not to specifically mention ITRF in the manuscript (whose description would divert the flow of the paper), because the theoretical formalism presented here is valid as long as both the bedrock and MSL are measured with respect to the same reference frame **(page 3 lines 20-21)**.

p. 2, l. 32: It is stated first that S is highly variable in space and time, and then it is stated that S is quasi-static and does not, in fact, include short-term dynamic processes. Please use S only to refer to quasi-static mean sea level, and use a different term when discussing short-term dynamics.

These sentences do appear in the revised manuscript. We have now provided a more precise definition of S (**page 3 lines 22-33**), which generally complies with the definition given by Gregory et al. (2019).

p. 3, l. 5: "Sea level" is another ambiguous term, as discussed by Gregory et al. I suggest "mean sea level."

Agreed. We now use mean sea level (MSL) **(page 3 line 20)**. See also the revised paragraph (**page 3 lines 22-33**) in the manuscript.

p. 3, l. 5: "represents an equipotential surface whose spatial pattern mimics the geoid." This is confusing. First, how is the geoid defined? Gregory et al. define it as the geopotential surface chosen so that the volume between the geoid and the sea floor is equal to the time-mean volume of sea water (including the liquid-water equivalent of floating ice) in the ocean. Second, what is meant by "mimics" the geoid? Does "mimics" mean "is equivalent to", or "is similar to"? If the latter, in what way does $S(\omega, t)$ differ? If S is mean sea level, then it is not an equipotential surface; for instance, mean sea level has a higher geopotential on one side of the Gulf Stream or ACC than the other. (Though it could be convenient to define S as an equipotential surface in areas not covered by ocean.)

The mean sea level (MSL), as defined in the revised manuscript, is familiar in GIA modeling, which requires to solve the so-called "sea level equation". This particular definition, which complies with general definition of MSL (Gregory et al., 2019), represents the equipotential surface and it differs from the geoid only by a spatial invariant for the sake of mass conservation in sea-level equation (Tamisiea, 2011). See **page 3 lines 30-33**.

p. 3, l. 11: Since ocean and ice have variable density, it would be clearer to refer to ρ_o and ρ_i as reference densities.

Agreed. See **page 4 line 7**.

p. 3, l. 12: "sea surface relative to the seafloor". I suggest "local mean sea level relative to the seafloor".

Agreed. This sentence has been moved to Section 4 **(page 12 line 8)**.

p. 4, Eq. (3): Why are the ocean and land functions undefined at coastlines and grounding lines? Is it problematic not to include them in one domain or the other?

The coastlines and grounding lines are neither part of the land nor of the ocean. They are the interfaces between the two. However, we understand the reviewer's concern given the lack of a precise description of these interfaces. We have included a mathematical description of a generic land-ocean boundary, including grounding lines **(page 4 line 16 – page 5 line 14)**. We have also given a specific example of how one can carry information about the domeain interfaces in numerical models **(page 5 lines 22-26)**.

p. 4, l. 7: Please say precisely what is meant by "connected to the open ocean". I would guess that the connected regions include marginal seas (e.g., the Mediterranean) but not inland lakes (e.g., the Great Lakes). Also, one needs an operational definition of the open ocean before defining a connection to the open ocean.

With new materials that present a mathematical description of level set function $\mathcal{F}(\omega, t)$ and its zero level set $\mathcal{T}(\mathcal{F})$ **(page 4 line 16 – page 5 line 14)**, we do not need to invoke the (relatively subjective) concepts of "connectivity" and "open ocean". The ocean function as defined by **equation (5)** is free of any ambiguity.

p. 4, l. 17: "is connected to" is more precise than "is in direct contact with"?

As noted in our response to the previous comment, the concept of "connectivity" has been discarded altogether by providing a mathematical description of the level-set function. Related phrases do not appear in the revised manuscript.

p. 4, l. 23: Are there published examples of frameworks (i.e., "traditional theory") that cannot handle pinning points? It seems natural to define the ice domain as in Eq. (4), and I'm not aware of frameworks with a different or less natural definition.

No doubt that Eq. 4 (now **equation 6**) is a simple and perhaps the most generic definition of ice domains (**page 6 line 5**). And, there is nothing new about it here, compared to the previous studies. To avoid a confusion, we have deleted the referred sentence. However, what we meant to imply by referring to the previous (aka traditional) studies is that they consider simplified geometries of land, ocean and ice sheets, perhaps due to the lack of constraining data or computational resources (**page 2 lines 5-9**). Consequently, these studies do not capture kilometer-scale features such as rugged fjord geometries and pinning points (**page 2 lines 11-15**) that are critical for accurate modeling of the grounding-line migration and hence ice-sheet dynamics. Also, see our response to your comment #p.2,l.8.

p. 4, l. 24: Which "employed assumption" is being referenced here? Maybe the assumption that only ice that is part of the ocean domain is included within the floating ice mask?

Correct. This has been clarified **(page 6 lines 7-8)**.

p. 4, l. 27: "the first generation of Earth system models". I'm not sure we are still in the first generation, since ESMs have been around for about a decade. Maybe "current Earth system models".

Agreed. See **page 6 line 10**.

p. 5, Eq. (6): Why is the grounded mask needed in this expression, if it is true that $H = H_0$ for floating ice shelves?

We define the flotation height for ice H_0 **(equation 7)** based on the height of the oceanic water rather than the ice thickness, and we interpret H_0 as the fraction of ice thickness that can potentially contribute to sea level by channging the mass of the oceanic water **(page 6 lines 24)**. Ice shelves must have smaller thickness than H_0, implying that the ice shelves can potentially contribute to the barystatic sea-level change, which obviously is not correct. Therefore we must invoke the grounded ice mask in **equation (8)**.

p. 5, l. 8: What is the referent in "it can be negative"?

It is H_F. Now clarified on **page 6 line 29**.

p. 5, l. 21: What is meant by "directly affects"? Is this just the (fairly trivial) statement that some, but not necessarily all, of the net change in ice mass results in a change in ocean mass?

Yes to your second question. We have rephrased the statement **(page 7 line 12)**, and the quoted phrase does not appear anymore.

p. 5, ll. 21-22: "Quantifying the fraction of ice mass change that contributes to sea level" I thought that this was the main point of the section. In what sense is it analytically unapproachable and beyond the scope of the study?

We meant to imply that not all of ice mass loss makes it to the ocean. A fraction of it, for example, may get impounded in proglacial lakes. However, the quoted statement is in direct conflict with the assumption that "the net change in grounded ice mass results in the equivalent change in ocean mass..." (**page 7 line 12**), so we have removed the referred sentence. The reviewer is correct that ΔH_S (well, in fact, its one component ΔH_M) provides the accurate estimate of ice mass loss that contributes to the barystatic sea level (see **page 7 equation 10 and the text that follows**).

p. 5, l. 23: "Despite the assumption"? Which assumption? If the reference is to the assumption that "the net change in ice sheet mass directly affects the ocean mass", I'm not sure there is a contradiction, but I'm not clear on the precise meaning of the assumption.

Yes, it was in reference to the quoted assumption. We agree that there is no contradiction either. We have deleted the quoted phrase and revised the sentence (**page 7 line 13**).

p. 5, l. 27: "...yields some error." After reading this statement, I was expecting to see quantitative error estimates later in the text. There is an illustration in Fig. 3, but is it possible to state the typical order of magnitude of the error? For instance, is it closer to

1% or 10%?

This is an excellent point. We have now added two new panels in the figure **(Figures 3d-e)** to show the time series of ice-volume change that is attributable to the sea-level change. As shown in **Figure 3e**, we find that the new method predicts much larger sea-level contribution (on the order of 10-15%) than the usual method (the height above flotation, HAF, method) throughout the model simulation. This has been discussed in **Section 3.3 (page 10 lines 24-31)** and highlighted in the Conclusion **(page 14 line 8)**.

p. 5, l. 28: Please say precisely what is meant by "contributes to sea level". For example, if an ice sheet loses mass, the geoid will change because of gravitational and rotational effects, but these changes aren't part of ΔH_S. What is meant, I think, is the part of the ice loss that adds to the mass of the ocean, i.e. the barystatic sea-level component. If so, then barystatic SLR and related terms should be defined here or earlier.

We agree with the reviewer's assessment. We have now explicitly stated that ΔH_S contributes to the sea-level change by modulating both the mass and volume of the oceanic water (**page 7 lines 27-28**). In fact, we have restructured the original equation to isolate "mass" and "volume" component of ΔH_S so that it can be easily interpreted in light of GMSL estimates or GRD computations (**page 7 line 30 – page 8 line 7**).

p. 5, l. 30: Since this is a central equation in the paper, I would like to see a clearer description of the physical meaning of each term, and when appropriate a derivation. I convinced myself that the first two terms on the RHS are correct, but I was not able to derive the third term or understand the physical motivation. In the text, the closest thing to an explanation is on p. 7, l. 20: "The last term in the equation accounts for the fact that fresh water density evolves during the accretion and ablation of ice, whereas the average ocean water density in the vicinity of the grounding line acts to determine the ablation height." This is confusing, in part because freshwater density ρ_w is a physical constant that does not evolve. Please provide a clearer explanation and, if possible, a supporting figure.

The last-term of equation (7) in the original manuscript was incorrect, which is now corrected and is given as a separate equation **(Equation 12)** because this term only modulates the ocean volume (not mass) and does not participate in GRD calculations. We have now added an **Appendix (page 15 line 3 – page 18 line 5)** wherein we have derived individual terms appearing in **Equations (11-12)** and provided their physical interpretation. We have separated three regimes of the ice domain **(page 9 lines 4-31)** to summarize such interpretation in the main text. The quoted sentence does not appear in the manuscript. See **page 9 lines 20-24 and 27-29** for the revised statements.

p. 6, Figure 2: The figure and caption are helpful, especially panels a and b with the four different regimes.

Thank you.

p. 7, ll. 3ff: The text refers to three distinct "regimes", whereas Fig. 2 refers to four regimes that are defined differently from the regimes in the text. Please use the term "regimes" consistently. In the text, paragraph 1 corresponds to the first term on the RHS of Eq. (7), and paragraph 2 to the second and third terms. But as stated above, the explanation of the third term is not clear. Perhaps revise so that paragraph 2 addresses the second term and paragraph 3 the third term. Then the current paragraph 3 would become a short paragraph 4.

Indeed, we also realize that the 4 regimes originally defined to interpret H_F (Figure 2a) may be confused with the 3 regimes defined to interpret ΔH_S (Figure 2b). To avoid this confusion, we have labelled the formers as Sectors A, B, C and D, and the latter as Regimes 1, 2, and 3 **(see Figure 2)**. The description of three regimes has been restructured, as suggested when possible **(page 9 lines 4-31)**. The explanation of third term appearing in equation 7 of the original manuscript **(now Equation 12)** has been considerably improved **(page 9 lines 20-24 and 27-29)**. Also see our response to your comment #p.5,l.30.

p. 7, l. 19: Could you give an example of when the magnitude of the change in H_F would be equal to the magnitude of the change in H, and when it would be less?

We have now provided a detailed interpretation of the ΔH-ΔH_F-ΔH_S relationship in the **Appendix**, in light of evolving ice thickness, bedrock elevation and mean sea level. As described in **Appendix A2 (page 16 line 8 – page 17 line 29)**, the inequality (not equality) must hold in the mentioned relation for a grounded ice colume to float, or the reverse, in the absence of the externally-forced bedrock and mean sea level change **(see Figure A1b)**. This has now been corrected **(page 9 line 20)**. We thank the reviewer for this comment.

p. 7, ll. 29ff: I am confused about the difference between events in regimes 1 and 2. My understanding is that regime 1 consists of regions that are grounded at both the start and end of the simulation, whereas regime 2 consists of regions that transition from grounded to floating during the simulation. If so, this should be stated clearly. For regime 1, it is stated that ΔH_S is different from ΔH_F because of "evolving bedrock and sea level". Are bedrock and sea level not evolving in regime 2? Or is the point rather that in region 1, the ice remains grounded throughout the simulation, and therefore the entire DeltaH contributes to ΔH_S, whereas ΔH differs from ΔH_F because of bedrock changes? For regime 2, it is stated that the discrepancy is due to the ?missing fraction of newly grounded or newly floating ice.? I am not sure what this means. I understand why ΔH_S differs from ΔH in this region, but not why ΔH_F differs from ΔH_S.

The reviewer is correct about how we separate three regimes in **Figures 3a-c**, which has now been mentioned explicitly in the manuscript **(page 10 lines 18-19; also see the figure caption)**. It is not that we consider the effects of evolving bedrock and mean sea level in one regime, and not in others. We first define ΔH_S by accounting for ΔH, ΔB and ΔS **(Equations 10-12)**. Only then we separate 3 regimes for the ease of interpretation. We agree that our interpretation of the results, particularly Figure 3c, is too brief and perhaps confusing, which has been significantly improved now **(page 10 lines 20-23)**. The reviewer's second interpretation about the results in Regime 1 is accurate **(page 10 lines 20-21)**. Mathematically, the difference between ΔH_S and ΔH_F in Regimes 2 and 3 is simply ΔH_V **(equation 12)**, which accounts for the volumetric contribution of ice-thickness change in excess of the change in HAF **(page10, lines 22-23)**.

p. 7, l. 30: Please say more precisely what is meant by the "customary approach of using ΔH_F". Can you cite specific examples in the literature in which the ice-sheet contribution to SLR was derived from ΔH_F, yielding a significant error? In the literature (beyond this specific example from Larour et al. (2019)), do the errors have a systematic sign? Are these errors prevalent in ice sheet models that include isostatic adjustment (i.e., where ΔH_S could have been computed accurately, but ΔH_F was reported instead)? Or is the problem that most ice sheet models ignore isostatic adjustment, so that they are missing a key term needed to compute ΔH_S?

By "customary approach" we meant to imply the methods that use the change in HAF to estimate the sea-level contribution (**page 7 line 17**). The phrase does not appear in the manuscript anymore (see **Section 3.3** for the revised text).

Regarding the second question: Most of the stand-alone ice sheet models (e.g., those contributing to SeaRISE and ISMIP6 projects) derive ice sheets' contribution to sea-level change from ΔH_F (**page 7 lines 16-17**). Unless we compare these estimates with those derived from better/improved methods (e.g., our methods or Goelzer et al., 2020), we cannot quantify/cite errors associated with these estimates. This is why the results presented in Figure 3 is useful. We have now included two additional panels in the figure **(Figures 3d-e)** that show the time series of Antarctic ice-volume change that is attributable to sea-level change. We find a significant difference between ours and the HAF method, which is on the order of 10-15% (see **Figure 3e**). This is now reported in the main text **(page 10 lines 24-31)** and highlighted in the Conclusion (**page 14 line 8**).

Regarding the third question: As noted above there are no error estimates available in the literature, so we cannot comment on whether they have a systematic sign. But, for the particular case consider in our study, we find that the HAF method systematically underpredicts the sea-level contribution (**page 10 line 25, also see the figure caption**).

Regarding the fourth question: It may be possible. Goelzer et al. (2020), for example, write: "In our own ice-sheet modelling experience and from exchange with colleagues in different groups, it is not always clear how the sea-level contribution should exactly be calculated and what corrections need to be applied."

Regarding the fifth question: The reviewer's speculation is absolutely correct. Most of the participating models of SeaRISE and ISMIP6 experiments are stand-alone ice models.

Also, can you state the magnitude of the systematic error? That is, what is the magnitude of the integrated error in Fig. 3 panel c, relative to the integrated value of ΔH_F?

Ok. This error, at least for the example case considered in our study, is on the order of 10-15%, now reported in the main text (**page 10 line 26, see also Figure 3e)** as well as in the Conclusion (**page 14 line 8**).

p. 8, Fig. 3: In addition to the 2D fields, it would be useful to show a graph consisting of time series of the area-integrated values of H_S and H_F. This graph could show not only the total values, but also the values computed separately for regions 1, 2, and 3. Also, please cite Larour et al. (2019) in the caption.

We appreciate the comment. We have now added two new panels as suggested by the reviewer **(Figures 3d-e)**. Instead of partitioning the results by the regime, we believe that it is more instructive to partition them by the "mass" and "volume" components as they have direct implications for interpreting results in terms of GMSL or GRD computations. It is interesting that the two methods differ from each other by about 5% consistently throughout the model simulation in terms of their mass contribution to the ocean, the component that drives the GRD response **(page 10 lines 27-29)**. We have now cited Larour et al. (2019) in the caption, as suggested.

p. 8, l. 6: Here, barystatic sea level change is finally defined. I suggest introducing and defining this concept earlier in the paper. Also "ocean mass-related" is a bit vague; I suggest phrasing similar to that of N19 in Gregory et al. (2019): e.g. "the part of global-mean sea-level rise which is due to the addition to the ocean of water mass that formerly resided within the land area as land water storage or land ice." Then ΔR^I, introduced below, would be the land-ice contribution, and ΔR^L would be the contribution from other land terms.

We have defined the barystatic sea level much earlier in the manuscript **(page 3 lines 27-28)**. The quoted phrase does not appear in the manuscript (**page 9 lines 31-32**). See **Section 4** for improved definitions of the barystatic components that contribute to RSL change **(page 12 lines 15-31)**.

p. 8, Eq. (8): The term on the LHS includes a subscript, a superscript, and an overbar, without immediately saying what these things mean. I suggest a more gradual and systematic introduction to the notation. Also, could you explain why the denominator contains ρ_w instead of ρ_o? At first, I assumed that the denominator represents the mass of the ocean, but I think the reason for ρ_w is that we are converting a mass of fresh ice into an equivalent ocean volume, ignoring halosteric effects. Again, a more detailed physical explanation would be helpful.

As we have restructured the materials, largely under **Section 4 (page 12 line 1 – page 13 line 21)**, the cited equation does not appear in the manuscript. We have rather explicitly stated in the text that the spatial integration of $-\rho_i/\rho_w \Delta H_S$ gives the total freshwater volume being added to the ocean **(page 9 line 33)**. Also see our response to your comment that follows.

p. 8, Eq. (9): This equation introduces several more terms without preamble, and the reader has to study the following paragraph carefully to translate each term. Please rewrite in a way that is gentler for the reader.

We appreciate the comment. To address this and similar comments, we have introduced a new section (**Section 4**) and restructured (and expanded) the original materials as follows.

1. We contextualize why we need to consider all contributors of relative sea-level (RSL) change in a mass conserving Earth System framework in order to compute ΔH_S and $\mathcal{T}$ **(page 12 lines 2-14)**;
2. We introduce non-steric components of RSL including the one induced by ΔH_S itself

over the period Δt **(page 12 lines 15-27)**, and interpret them in light of space observations and existing GRD models **(page 12 lines 27-31)**;

3. We dissect ΔH_S into two parts: the component that modulate both the mass and volume of the oceanic water and drives the GRD response of solid Earth, and the (smaller) component that only modulates the ocean volume **(page 12 line 32 – page 13 line 6)**;
4. We summarize how the spatial patterns of the "mass" component is computed, which necessitates conservation of mass in the Earth system **(page 13 lines 9-18)**.

We hope that the reader finds it more accessible.

p. 9, ll. 7-9. I am not clear on the meaning of the third and fourth ("past") terms, and how these terms change the ocean mass. I understand that past ice-sheet changes affect sea level through ongoing glacial isostatic adjustment, but isn't GIA included in term 6, the vertical-land-motion term?

The third and fourth terms are indeed related to GIA processes **(page 12 lines 30-31)**, that capture the ongoing viscous response of the solid Earth to the ice-ocean mass exchange since the Last Glacial Maximum. They modulate not only the present-day bedrock elevation but also the geoid field, hence their imprints are imbedded both in ΔB and ΔS. The GIA processes are part of what we call the "barystatic components" **(page 12 line 19)**. The last term ΔR_V **(now ΔR_O)** is reserved for other non-barystatic processes that at least modulate the ocean bathymetry or coastal geometry (e.g., earthquakes, landslides, etc.), now clarified on **page 12 lines 25-27**.

A more general comment: It could be easier for the reader if the text were organized from general to specific, instead of specific to general. That is, first define the various kinds of sea level rise, introduce notation, and state the various source terms. Then state that this paper is focused on R_C^I, as computed in equation (8) based on ΔH_S. Finally, show how to compute ΔH_S. This would be a fairly major rewrite, and I don't want to be too prescriptive, but it is challenging for readers to introduce basic concepts just a page or two before the conclusions.

We really appreciate this comment. We have restructured and expanded the content of the original materials, now under **Section 4 (page 12 line1 – page 13 line 21)**. See our response to your commennt #p.8,Eq.9 for an overview of Section 4.

p. 9, l. 16: When ΔM is described as a mass-conserving field, is this equivalent to saying that its global integral is zero? Also, is it strictly true that $\rho_i \Delta H_S$ is equal to the change in ice mass at each location? Here, I'm wondering about the third term in Eq. (7); is that term associated with a change in the local mass per unit area?

Yes to the first question. See **page 13 line 14**. The change in $\rho_i \Delta H_M$ **(Equation 14)** is equal to the change in ice mass per unit area that contributes to the change in ocean mass. The correct version of the third term of Eq. (7) in the original manuscript now appears in **(Equation 12)**, whose units are ice-equivalent height, not mass per unit area. See **page 9 lines 20-23 and lines 27-29** for its physical interpretation.

p. 9, Conclusions: As mentioned above, it would be helpful to quantify the benefits of the new methods, e.g. by estimating the errors associated with the older methods.

Agreed. The level of improvements are quite large (about 10-15%, see **Figure 3e**), now highlighted in the Conclusion **(page 14 line 8)**. We appreciate the comment.

Minor corrections

p. 1, l. 6: "and include the ice shelves and adjacent ocean mass." The phrasing is awkward. Please use a parallel grammatical construction.

We have completely rewritten the second half of the Abstract, highlighting the two main goals of the paper **(page 1 lines 5-8)** and their utility and implications **(page 1 lines 8-10)**. The referred sentence does not appear in the revised manuscript.

p.1, l. 7: "is" → "can be"?
p. 1, l. 15: "grounding line" → "grounding lines"
p. 1, l. 17: Delete "involved"
p. 2, l. 2: "first order" → "first-order"
p. 2, l. 28: Delete "for"
p. 3, l. 5: "refer" → "refer to"
p. 4, l. 30: "ice sheet driven" → "ice-sheet-driven"
p. 4, l. 32: "far field" → "far-field"
p. 5, l. 2: Add "the" after "estimate"
p. 5, l. 20: "farfield" → "far-field"
p. 5, l. 26: Add a comma after "below"
p. 6, l. 7: "predict" → "predicts"
p. 9, l. 15: Insert "the" before "ice sheet"
p. 10, l. 6: "analyses" → "analysis"

We have implemented all of these changes.

Referee #3

The paper by Adhikari et al. presents a formalism to calculate the contribution of dynamic ice sheets to mean sea level by considering a changing sea-level, bedrock and land-ice-ocean mask. Compared to standard methods the contribution to mean sea level is computed from ice thickness and not from the height above flotation. The formalism is valid on all timescales, but the full benefit is on longer timescales. As there are currently huge efforts to include dynamic ice sheets in Earth system models the presented paper is a good reference.

Generally, I found the paper well written and structured with illustrative and clear figures. The paper is worth publishing in The Cryosphere but as the focus is rather technical it would fit much better to GMD. The current form of the manuscript lacks a bit in the presentation and clarity. Therefore, I have a few suggestions that should be addressed in a revised version.

We appreciate the positive and constructive review by the referee. Find our response to individual comments below. The changes made in the revised manuscript are pointed by the **bold text**.

1) I found the title a bit misleading to the content of the paper. I am missing the definition of "mass conservation" in the text. I also would like to see (e.g. with an example), if the formalism is mass-conserving (or better mass-conserving than traditional or other methods). Also, the only example in the manuscript was about calculating SLE relevant thickness changes rather than mass-conservation.

Although "mass conservation" is still an important element of the paper (see, for example, **page 1 line 10**), we also think that the original title was a bit too vague. We have now suitably modified the title, **A kinematic formalism for tracking ice-ocean mass exchange on the Earth's surface and estimating sea-level change**, so that it reflects the key goals of the paper **(page 1 lines 5-8)**. As a compensation, we have now introduced **Section 4** wherein we place the presented formalism in a broader context of sea-level change and mass conservation in the Earth System. We have now defined what mass conservation means in the context of our formalism **(page 2 lines 32-35)** and also provided example cases where mass conservation is violated **(page 13 lines 18-21)**. In terms of results, there is nothing really to show to justify mass conservation except to state that **Equation (14)** takes ΔH_M from equation (11) and ΔR_C^I is computed using a global GRD (gravitational, rotational, deformational) solid Earth model, such that the global integral of ΔM equals zero **(page 13 lines 9-18)**.

2) The Introduction should refer to Goelzer et al. (2020). You do very quick comparisons to Goelzer et al. (2020) on page7,line6 and 23, but I think the Introduction should clearly say what you are doing differently and why. This could be a motivation to release your new formalism. An appropriate discussion to Goelzer et al. (2020) is also missing. Additionally, from the Introduction it was not really clear to me what is actually wrong with the traditional methods, the order of error on SLE they could introduce and what you are now aiming to improve.

In the Introduction, we have briefly discussed the common method used in estimating sea-level contribution from ice sheets and its key limitations and summarized the effort by Goelzer et al. (2020) to provide appropriate corrections **(page 2 lines 24-27)** . We then contrast the goal of our paper **(page 2 lines 28-35)**. A detailed discussion about the common method, which is based on the concept of ice-height above flotation (HAF), is given on **page 7 lines 16-25**. Regarding ours versus the Goelzer method, kindly see our response to General Comments by Referee #1 on pages 2-3 of this document. As for the knowledge about errors associated with the existing methods, kindly see our response to Referee #2's Summary on page 15 of this document. For the example case that we have considered in this study **(see Figure 3e)**, however, we find that our method predicts much larger sea-level contribution compared to the common HAF-based approach (by about 10-15%). This has now been discussed in the main text **(page 10 lines 24-31)**, and highlighted in the Conclusion as well **(page 14 line 8)**.

3) Not sure if this could be really addressed, but it would be interesting to estimate the errors (i.e. traditional versus your formalism) of current projections of SLE from the two big ice sheets e.g. within the ISMIP6 framework (Antarctica: Seroussi et al., 2020; Greenland: Goelzer et al., 2020b). Or from current remote sensing products like IMBIE (Shepherd et al., 2019). My point here is, that I would get a better feeling for the error on e.g. different timescales, regional settings and how it differs for Greenland and Antarctica. The example based on the Larour et al. (2019) simulation is very helpful (see also my comment to P7,l29ff) but very specific – and, as I understood – not in line with current projections efforts. I do not strictly insist that you show an error for ISMIP6 or IMBIE, but as also commented below, I would like to have a better error estimate and its impact on current research (compare Fig. 3 in Goelzer et al. (2020)).

We appreciate the comment. This sounds like a great idea, but due to the lack of enough information in these datasets and model results, i.e. ΔB and ΔS, we really cannot evaluate ΔH_S **(equation 10)**. Computing ΔB and ΔS is beyond the scope of this theoretical paper. That said, based on Larour et al. (2019), we have shown the results to highlight the improvement that our method makes over the more common HAF method (Figure 3). We have now included two additional panels **(Figures 3d-e)** wherein we compare the two methods in terms of estimated Antarctic ice-volume change that is attributable to the global-mean sea-level (GMSL) change. The figure **(panel e)** also shows the level of improvements possible by employing the new method over the HAF method, which is quite large: on the order of 10-15%.

Minor comments:

P1,l8: I have not found in the text, which computational strategies you have simplified. What do you mean with computational strategies?

For improved clarity and focus, we have completely rewritten the latter half of the Abstract (**page 1 lines 5-10**). The cited sentence does not appear anymore.

P2,l11: Why are "floating ice shelves, ice rises and rumples, and retrograde bedrock slopes" complex features?

We have slightly modified the phrasing "...kilometer-scale geometric features, such as ice rises and rumples, rugged fjord geometries..." **(page 2 line 12)**. We do not call these features complex in the revised text. What we really meant to imply in this sentence, however, is that these kilometer-scale geometric features are not generally captured in the previous studies of evolving ice-bed-ocean interfaces **(page 2 lines 13-15)**, although they are absolutely essential to understand the dynamics of marine-based ice sheet.

P2,l13: What is "traditional theory for ice-bedrock-ocean interface changes"?

This question is fully addressed in our response to comment #p.2,l.8 on page 17 of this document.

P2,l18: "... that can be straightforwardly employed in any Earth System model"?. I think it would be worth to mention (somewhere), if this new formalism could be adopted to other disciplines (e.q. remote sensing, standalone ice sheet modelling). In the current form, it sounds the formalism in only valid/applicable in ESMs.

By design, the proposed formulation is very generic and we do not see why it cannot be applied for both (stand-alone) modeling and observational data (**page 2 lines 18-19**). For example, definitions of land, ocean, and ice domains, as well as their interfaces are valid in both realms. However, due to lack of modeled/observed data for some fields (e.g., ΔB and ΔS) some aspects of the formalism (e.g., equation 10) are best suited for ESMs **(page 1 lines 9-10, page 2 lines 29-35)**.

P4,l23: I cannot see from Eq. 4 that your new setup diverges from traditional approaches. Eq. 4 is a very common equation to define an ice-mask. On page5,line 25 you give another example of how the traditional setup differs from your setup. Maybe outlining the differences could be gathered together.

Agreed that Eq. 4 is a simple and perhaps the most generic mathematical description of ice domains **(page 6 line 5)**, and there is nothing special about it here. To avoid any confusion, we have simply deleted the referred sentence. Regarding "traditional" versus our setup: kindly see our response to comments #p.2,l.8 on page 17 and #p.4,l.23 on page 19 of this document.

P5,l8: I don't understand this sentence. What is negative?

H_F is negative. It is clarified now **(page 6 line 29)**.

P5,l9: "...hence contribute to sea level inversely." Maybe say sea-level drop/fall to avoid confusion.

Agreed. See **page 6 line 30**

P5,l10: I am not a native speaker, but are both "evolving" really needed?

We have rephrased the sentence **(page 7 line 1)**.

P5,l25: Can you add a reference to a Figure after "show"?

We believe that line 26 is being refereed here, not 25. Considering this and Referee #1's comment (#p5.26 on page 10 of this document), we have decide to remove the phrase containing the quoted word. See **page 7 lines 24-25** for the revised sentence.

P7,l4: "...and the elevations"?

Should be "at the elevations" as is in the manuscript. But, we understand the reviewer's confusion. We have suitably revised the sentence **(page 9 lines 5-6)**.

P7,l29ff: I would first describe the Larour et al. (2019) setup and then present the results. Can you give an integrated value (e.g. SLE) for both approaches to get a better feeling for the error? The simulations were run over 500 years. Why do you choose to present the results after 350 years?

We appreciate the comment. We have expanded and restructured the paragraph as advised, under the new Subsection 3.3 **(page 10 lines 5-31)**. There is no particular reason as to why we chose 350 years, but we keep it as is. However, we now include time series of the total Antarctic ice-volume change over 500 years that is attributable to the sea-level change **(Figures 3d-e)**. The **panel e** shows that the difference between ours and the more common HAF method is significant: on the order of 10-15%.

P9,l6ff: The following paragraphs and Equations appeared very suddenly and without introducing their purpose. According to the title, I would expect Eq. 10 (the mass conserving field M) is the main point in your paper. But this is not illustrated and somehow contradicts with your statement in the conclusion (p9,l31-32); here you say ΔH_s is the main point. This is perhaps personal matter, but I found it a bit brutal to stop the results of the paper with these equations. An illustrative example on "implication of this new geometrical setup for sea level and solid Earth loading studies" (your comment on p2,l20) would make more sense to me.

We have restructured the materials under the new **Section 4**, where we

- contextualize why we need to consider all contributors of relative sea-level (RSL) change in a mass conserving Earth System framework in order to compute ΔH_S and $\mathcal{T}$ **(page 12 lines 2-14)**;
- introduce non-steric components of RSL including the one induced by ΔH_S itself over the period Δt **(page 12 lines 15-27)**, and interpret them in light of space observations and existing GRD models **(page 12 lines 27-31)**;
- dissect ΔH_S into two parts: the component that modulate both the mass and volume of the oceanic water and drives the GRD response of solid Earth, and the (smaller) component that only modulates the ocean volume **(page 12 line 32 – page 13 line 6)**;
- summarize how the spatial patterns of the "mass" component is computed, which necessitates conservation of mass in the Earth system **(page 13 lines 9-18)**.

As noted earlier, we have also changed the title suitably. And, we believe that the newly added two panels in Figure 3 (**Figures 3d-e**) further strengthen the utility of the proposed method. We have also revised the Conclusion to even out the importance of the generic level-set for coastlines and grounding lines and the new method for estimating ice sheets' contribution to the sea-level change.

Fig.3: What is the grey line? And in the caption: Is “conventional”==“traditional”? I guess yes. Please use the same wording in the whole text. Eq. 3 and 4: consider rewriting with “latex-cases”.

The gray (now black) line shows the present-day ice-ocean interface, now mentioned in **the figure caption**. Yes, the quoted words are equivalent. We have accommodated suggested changes when possible.

A [..*] kinematic formalism for [..†] tracking ice-ocean mass exchange on the Earth's surface and [..‡] estimating sea-level change

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Abstract. Polar ice sheets are important components of [..4] the Earth System. As the [..5] geometries of land, ocean, and ice [..6] sheets evolve, they must be consistently [..7] captured within the lexicon of geodesy. Understanding the interplay between the processes such as [..8] ice-sheet dynamics, solid-Earth deformation, and [..9] sea-level adjustment requires both geodetically consistent and mass conserving descriptions of evolving land and ocean domains, grounded ice sheets and floating ice [..10] shelves, and their respective interfaces. Here we present [..11] mathematical descriptions of a generic level set that can be used to track both the grounding lines and coastlines, in light of ice-ocean mass exchange and complex feedbacks from the solid Earth and [..12] sea level. We next present a unified method to accurately compute the sea-level contribution of evolving ice sheets based on the change in ice thickness, bedrock elevation and mean sea level caused by any geophysical processes. Our formalism can be applied to arbitrary geometries and at all time scales. While it can be used for applications with modeling, observations and the combination of two, it is best suited for Earth System models, comprising ice sheets, solid Earth and sea level, that seek to conserve mass[..13].

*removed: mass conserving

†removed: ice sheet, solid

‡removed: sea level interaction

[4]removed: any Earth Systemmodel

[5]removed: domains

[6]removed: sheet change

[7]removed: defined

[8]removed: ice sheet dynamics, solid Earth

[9]removed: sea level

[10]removed: masks, coastlines and grounding lines, and bedrock and geoid height as viewed from space

[11]removed: a geometric description of an evolving ice sheet margin and its relations to sea level change, the position and loading of

[12]removed: include the ice shelves and adjacent ocean mass. We generalize the formulation so that it is applied to arbitrarily distributed ice , bedrock and adjacent ocean, and their interactive evolution. The formalism simplifies computational strategies

[13]removed: in Earth System models.

1 Introduction

Recently there has been intense interest in defining the physics involved in determining multidecadal change in the location and the migration rate of the grounding line, a boundary separating a grounded ice sheet from its floating extension, usually a floating ice shelf (e.g., *Nowicki and Wingham*, 2008; *Schoof*, 2012; *Sergienko and Wingham*, 2019). Indeed, how well a numerical [..[14]]model of marine ice sheets predicts the sea-level contribution largely depends on its ability to capture the subtle migration of grounding [..[15]]lines. The non-equilibrium thermodynamics, fluid dynamics, plastic failure criteria, and conditions governing nonlinear stability of ice sheets are, quite generally, up for lively debate. In order to better tackle the difficult nonlinear physics [..[16]]and to better address the associated numerical challenges (e.g., *Schoof*, 2007; *Durand et al.*, 2009; *Sayag and Grae Worster*, 2013; *Favier et al.*, 2016; *Seroussi and Morlighem*, 2018) as well as to define proper observational criteria for locating the grounding lines and their migrations (e.g., *Hogg et al.*, 2018; *Milillo et al.*, 2019), it is important to agree on some of the baseline variables and boundary conditions. Direct interactions with the ocean [..[17]](e.g., *Seroussi et al.*, 2017; *Nakayama et al.*, 2018) and the solid Earth (e.g., *Gomez et al.*, 2010; *Larour et al.*, 2019) are now seen as critical elements that must be incorporated into projections[..[18]], or retrospective paleoclimate simulations, of the rate of grounding line retreat in a warming climate (e.g., *Jones et al.*, 2015; *Whitehouse et al.*, 2017). Given the computational complexity of this problem, however, it is essential to properly define the simple geometrical parameters, primarily moving boundaries at the [..[19]]ice/bedrock/ocean interfaces, for there to be rationally organized intercomparison among various research teams and their results. [..[20]]

[..[21]]A general description of mechanical analysis of [..[22]]ice-sheet evolution at the [..[23]]ice/bedrock/ocean interfaces has been [..[24]]given for a set of simplified geometries (for example in Chapter 3 of *Hutter*, 1983), owing to the lack of constraining data or computational resources. A similar geometric approach is also familiar in the development of glacial isostatic adjustment (GIA) theory for ice sheets, sea level and bedrock evolution following the Last Glacial Maximum [..[25]]with migrating grounding lines and coastlines (e.g., *Milne*, 1998; *Lambeck et al.*, 2003; *Mitrovica and Milne*, 2003). Modern satellite techniques have allowed us to gain knowledge of both the [..[26]]present locations and migration rates of the

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[14]removed: ice sheet model projects its future sea level

[15]removed: line

[16]removed: involved

[17]removed: (e.g., *Nakayama et al.*, 2018) and

[18]removed: based on physical models

[19]removed: ice-bedrock-ocean

[20]removed: Defining geometry is a first order step for proper construction of models for tracking grounding line migration (e.g., **?**) and in this paper we propose a simple set of definitions that the field may find useful for future debate and reconciliation.

[21]removed: Much of the basic configuration setup for

[22]removed: ice sheet

[23]removed: ice-bedrock-ocean

[24]removed: treated with great rigor, for example, in Chapter 3 of *Hutter* (1983). Similar setups are

[25]removed: (LGM) with migrating coast lines (e.g., *Milne*, 1998; *Lambeck et al.*, 2003; *Mitrovica and Milne*, 2003). We refer to these as traditional configurations.

[26]removed: present-day

grounding line (e.g., *Rignot et al.*, 2011; *Milillo et al.*, 2019). However, both [..[27]]observations and numerical simulations of subtle change in grounding line positions are complicated by the presence of [..[28]]kilometer-scale geometric features, such as ice rises and rumples, [..[29]]rugged fjord geometries, and uneven bedrock topography. These features complicate the required geometrical simplifications used in [..[30]]the previous studies of ice/bedrock/ocean interface changes, especially [..[31]]when the system of ice sheets, solid Earth and sea level is fully interactive (e.g., *Lingle and Clark*, 1985; *Gomez et al.*, 2013; *de Boer et al.*, 2014; *Konrad et al.*, 2015; *Larour et al.*, 2019). Here we consider a simple level-set method, which has been previously applied for tracking grounding lines (e.g., *Seroussi et al.*, 2014) and calving-front positions (e.g., *Bondzio et al.*, 2017), and generalize it to facilitate a precise tracking of both the grounding lines and coastlines of arbitrary geometries in a seamless manner. The method is very generic and can be used for applications based on modeling, observations, or combination of models and observations.

Evolving bedrock and sea level impact the ice-sheet dynamics via the modulation of bedrock slope, grounding-line positions, and gravitational driving stress. For the marine portions of the ice sheet having retrograde bedrock slopes, this effect promotes the stability, as has been demonstrated by both the observation-based (e.g., *Barletta et al.*, 2018; *Kingslake et al.*, 2018) and the model-based studies (e.g., *Lingle and Clark*, 1985; *Gomez et al.*, 2010, 2015; *Adhikari et al.*, 2014; *Konrad et al.*, 2015; *Larour et al.*, 2019). The inclusion of evolving bedrock and sea level in a dynamical ice-sheet model, however, requires a modification to the common method of estimating sea-level contribution. The method, based on the concept of ice-height above floatation (e.g., *Bindschadler et al.*, 2013), yields inaccurate results for the marine portions of the ice sheet. *Goelzer et al.* (2020) recently provide appropriate corrections for the effects of bedrock elevation change and externally-forced sea level. Our goal here is to formulate a unified method to calculate the exact fraction of ice thickness that contributes to the sea-level change over a given period by considering evolving bedrock and sea level driven by any geophysical processes. In conjunction with observational data, the method can be applied to a variety of models such as stand-alone ice-sheet models and those that account for isostatic bedrock adjustment (e.g., *Le Meur and Huybrechts*, 1996; *Pattyn*, 2017) or a self-consistent GRD (gravitational, rotational, deformational) response of the solid Earth (e.g., *Gomez et al.*, 2013; *Larour et al.*, 2019). In the latter set of models, the presented formalism ensures mass conservation in the Earth System by exchanging mass between the land and the ocean, accounting for the induced GRD response of solid Earth, and adjusting the ocean area through migration of grounding lines and coastlines, simultaneously.

In the following, we begin by presenting a generalized [..[32]]description of land, ocean and ice [..[33]]domains and their respective interfaces (Section 2). We consider a global ocean, composed of an interconnected system of oceanic basins,

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[27]removed: observation and numerical simulation

[28]removed: complex geometric featuresincluding floating ice shelves,

[29]removed: and retrograde bedrock slopes.

[30]removed: traditional theory for ice-bedrock-ocean

[31]removed: if the interactions are two-way (e.g., *Lingle and Clark*, 1985; *Gomez et al.*, 2013; *de Boer et al.*, 2014). For example, estimating sea level contribution of an evolving ice sheetin such cases becomes non-trivial (**?**).

[32]removed: definition

[33]removed: sheet domains and that of coastline and grounding line positions

and distributed system of ice domains[..[34]], comprising glaciers, ice sheets and ice shelves, that can be straightforwardly employed in any Earth System model in order to track the global mass transport and assess the evolution of a dynamic system of ice sheets, solid Earth and sea level. [..[35]]In Section 3[..[36]], we briefly review the common method of estimating sea-level contribution of ice sheets and present a new method, wherein we isolate mass and volume contributions to the ocean, which is critical to accurately drive the GRD response of the solid Earth. In Section 4, we assess our formalism in a broader context of sea-level change and mass conservation in the Earth System. Finally, in Section [..[37]]5, we summarize the key conclusions[..[38]].

2 Land, ocean and ice [..[39]]domains and their [..[40]]interfaces

To begin our discussion[..[41]], we consider a spherical planet whose surface is divided into complementary domains of land and ocean. The ocean may be thought of as an interconnected system of oceanic basins – just like Earth's ocean that also includes fjords and marginal seas such as Mediterranean – that are able to freely exchange and redistribute mass between them. This assumption simplifies what would otherwise be an arduous task for mass attribution and conservation in the Earth System. Distributed ice domains including glaciers[..[42]], ice sheets and ice shelves exist on the land or the ocean (Figure 1). We [..[43]]generally consider ice domains as part of the land, except where they float on the oceanic water as ice shelves. In order to present mathematical descriptions of these domains and their interfaces at time t, we denote 2-D spatial coordinates on the planetary surface [..[44]]by ω. Depending upon the spatial scale [..[45]](e.g., the ocean versus glaciers), we interchangeably use ω to represent [..[46]]geographic coordinates (θ, ϕ) or Cartesian [..[47]](x, y), assuming that an appropriate coordinate transformation is applied. [..[48]]

[34] removed: comprised of glaciersand ice sheets of arbitrary geometric configurations

[35] removed: We then explore the implication of this new geometrical setup for sea level and solid Earth loading studies (Section

[36] removed:)

[37] removed: 4

[38] removed: of the study

[39] removed: sheet

[40] removed: boundaries

[41] removed: of a proposed geometrical setup

[42] removed: and ice sheets exist on both land and

[43] removed: define $B(\omega,t)$ and $S(\omega,t)$ to be the land/bedrock/seafloor and sea surface elevations, respectively, measured relative to the same datum, preferably consistent with the International Terrestrial Reference Frame (e.g., *Altamimi et al.*, 2016). Here ω denotes

[44] removed: at time t

[45] removed: ,

[46] removed: for

[47] removed: coordinates

[48] removed: We define $H(\omega,t)$ as the thickness of ice bounded between its surface and the base.When the base of ice is in contact with the underlying bedrock , ice is grounded. Ice may float on subglacial or proglacial lakes or on the ocean, with its base above the bedrock. Our focus here is on marine portions of the ice sheet, with $B(\omega,t) < S(\omega,t)$.

[..[49]]The entire formalism presented in this study can be derived from three field variables: the solid Earth surface (i.e., land surface or sea floor) or simply bedrock $B(\omega,t)$, mean sea level (MSL) $S(\omega,t)$[..[50]], and ice thickness $H(\omega,t)$. The first two fields must be defined relative to the same reference ellipsoid (e.g., *Altamimi et al.*, 2016).

Our definition of MSL complies with that given by *Gregory et al.* (2019): Time-mean of sea surface over a sufficiently long period so that the effects of waves, tides, or meteorologically-driven high-frequency fluctuations are eliminated. The period of time-mean may be on the order of 20 years or longer, [..[51]]a timescale over which interactions between sea level and ice sheet may become important (e.g., *Hillenbrand et al.*, 2017; *Larour et al.*, 2019). [..[52]]One key difference, however, is that the change in MSL in the present context does not account for the steric component that is due to the change in the ocean density. Here, in the strict sense of the word, the change in global-mean of MSL is given by the so-called barystatic sea-level change, which is the global-mean sea-level (GMSL) change due to the exchange of water between the land [..[53]]and the ocean[..[54]], and the evolving spatial pattern of MSL is dictated by the GRD response of the solid Earth to land-ocean (water or sediment) mass exchange and the tectonic activities. This definition of evolving MSL is familiar in GIA modeling wherein there is a requirement to solve for a gravitationally self-consistent solution of evolving bedrock and (non-steric) MSL driven by the ice-ocean mass exchange following the Last Glacial Maximum (*Farrell and Clark*, 1976; *Milne and Mitrovica*, 1998). The MSL as defined above represents an equipotential surface whose spatial pattern [..[55]]matches the geoid (*Tamisiea*, 2011).

[..[58]]

2.1 Coastlines and grounding lines as a seamless interface

We develop our formalism based on the principle of hydrostatic equilibrium for a system of ice and ocean. Since $H(\omega,t)$ may be considered as a globally-defined field, with $H(\omega,t)=0$ outside the ice domains, this concept can be generalized to deduce a criterion for delineating boundaries between the land and the ocean, and the floating and the grounded ice.

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[49] removed: It is important to note that

[50] removed: may be highly variable both in space and time due to relatively short-term dynamic processes such as tides, wind stress, atmospheric pressure variability, and associated ocean circulation. At timescales

[51] removed: changing

[52] removed: Hence, any long-term change in $S(\omega,t)$ caused, for example, by a sustained water and heat exchange

[53] removed: ice

[54] removed: is of central interest to our formulation. Here, $S(\omega,t)$ strictly refers to this quasi-static component of sea surface that is free from ocean dynamic signal and high-frequency signals of waves or meteorologically driven fluctuations. We hereafter refer $S(\omega,t)$ to as "sea level" for brevity. Sea level as

[55] removed: mimics the geoid (*Tamisiea*, 2011; *Gregory et al.*, 2019). Under this definition, we may include areas that are non-oceanic, including the interior of marine ice sheets.

[58] removed: Figure 2a shows a cross-sectional view of land, ocean, and the grounded and floating portions of icesheet domains. We introduce a function $F(\omega,t)$ that is used to definethese domains in terms of the principles of hydrostatic equilibrium

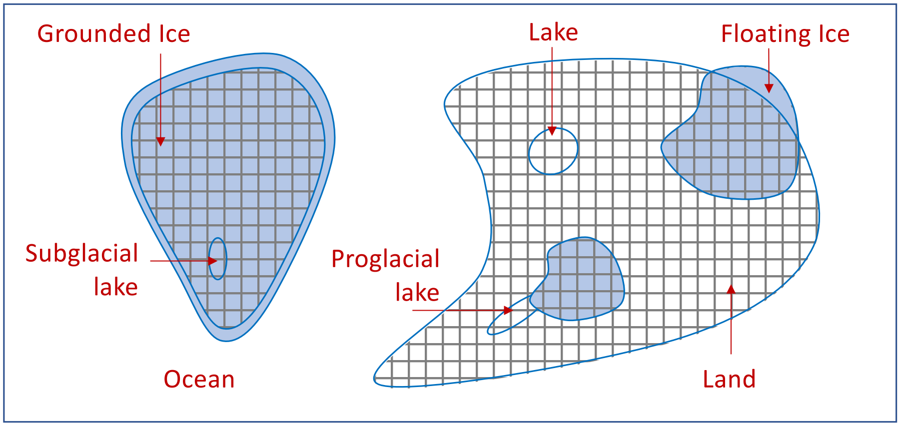

Figure 1. Conceptual depiction of land, ocean and ice domains in the Earth System. Gridded areas represent land and the rest ocean. Lakes are considered as part of the land. Ice can have multiple domains, shown here with blue sheds. The land-ocean boundary is generally defined as the coastline, which is called grounding line when it is part of the ice domain. Because our focus is on grounding line migration in marine portions of [..[56]]an ice-sheet/shelf system, we assume that all of ice on land (gridded portions of blue sheds) is grounded. Consequently, [..[57]]floatation of ice on subglacial and proglacial environments are not considered in this study.

We define: [..[59]]

$$\mathsf{F}(\omega,\mathsf{t}) = \mathsf{H}(\omega,\mathsf{t}) - \frac{\rho_\mathsf{o}}{\rho_\mathsf{i}}\Big[\mathsf{S}(\omega,\mathsf{t}) - \mathsf{B}(\omega,\mathsf{t})\Big], \tag{1}$$

such that $F(\omega,t) = 0$ satisfies the hydrostatic equilibrium between ice and the oceanic water in the marine sectors where $B(\omega,t) < S(\omega,t)$. Here ρ_i [..[60]]and ρ_o are the average densities of ice and ocean water, respectively. [..[61]]Our goal here is to use equation (1) as a basis for defining the land-ocean boundaries consistently. The equation, at first glance, suggests that the ocean (land) takes negative (positive) values of $F(\omega,t)$ and their interfaces have zero values. However, a few aspects should be further clarified. To simplify a mathematical description of the land-ocean boundaries, we ensure the absence of marine ice cliffs that have larger thickness than the floatation height (i.e., negative of the second term on the right-side of the equation) by assuming that $F(\omega,t) \le 0$ at the "[..[62]]ice front"[..[63]]. The ice front that satisfies the equality (inequality) here represents the calving face of a tidewater glacier (an ice shelf). Along the same lines, we

[59] removed:

$$F(\omega,t) = H(\omega,t) + \frac{\rho_o}{\rho_i}\Big[B(\omega,t) - S(\omega,t)\Big],$$

where ρ_o and

[60] removed: are the densities of ocean water and ice

[61] removed: Note that , by definition, $R(\omega,t) = S(\omega,t) - B(\omega,t)$ where $R(\omega,t)$ is sea surface relative to the seafloor, termed the

[62] removed: relative sea level

[63] removed: (*Gregory et al.*, 2019). By design, the position with $F(\omega,t) = 0$ represents a boundary between the land and the ocean, provided that ω is in direct contact with the open ocean .

assume that the terrestrial ice cliffs are not present where $B(\omega,t) = S(\omega,t)$. These assumptions are generally valid, as the flow is diffusive on the timescale (decades or longer) we are interested in.

We now define a level set of the function $F(\omega,t)$ such that:

$$\mathrm{T(F)} = \{(\omega,\mathrm{t}) \mid \mathrm{F}(\omega,\mathrm{t}) = 0\}. \tag{2}$$

This zero-level set may consist of several simple curves, $T_i(F)$, that divide the planetary surface into several non-overlapping regions, $\Omega_i(F)$. Let $\Omega_i^-(F)$ denote the regions in which the function $F(\omega,t)$ takes negative values, and are therefore the candidates of the ocean domain. Since we consider the ocean to be an interconnected water volume, termed the global ocean, as in traditional physical oceanography and sea level studies, only the largest amongst $\Omega_i^-(F)$ forms the ocean domain. Smaller $\Omega_i^-(F)$, if there are any, and their boundaries $T_i(F)$ are considered to be part of the land, meaning they are unable to freely exchange mass with the global ocean by GRD processes. One obvious example of the region that does not belong to the ocean in spite of having $F(\omega,t) < 0$ is a continental trough with bathymetry below MSL. Unless this trough is physically connected to the global ocean via oceanic water, we consider this to be part of the land rather than the ocean. Let $\Omega_S^-(F)$ be the union of all these small non-oceanic regions and $T_S(F)$ be the union of corresponding boundaries. We modify $F(\omega,t)$ to define a new function

$$\begin{aligned} \mathcal{F}(\omega,t) &= |F(\omega,t)| + \epsilon \quad \text{if } \omega \in \Omega_S^-(F) \text{ and } \omega \in T_S(F) \\ &= F(\omega,t) \qquad\qquad \text{otherwise,} \end{aligned} \tag{3}$$

so that its zero-level set

$$\mathcal{T}(\mathcal{F}) = \{(\omega,\mathrm{t}) \mid \mathcal{F}(\omega,\mathrm{t}) = 0\} \tag{4}$$

represents the land-ocean [..[64]]boundaries. Here ϵ is a positive number to ensure $\mathcal{F}(\omega,t) > 0$ at $\omega \in T_S(F)$.

The land-ocean boundaries are generally known as [..[65]]coastlines. Given the definition of the level-set function (equation 3), coastlines are free of ice where [..[66]]$B(\omega,t) = S(\omega,t)$. No coastline exists with [..[67]]$B(\omega,t) > S(\omega,t)$. Only in the marine sectors where $B(\omega,t) < S(\omega,t)$, does a coastline have finite ice thickness and then is replaced by the term grounding line.

[64] removed: boundary is

[65] removed: a coastline. It follows from equation (1)that ice thickness at the coastline, $H_C(\omega,t)$, is given by:

$$H_C(\omega,t) = -\frac{\rho_o}{\rho_i}\Big[B(\omega,t) - S(\omega,t)\Big],$$

and that the coastline is

[66] removed: $B(\omega,t) = S(\omega,t)$. Since ice thickness cannot be negative, no

[67] removed: $B(\omega,t) > S(\omega,t)$

2.2 Definitions of land, ocean, and ice domains

Given the definition of [..[68]]coastlines and grounding lines, we may define the ocean domain [..[69]]as follows:

$$\begin{aligned} \mathcal{O}(\omega,t) &= 1 \quad \text{if } \mathcal{F}(\omega,t) < 0; \\ &= 0 \quad \text{otherwise, except when } \omega \in \mathcal{T}(\mathcal{F}). \end{aligned} \tag{5}$$

The [..[70]]land domain is simply given by $\mathcal{L}(\omega,t) = 1 - \mathcal{O}(\omega,t)$. Surface areas of these complementary domains together make up the total area of the planetary surface, a necessary condition for mass conservation in the Earth System. Note that neither $\mathcal{O}(\omega,t)$ nor $\mathcal{L}(\omega,t)$ is defined at the [..[71]]coastlines or grounding lines. These interfaces rather form their own level set, $\mathcal{T}(\mathcal{F})$, as defined in Section 2.1. For practical purposes, however, one may carry all these masks and level sets as a single field. For example, the Ice-sheet and Sea-level System Model (ISSM; https://issm.jpl.nasa.gov/) uses the field: `md.mask.ocean_levelset`, which takes -1 in the ocean, 1 in land, and 0 at the coastlines and grounding lines. [..[72]]

We define $\mathcal{I}(\omega,t)$ to be a globally distributed system of ice domains, such that

$$\begin{aligned} \mathcal{I}(\omega,t) &= 1 \quad \text{if } H(\omega,t) > 0; \\ &= 0 \quad \text{otherwise.} \end{aligned} \tag{6}$$

For many applications, it may be useful to decompose $\mathcal{I}(\omega,t)$ into a number of sub-domains: $\mathcal{I}(\omega,t) = \{\mathcal{I}_1, \mathcal{I}_2, ..., \mathcal{I}_i, ...\}$, where $\mathcal{I}_i(\omega,t)$ represents the i-th ice domain. Individual ice sheets and glaciers can be thought of individual ice domains. As defined [..[73]]in Section 2.1, the grounding line within a given ice domain is [..[74]]given by the level-set $\mathcal{T}(\mathcal{F})$. Using equations ([..[75]]5) and ([..[76]]6), we may define the grounded ice mask simply as $\mathcal{G}(\omega,t) = \mathcal{I}(\omega,t)\mathcal{L}(\omega,t)$ and the floating ice mask as $\mathcal{I}(\omega,t)\mathcal{O}(\omega,t)$. [..[77]]Equation (6) is a simple, and perhaps the most generic, definition for ice domains, which can accommodate any geometric features such as kilometer-scale pinning points or rugged fjords that can modulate marine

[68] removed: coastline and grounding line

[69] removed: , $\mathcal{O}(\omega,t)$,

[70] removed: complementary

[71] removed: coastline or grounding line. The concept of "connectivity" is introduced because the criterion $F(\omega,t) < 0$ in and of itself is not always sufficient to define the ocean domain. One obvious example where ω may not belong to ocean in spite of having $F(\omega,t) < 0$ is a deep continental trough with bathymetry well below sea level

[72] removed: Unless this trough or any such locations is physically connected to the open ocean via oceanic water, we consider this to be part of land rather than the ocean.This assumption ensures an interconnected system of Earth's oceanic waters, termed global ocean, as has been considered traditionally in physical oceanography and sea level studies.

[73] removed: above

[74] removed: where $F(\omega,t) = 0$, provided again that ω is in direct contact with the open ocean

[75] removed: 3

[76] removed: 4

[77] removed: Note that $F(\omega,t) \leq 0$ can exist even within the grounded ice mask. A condition where this may occur is when the interior of a marine ice sheet resides on a deep trough (see Figure 2a) . If a column of ice is shielded by surrounding bathymetric highs from the open ocean, we expect it to remain grounded. The ice sheet domain as defined by equation (4) can accommodate complexities such as a pinning point

[..[78]]ice-sheet instability on retrograde slopes (e.g., *Matsuoka et al.*, 2015; *Whitehouse et al.*, 2017). [..[79]]The employed definition of floating ice mask, however, limits us from capturing the floating ice on subglacial and proglacial lakes that are not part of the global ocean ([..[80]]Figure 1). We believe that the localized processes of ice-lake interactions are of secondary importance, at least, for the purpose of capturing large-scale interplay between the continental ice sheets, solid Earth and sea level in the [..[81]]current Earth System models.

Our definition of [..[82]]coastlines and grounding lines, and hence that of [..[83]]the land and ocean and the grounded and floating ice, facilitates direct evaluation of the interaction between a dynamic system of ice sheets, solid Earth and sea level, as well as the estimation and interpretation of [..[84]]ice-driven global and regional [..[85]]sea-level change by conserving mass in the Earth system. Although a distributed system of ice domains is an integral part of the Earth System, in the following we consider, for brevity, a single domain as an ice sheet, while other ice domains are collectively referred to as [..[86]]far-field ice.

3 [..[87]]Sea-level contribution [..[88]]from an ice sheet

[..[89]]The estimation of the sea-level contribution from an evolving ice sheet, [..[90]]featuring marine-based grounded and floating ice, is not trivial, particularly in light of evolving bedrock and MSL. Here we review the common method and its limitations, and present a new method that is applied to arbitrary ice geometries, all kinds of bedrock and MSL forcings, and at all time scales.

3.1 Change in ice-height above floatation

We use the bedrock and MSL to define a floatation height for ice

$$H_0(\omega,t) = \frac{\rho_o}{\rho_i} \max\left[\Big\{S(\omega,t) - B(\omega,t)\Big\},0\right], \tag{7}$$

[..[91]]such that the ice thickness in excess of $H_0(\omega,t)$ represents the so-called [..[92]]height above floatation (HAF). We may interpret HAF as the fraction of ice thickness that can potentially contribute to sea level by changing the mass of the

[78] removed: ice sheet

[79] removed: This is where the new setup diverges from traditional theory. The employed assumption in our description

[80] removed: see

[81] removed: first generation of

[82] removed: coastline and grounding line

[83] removed: land, ocean and ice domains

[84] removed: ice sheet driven

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[90] removed: we define a flotation height of ice

[91] removed: so

[92] removed: "height above flotation", $H_F(\omega,t)$.

oceanic water, and is therefore only defined in the grounded ice domain. Mathematically, [..[93]]

$$H_F(\omega,t) = \mathcal{G}(\omega,t)\Big[H(\omega,t) - H_0(\omega,t)\Big]. \tag{8}$$

The physical interpretation of HAF is illustrated in Figure 2a. It is clear from the above equations and the figure that $H_F(\omega,t) = H(\omega,t)$ for the grounded ice sheet that rests on the bedrock whose elevation is at or above [..[94]]MSL. For grounded portions of the marine ice sheet, $H_F(\omega,t) < H(\omega,t)$[..[95]]. In fact, $H_F(\omega,t)$ can take negative values (see Sector D in Figure 2a)[..[96]]. Such a region, when physically connected to the [..[97]]ocean by oceanic water, can take up water and contribute to sea-level fall.

The evolving [..[98]]ice-sheet geometry is usually described in terms of [..[99]]ice thickness and [..[100]]ice-sheet margins. Indeed, prognostic simulations of [..[101]]ice-sheet models track the transport of mass in terms of equivalent [..[102]]ice-thickness distribution. The transport of mass within the ice domain and ice-ocean mass exchange induce a GRD response of the solid Earth, which further redistributes the mass in the Earth System. This modulates the bedrock topography as well as [..[103]]MSL. These evolving fields may also have components that are forced by external processes such as contemporaneous melting of far-field ice, or GIA, or tectonics. We may describe the evolving [..[104]]ice-sheet geometry in terms of ice thickness, bedrock elevation, and [..[105]]MSL (see Figure 2b-e). We denote $\Delta H(\omega,\Delta t)$, $\Delta B(\omega,\Delta t)$, and $\Delta S(\omega,\Delta t)$ to be the change in respective fields over the time interval Δt. For the new [..[106]]ice-sheet geometry at time $t+\Delta t$, equation ([..[107]]8) gives $H_F(w,t+\Delta t) = \mathcal{G}(\omega,t+\Delta t)\Big[H(\omega,t)+\Delta H(\omega,\Delta t) - H_0(\omega,t+\Delta t)\Big]$, where [..[108]]$H_0(\omega,t+\Delta t) = \rho_o/\rho_i \ \max\Big[\Big\{S(\omega,t)+\Delta S(\omega,\Delta t) - B(\omega,t) - \Delta B(\omega,\Delta t)\Big\},0\Big]$ is given by equation ([..[109]]7). Similarly, we may rewrite

[93] removed:

$$H_F(\omega,t) = \mathcal{G}(\omega,t)\Big[H(\omega,t) - H_0(\omega,t)\Big].$$

Here we invoke a grounded ice mask $\mathcal{G}(\omega,t)$ to ensure no net sea level contribution from the floating ice shelf. Clearly,

[94] removed: sea level.

[95] removed: and it can be negative (see regime 4

[96] removed: , implying that this portion can take up ocean water when it is

[97] removed: open ocean and hence contribute to sea level inversely

[98] removed: ice sheet

[99] removed: evolving

[100] removed: ice sheet domain

[101] removed: ice sheet

[102] removed: ice thickness distribution. This redistribution of ice mass and associated relative sea level change induce a solid Earth response in terms of its gravitation, rotation and viscoelastic deformation. This , in turn,

[103] removed: the geoid. We may therefore

[104] removed: ice sheet

[105] removed: sea level (

[106] removed: ice sheet

[107] removed: 6

[108] removed: $H_0(\omega,t+\Delta t) = -\rho_o/\rho_i \ \min\Big[\Big\{B(\omega,t)+\Delta B(\omega,\Delta t) - S(\omega,t) - \Delta S(\omega,\Delta t)\Big\},0\Big]$

[109] removed: 5

equation (1) for $F(\omega,t+\Delta t)$ and define the new ocean domain $\mathcal{O}(\omega,t+\Delta t)$, land domain $\mathcal{L}(\omega,t+\Delta t)$, [..[110]]and grounded ice [..[111]]domain $\mathcal{G}(\omega,t+\Delta t)$ as described in Section 2. [..[112]]

In what follows, we assume that the net change in [..[113]]grounded ice mass [..[114]]results in the equivalent change in ocean mass, ensuring mass conservation in the Earth System. On the one hand not all of $\Delta H(\omega,\Delta t)$ contributes to [..[115]]change in mass of oceanic water, but on the other hand, in response to the externally forced bedrock and [..[116]]MSL change, the ice sheet may still contribute to [..[117]]change in ocean mass even when $\Delta H(\omega,\Delta t)=0$ as we count floating ice in the ocean mass (see Appendix A). The stand-alone ice-sheet models evaluate the change in HAF in order to calculate the sea-level contribution of an ice sheet[..[118]], termed the HAF method for brevity (e.g., *Bindschadler et al.*, 2013; *Nowicki et al.*, 2016):

$$\Delta \mathrm{H_F}(\omega,\Delta \mathrm{t}) = \mathrm{H_F}(\omega,\mathrm{t}+\Delta \mathrm{t}) - \mathrm{H_F}(\omega,\mathrm{t}). \tag{9}$$

These models generally (but incorrectly) calculate the equivalent oceanic water volume (rather than the freshwater volume) by spatially integrating $-[\rho_i/\rho_o]\Delta H_F(\omega,\Delta t)$ and divide it by the ocean surface area to estimate the GMSL change. Apart from this water density related error, the HAF method in absence of evolving bedrock and MSL yields the correct estimates of the sea-level contribution (see Appendix A). In fact, the effects of $\Delta B(\omega,t)$ and $\Delta S(\omega,t)$ may be negligible over the timescale of a few decades or shorter. The stand-alone ice-sheet models typically inherit this assumption, even though simulation timescales can be on the order of centuries. Over such relatively longer timescales, this simplistic approach, yields some error, [..[119]]especially in the marine portions of an ice sheet (*Larour et al.*, 2019; *Goelzer et al.*, 2020).

[..[158]]

3.2 A new field for estimating sea-level contribution

In order to overcome the limitations of the HAF method, we define a unified field, $\Delta H_S(\omega,\Delta t)$, that contributes to [..[159]]sea-level change by modulating both the mass and volume of oceanic water over the period Δt. [..[160]]This field captures

[110]removed: ice sheet domain $\mathcal{I}(\omega,t+\Delta t)$,

[111]removed: mask

[112]removed: Note that $\Delta B(\omega,\Delta t)$ and $\Delta S(\omega,\Delta t)$ may have components that are forced by external processes such as farfield ice melting or tectonics.

[113]removed: ice sheet mass directly affects the ocean

[114]removed: . Quantifying the fraction of ice mass change that contributes to sea level is a non-trivial problem. It cannot be approached analytically and is beyond the scope of this study. Despite the assumption,

[115]removed: sea level change all the time. On the

[116]removed: sea level

[117]removed: sea level change even when $\Delta H(\omega,\Delta t)=0$. Traditionally, change in ice thickness above flotation, i.e. $\Delta H_F(\omega,\Delta t) = H_F(\omega,t+\Delta t) - H_F(\omega,t)$, is used to calculate sea level

[118]removed: (e.g., *Bindschadler et al.*, 2013). As we show below this simplistic approach

[119]removed: particularly when evolving bedrock and sea level are considered (*Larour et al.*, 2019; **?**)

[158]removed: We define $\Delta H_S(\omega,\Delta t)$ to be a portion of ice thickness

[159]removed: sea level

[160]removed: The following relationship holds

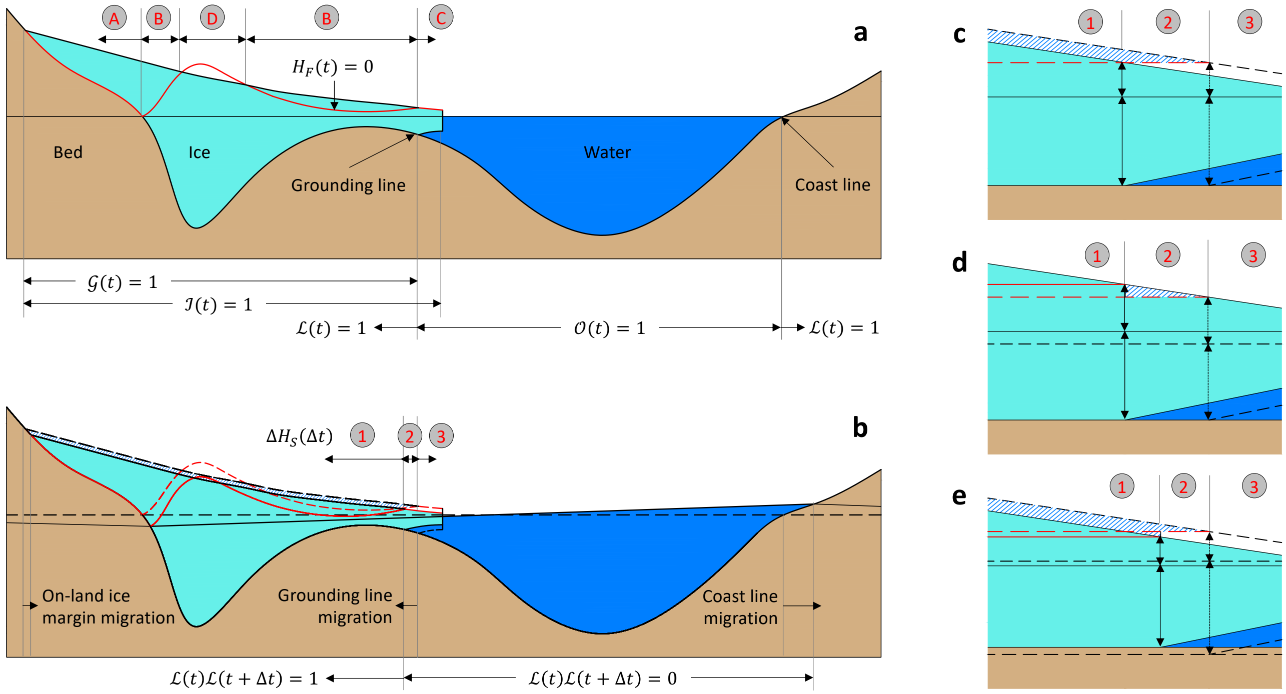

Figure 2. [..[120]]**Conceptual depiction of an evolving ice-sheet geometry.** **(a)** Domains of ocean [..[121]]$\mathcal{O}$, land [..[122]]$\mathcal{L}$, ice [..[123]]$\mathcal{I}$, and its grounded [..[124]]portion $\mathcal{G}$ at time t. [..[125]]The floatation height, having [..[126]]$H_F = 0$, is represented by [..[127]]the red line. [..[128]]Ice-height above [..[129]]floatation satisfies the condition $H_F = H$ in [..[130]]Sector A, [..[131]]$0 < H_F < H$ in [..[132]]Sector B, [..[133]]$H_F = 0$ in [..[134]]Sector C, and [..[135]]$H_F < 0$ in [..[136]]Sector D. **(b)** [..[137]]Ice-sheet geometry at time $t + \Delta t$ after changes in ice thickness and [..[138]]MSL. (For simplicity, bedrock change is not considered.) Old geometry and field variables are shown with dashed lines. Ice thickness that contributes to [..[139]]sea-level change[..[140]], [..[141]]ΔH_S, is given by [..[142]]ΔH in [..[143]]Regime 1 and [..[144]]by $\Delta H_F + (1 - \rho_w/\rho_o)(\Delta H - \Delta H_F)$ in [..[145]]Regimes 2 and 3. The hatched area contributes to the ocean mass change ([..[146]]equation [..[147]]11)[..[148]]. Since $\Delta H_F = 0$ in [..[149]]Regime 3, it contributes to sea level by modulating the ocean volume (not mass) alone (equation 12). We zoom in around the grounding line to assess [..[150]]different scenarios: **(c)** when ice thickness changes but [..[151]]the bedrock and MSL do not, typically assumed in stand-alone [..[152]]ice-sheet models; **(d)** when [..[153]]externally-forced MSL changes but ice thickness does not; and **(e)** when [..[154]]ice thickness[..[155]], [..[156]]bedrock [..[157]]and MSL all evolve simultaneously. Sketches are not to scale.

the effects of evolving bedrock and MSL induced by any geophysical processes and is applied to arbitrary ice geometries and at all timescales. We find it convenient to partition $\Delta H_S(\omega, \Delta t)$ upfront into two components:

$$\Delta \mathrm{H_S}(\omega, \Delta \mathrm{t}) = \Delta \mathrm{H_M}(\omega, \Delta \mathrm{t}) + \Delta \mathrm{H_V}(\omega, \Delta \mathrm{t}), \tag{10}$$

such that the first component $\Delta H_M(\omega, \Delta t)$ modulates both the mass and volume of the oceanic water, while the second component $\Delta H_V(\omega, \Delta t)$ can only modulate the ocean volume. The following relationships hold for generalized ice geometries, and bedrock and [..[161]]MSL forcings:

$$\Delta \mathsf{H_M}(\omega,\Delta \mathsf{t}) = \Delta \mathsf{H}(\omega,\Delta \mathsf{t})\ \mathcal{L}(\omega,\mathsf{t})\ \mathcal{L}(\omega,\mathsf{t}+\Delta \mathsf{t})\ +\ \Delta \mathsf{H_F}(\omega,\Delta \mathsf{t})\ \Big[1-\mathcal{L}(\omega,\mathsf{t})\ \mathcal{L}(\omega,\mathsf{t}+\Delta \mathsf{t})\Big]\ , \tag{11}$$

$$\Delta \mathsf{H_V}(\omega,\Delta \mathsf{t}) = \left(1-\frac{\rho_\mathsf{w}}{\rho_\mathsf{o}}\right)\Big[\Delta \mathsf{H}(\omega,\Delta \mathsf{t})-\Delta \mathsf{H_F}(\omega,\Delta \mathsf{t})\Big]\ \Big[1-\mathcal{L}(\omega,\mathsf{t})\ \mathcal{L}(\omega,\mathsf{t}+\Delta \mathsf{t})\Big], \tag{12}$$

where ρ_w is the fresh water density. [..[162]]For grounded ice sheets, the mass component makes up about 97% of $\Delta H_S(\omega,\Delta t)$, which loads the solid Earth and induces its GRD response and sea-level adjustment, which will be further discussed in Section 4.

While a detailed interpretation of individual terms appearing in equations (11-12) is given in the Appendix by considering all possible scenarios of evolving ice thickness, bedrock elevation and MSL, Figure 2 [..[163]]illustrates a few representative scenarios. In reference to [..[164]]this figure and equations (11-12), we outline three distinct regimes[..[165]]:

– [..[166]]**Regime 1: Where ice remains grounded at both times t and $t+\Delta t$.**

  All of $\Delta H(\omega,\Delta t)$ [..[167]]in this regime contributes to sea-level change by modulating both the mass and volume of the ocean (first term on the [..[168]]right-side of equation 11), irrespective to the elevation of bedrock upon which the ice is grounded. It turns out [..[169]]$\Delta H(\omega,\Delta t) \neq \Delta H_F(\omega,\Delta t)$ only in the marine portions of the [..[170]]regime and only when evolving bedrock and [..[171]]MSL are considered (see Appendix A1). *Goelzer et al.* (2020) present a method to backtrack $\Delta H(\omega,\Delta t)$ from $\Delta H_F(\omega,\Delta t)$ in such situations, assuming that $\Delta B(\omega,\Delta t)$ and $\Delta S(\omega,\Delta t)$ are known. [..[172]]

[161] removed: /or sea level forcings:

$$\Delta H_S(\omega,\Delta t) = \Delta H(\omega,\Delta t)\ \mathcal{L}(\omega,t)\ \mathcal{L}(\omega,t+\Delta t) + \Delta H_F(\omega,\Delta t)\ |\Delta\mathcal{L}(\omega,\Delta t)|\ + \left(1-\frac{\rho_w}{\rho_o}\right) H_0(\omega,t+\Delta t)\ \Delta\mathcal{L}(\omega,\Delta t),$$

[162] removed: The change in land domain is simply given by $\Delta\mathcal{L}(\omega,\Delta t) = \mathcal{L}(\omega,t+\Delta t)-\mathcal{L}(\omega,t)$. Since $\Delta H_F(\omega,\Delta t)$, unlike $H_0(\omega,t+\Delta t)$, can track the direction of margin migration (i.e., advance or retreat), absolute value of $\Delta\mathcal{L}(\omega,\Delta t)$ is used in the second term on the right-hand side. This method requires bookkeeping the global land domain. This is crucial for considering distributed ice-ocean domains with complex geometries in Earth System models. For the treatment of an individual ice sheet, however, it is sufficient to track a continental or regional land domain, provided that there is an understanding that ocean water may recede from, or reinundate, continental land (e.g., *Johnston*, 1993; *Milne*, 1998). In fact, often it may be possible to replace all $\mathcal{L}$'s appearing in equation (7) by corresponding $\mathcal{G}$'s, the grounded ice masks. However, in an exceptional case with $H_0(\omega,t+\Delta t)>0$ holding in the areas of on-land ice margin migration, using $\mathcal{G}$'s rather than $\mathcal{L}$'s incorrectly predict a non-zero contribution from the last term in equation (7).

[163] removed: shows a few schematics of evolving ice sheet geometries and their sea level contributions. We consider all plausible cases by combining scenariosof ice thickness and relative sea level changes

[164] removed: these sketches and equation (7

[165] removed: of an evolving ice sheet and their sea level contribution over the period Δt

[166] removed: Where ice remains grounded on the bedrock at the elevations above, at, or below corresponding sea level at both times t and $t+\Delta t$, all

[167] removed: contributes to sea level

[168] removed: right-hand side)

[169] removed: that

[170] removed: grounded ice

[171] removed: sea level are considered . **?**

[172] removed: Over the timescale of a few decades or shorter, it may be that relative sea level does not evolve significantly, and in these cases $\Delta H_F(\omega,\Delta t) \approx \Delta H(\omega,\Delta t)$. Stand-alone ice sheet models typically inherit this assumption even though simulation timescales can be on the order of centuries (e.g., *Bindschadler et al.*, 2013).

This regime also includes land areas covered by the evolving [..[173]]ice-sheet margins. When ice margin advances over the period Δt, newly glaciated areas must satisfy $H(\omega,t) = 0$ and $\Delta H(\omega,\Delta t) > 0$. When it retreats, $\Delta H(\omega,\Delta t) = -H(\omega,t)$ must hold in the recently deglaciated areas. In both cases, all of $\Delta H(\omega,\Delta t)$ contributes to [..[174]]the sea-level change.

[..[175]]Externally-forced $\Delta B(\omega,\Delta t)$ or $\Delta S(\omega,\Delta t)$ does not affect the estimate of $\Delta H_S(\omega,\Delta t)$ in this regime although it may [..[176]]alter bedrock slope or gravitational driving stress and possibly modulate the [..[177]]ice-flow dynamics. While the effects of far-field ice melting and associated ocean loading may be negligible due to their relatively long-wavelength imprints, $\Delta B(\omega,\Delta t)$ due to large earthquakes beneath the ice sheet may have some impact on ice dynamics.

– [..[178]]**Regime 2: Where ice transitions from grounded to floating, or the reverse, over the period Δt.**

The sea-level contribution from this regime [..[179]]mainly depends on the change in HAF (second term on the right-side of equation 11), which modulates both the mass and volume of oceanic water. In the absence of [..[180]]externally-forced bedrock and MSL, it follows that [..[181]]

[..[182]]$|\Delta H_F(\omega,\Delta t)| < |\Delta H(\omega,\Delta t)|$ (see Appendix A2). The change in ice thickness in excess of the change in HAF (right-side term in equation 12) nominally modulates the volume of the oceanic water. This is due to the [..[183]]difference in volume between the freshwater that would be produced when ice melts and the oceanic water that would be displaced when it floats.

This is the only regime [..[184]]where, in response to the externally-forced $\Delta B(\omega,\Delta t)$ [..[185]]or $\Delta S(\omega,\Delta t)$[..[186]], an ice sheet may modulate both the mass and volume of the ocean even when $\Delta H(\omega,\Delta t) = 0$[..[187]]. Specific examples are given in Appendix A2.

[173]removed: ice sheet
[174]removed: sea level
[175]removed: Externally forced
[176]removed: yield nonzero $\Delta H_F(\omega,\Delta t)$
[177]removed: ice flow dynamics.
[178]removed: Where ice has transitioned from the grounded to floating state or vice versa over the period Δt, sea level
[179]removed: depends on both $\Delta H_F(\omega,\Delta t)$ and $H_0(\omega,t+\Delta t)$ (the last two terms on the right-hand side)
[180]removed: externally forced bedrock and sea level
[181]removed: $|\Delta H_F(\omega,\Delta t)| \leq |\Delta H(\omega,\Delta t)|$ in this regime.
[182]removed: The last term in the equation accounts for the fact that fresh water density evolves during the accretion and ablation of ice , whereas the average ocean water density in
[183]removed: vicinity of the grounding line acts to determine the flotation height. For typical values of ρ_w and ρ_o, we find that about 2.5% of $H_0(\omega,t+\Delta t)$ contributes to sea level change. This particular contribution has been considered by **?**.
[184]removed: in which responses to the externally forced
[185]removed: and/
[186]removed: contribute to sea level change,
[187]removed: (see Figure 2d).

– [..[188]]**Regime 3: Where ice remains floating at both times t and $t+\Delta t$.**

Since $\Delta H_F(\omega,\Delta t)=0$ holds true in this regime, the change in ice thickness does not modulate the ocean mass itself but it releases or takes up freshwater that has slightly larger volume than the oceanic water upon which it floats. This minor difference in water volume (right-side term in equation 12) contributes to the sea-level change (see Appendix A3). More importantly, the change in ice [..[189]]thickness in this regime can affect the interior-ice sheet dynamics via modulation of buttressing force (e.g., *Gudmundsson et al.*, 2019) and may amplify the future sea-level change.

[..[190]]Given the new field for estimating sea-level contribution (equation 10), we may readily calculate the GMSL change by spatially integrating $-[\rho_i/\rho_w]\Delta H_S(\omega,\Delta t)$, which yields the total freshwater volume being added to the ocean over the period Δt, and dividing it by the ocean surface area at time $t+\Delta t$. Assume that an ice sheet collapses instantaneously and that all of the melt water makes it to the ocean. Resulting GMSL change represents the "potential sea level" of the ice sheet at time t, and it can be readily derived from equation (10) by setting $\Delta H(\omega,\Delta t)=-H(\omega,t)$ and [..[191]]$\mathcal{G}(\omega,t+\Delta t)=0$ in the [..[192]]limit of $\Delta t\to 0$. Note that $\Delta H(\omega,\Delta t)$ and $\mathcal{G}(\omega,t+\Delta t)$ are implicit via equations (9, 11-12).

3.3 Quantitative comparison of the two methods

Here we present a case study to demonstrate the level of improvements possible by employing the new method (equation 10) over the HAF method (equation 9). For a quantitative comparison, we rely on the recent work of *Larour et al.* (2019), who provide consistent solutions of evolving $H(\omega,t)$, $B(\omega,t)$ and $S(\omega,t)$ for [..[193]]the Antarctic Ice Sheet over the next 500 years. They simulate a high-resolution dynamical ice-flow model (*Larour et al.*, 2012) that is fully coupled with a global solid-Earth deformation and sea-level adjustment model (*Adhikari et al.*, 2016) under the present-day surface climatology and a realistic sub-ice shelf melting scenario [..[194]](*Seroussi et al.*, 2017). They also account for the effects of far-field ice-mass change on the evolution of bedrock and MSL in Antarctica. To this end, they consider mass balance of the Greenland Ice Sheet and global glaciers, extrapolated into the next 500 years based on the [..[195]]space gravimetry-based measurements. The ongoing change in bedrock and MSL due to the viscous response of the solid Earth to the global deglaciation since the Last Glacial Maximum is also accounted for through an off-line coupling of a GIA model (*Caron et al.*, 2018).

[188] removed: Where ice is floating at both times t and $t+\Delta t$, there is no net direct contribution from this regimeof the ice sheet to sea level change . However,

[189] removed: shelf thickness can greatly affect the interior ice

[190] removed: In Figure 3, we present a quantitative assessment of $\Delta H_S(\omega,\Delta t)$

[191] removed: $\Delta H_F(\omega,\Delta t)$ over the next 350 years for Pine Island and Thwaites glaciers. Notice the systematic error associated with a customary approach of using $\Delta H_F(\omega,\Delta t)$ to quantify sea levelcontribution from an ice sheet (Figure 3c) . The discrepancy in regime 1 is due to the evolving bedrock and sea level, and that in regime 2 is due to

[192] removed: missing fraction of newly grounded or floating ice. These results are based

[193] removed: Antarctica

[194] removed: . The effect of farfield ice mass change ,

[195] removed: contemporary trend, on the evolution of bedrock and sea level in Antarctica

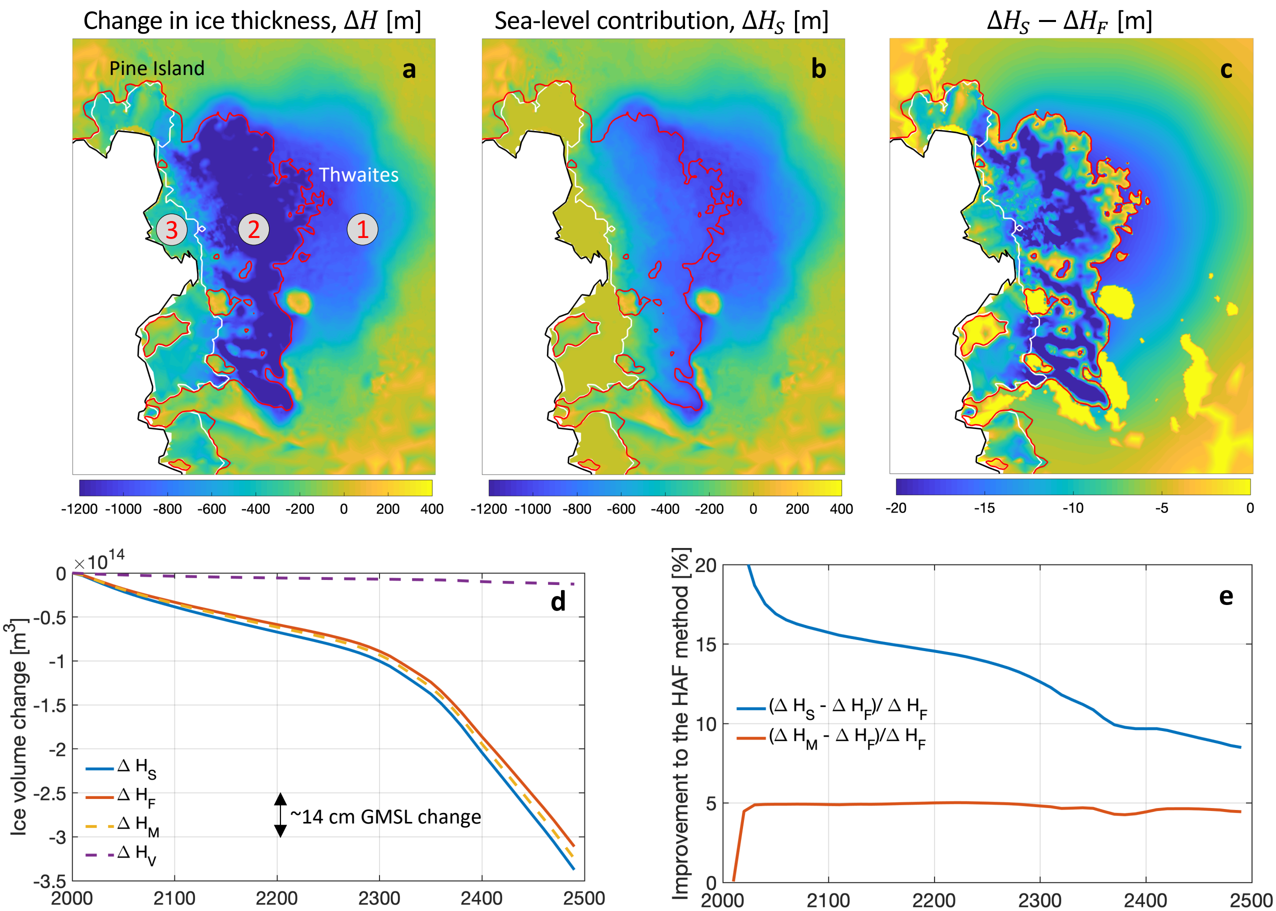

Figure 3. [..[196]]**Example of ice-thickness change and its contribution to the sea level.** **(a)** Modeled change in ice thickness [..[197]]at AD 2350 for portions of Pine Island and Thwaites glaciers adjacent to the Amundsen Sea (*Larour et al.*, 2019). [..[198]]The black line denotes the ice-ocean interface at AD 2000 and the white ([..[199]]red) line denotes the land-ocean interface i.[..[200]]e.[..[201]], [..[202]]grounding lines, at AD 2000 (AD 2350). These interfaces are used to separate three regimes of the ice sheet as defined in Section 3 ([..[203]]see [..[204]]also Figure 2b). **(b)** Estimation of ice thickness that contributes to the sea level over the next 350 years based on the new method proposed in [..[205]]this study (equation 10). **(c)** Comparison of our method with respect to the [..[206]]HAF method (equation 9). Note that only in [..[207]]portions of [..[208]]Regime 1 [..[209]]where the bedrock elevation is higher than the MSL, the two methods agree (yellow patches). **(d)** The total volume change of the Antarctic Ice Sheet that is attributable to the sea-level change. While ΔH_M and [..[210]]ΔH_F modulate both the mass and volume of the ocean, ΔH_V only modulates the [..[211]]ocean volume. **(d)** Difference between the new method and the HAF method. The latter method underpredicts [..[212]]the sea-level contribution [..[213]]of the ice sheet [..[214]]throughout the [..[215]]model simulation. The mass component of the new method alone consistently predicts 5% more sea-level contribution than the HAF method.

[..[216]]In Figure 3, we compare the two methods both in terms of their spatial and temporal patterns. We show $\Delta H_S(\omega,\Delta t)$ and $\Delta H_F(\omega,\Delta t)$ computed at AD 2350 relative to AD 2000 for Pine Island and Thwaites glaciers (Figures 3a-c). To facilitate the interpretation, we separate the model domain into three regimes as in Figure 2b: Regime 1 (Regime 3) consists of regions that are grounded (floating) at both times; and Regime 2 consists of regions that transition from grounded to floating over the course of simulation. Only $\Delta H(\omega,\Delta t)$ contributes to $\Delta H_S(\omega,\Delta t)$ in Regime 1 (see equation 11). In the marine portions of this regime, $\Delta H(\omega,\Delta t)$ and hence $\Delta H_S(\omega,\Delta t)$ differs from $\Delta H_F(\omega,\Delta t)$ due to the [..[217]]effects of evolving bedrock and MSL in the latter field. The difference between $\Delta H_S(\omega,\Delta t)$ and $\Delta H_F(\omega,\Delta t)$ in Regime 2 and Regime 3 are due to $\Delta H_V(\omega,\Delta t)$ (equation 12), which accounts for the volumetric contribution of ice-thickness change in excess of the change in HAF (see Appendix A2). We also show the time series of the total Antarctic ice-volume change that is attributable to the GMSL change (Figures 3d-e). We find that the new method predicts systematically larger sea-level contribution, compared to the HAF method, throughout the model simulation. The difference in the first 100 years is more than 15% and in the last 100 years is about 8-10%. We isolate the mass (equation 11) and the volume component (equation 12) of the new method to show that the former component alone, which drives the GRD response of the solid Earth, consistently predicts larger sea-level contribution than the HAF method by about 5%. Note that the HAF method usually converts the ice-volume change (Figure 3d) into the equivalent oceanic water (rather than the freshwater) volume change (e.g., *Bindschadler et al.*, 2013) and hence systematically underpredicts the amplitude of GMSL change by additional 2-3%, which is not accounted for in the above analysis.

4 Sea-level change and mass conservation in the Earth System

Delineation of evolving coastlines (and grounding lines) and estimation of $\Delta H_S(\omega,\Delta t)$ require the knowledge of $\Delta H(\omega,\Delta t)$, $\Delta B(\omega,\Delta t)$, and $\Delta S(\omega,\Delta t)$ along with accurate information of the solid Earth surface (e.g., bedrock topography in ice domains, and land-surface topography and ocean bathymetry in the vicinity of coastlines). Parts of $\Delta B(\omega,\Delta t)$, and $\Delta S(\omega,\Delta t)$ are induced by $\Delta H_S(\omega,\Delta t)$ and associated ocean-mass change themselves. These fields are therefore intertwined with each other, and only by using a mass conserving Earth System model that can capture ice-sheet dynamics, solid-Earth deformation and sea-level adjustment may we find self-consistent solutions. We find it convenient to treat the change in bedrock and MSL collectively in terms of the change in relative sea level (RSL), which by definition is the MSL relative to the sea floor or bedrock (*Gregory et al.*, 2019). Mathematically, $\Delta R(\omega,\Delta t) = \Delta S(\omega,\Delta t) - \Delta B(\omega,\Delta t)$. Since $\Delta R(\omega,\Delta t)$ may be induced by processes other than $\Delta H_S(\omega,\Delta t)$, we must consider them as they impact the estimate of evolving $\mathcal{T}(\mathcal{F})$, and hence the ocean surface area, and $\Delta H_S(\omega,\Delta t)$ itself. In fact, we should also consider the change in steric MSL, which is not accounted for in $\Delta S(\omega,\Delta t)$ (see Section 2.1). The inclusion of this component, however, must

[216]removed: The barystatic sea level change, defined as

[217]removed: ocean mass related global mean sea level change (*Gregory et al.*, 2019), due to ice mass change over the period Δt is given by

$$\overline{\Delta R_C^I}(\Delta t) = \left[-\rho_i \int \Delta H_S(\omega,\Delta t)\,\mathrm{d}\omega\right] \Big/ \left[\rho_w \int \mathcal{O}(\omega, t+\Delta t)\,\mathrm{d}\omega\right].$$

The numerator in the equation owes to

be accompanied by the [..[218]]spatially- and temporally-varying ocean density (see, for example, equation 1), which modulates the coastlines, and hence the ocean surface area, but does not affect the buoyant force on the ice, grounding-line migration, and the estimate of $\Delta H_S(\omega,\Delta t)$.

[..[219]]To further diagnose $\Delta R(\omega,\Delta t)$, especially in light of space-based observations and existing GRD models, we present a synopsis of contributing processes as follows:

$$\Delta R(\omega,\Delta t) = \Delta R_C^I(\omega,\Delta t) + \Delta R_C^L(\omega,\Delta t) + \Delta R_P^I(\omega,\Delta t) + \Delta R_P^L(\omega,\Delta t) + \Delta R_{[..^{220}]}\mathrm{O}(\omega,\Delta t). \tag{13}$$

The first four terms on the right-side of the equation represent the processes that [..[221]]exchange water between the land and the ocean and contribute to sea-level change by inducing GRD response of the solid Earth (termed, for brevity, the barystatic components)[..[222]]. We use the superscript I to refer to the *ice sheet* under consideration and superscript L to [..[223]]other parts of the *land*, including [..[224]]far-field ice and hydrological basins[..[225]]. When these sources of freshwater contribute to [..[226]]sea-level change over a *contemporaneous* period $[t, t+\Delta t]$, corresponding changes in [..[227]]RSL are denoted with the subscript C. [..[228]]The land-ocean water exchange may have occurred in the *past* [..[229]]i.e., over the period $(-\infty, t]$, and the induced viscous response of the solid Earth may still contribute to the [..[230]]RSL change over the period $[t, t+\Delta t]$. These components [..[231]]are denoted with the subscript P. The last term appearing in the equation captures other non-barystatic processes that may or may not induce GRD response of the solid Earth, but at least modulate the ocean bathymetry or coastal geometry. These processes include earthquakes, landslides, sediment transport and coastal subsidence, amongst others. Assuming that the contemporaneous period Δt is on the order of 10 years, we may interpret $\Delta R(\omega,\Delta t)$ as [..[232]]the non-steric component of ongoing RSL change monitored by the satellite gravimetry and altimetry [..[233]](*WCRP Global Sea Level Budget Group*, 2018). We may interpret $\Delta R_C^I(\omega,\Delta t) + \Delta R_C^L(\omega,\Delta t)$ and

[218]removed: net change in ice mass over Δt, and the integral in denominator represents the ocean surface area at time $t+\Delta t$. Assume that an ice sheet collapses instantaneously and that all of melt water contributes to sea level. Resulting $\overline{\Delta R_C^I}(\Delta t)$ represents the "potential sea level" of the ice sheet at time t, and it can be readily derived by setting $\Delta H(\omega,\Delta t) = -H(\omega,t)$ and $\mathcal{G}(\omega, t+\Delta t) = 0$ in the limit of $\Delta t \to 0$ in equation (7).

[219]removed: Ocean surface areachanges as grounding line and coastline positions migrate. Such a migration over Δt is controlled by a number of processes that contribute to $\Delta R(\omega,\Delta t)$. A comprehensive synopsis of these processes can be stated

[221]removed: contribute to sea level by changing the ocean mass (termed

[222]removed: , while $\Delta R_D(\omega,\Delta t)$ represents the density related steric sea level change and $\Delta R_V(\omega,\Delta t)$ represents a component due to vertical land motion that is not captured by the barystatic processes

[223]removed: the

[224]removed: farfield ice sheets, glaciers

[225]removed: , that contribute to barystatic sea level change

[226]removed: ocean mass

[227]removed: relative sea level fields

[228]removed: The ice sheet and other parts of the land may have contributed to the evolution of barystatic sea level

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[230]removed: change in relative sea level

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[232]removed: ongoing change in relative sea level monitored by

[233]removed: (e.g., *WCRP Global Sea Level Budget Group*, 2018). Along the same lines, we

$\Delta R^I_P(\omega,\Delta t) + \Delta R^L_P(\omega,\Delta t)$ as ongoing [..[234]]RSL change driven by contemporary global surface mass redistribution (e.g., *Adhikari et al.*, 2019) and by global GIA processes (e.g., *Peltier et al.*, 2015; *Caron et al.*, 2018), respectively.

As [..[235]]defined in equation (10), only part of $\Delta H_S(\omega,\Delta t)$ potentially contributes to the ocean mass change, loads the underlying solid Earth, and induces its GRD response and contributes to sea-level adjustment. Because the GRD effect is applied to the entire column of the ocean water, only $[\rho_i/\rho_o]\Delta H_M(\omega,\Delta t) \approx 0.892 \times \Delta H_M(\omega,\Delta t)$ induces the barystatic component of the RSL change, which in reference to equation (13) is equivalent to $\Delta R^I_C(\omega,\Delta t)$. The fresh-water equivalent of other parts of $\Delta H_S(\omega,\Delta t)$, i.e. the sum of $[\rho_i/\rho_w - \rho_i/\rho_o]\Delta H_M(\omega,\Delta t) \approx 0.025 \times \Delta H_M(\omega,\Delta t)$ and $[\rho_i/\rho_w]\Delta H_V(\omega,\Delta t)$, contributes to the RSL change by modulating oceanic water density, and hence it may be considered as part of the steric MSL. For grounded ice sheets, this component of RSL change is about 97% smaller that the barystatic component. The remaining terms of equation (13) are what we collectively refer to as the "externally-forced" RSL change. In other words, these are the RSL components not directly induced or contributed by the ice sheet under consideration over the period Δt. [..[236]]

To solve for the spatial pattern of the steric MSL due to $\Delta H_S(\omega,\Delta t)$, one must consider a dynamic ocean circulation model. Such computations are generally not warranted in the longer-term (decadal or longer timescale) sea-level studies, owing to their smaller amplitudes compared to those of the barystatic component. The spatial pattern of the barystatic RSL due to $\Delta H_S(\omega,\Delta t)$ can be obtained by solving the so-called "sea-level equation" on a self-gravitating, viscoelastically compressible, rotating Earth (*Farrell and Clark*, 1976; *Milne and Mitrovica*, 1998). To this end, we must consider a mass conserving field [..[237]]that describes the net change in mass per unit area on the solid Earth surface[..[238]]:

$$\Delta M(\omega,\Delta t) = \rho_i \, \Delta H_{[..239]}\mathrm{M}(\omega,\Delta t) + \rho_{[..240]}\mathrm{o} \, \Delta R^I_C(\omega,\Delta t) \, \mathcal{O}(\omega,t+\Delta t) \qquad (14)$$

[..[241]], such that its global integral is zero. Here $\Delta H_M(\omega,\Delta t)$ is given by equation ([..[242]]11) and $\Delta R^I_C(\omega,\Delta t)$ [..[243]]is precisely the same as the first term on the right-side of equation ([..[244]]13). Because the RSL is defined globally, including [..[245]]in land, we [..[246]]must invoke the ocean mask in [..[247]]the equation. Solving the sea-level equation, in essence, means that we load the solid Earth by the mass-conserving surface load (equation 14) and let its GRD response dictate the self-consistent patterns of RSL, MSL and bedrock changes as well as the new positions of coastlines and grounding lines. The ice-sheet models that account for local or regional isostatic adjustment of bedrock (e.g., *Le Meur and Huybrechts*,

[234] removed: sea level

[235] removed: ice sheet evolves, it not only contributes to sea level but also

[236] removed: For the period $[t, t+\Delta t]$, we may define

[237] removed: $\Delta M(\omega,\Delta t)$

[238] removed: as follows:

[241] removed: Here $\Delta H_S(\omega,\Delta t)$

[242] removed: 7

[243] removed: represents the associated relative sea level change, whose spatial pattern is dictated by the perturbation in Earth's gravitational and rotational potentials and associated viscoelastic deformation of the solid Earth (*Farrell and Clark*, 1976; *Milne and Mitrovica*, 1998). This component of sea level is

[244] removed: 9), whose ocean averaged value is given by equation (8). Because sea level

[245] removed: on

[246] removed: ensure mass conservation by use of an

[247] removed: equation (10)

1996; *Bueler et al.*, 2007; *Pattyn*, 2017) do not consider $\Delta R_C^I(\omega,\Delta t)$ as part of the surface load. As a result, these models violate mass conservation in the Earth System and capture incomplete signals of $\Delta B(\omega,\Delta t)$ and $\Delta S(\omega,\Delta t)$ in the estimation of $\Delta H_S(\omega,\Delta t)$.

5 Conclusions

We have divided the Earth's surface into complementary domains of land and ocean, which are separated by [..[248]]coastlines. While there may be multiple land domains, we maintain a single global ocean [..[249]]of interconnected oceanic water as in the majority of studies in physical oceanography and sea level. Distributed bodies of ice intersect [..[250]]the land and the ocean to form glaciers, ice sheets and ice shelves. Grounding lines are defined as the coastlines that belong to the ice domains. The set of generic, and quite simple, [..[251]]mathematical descriptions presented here can handle the complex geometries of both the coastlines and [..[252]]grounding lines, complementary to those of [..[253]]far-field land, ocean and ice domains and their respective overall evolutionary history.

[..[254]]Based on this formalism of evolving coastlines and grounding lines, we present a unified method [..[255]]to calculate the exact fraction of ice thickness [..[256]]that contributes to [..[257]]the sea-level change over a given period. The method is a function of evolving ice thickness, bedrock elevation and mean sea level driven by any geophysical processes. Along with its obvious application to estimate [..[258]]the global-mean sea-level change, it is absolutely critical to track the global mass transport, and assess the response of a dynamic system of ice sheets, solid Earth and sea level, while accounting for [..[259]]kilometer-scale features in ice/bedrock/ocean geometries. Our method requires bookkeeping the global land and ocean domains. This is crucial for considering distributed ice and land domains with complex geometries in Earth System models. For the treatment of an individual ice sheet, however, it is sufficient to track a continental or regional land domain, provided that there is an understanding that ocean water may recede from, or reinundate, continental land (e.g., *Johnston*, 1993; *Milne*, 1998). In fact, it may often be possible to use grounded ice masks in place of the land domains. In the most simplified case when [..[260]]the bedrock and mean sea level do not evolve, our method reduces to the [..[261]]common method that is based on the concept of ice-height above floatation. For an example model simulation considered in this study (*Larour et al.*, 2019), we find that the new method systematically yields 10-15% more sea-level contribution from the

[248] removed: coast lines
[249] removed: as in traditional studies of
[250] removed: land and
[251] removed: descriptions given in this paper
[252] removed: evolving
[253] removed: farfield
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[255] removed: is outlined to determine
[256] removed: change
[257] removed: sea level change , $\Delta H_S(\omega,\Delta t)$, over the period$\Delta t$.
[258] removed: global mean sea level, this field
[259] removed: fine-scale complexities in geometry. In the
[260] removed: bedrock and
[261] removed: traditional approach that assumes that change in ice height above flotation, $\Delta H_F(\omega,\Delta t)$, directly contributes to sea level

Antarctic Ice Sheet. We recommend that the [..[262]]ice-sheet modeling community consider the proposed method as a metric to quantify the [..[263]]sea-level contribution of evolving ice sheets. This is especially appropriate for model [..[264]]analysis that is informed by ice and ocean mass monitoring from space assets, such as ocean and ice altimetry, radar interferometry and space gravimetry (e.g., *Bentley and Wahr*, 1998).

Notation

B	[..[265]]Solid Earth surface (i.e., land surface or sea floor), or simply bedrock, elevation
ΔB	Change in bedrock elevation over the period Δt
F	A function [..[266]]such that $F = 0$ [..[267]]satisfies the hydrostatic equilibrium between ice and the oceanic water
$\mathcal{F}$	A function such that $\mathcal{F} = 0$ represents the grounding lines or coastlines
$\mathcal{G}$	Mask of the grounded portions of [..[268]]an ice sheet
H	Ice thickness
ΔH	Change in ice thickness over the period Δt
H_0	[..[269]]floatation height for ice
[..[270]][..[271]]H_F	Ice height above [..[272]]floatation (HAF) defined for grounnded ice
ΔH_F	Change in [..[273]]HAF over the period Δt
ΔH_M	A component of ΔH_S that modulates both the mass and volume of the ocean over the period Δt
ΔH_S	[..[274]]A new field for estimating the sea-level contribution of ice sheets over the period Δt
[..[275]]ΔH_V	[..[276]]A component of ΔH_S that can only modulate the ocean volume over the period Δt
[..[277]]$\mathcal{I}$	[..[278]]A globally distributed system of ice domains
[..[279]]$\mathcal{L}$	[..[280]]A globally distributed system of land domains

[262] removed: ice sheet modeling community considers $\Delta H_S(\omega, \Delta t)$ rather than $\Delta H_F(\omega, \Delta t)$
[263] removed: sea level
[264] removed: analyses
[265] removed: Bedrock
[266] removed: defined
[267] removed: represents a grounding line or coastline
[268] removed: the
[269] removed: Flotation height of grounded
[270] removed: H_C
[271] removed: Ice thickness at a coastline or a grounding line
[272] removed: flotation
[273] removed: (ice) height above flotation
[274] removed: Change in ice thickness that directly contributes to sea level
[275] removed: $\mathcal{I}$
[276] removed: Ice sheet domain
[277] removed: $\mathcal{L}$
[278] removed: Land domain
[279] removed: $\Delta\mathcal{L}$
[280] removed: Change in land domain over the period Δt

ΔM	Change in mass per unit area on the solid Earth surface over the period Δt
$\mathcal{O}$	[..[281]]The global ocean domain
[..[282]]ΔR	[..[283]]Non-steric component of the relative sea level [..[284]][..[285]][..[286]](RSL) change over the period Δt
ΔR_C^I	[..[287]][..[288]][..[289]]A component of ΔR due to the contemporary ice-sheet mass change
ΔR_C^L	[..[290]]A component of ΔR due to the contemporary change in far-field ice and land water storage
ΔR_P^I	[..[291]]A component of ΔR due to the past ice-sheet mass change
ΔR_P^L	[..[292]]A component of ΔR due to the past change in far-field ice and land water storage
[..[293]]ΔR_O	[..[294]]A component of ΔR due to other non-steric processes
[..[295]][..[296]]ρ_i	Density of ice
ρ_o	Mean density of the [..[297]]oceanic water
ρ_w	Density of [..[298]]freshwater
S	[..[299]]Mean sea level (MSL), excluding its steric component
ΔS	Change in [..[300]]S over the period Δt
T	The zero-level set of F that satisfies the hydrostatic equilibrium between ice and the oceanic water
$\mathcal{T}$	The zero-level set of $\mathcal{F}$ that represents the coastlines and grounding lines
t	time
Δt	time [..[301]]period
ω	2-D spatial coordinates[..[302]], geographic (θ, ϕ) or Cartesian [..[303]](x, y), on the planetary surface

[281] removed: Ocean
[282] removed: R
[283] removed: Sea level relative to the bedrock, termed "
[284] removed: "
[285] removed: ΔR
[286] removed: Change in relative sea level
[287] removed: Change in relative sea level over the period Δt due to contemporary ice sheet mass change
[288] removed: $\overline{\Delta R_C^I}$
[289] removed: Change in barystatic sea level over the period Δt due to contemporary ice sheet
[290] removed: Change in relative sea level over the period Δt due to contemporary land water/ice mass change
[291] removed: Change in relative sea level over the period Δt due to past ice sheet
[292] removed: Change in relative sea level over the period Δt due to past land water/ice mass change
[293] removed: ΔR_D
[294] removed: Change in relative sea level over the period Δt due to density related steric change
[295] removed: ΔR_V
[296] removed: Change in relative sea level over the period Δt due to vertical land motion
[297] removed: ocean
[298] removed: fresh water
[299] removed: Sea surface elevation
[300] removed: sea surface elevation
[301] removed: interval
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Code and data availability. The data and code used to produce Figure 3 are available online at https://doi.org/10.7910/DVN/9LUJTD (*Adhikari et al.*, 2020).

Appendix A: Interpretation of $\Delta H_F(\omega,\Delta t)$ and $\Delta H_S(\omega,\Delta t)$

Here we provide an in-depth comparison between the HAF method (equation 9) and ours (equation 10) in light of evolving ice thickness, bedrock elevation and MSL. The latter two fields may be treated collectively in terms of the relative sea level (RSL) which, by definition, is the MSL relative to the bedrock or sea floor. In the following, we consider all plausible scenarios by combining the change in ice thickness, $\Delta H(\omega,\Delta t)$, and relative sea level, $\Delta R(\omega,\Delta t)$, over the period Δt.

A1 Where ice remains grounded at both times t and $t+\Delta t$

In our method, all of $\Delta H(\omega,\Delta t)$ irrespective to $\Delta R(\omega,\Delta t)$ contributes to sea-level change by modulating both the mass and volume of the oceanic water (see the first term on the right-side of equation 11). The same is true for the HAF method as long as ice remains grounded on the bedrock whose elevation is at or above MSL at both times t and $t+\Delta t$, in which case $\Delta H_F(\omega,\Delta t)=\Delta H(\omega,\Delta t)$. There is nonetheless a minor density-related difference between the two methods: our method evaluates the freshwater equivalent height $[\rho_i/\rho_w]\Delta H(\omega,\Delta t)$ whereas the HAF method generally evaluates the oceanic-water equivalent height $[\rho_i/\rho_o]\Delta H(\omega,\Delta t)$. As a result, the HAF method systematically underestimates the amplitude of the global-mean sea-level (GMSL) change by about 2 to 3%.

If the ice is grounded on the marine bedrock (whose elevation is below MSL) at least at time t or $t+\Delta t$, the HAF method generally yields incorrect solution in addition to the density-related error noted above. In this case, $\Delta H_F(\omega,\Delta t)\neq\Delta H(\omega,\Delta t)$ generally holds true, and depending upon the relative amplitudes and signs of $\Delta H(\omega,\Delta t)$ and $\Delta R(\omega,\Delta t)$, the HAF method may over- or under-predict the sea-level contribution compared to our method. Two special cases are worth mentioning:

- Case A1.1: $\Delta R(\omega,\Delta t)=0$. When the effect of evolving RSL is not considered, we find that $\Delta H_F(\omega,\Delta t)=\Delta H(\omega,\Delta t)$ and, consequently, the two methods are equivalent.
- Case A1.2: $\Delta H(\omega,\Delta t)=0$. When the thickness of grounded ice does not change, the HAF method may incorrectly predict non-zero sea-level contribution in cases when $\Delta R(\omega,\Delta t)\neq 0$ (see Figure A1a). In this case, the HAF method systematically overestimates (underestimates) GMSL change when $\Delta R(\omega,\Delta t)$ is greater (less) than zero.

A2 Where ice transitions from grounded to floating, or the reverse, over the period Δt

Here the working principle of both methods is same and as follows. We derive the potential sea level (PSL) contributions of ice thicknesses at time t and $t+\Delta t$. We then compute their difference to derive the actual sea level (ASL) contribution

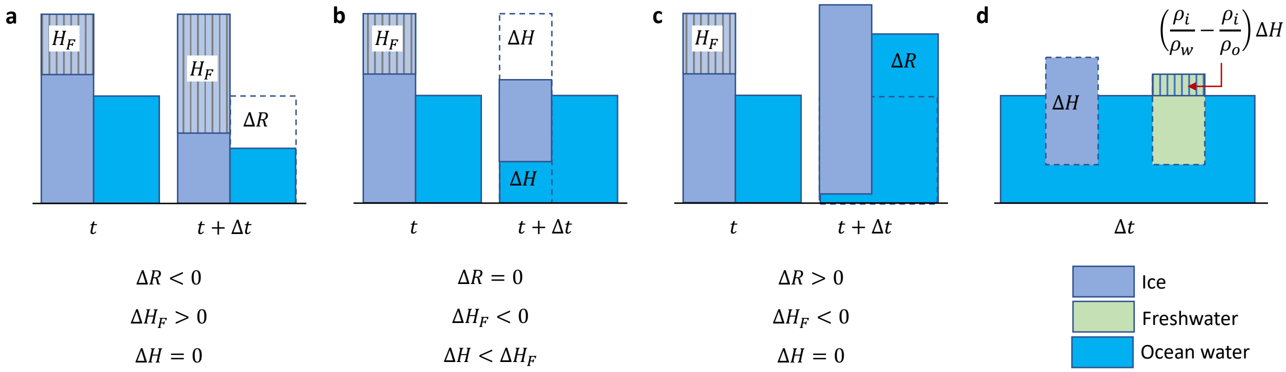

Figure A1. **Scenarios of ice-thickness and RSL change and sea-level contributions. (a)** Since the column of ice remains grounded at both times and its thickness does not change over the period, the ice column does not contribute to GMSL change. The HAF method incorrectly predicts GMSL drop because of a positive value of ΔH_F in response to the imposed drop in RSL. **(b)** For the grounded ice to float in the case of fixed RSL, it must thin sufficiently, such that $\Delta H < \Delta H_F < 0$ holds true, contributing to the GMSL rise. **(c)** Significant rise in RSL may cause the grounded ice to float even if its thickness does not change. As a result, the column of ice contributes to the GMSL rise. **(d)** Melting of the floating ice produces freshwater that occupies slightly larger volume than that of the ocean water that was replaced by the ice. This excess volume, the hatched portion of the freshwater column, causes the GMSL to rise. Sketches are not to scale.

over the period Δt. We may define PSL and ASL in terms of freshwater equivalent height as follows (see Figure A1):

$$\mathrm{PSL}(t) = \frac{\rho_i}{\rho_w} H_F(t) + \left(\frac{\rho_i}{\rho_w} - \frac{\rho_i}{\rho_o}\right)\Big[H(t) - H_F(t)\Big], \tag{A1}$$

$$\mathrm{PSL}(t+\Delta t) = \frac{\rho_i}{\rho_w} H_F(t+\Delta t) + \left(\frac{\rho_i}{\rho_w} - \frac{\rho_i}{\rho_o}\right)\Big[H(t) + \Delta H(\Delta t) - H_F(t+\Delta t)\Big], \tag{A2}$$

$$\mathrm{ASL}(\Delta t) = \frac{\rho_i}{\rho_w} \Delta H_F(\Delta t) + \left(\frac{\rho_i}{\rho_w} - \frac{\rho_i}{\rho_o}\right)\Big[\Delta H(\Delta t) - \Delta H_F(\Delta t)\Big]. \tag{A3}$$

Ice equivalent height of the terms appearing on the right-side of equation (A3) are reported in the main text: the second term on the right-side of equation (11) and the term on the right-side of equation (12), respectively. Since the HAF method deals with the oceanic water density, rather than the freshwater density, both PSL and ASL in this method can be deduced from the above equations by replacing ρ_i/ρ_w by ρ_i/ρ_o. The second terms in these equations vanish, and the ASL is given in terms of oceanic water equivalent height by $[\rho_i/\rho_o]\Delta H_F(\Delta t)$ whose ice equivalent height is reported in the main text (equation 9).

When ice transitions from grounded to floating, $H_F(t+\Delta t) = 0$ and the first term appearing in equation (A3) always takes a negative value. Three distinct scenarios of evolving ice thickness and RSL may be of interest:

- Case A2.1: $\Delta R(\omega, \Delta t) \le 0$ and $\Delta H(\omega, \Delta t) < 0$. In this case, the only condition for the grounded ice to float is through its sufficient thinning such that $\Delta H(\omega, \Delta t) < \Delta H_F(\omega, \Delta t) < 0$ (see Figure A1b). Both terms appearing in equation (A3) take negative values, causing the GMSL to rise.

– Case A2.2: $\Delta R(\omega,\Delta t) > 0$ and $\Delta H(\omega,\Delta t) = 0$. Here the condition $\Delta H_F(\omega,\Delta t) < \Delta H(\omega,\Delta t) = 0$ holds true (see Figure A1c). Since the first term appearing in equation (A3) is about 97% larger in magnitude than the second term (which takes a positive value), it causes the GMSL to rise. In other words, the externally forced RSL rise causes the ice to contribute to GMSL rise even though its thickness does not change.

– Case A2.3: $\Delta R(\omega,\Delta t) > 0$ and $\Delta H(\omega,\Delta t) \neq 0$. Only when $\Delta H(\omega,\Delta t) > 0$ and its amplitude is significantly larger (by about a factor of 35) than that of $\Delta H_F(\omega,\Delta t)$, the second term appearing in equation (A3) that takes a positive value dominates and causes the GMSL to fall. Otherwise, the GMSL rises even when ice thickens over the period Δt.

When ice transitions from floating to grounded, $H_F(t) = 0$ and the first term appearing in equation (A3) always takes a positive value. Three distinct scenarios of evolving ice thickness and RSL may be of interest:

– Case A2.4: $\Delta R(\omega,\Delta t) \geq 0$ and $\Delta H(\omega,\Delta t) > 0$. In this case, the only condition for the floating ice to be grounded is through its sufficient thickening such that $0 < \Delta H_F(\omega,\Delta t) < \Delta H(\omega,\Delta t)$. Both terms appearing in equation (A3) take positive values, causing the GMSL to fall.

– Case A2.5: $\Delta R(\omega,\Delta t) < 0$ and $\Delta H(\omega,\Delta t) = 0$. Here the condition that $\Delta H(\omega,\Delta t) = 0 < \Delta H_F(\omega,\Delta t)$ holds true. Since the first term appearing in equation (A3) is about 97% larger in magnitude than the second term (which takes a negative value), it causes the GMSL to fall. In other words, the externally forced RSL drop causes the ice to further contribute to GMSL drop even though its thickness does not change.

– Case A2.6: $\Delta R(\omega,\Delta t) < 0$ and $\Delta H(\omega,\Delta t) \neq 0$. Only when $\Delta H(\omega,\Delta t) < 0$ and its amplitude is significantly larger (by about a factor of 35) than that of $\Delta H_F(\omega,\Delta t)$, the second term appearing in equation (A3) that takes a negative value dominates and causes the GMSL to rise. Otherwise, the GMSL falls even when ice thins over the period Δt.

A3 Where ice remains floating at both times t and $t + \Delta t$

We may evaluate PSL and ASL contributions from this region based on equations (A1)-(A3). Since $H_F = 0$ at both times t and $t+\Delta t$, the evaluation of the sea-level contribution does not depend on the evolving RSL. In this scenario, the ASL is given in terms of freshwater equivalent height by $[\rho_i/\rho_w - \rho_i/\rho_o]\Delta H(\omega,\Delta t)$ whose ice equivalent height can be deduced from equation (12) by setting $\Delta H_F(\omega,\Delta t) = 0$ (see Figure A1d). When the ice thins (thickens), it causes the GMSL to rise (fall) by modulating the volume (not mass) of the oceanic water. The ASL contribution in the HAF method can be deduced by replacing ρ_i/ρ_w by ρ_i/ρ_o and is effectively zero and, therefore, does not appear explicitly in equation (9). This suggests that the HAF method systematically underestimates the amplitude of GMSL change.

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