Response letter to reviewers for "Modelling steady states and the transient response of debris-covered glaciers" by J.Ferguson and A.Vieli

Dear Dr. Zekollari,

Thank you for handling the review process for our manuscript. We have addressed all the reviewers' comments below.

Best wishes,
James Ferguson and Andreas Vieli

1 Summary

We would like to thank both reviewers for two very helpful reviews. In order to address the many valuable suggestions, feedback, and proposed improvements, we have substantially rewritten significant portions of the manuscript, added additional figures, cited additional literature, and included additional animations in the supplementary material. In what follows, we show first the reviewer's comments in black and then our response in blue. At the end of this document, we provide a marked-up version of the manuscript which shows the changes that were made between the original and the newly revised.

2 Response to Reviewer 1: Leif Anderson

Major comments

This work is emerging from a dialogue between other DCG modeling and observational efforts. As the paper reads now, that dialogue is not yet properly developed. Often times previous work immediately relevant to points being developed in this manuscript are cited (or not) as having worked in the general topic at the start of a paragraph. But the insights gained from past efforts are not yet allowed to be in dialogue with the results from this work.

This partly means that a stronger foundation should be laid in the manuscript (in the introduction) with regards to what insight has already been gained from previous work and how this effort builds off of those previous efforts. This also means that the writing does not clearly delineate between conclusions made by previous work and the new findings here (especially in the discussion section and the toe parameterization appendix). I raise this point not to diminish the important contributions made here. On the contrary engaging past work with the new insights will highlight the work done here more clearly and make for an even more valuable contribution to the community.

We rewrote the introduction to better incorporate previous work into the manuscript both for context and to explain what is new (and what is not) in this paper. See the line-by-line comments for more details.

Because the model developed in Anderson and Anderson (2016) and the one presented here very similar I think it is appropriate to be a bit more explicit about how the models are different. As it reads now it is not always clear what was originally derived by A & A 2016 and what is new here. A bit more care should be taken when discussing the differences between the toe parameterization approaches. It is unclear how different the approach derived here is different from the range of parameterizations explored in A & A 2016. A more explicit statements about the toe method will allow the method developed here to be reproduced.

We rewrote both the Methods section and Appendix A to clarify the details of the boundary condition used at the terminus and how it differs from that used in Anderson and Anderson (2016). See the line-by-line comments for more details.

Figures in general are well composed, though a few more simple figures will expand accessibility to a broader audience. It would be helpful to see some modeled mass balance profiles plotted in the main paper since they are essential for showing the difference between debris-free and debris- covered glaciers. And the added effect of cryokarst on DCG mass balance profiles.

We included additional surface mass balance plots in the manuscript (Fig. 2b and 2e, Fig B1d) and have also included some animations in the supplements for illustrating the differences in transient response. Please see the line-by-line comments for more details.

A figure or schematic showing how the cryokarst formulation is implemented in the model would be helpful. Maybe driving stress could be plotted and an example of the effect of the cryokarst features on the mass balance profile would aid the reader in understanding the new parameterization and its effect on the glacier. As the manuscript is now I have trouble visualizing the pattern of cryokarst features on the glacier at any one time. I ultimately think this parameterization is an important and useful contribution and showing a bit more how it works will only benefit the manuscript and the community. This is an important contribution! The authors might also consider shifting from the use of ?cryokarst,? as ice cliffs themselves are not necessarily the result of the collapse of englacial tunnels.

We have included additional figures in the manuscript (Fig. 1 and Fig. 8) that aid in visualizing the link between driving stress and surface mass balance in the parameterization of cryokarst features, as discussed in detail below.

The authors might consider adjusting the use of the term ‘white noise’ as it refers to the climate forcing. In terms of climate, white noise forcing almost always refers to year-to-year variability in the climate. The climate forcing applied here is actually red noise because the timestep is 100 years and there is therefore autocorrelation from year-to-year. This manuscript uses persistent climate changes to fore the model. I am not actually sure of the correct phrasing but maybe climate changes that are randomly sampled from a normal distribution would interface better with previous work. Or just the response of a DCG to variable climate?

We have changed this wording in the text to ‘random climate forcing’ instead.

The discussion section would be improved with a more thorough discussion of the uniform englacial debris concentration assumption. It is very important to consider what a steady, uniform englacial debris concentration implies for headwall erosion rates when glacier geometry is changing. I have included/expanded on some points lower down.

We agree – this is an important point. We have addressed this by included a discussion of the issue in the section on Model Limitations. See below for further details.

The manuscript in general should be streamlined and repeated statements should be cut out. Individual sentences are well composed, but I find myself a bit overwhelmed at times in the text. The modeling results section will benefit the most from some textual work. The number of experiments and the changing focus from various parts of the DCG system make it hard to follow. Anything that can be done to simplify and distill the description of these experiments will help the reader.

We attempted to better streamline the text and to remove any repetitions. We further provided an overview of the experiments at the end of the Introduction as well as a more detailed summary of the experiments at the start of Section 3. In addition, we moved Section 3.6 (Terminus behaviour during transient response) to the Discussion, as it does not present any new experiments but instead consists of further analysis of the step change experiments with regards to the surface evolution equation.

Line-by-line comments

Line 13. "as is also observed in remote sensing" this could be a little clearer. Maybe just remove 'in remote sensing'

Agreed - we removed it.

Line 40. "the relatively recent advent of remote sensing data" consider re-phrasing here.

Agreed - we changed this to "the lack of long-term remote sensing data..."

Line 44. The introduction is a bit parsimonious towards previous efforts. What are the contributions of previous debris-covered glacier models? What have we learn up to now? By setting the stage more the novel and interesting contributions of this work, which there are many, will be better highlighted.

Agreed. We modifed the introduction to include more references, a clearer description of what advances were made previously, and a better context for our contribution in the following way (starting at Line 44 in the revised manuscript):

> Given the paucity of longterm data it is therefore essential to use advanced numerical models in order to investigate the role of glacier dynamics on glacier evolution and mass loss, allowing for the study of interacting processes over longer timeframes. Recent progress with numerical simulations of debris-covered glaciers include: Konrad and Humphrey (2000), where an early steady-state model of debris-covered glaciers was developed; Vacco et al. (2010) and Menounos et al. (2013), both of which studied the effect of a rock avalanche on glacier evolution using a coupled debris transport and ice dynamics model; Banerjee and Shankar (2013), which suggested that the transient response of a debris-covered glacier to changes in climate has two distinct timescales; Rowan et al. (2015), which was the first modern model of coupled debris-ice dynamics to study the longterm evolution of a debris-covered glacier; and Wirbel et al. (2018), which tested both 2-D and 3-D advective debris transport using a full-Stokes solver for the ice dynamics. Perhaps the most significant modelling study to date is Anderson and Anderson (2016), where certain technical issues are addressed in detail for the first time, such as how to handle both the boundary condition at the glacier terminus and the possibility of a variable debris source in the accumulation area. The body of work that uses essentially the same model has examined diverse topics relating to the feedbacks that exist between debris flux, debris thickness patterns, and steady-state glacier extent; has also studied the relationship between debris-covered glaciers and rock glaciers; and has been used to explore the processes that govern the age of ice-cored moraines (Anderson and Anderson, 2016; Crump et al., 2017; Anderson and Anderson, 2018; Anderson et al., 2018).
>
> However, to date no study has used a coupled ice flow-debris transport model to systematically and in detail study the transient response and characteristic response times of a debris-covered glacier. A better understanding of how a debris-covered glacier responds to changes in climate, and what role the debris concentration and the prevalence of cryokarst features play in determining the magnitude of this response, is critical to predicting how today's debris-covered glaciers will evolve as the Earth's climate changes. Therefore, this study aims to fill the gap by investigating the difference in transient response of debris-covered glaciers from their debris-free counterparts. In particular, we: (1) examine how debris cover changes the transient response of an idealized glacier to step changes in climate, quantifying both the volume and length response; (2) examine the response to a fluctuating climate

> signal on the long-term evolution of am idealized debris-covered glacier as a function of debris concentration; and (3) examine the impact on mass loss and surface evolution when cryokarst features are included, quantifying this impact as a function of cryokarst area.

Line 46 and 47. Recognizing that you have cited several of our papers here, but there are additional transient simulations of debris-covered glaciers responding to climate change using essentially the same model as A &A 2016 in Crump et al., 2017 and Anderson et al., 2018. The references are fully written at the end of the manuscript. Also Anderson et al., 2019a does not include any model simulations.

The references are included / corrected as noted above.

Line 57-59. The way this sentence is written it is fuzzy what the actual differences are between the models. Is it the same besides the differences listed here? If not it would be good to make it a bit more clear what the other differences are in the methods section.

We go into more detail on the model differences in the Methods section, as discussed below.

Line 63-65. This is a great way to support the use of SIA!

Thanks.

Line 90. Reading this sentence makes it seem like this melt formulation (equation 6) was derived by Nicholson and Benn (2006) but it was actually derived in Anderson and Anderson (2016). It is appropriate to cite that work here.

Agreed, we changed the reference to reflect this.

Line 99-100. How is it different? It seems to be nearly the same. It might be more appropriate to state that you 'improve upon' or 'start from.' How is this different from Anderson and Anderson (2016)? Explicitly stating what they do will make it more clear what the new contributions of this work are.

Also how was your value of D0 chosen?

We added an explanation of the choice of D_0 in the Methods (Line 109). To address the others concerns, we completely rewrote this section in the Methods as follows (starting at Line 112):

> The choice of boundary condition at the terminus is of critical importance, since the rate at which the supraglacial debris covering the ablation zone leaves the glacier significantly affects the glacier extent (Anderson and Anderson, 2016) and if this is handled incorrectly, it can lead to runaway glacier growth (Konrad and Humphrey, 2000). To ensure that the boundary condition makes sense, it should be consistent with observations and also grid size independent, since the laws of physics should not depend on the choice of discretization.
>
> With these requirements in mind, we set the boundary condition such that the debris leaves the system via a terminal ice cliff, as typically observed at termini of debris covered glaciers (Ogilvie, 1904, see Fig A1 in the appendix). This can be achieved most easily through an adjustment of the surface mass balance, which we adjust at the point where the ice reaches a critical thickness H^*. All debris melted out or transported past this point is assumed to slide off of the glacier relatively quickly and therefore does not play a role in the surface mass balance. This implies that the glacier will always have a small debris-free cliff area at the terminus with clean ice melt. Therefore near the terminus, the surface mass balance a is given by
>
> $$a = \begin{cases} \tilde{a}\frac{D_0}{D_0+D}, & \text{for } x < x^* \\ \tilde{a}, & \text{for } x \geq x^* \end{cases}, \qquad (1)$$

> where as above, $\tilde{a}$ is the debris-free surface mass balance, and x^* is the location at which the ice thickness $H = H^*$. It is important to note here that although H^* is a parameter chosen to reflect the height at which the terminal ice cliff forms, we accounted for the steep surface gradient of the SIA at the terminus by taking a higher value of H^* than normally observed. In addition, we accounted for the fact that the terminal ice velocity in SIA goes zero, which is not physically realistic, by taking an averaged velocity over several ice thicknesses near the terminus when computing the debris transport.
>
> We note that although the terminal boundary condition used here is similar to the one implemented in Anderson and Anderson (2016), since in both cases the glacier ends with a mall debris-free terminus, our choice of boundary condition has the subtle but possibly important feature of being essentially independent of grid size. Further details of the boundary condition, including the interpolation scheme between grid points and convergence tests for different grid sizes, is found in Appendix A.

Line 102. This sentence could be simplified right now it is a little more complicated than it needs to be.

We simplified this (Line 134) by writing instead: "For some experiments, we attempt to include the effects of melt enhancement from cryokarst features."

Lines 103-105. Your case would be stronger if you develop the justification for the parameterization a bit better here. It seems to me that there are some more citations here for work that has linked ice flow with these features. Like Kraaijenbrink et al., 2016 and/or Watson et al., 2017. It is a clever approach though.

Lines 105-110. It would be helpful for the reader to include the equation for driving stress here. That way readers can connect to the fact that driving stress scales with ice thickness and surface slope. Is there a physical mechanism why cryokarst features might follow driving stress? Would be good to include that.

We have added the additional references you have suggested. We have also included the equation for driving stress and an additional figure depicting the relationship between driving stress and cryokarst fraction.

Line 122. Just need to clarify what the CFL condition is here as this is the first time this acronym shows up in the text.

Agreed – we clarified this in the text (Line 158).

Section 3 Modelling results

This section is rather difficult to follow and I am quickly overwhelmed by the number of simulations and how quickly the writing moves between them.

Lines 132-135. It could be beneficial to include a what you refer to as a 'baseline' case (with base debris concentration) so the reader has a single simulation to compare the others to. Reading below it is easy to get lost in all of the simulations. Maybe this baseline case could be bolded in the figures below?

We do not fully agree with this point. However, we also added a short overview of the experiments at the end of the Introduction (Lines 65-69), as well as a detailed summary of the experiments at the start of Section 3 (Lines 174-182). We also moved the subsection on transient terminus behaviour (originally Subsection 3.6) to the Discussion (now Subsection 4.1). While there are a number of simulations and experiments, we think that the modifications we have made to the text and additionally the outline in Table 1, together with the section headings, are enough to help the reader navigate through them.

Figure 1. Nice figure. What part of the glacier is covered with debris? How does that relate to the ELA. Perhaps adding these would be helpful to bring the various components of the model together for the reader.

We have modified the figure to include surface mass balance plots, as suggested above, and we more clearly denoted the ELA position. Note this is now Fig. 2 in the revised manuscript.

Line 157. need a hyphen between 'debris' and 'covered'

Agreed, we corrected this.

Section 3.2 I think these are all important interesting simulations. This section would be improved though with a bit more synthesis. It is a bit difficult to follow because of the number of different experiments. Maybe more clear topic sentences clearly keying on what each experiment the paragraphs correspond to would help? Or sub-section titles for each experiment?

It might also be that the description moves between simulations using different englacial debris concentrations quickly. Perhaps it would be easier to follow if the descriptions of the experiments use one concentration case?

As noted above, we feel that the experiments are well delineated, especially with the additional changes to the manuscript that we have noted above.

Figure 2 is really a great future!

Lines 223-224. Might be good to have a citation here.

Agreed. We have now cited Benn et al. (2012) here (Line 278 in the revised manuscript).

Figure 4 is also really clear.

Thanks!

Table 4. The table looks very clean but maybe adding in text at the top the definition of each variable again would help the reader follow.

Agreed – we added this.

Figure 6. The introduction of 'Bare ice %' is hard to wrap my head around since it seems to be a new way of describing the cryokarst features. Maybe just label it % of the surface composed of cryokarst features. Consider finding another way to represent the contribution of cryokarst that is more clear.

Good point. We added additional explanation in the text and an additional figures showing the link with driving stress – Fig. 1 in the Methods and Fig. 8 in the Results.

Figure 7. You might consider moving this figure into the supplemental and just describing the effect of cryokarst on long term evolution in the text. Just so the reader does not feel overwhelmed.

We feel the inclusion of cryokarst is an essential and new aspect of this work and in particular, showing the impact for a variable climate is central to the manuscript. Hence, we have left this figure in the main body of the paper.

4.1 Debris-covered glacier memory

This section highlights some interesting findings. The section, though, would be improved by stating what past studies have concluded related to this topic and then emphasizing showing how your results/conclusions differ. This is especially relevant to interface a bit with past transient glacier model simulations. Do they show a similar effect that support your discussion here?

We have added more details of previous work here in the first paragraph of Section 4.2, which we rewrote as follows (starting on Line 354):

> We showed that the memory of a debris-covered glacier is selective, exhibiting an effective hysteresis, with periods of relatively cold climate having a sustained effect on the volume and in particular on the length. Strictly speaking this is not a true hysteresis since if the glacier is allowed a lengthy relaxation period of several centuries, the resulting equilibrium is independent of history. Previous numerical simulations of the transient response of debris-covered glaciers focused only on the effects of sudden debris input in the form of an avalanche (Vacco et al., 2010; Menounos et al., 2013). This one-time debris input leads to an advance in glacier extent and foreshadows the results of our study, where a constant debris source and changing climate forcing gives rise to a more complex response.

Lines 299-302. This paragraph would benefit from a look at the past literature on the subject, as this point has been raised previously. Additionally Clark et al., 1994 et al. also discuss this effect.

We added a reference to the paper by Clark et al., 1994.

Line 347-351. There are studies that do connect ice cliff occurrence to ice dynamics, including Benn et al., 2012; Kraaijenbrink et al., 2016, .

We already cite Benn et al., 2012 immediate above this. We added the reference to Kraaijenbrink et al., 2016. However, it does not change the point we make here – that the onset of cryokarst features is quantitatively not well (or not very explicitly) linked to observations of glacier dynamics, hence the need for a model that links it to evolving dynamics.

Section 4.4 Steady state velocity–debris thickness relationship

I think this is a very interesting section. I do think it would be improved if it interfaced with the previous literature on the topic. Especially emphasizing how this work has expanded on those previous insights.

Lines 392-393. A & A 2018 also do a compilation of 8-10 glaciers that show that debris thickness patterns follow this same pattern. These observed profiles can also be referenced with the Mlg study as well.

We have added these references and we have rewritten the text to address these points in Subsection 4.5, and the ones that follow below, in the following way (starting on Line 474):

> Note that we have not assumed SIA or any other ice flow model here, so there is no issue with vanishing velocity at the terminus, although even in that case one can require nonzero velocity at the terminus as in our debris transport model. Equation (14) is similar to an equation derived by Anderson and Anderson (2018) but with the important difference that we have allowed variable velocity in this derivation, rather than assuming a constant velocity along the entire glacier.
>
> Consistent with Anderson and Anderson (2018), Eq. (14) suggests that debris thickens towards the terminus as the velocity necessarily decreases. An interesting consequence of our formulation makes use of the fact that we have allowed for a variable velocity. In the case of a debris-covered glacier near steady state with an approximately uniform debris concentration, one can infer this concentration by measuring the representative velocity and debris thickness at the terminus and the velocity and ice thickness at the emergence location.

Line 395. “It is natural to ask to what extent the debris thickness profile depends on the ice flow model and the debris transport model used. That question can be answered for the steady state case without assuming anything about the ice flow and considering only?conservation of mass.”

It is unclear how the statement above relates to the rest of the paragraph. This seems like an interesting topic though.

This statement is directly connected to what follows, as we do not assume an ice flow model. The text was rewritten as shown above to address this point.

Equation 11 is very similar to one derived by Anderson and Anderson (2018) who follow a similar approach. It seems appropriate to cite that you are following that line of logic or interface with that work here.

As shown above, the text in Subsection 4.5 was rewritten to make the connection to the work in A & A 2018 more clear. However, we note that our approach has some subtle but important differences, in that we do not assume velocity is constant (as they do) but rather start with conservation of mass.

Line 404. How is it possible that there is ice flow at the terminus that is not 0? The SIA is based solely on internal deformation which is requires that ice thickness is larger than 0 which is not the case at the terminus. Just a bit of clarification will help.

This was clarified in the text as shown above.

Line 409. There is an interesting discussion to be had between the insights from A &A 2018 (Fig. 9) and what is discussed in this paragraph, especially regarding the zone of englacial debris emergence as described there. How does this discussion mesh/build off of with what was discussed in A &A 2018?

As we have already made the connection between this manuscript and A & A 2018 clearer, we leave it to the interested reader to investigate the very interesting points raised in that study.

4.5 Model limitations

Lines 421-425. This is a repeat from a point made above. My sense is that this only needs to be stated once.

Good point– we removed this reference here.

Lines 426 to 431. The authors should discuss the implications of the assumption of uniform englacial debris concentration further. From my view it seems more fair to say ‘that the effect of a uniform englacial debris concentration should be explored further.’ I mention this because there are a number of simplifications that go into this assumption.

I think its is a reasonable first order approach, but this means the entire ablation area will be covered with debris.

It needs to be added here that different ice flow paths will change the englacial debris concentration even with a uniform input of debris everywhere on the glacier. It is really impossible to have a glacier with a uniform englacial debris concentration because of the straining of ice and the inevitable variability of debris input (in space and time) to the glacier surface.

One additional point that should be discussed is how applying a uniform englacial debris concentration relates to headwall erosion rates. If the headwall erosion rate is constant in time then as a glacier gets bigger the englacial debris concentration by definition must become smaller.

This effect is not included in this model. By keeping the englacial debris concentration uniform and steady there must be a requisite increase in headwall erosion rate as the glacier grows in size. If the glacier doubles in size then the headwall erosion rate would need to also double. I think this is simply an underlying assumption of this approach that should be clear to the reader and if possible should be quantified and placed in the supplemental material.

This is an important point – we agreed completely. To address this, we included a discussion of this point in Subsection 4.6 in the following way (beginning at Line 497):

> Our debris transport model did not resolve the englacial transport of debris and assumed that all debris immediately starts melting out at the ELA. This approximation is a reasonable first order approach, especially for debris that is deposited onto the glacier surface near the effective ELA or during transient simulations. This assumption works less well for glaciers whose debris deposition zone is far above the ELA, since the resulting emergence location will be located much further down glacier. It will likely not change any of our qualitative results but just shift the debris emergence location downstream from the ELA, resulting in the same general pattern of thickening of debris along the glacier and therefore essentially the same transient behaviour. It is also not an accurate method for determining the steady-state glacier extent for different climate scenarios, since due to the assumption of constant debris concentration, the total debris input into the system necessarily scales with the size of the accumulation area and accumulation rate. This is a significant shortcoming of the model, since it results in larger glaciers associated with colder climates possessing stronger debris source terms, which is not supported by observations (Banerjee and Wani, 2018). However, the general conclusions with regard to transient behaviour and delay in response of debris-covered glaciers still holds. Further, since the changes in debris input stay in general relatively small and since the glacier does not exhibit a response until the additional debris has worked its way from the accumulation zone to the surface of the ablation zone, which can take decades to centuries, this becomes more important for final steady-state length and volume changes and is not expected to appreciably change the transient response of a debris-covered glacier far from equilibrium. To address this issue a model with the capability to track englacial debris transport would be required, which is beyond the scope of this study and is computationally much more expensive.

Line 427. missing period.

Thanks – we corrected this.

Line 434. It should be made explicitly clear what the differences are between the toe condition applied here and the one presented in A and A (2016) in the main text. Is it simply a modification of the approach presented in A and A 2016? Are they not also quantitatively similar? See the text regarding the toe parameterization in the Appendix below.

Agreed. We addressed this point already by modifications to the Methods and Appendix A.

Lines 436-437. I could not find where this statement is discussed in the appendix. Is there a citation that notes this or is it a new observation? I am unaware of this effect.

We have clarified this in Appendix A (starting at Line 557).

Lines 437-438. The way this paragraph is written implies that A & A 2016's approach would not capture the effects of a stagnating tongue. Is this actually true? Looking at the other publications after A & A 2016 like Crump et al., 2017 and Anderson et al., 2018 the length change curves are similar to those presented here and based on the toe parameterizations the dynamics should be represented similarly to the work here.

We do not believe this paragraph implies anything about the limitations of the boundary condition used in A & A 2016. The context of this paragraph is on limitations of our model, not the model used in any other studies.

Appendix A and the toe parameterization in general

It is a substantial effort to develop a toe parameterization and any improvements on the exploration from A & A 2016 are welcome, important, and vital for the future development of debris-covered glacier models. It is also important that the method presented here also be reproducible. It would be good for the authors to describe the sub-grid interpolation scheme in detail. What shape/formula do you assume? What H* terms are viable?

Lines 465-475. It seems like there should also be some discussion of how this formulation relates to the original terminal condition described by A and A 2016. How are the approaches different?

A & A 2016 explore a range of possibilities for the terminal parameterization which the toe is drowned in debris because it cannot leave the glacier and also a case in which an ice cliff persists at the terminus and debris is effectively rapidly removed from the glacier. See Figure B1 and section 5.2 in A & A 2016. Ultimately, A & A 2016 use a scheme where debris is removed based on the bare ice melt rate which is basically the same as is implemented here. I think it would benefit the readership to have a more complete description of the differences between the two schemes and how different they actually are.

It seems that the parameterization presented here is a smart approach. Despite the way the text is written it seems the approach follows the A & A 2016 formulation closely and fits almost within the range of parameters explored there. The new approach presented here essentially sets no limit on the d_flux term from A & A 2016, and the formulation presented here would be close to the c =10 case in Figure B1 from A & A 2016 for debris removal. The main difference is that this approach keeps the ice cliff backwasting at the bare ice melt rate despite the removal of more debris than that backwasting of the ice cliff actually would allow.

The down side of the approach presented here is that the removal of debris from the glacier is not necessarily physically representative of the process of debris removal at the terminus of real debris- covered glaciers.

The A & A 2016 scheme honors that the removal of debris from the toe in the ice cliff case is determined by the backwasting rate, but this in turn leads to a greater grid scale dependence than the scheme presented here. From my view the benefit of either of these schemes depends on the decision to value either grid-scale dependence or the physical representativeness of debris removal from the toe.

Either way a more nuanced description of this toe scheme and how it relates to the work of A & A 2016 is needed to ensure the community can follow these methodological differences.

We have added additional details in both the Methods and Appendix A to address these points, more clearing describing our boundary condition and directly referencing the work of A & A 2016. While we think it is outside the scope of this manuscript to do a full comparison of the two approaches, it is likely that they behave in a fairly similar way. We leave a detailed comparison for possible future work and state in the Appendix these words (starting at Line 574):

> A similar boundary condition was used in Anderson and Anderson (2016), where a debris-free region was also employed at the terminus. In their approach, the size of the terminal ice cliff depends directly on the grid spacing but the debris flux can be varied. In contrast, our terminal ice cliff is roughly independent of grid size, since it depends on the critical ice thickness H^*, but the debris flux is fixed by the local ice velocity.

Figure B1. It seems like this figure should plot the mass balance curve with time, since the the cryokarst parameterization adjusts that directly. I would also like this figure with the SMB curves included in the main manuscript since the cryokarst parameterization is a central, new contribution of this work.

Agreed – we added a SMB plot in the main text Fig. 8, and another here in Fig. B1.

3 Response to Reviewer 2: Fabien Maussion

General comments

At the end of the introduction, you write: ?to date no study has used a coupled ice flow-debris transport model to study in detail the transient response and characteristic response times of a debris-covered glacier. This study aims to fill this gap...?. ?It has never been done before? is not a good motivation for a study, and I think that the paper would gain from clearly stated research questions. In particular, it would help to understand what motivated the model design and the design of the idealized experiments (why this bed profile, why this model design, etc.). Research questions will also help to place the study in the context of previous literature, and prepare the reader to understand what you are trying to achieve with this paper.

Agreed. We have added a clearer and more detailed description of the work that has gone before our study in the Introduction in order to better motivate our approach. We have also made our research aims more precise as follows (starting at Line 64):

> Therefore, this study aims to fill the gap by investigating the difference in transient response of debris-covered glaciers from their debris-free counterparts. In particular, we: (1) examine how debris cover changes the transient response of an idealized glacier to step changes in climate, quantifying both the volume and length response; (2) examine the response to a fluctuating climate signal on the long-term evolution of am idealized debris-covered glacier as a function of debris concentration; and (3) examine the impact on mass loss and surface evolution when cryokarst features are included, quantifying this impact as a function of cryokarst area.

The word ”idealized” does not show up in the title, abstract, or introduction. I think it should be clearly stated much earlier (maybe not in the title, but at least in the abstract).“Numerical modeling” could be understood as “applied to real glacier”.

Good point. We have added the word ‘idealized’ in the Abstract and in the Introduction.

This may be subjective, but I don’t find any of the comparisons with Jóhannesson’s response times informative or useful. Even without debris cover, you can find numerical response times of glaciers which are widely different than the analytical ones, since the e-folding times are highly dependent on parameters such as bed depressions, mass-balance (MB) gradients, etc. (see e.g. Zekollari et al. 2015 or Schuster, 2020 - unpublished thesis work).

While it is certainly true that one can generate widely different response times due to different geometry and/or climate forcing, here we have the same bed geometry (and very similar upstream surface geometry) and the same climate forcing. However, the response is quite different. Since the Jóhannesson time scale is often used to give a general idea of how changing the geometry or forcing changes the response time for debris-free glaciers, we still feel that a comparison here is warranted. The purpose of the Jóhannesson time scale is actually to be able to compare glacier dynamics between different glaciers and it is a useful measure that characterizes the dynamic response.

Your code availability statement (“available upon request”) is against this journal’s data and open science policies: https://www.the-cryosphere.net/policies/data_policy.html. I strongly recommend to make your code available (under a clear license), which will increase the visibility and re-usability of your work.

We have included a version of our code as part of the supplementary material.

Abstract. I'm not very familiar with the debris-covered glacier literature, but I had to search for “cryokarst” online

We have clarified this in the Introduction by writing (starting on Line 32):

> The formation of features such as ice cliffs and supraglacial ponds, which we refer to collectively here as cryokarst features, has been suggested as a potential explanation for this anomalous thinning and therefore the occurrence of such features and their enhancing effect on surface melt have been intensively studied (Sakai et al., 2000, 2002; Steiner et al., 2015; Buri et al., 2016; Miles et al., 2016).

Abstract. add “idealized numerical simulations”

Agreed – we modified the Abstract to read:

> Due to a lack of observations, here we use numerical modelling to explore the dynamic interactions between debris cover and glacier evolution for an idealized glacier over centennial timescales.

Eq. (1) consider using b instead of a for mass-balance (more common I believe)

We used a instead of b or $\dot{b}$ so as not to confuse with the bed, which is denoted here as $b(x)$.

L72 “for a given a bed elevation” - remove ”a”

Agreed – we fixed this.

L96 having read section 2.1.4 and the appendix, it's still not clear to me how you compute H^* (and I don?t want to check up on Anderson et al 2016). I notice later that H^* is a constant and a model parameter: mention this earlier in the text.

Good point. We adjusted this in the Methods section to clearly state that H^* is a model parameter (Line 125).

L100 specify which appendix.

Good point - we fixed this.

Appendix A despite of your valid attempts to show that this boundary condition may be found in the real-world, I still believe that the ice-free terminus condition is more a model necessity (trick) than a real-world feature. You don't have to change anything in the text here, I just wanted to comment on that.

That is an interesting comment. While we admit that we do not know exactly what the best boundary condition is, we believe our formulation is consistent with a number of observations and importantly it is also grid-size independent. But we are happy to admit that there might be a better way to formulate the boundary condition here. In any case, we added more details in the Methods and in Appendix A which we hope have helped to better motivate the choice.

Sect. 3.1 (steady states) I really had to think twice about how you can reach steady state with such a model. I think that it would help to write more about it. E.g. by saying again that (i) steady state can be reached only because the MB doesn?t go too close to 0 and (ii) that this is only possible by removing debris at the terminus and effectively capping the debris thickness to a reasonable value. You can refer to Fig. S1 in this section (or mention typical values of MB at the terminus in the model) to help understanding.

To address this, we have added the following (Lines 184-188):

> Although in reality, glaciers never attain a true steady state due to a constantly changing climate, equilibrium conditions are useful for theoretical studies because they provide well-understood rest states around which glacier fluctuations can be more easily studied. As for debris-free glaciers, a steady state is defined as the point at which, for a fixed climate, the glacier geometry no longer changes with time. Additionally, in our modelling there is a further requirement that the debris flux entering the glacier also must leave the glacier surface at the terminus.

L155 to our knowledge, Jóhannesson et al, (1989) wrote: "The volume time-scale tau can be computed from the volume differences between two steady-state profiles scaled to the causal mass- balance change", but did not mention the e-folding volume response time (yet). Maybe refer to another paper as well: e.g. Oerlemans (1997) or Jóhannesson (1997)

Good point. We have now cited Oerlemans (2001) here.

L185 volume-area scaling: since it might be unclear to some of your readers, add here that (in your model) area is directly proportional to length

Good point. We added the following line to clarify this:

> Note that for our flowline model, the equivalent scaling law is a relationship between area and length.

L190 is "stagnant" the correct term here? I was confused several times in the manuscript about this, because you seem to use "stagnant" for when the glacier length does not change. Personally, I understand "stagnant" as "ice that is not moving" ($u = 0$). You cannot have "stagnant" ice with your numerical model setup. I would argue for using "stable terminus" in place of "stagnant", or clearly state in the text what you mean with "stagnant". At the very end of the paper there is a sentence going in this direction ("stagnation or more specifically the cessation in local dynamic replacement of ice.").

L277 "stagnated": same here. Is it the correct way to say that? Non-divergence is still happening with u constant and non-zero, i.e. moving ice.

We have clarified our use of the word 'stagnate', which is used in numerous other studies of debris-covered glaciers in the same way as we mean it. by adding this (Line 244):

> Although the velocity is nonzero, this type of terminus is often described as stagnating because dynamic replacement of ice is close to zero.

Section 3.3 "white noise" traditionally, white noise climate should be applied on a year to year basis and the periods of cold and warm climates would occur "naturally", as a result of random sampling. I wonder how this would affect your results. Additionally, I wonder if an annually varying MB would still work with your debris cover formulation, since you don't deal with temporary ice/snow cover on debris as for now.

Agreed - we have changed the wording to "random climate forcing" where appropriate. We have run simulations with annually varying ELA as well and it works fine with our model but whether this is physically realistic is of course open to discussion.

Fig. 5 while this figure carries well your main message, I think that it can be misinterpreted. In particular, the blue line in Fig. 5b gives the impression that the glacier will always grow, i.e. never reach a "steady state" (i.e. an average length around which it oscillates - albeit in a strange, debris covered way). What you could do here is continue the simulation for an additional 5k years (at least) and see what happens. It might have an interesting consequence: the "average length" of a debris covered glacier under a random climate might be longer than the length of the same glacier under constant forcing. I expect the average length to be some- where between the steady state lengths with the two ELAs (although it might even be longer than that, which would be very interesting to discuss further).

We attempted to raise these very points by showing these plots in Fig. 5. There are a number of further points that could be raised here and indeed, further studies one could do with our model to examine the volume and length response given different climate forcing. We added the following to address your last point (Line 273):

> This also suggests that time-averaged length of a debris-covered glacier under random climate forcing will be longer than the steady state length for the equivalent constant climate forcing.

Figure S1 : write that "SS" stands for "steady state" in the legend.

Agreed - we adjusted this, though it is Fig. B1 in the Appendix, not S1 in the supplementary material.

Modelling steady states and the transient response of debris-covered glaciers

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Abstract. Debris-covered glaciers are commonly found in alpine landscapes of high relief and play an increasingly important role in a warming climate. As a result of the insulating effect of supraglacial debris, their response to changes in climate is less direct and their dynamic behaviour more complex than for debris-free glaciers. Due to a lack of observations, here we use numerical modelling to explore the dynamic interactions between debris cover and ~~glacier evolution~~ geometry evolution for an idealized glacier over centennial timescales. The main goal of this study is to understand the effects of debris cover on the glacier's transient response. To do so, we use a numerical model that couples ice flow, debris transport and its insulating effect on surface mass balance and thereby captures dynamic feedbacks that affect the volume and length evolution. In a second step we incorporate the effects of cryokarst features such as ice cliffs and supraglacial ponds on the dynamical behaviour. Our modelling indicates that thick debris cover delays both the volume response and especially the length response to a warming climate signal. Including debris dynamics therefore results in glaciers with extended debris-covered tongues and that tend to advance or stagnate in length in response to a fluctuating climate and hence remember the cold periods more than the warm. However, when including even a relatively small amount of melt enhancing cryokarst features in the model, the length is more responsive to periods of warming and results in substantial mass loss and thinning on debris covered tongues, as is also observed~~in remote sensing.~~.

1 Introduction

Debris-covered glaciers are commonly found in alpine landscapes of high relief, often when a primary source of mass input to the glacier comes from avalanching. Steep headwalls and slopes deliver debris consisting of loose rocks onto the glacier surface, mixed in with ice and snow. This debris becomes entrained in the ice and emerges on the surface further down-glacier in the ablation zone after it is left behind as the ice melts.

A debris-covered glacier is commonly defined as any glacier with a continuous debris cover across its full width for some portion of the glacier (Kirkbride, 2011). For a thin layer of debris, the resulting decrease in surface albedo leads to an elevated melt rate of the underlying ice; however, when the debris cover exceeds a thickness of a few centimetres, it reduces the ablation of the underlying ice (Østrem, 1959; Nicholson and Benn, 2006). For highly debris-covered glaciers, this reduced melt rate leads to glaciers with larger volumes and greater extents than would be expected for the corresponding debris-free case

(Scherler et al., 2011).

Debris-covered glaciers exhibit a wide range of responses to changes in climate, some of which are counterintuitive (Scherler et al., 2011). Many debris-covered glaciers globally are retreating, particularly in the Himalayas (Bolch et al., 2012), though more slowly and with stagnant termini. Additionally, some debris-covered glaciers exhibit mass loss rates that are similar to those observed for nearby debris-free glaciers (Kääb et al., 2012; Gardelle et al., 2013; Pellicciotti et al., 2015; Brun et al., 2018) and that have been related to enhanced thinning rates on their debris covered glacier tongues. The formation of ~~cryokarst features ,~~ features such as ice cliffs and supraglacial ponds, which we refer to collectively here as cryokarst features, has been suggested as a potential explanation for this anomalous thinning and therefore the occurrence of such features and their enhancing effect on surface melt have been intensively studied (Sakai et al., 2000, 2002; Steiner et al., 2015; Buri et al., 2016; Miles et al., 2016). However, the influence of dynamic effects on the thinning rates and glacier evolution has so far largely been neglected and such dynamic effects remain poorly understood. Further, from the reduction in ablation on debris covered glaciers a more delayed and dampened response is expected (Benn et al., 2012). Ideally one would use observational data across a greater temporal and geographical spectrum so that process feedbacks can be observed and examined over relevant time scales. However, the ~~relatively recent advent of~~ lack of long-term remote sensing data means that we have severe constraints on the availability of such long observational time series and the recent reconstruction of Zmuttgletscher (Mölg et al., 2019) provides the only observable record of a debris covered glacier that goes beyond a century.

Given the paucity of longterm data it is therefore essential to use advanced numerical models in order to investigate the role of glacier dynamics on glacier evolution and mass loss, allowing for the study of interacting processes over longer timeframes. Recent progress with numerical simulations of debris-covered glaciers include~~Konrad and Humphrey (2000), Vacco et al. (2010), Banerjee and Shankar (2013), Rowan et al. (2015), Anderson and Anderson (2016)~~: Konrad and Humphrey (2000), where an early steady-state model of debris-covered glaciers was developed; Vacco et al. (2010) and Menounos et al. (2013), both of which studied the effect of a rock avalanche on glacier evolution using a coupled debris transport and ice dynamics model; Banerjee and Shankar (2013), which suggested that the transient response of a debris-covered glacier to changes in climate has two distinct timescales; Rowan et al. (2015), which was the first modern model of coupled debris-ice dynamics to study the longterm evolution of a debris-covered glacier; and Wirbel et al. (2018), which tested both 2-D and ~~Anderson et al. (2019a).~~ 3-D advective debris transport using a full-Stokes solver for the ice dynamics. Perhaps the most significant modelling study to date is Anderson and Anderson (2016), where certain technical issues are addressed in detail for the first time, such as how to handle both the boundary condition at the glacier terminus and the possibility of a variable debris source in the accumulation area. The body of work that uses essentially the same model has examined diverse topics relating to the feedbacks that exist between debris flux, debris thickness patterns, and steady-state glacier extent; has also studied the relationship between debris-covered glaciers and rock glaciers; and has been used to explore the processes that govern the age of ice-cored moraines (Anderson and Anderson, 2016; Crump et al., 2017; Anderson and Anderson, 2018; Anderson et al., 2018).

However, to date no study has used a coupled ice flow-debris transport model to ~~study in detail~~ systematically and in detail study the transient response and characteristic response times of a debris-covered glacier. ~~This~~ A better understanding of how a debris-covered glacier responds to changes in climate, and what role the debris concentration and the prevalence of cryokarst features play in determining the magnitude of this response, is critical to predicting how today's debris-covered glaciers will evolve as the Earth's climate changes. Therefore, this study aims to fill ~~this~~ the gap by investigating the difference in transient response of debris-covered glaciers from their debris-free counterparts. In particular, we~~focus on the~~: (1) examine how debris cover changes the transient response of an idealized glacier to step changes in climate, quantifying both the volume and length response; (2) examine the response to a fluctuating climate signal ~~and the impact of including cryokarst features, such as ice cliffs and supraglacial ponds,~~ on the long-term evolution of am idealized debris-covered glacier as a function of debris concentration; and (3) examine the impact on mass loss and surface evolution when cryokarst features are included, quantifying this impact as a function of cryokarst area.

2 Methods

2.1 Governing equations

In order to examine the essential features of the interaction between glacier dynamics and debris cover, we couple an ice flow model to a debris transport model that includes both the debris melt-out and its insulating effect on ice ablation. In this model, the debris evolution affects the geometry and ice flow through changes in the surface mass balance. Our model is similar to that used in Anderson and Anderson (2016) with the main differences being some simplifications in the description of the ice flow, no explicit englacial debris tracking within the ice, and the novel option of melt enhancement due to cryokarst features. This cryokarst component is coupled to the flow dynamics and switches on when the tongue becomes stagnant.

2.1.1 Ice dynamics

For ice flow, we use a flowline version of the Shallow Ice Approximation (SIA), a simple model that allows for a realistic qualitative study of a glacier's response to changing climatic condition. The SIA has been used for studying glacier evolution and response times for debris-free glaciers (e.g., Leysinger Vieli and Gudmundsson, 2004), where it achieved comparable results to a full-Stokes solver with significantly less computational time. For a glacier with evolving ice thickness $H(x,t)$ flowing along the down-glacier direction x with depth averaged velocity $\bar{u}(x,t)$ in response to a surface mass balance forcing $a(x,t)$, the equations for the thickness evolution and SIA ice flow are given by:

$$\frac{\partial H}{\partial t} + \frac{\partial(\bar{u}H)}{\partial x} = a, \tag{1}$$

$$\bar{u} = \frac{2A(\rho g)^n}{n+2} H^{n+1} \left|\frac{\partial h}{\partial x}\right|^{n-1} \frac{\partial h}{\partial x}, \tag{2}$$

where ρ is the density of ice, g is gravitational acceleration, A and n are the rate factor and exponent from Glen's flow law, respectively, and $h(x,t) = H + b$ is the glacier surface elevation for a given ~~a~~ bed elevation $b(x)$. The boundary conditions for the ice thickness H are handled by specifying a Dirichlet or Neumann boundary condition at $x = 0$ and requiring that H goes to zero at the glacier terminus, where the ice front position is a free boundary.

2.1.2 Debris dynamics

We assume that debris is homogeneously distributed within the ice with a spatially constant concentration c. The debris melts out when the ice melts at the surface and remains on the surface, where it is passively advected with the surface ice flow velocity $u_s = \frac{n+2}{n+1}\bar{u}$, until it reaches the terminus. The evolution of surface debris thickness $D(x,t)$ is represented by:

$$\frac{\partial D}{\partial t} + \frac{\partial (u_s D)}{\partial x} = \phi, \tag{3}$$

where ϕ is the debris source term at the surface given by

$$\phi(a,H) = \begin{cases} 0, & \text{if } a \geq 0 \\ -ca, & \text{if } a < 0 \end{cases}. \tag{4}$$

Note that for simplicity, we do not account for debris volume changes during melt due to density differences and debris porosity, hence our formulation is different by a constant factor compared to Naito et al. (2000) and Anderson and Anderson (2016).

2.1.3 Surface mass balance

We assume that debris-free ice has an elevation dependent surface mass balance $\tilde{a}$ given by

$$\tilde{a}(z) = \min\{\gamma(H + b - \text{ELA}), a_{max}\}, \tag{5}$$

where γ is the mass balance gradient, ELA is the equilibrium line altitude, and a_{max} is a maximum mass balance, which limits the accumulation to physically realistic values at very high elevations. A surface layer of debris enhances ice ablation when its thickness D is below a threshold D_0. We neglect the effect of enhanced ablation when $D < D_0$ and represent the inverse relationship of surface mass balance with debris thickness ~~(Nicholson and Benn, 2006)~~ (Anderson and Anderson, 2016) as

$$a = \tilde{a}\frac{D_0}{D_0 + D}. \tag{6}$$

The parameter D_0 is chosen based on Østrem curve data representative of a medium-sized Alpine glacier (e.g. Nicholson and Benn, 2006; M

.

2.1.4 Debris boundary condition at glacier front

~~Supraglacial debris that covers the glacier throughout the~~ The choice of boundary condition at the terminus is of critical importance, since the rate at which the supraglacial debris covering the ablation zone leaves the glacier significantly affects the glacier extent (Anderson and Anderson, 2016) and if this is handled incorrectly, it can lead to runaway glacier growth (Konrad and Humphrey, 2000). To ensure that the boundary condition makes sense, it should be consistent with observations and also grid size independent, since the laws of physics should not depend on the choice of discretization.

With these requirements in mind, we set the boundary condition such that the debris leaves the system via a terminal ice cliff, as typically observed at termini of debris covered glaciers (Ogilvie, 1904, see Fig A1 in the appendix). ~~Such an ice cliff with non-zero velocity is important for debris being removed from the system as otherwise there is a physically unrealistic piling up of debris at terminus. The location of the terminal ice cliff is given by the point at which ice thickness H~~ This can be achieved most easily through an adjustment of the surface mass balance, which we adjust at the point where the ice reaches a critical thickness H^*. All debris melted out or transported past this point is assumed to slide off of the glacier relatively quickly and therefore does not play a role in the surface mass balance. This implies that the glacier will always have a small debris-free cliff area at the terminus with clean ice melt. ~~The terminal boundary condition~~ Therefore near the terminus, the surface mass balance a is given by

$$a = \begin{cases} \tilde{a}\frac{D_0}{D_0+D}, & \text{for } x < x^* \\ \tilde{a}, & \text{for } x \geq x^* \end{cases}, \tag{7}$$

where as above, $\tilde{a}$ is the debris-free surface mass balance, and x^* is the location at which the ice thickness $H = H^*$, a model parameter. In addition, we accounted for the fact that the terminal ice velocity in SIA goes to zero, which is not physically realistic, by taking an averaged velocity over several ice thicknesses near the terminus when computing the debris transport.

We note that although the terminal boundary condition used here is similar to the one implemented in Anderson and Anderson (2016), since in both cases the glacier ends with a small debris-free terminal ice cliff, our choice of boundary condition has the subtle but important feature of being independent of grid size. Further details of the boundary condition, including the interpolation scheme between grid points and convergence tests for different grid sizes, is found in Appendix A. ~~For more details , see the appendix.~~

2.1.5 Terminus cryokarst features

For some experiments, we ~~use a simple and somewhat ad-hoc melt enhancement effect from ice cliffs and supraglacial ponds (denoted here as cryokarst features) that is coupled to ice dynamics~~attempt to include the effects of melt enhancement from cryokarst features. Observations indicate that ice cliffs and supraglacial ponds commonly occur near the termini of stagnating debris-covered glaciers ~~(Pellicciotti et al., 2015; Brun et al., 2016)~~ (Pellicciotti et al., 2015; Brun et al., 2016; Kraaijenbrink et al., 2016; W and are associated with regions that have low driving stresses ~~.~~ (Benn et al., 2012). The driving stress τ_d, representing the weight of the ice column, is given by

$$\tau_d = \rho g H \frac{\partial h}{\partial x}. \tag{8}$$

Using such a dynamic coupling as a first approximation, we couple the initiation of cryokarst features to driving stresses below a threshold value. Specifically, we define two driving stress thresholds, a maximum τ_d^+ and a minimum τ_d^- and we introduce a local cryokarst area fraction λ which represents the debris-free area associated with ice cliffs and supraglacial ponds. For a driving stress above τ_d^+, the local cryokarst area fraction λ is set to zero, which corresponds to no ice cliffs and no supraglacial lakes. For a driving stress below τ_d^-, the local cryokarst area fraction equals a maximum value λ_m. For driving stress values in between the thresholds, we assume the local cryokarst contribution is linear in λ, given by

$$\lambda = \begin{cases} 0, & \text{if } \tau_d^+ \leq \tau_d \\ \lambda_m(\tau_d^+ - \tau_d)/(\tau_d^+ - \tau_d^-), & \text{if } \tau_d^- < \tau_d < \tau_d^+ \\ \lambda_m, & \text{if } \tau_d \leq \tau_d^- \end{cases}. \tag{9}$$

This dependence of cryokarts area fraction on driving stress is illustrated in Fig. 1 for τ_d^- = 60 kPa, τ_d^+ = 110 kPa, and λ_m = 0.1.

For the fraction of area where cryokarst is present, we assume that there is no longer an insulating effect on the surface mass balance. Adjusting the local surface mass balance a to account for this gives

$$a = \lambda\tilde{a} + (1-\lambda)\tilde{a}\frac{D_0}{D_0 + D}. \tag{10}$$

The threshold values τ_d^+ and τ_d^- are based on the values of the driving stress during advance and retreat in the cryokarst–free case and are chosen such that τ_d only drops below the upper threshold τ_d^+ once the glacier begins to stagnate during retreat, with realistic values taken to be τ_d^+ between 100 and 125 kPa and τ_d^- between 50 and 75 kPa. For more details, see ~~the appendix~~Appendix B.

2.1.6 Model setup numerical implementation

The coupled dynamic system described above is solved using standard finite differences and discretizations for the ice flow, similar to that first described by Mahaffy (1976), coupled with centred differences for the debris transport. Care is taken to ensure that each time step fulfills the ~~CFL condition~~Courant-Friedrichs-Levy (CFL) condition, which is necessary to ensure that the numerical domain of dependence contains the mathematical domain of dependence. Importantly, the boundary condition at the glacier terminus requires interpolation to determine the exact location of the critical ice thickness H^* and to weight the surface mass balance forcing accordingly in the corresponding grid cell. Additionally, to avoid unwanted effects due to the steep gradient of the ice surface in SIA near the terminus which results in an unphysical drop in velocity, the velocity field in the vicinity of the terminus is adjusted by taking the mean velocity from the region upglacier averaged over ten ice thicknesses (here about 300 m), calculated using second order backward differencing.

In the results that follow, all computations are performed using a bed consisting of a headwall with a slope of 45° followed by a linear bed with slope of roughly 6°. All model constants are shown below in Table 2.

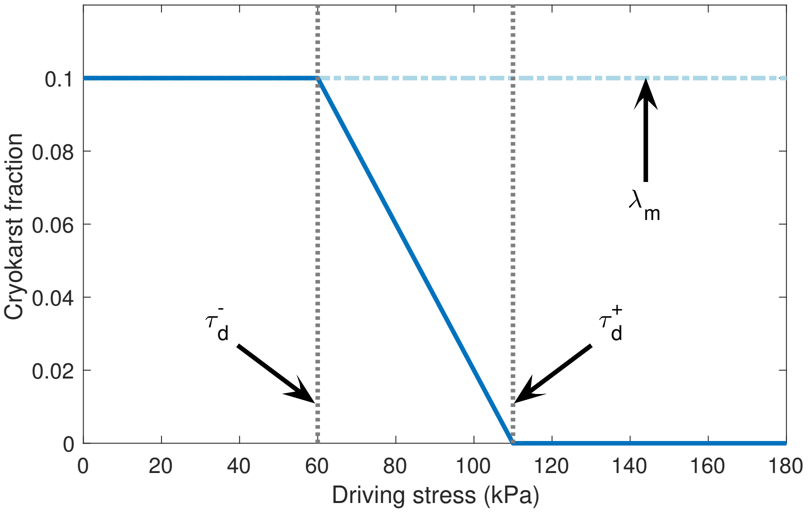

Figure 1. Cryokarst area fraction λ for a range of driving stress values τ_d for the case of τ_d^- = 60 kPa, τ_d^+ = 110 kPa, and λ_m = 0.1.

3 Modelling results

Our goal is to better understand how the transient response of a debris-covered glacier is different than that of a debris-free glacier and what effect cryokarst features have on this transient response. To do this, we perform a series of numerical experiments consisting of applying step changes or ~~white noise~~ random climate histories in the climate forcing for glaciers with variable debris concentration and hence various levels of surface debris and analyze the resulting volume and length response. ~~Table~~

First we examine the steady state case for two climates and four different debris concentrations in Section 3.1. Then in Section 3.2, we examine the transient behaviour of glaciers that move from one steady state to another after a step change in climate for the case of debris concentration $c = 0.25\%$. Next, we simulate a random climate forcing for a duration of 5000 years for glaciers with different debris concentrations and examine the resulting transient volume and length response, found in Section 3.3. We next examine the effect of introducing cryokarst features near the terminus on the transient response to a step change in climate in Section 3.4. Finally, we have a second look at the transient response to random climate forcing when cryokarst is present in Section 3.5. An overview of these experiments is found in Table 1~~gives an overview of the experiments .~~, with reference to the relevant figures in the text. Table 2 summarizes the parameter values used in the numerical model.

Table 1. Summary of modelling experiments performed.

No.	Description	Section	Figures
0	Baseline: steady-states at ELA = 3000, 3100 m	3.1	2
1	Transient response due to step change between steady-states	3.2	3,4,5
2	Random climate forcing	3.3	6
3	Transient response with cryokarst	3.4	7,8
4	Random climate forcing with cryokarst	3.5	9

Table 2. Values used for the model parameters.

Parameter	Name	Value	Units
ELA	Equilibrium line altitude	3000 – 3100	m
ρ	Density of ice	910	kg m^{-3}
g	Gravitational acceleration	9.80	m s^{-2}
c	Debris volume concentration	0 – 0.005	
A	Flow law parameter	1×10^{-24}	$\mathrm{Pa}^{-3}\ \mathrm{s}^{-1}$
n	Glen's constant	3	
D_0	Characteristic debris thickness	0.05	m
a_{max}	Maximum surface mass balance	2	m yr^{-1}
γ	Surface mass balance gradient	0.007	yr^{-1}
H^*	Terminal ice thickness threshold	30	m
λ_m	Maximum cryokarst fraction	0 – 0.2	
dt	Time step	0.01	yr
dx	Spatial discretization	25	m
τ_d^+	Upper cryokarst driving stress threshold	100–125	kPa
τ_d^-	Lower cryokarst driving stress threshold	50–75	kPa
θ	Bed slope	0.1	m m^{-1}
θ_c	Headwall slope	1	m m^{-1}

3.1 Steady state glacier extent

As a baseline for understanding the glacier's response to a changing climate, we first examine steady state features for the two climate extremes of our study, ELA = 3000 m and 3100 m. Although in reality, glaciers never attain a true steady state due to a constantly changing climate, equilibrium conditions are useful for theoretical studies because they provide well-understood rest states around which glacier fluctuations can be more easily studied. ~~Figure 1 shows the~~ As for debris-free glaciers, a steady 195 state is defined as the point at which, for a fixed climate, the glacier geometry no longer changes with time. Additionally, in our modelling there is a further requirement that the debris flux entering the glacier also must leave the glacier surface at the terminus.

Figure 2 shows the steady state glacier surface and bed profiles, velocity profiles, and debris thickness profiles for the debris-free case as well as for the debris concentrations of $c = 0.1, 0.25$, and 0.5%. The glacier profiles in Fig. ~~1a and 1c~~ 2a and 2d show the expected behaviour of higher debris concentration leading to longer, larger glaciers. A debris concentration of only 0.1% almost doubles the glacier length compared to the clean ice case. Note that the glacier geometry in the debris free part above the ELA is almost identical for all cases and hence independent of debris concentration. The surface mass balance for the debris-covered glaciers no longer decreases linearly with elevation but is instead controlled primarily by the debris thickness and strongly reduced over most of the ablation area , as shown in Fig. 2b and 2e. Surface velocities generally decrease with increasing debris thickness along the glacier, as seen in Fig. ~~1b and 1d~~2c and 2f.

An interesting observation here is that for a fixed climate, the debris thickness profile in steady state appears to be approximately independent of concentration, while the glacier extents differ strongly. This is discussed further in Sec. ~~4.4~~4.5.

3.2 Transient response between steady states

Next we analyze the response to a step change in the climate forcing. Figure ~~2~~ 3 shows the transient volume and length changes due to ELA step changes of ±100 m. In Fig. ~~2b~~3b, glacier volume time series show the response time dependence on debris concentration, with the expected result that higher debris concentration leads to longer volume response time. Here, filled-in squares denote the e-folding volume response time ~~(Jóhannesson et al., 1989)~~(Jóhannesson et al., 1989; Oerlemans, 2001) , which is the time it takes to reach $1 - 1/e \simeq 63\%$ of the total volume change. The ~~exact~~ values of the numerical volume response time are shown in Table 3 in the columns marked T_{num}. In general, the volume response times are strongly increased for ~~debris covered~~ debris-covered glaciers compared to the debris-free case. A more detailed discussion of volume response time follows in Section 4.3.

In Fig. ~~2c~~3c, the length times series allow for a comparison with the length response time. For the case of glacier advance, shown on the right side of the plot starting at $T = 1000$ years, the form of the length change is similar to that of the volume change: a slow but steady increase leading asymptotically to a steady state. However, the retreat phase, shown for $T = 0$ to 1000 yr, is contrasting this response behaviour. Here, we see a clear lag in length response, which gets stronger for larger debris concentrations. The lag is so pronounced that when the glaciers have reached their respective e-folding volumes, denoted as filled-in squares, they are still approximately at their pre-step change extent.

To show the difference in length versus volume response more clearly, we closely examine one debris-covered glacier, with $c = 0.25\%$ debris concentration, and contrast its response with the debris-free case. In Figure ~~3~~4, the normalized volume and

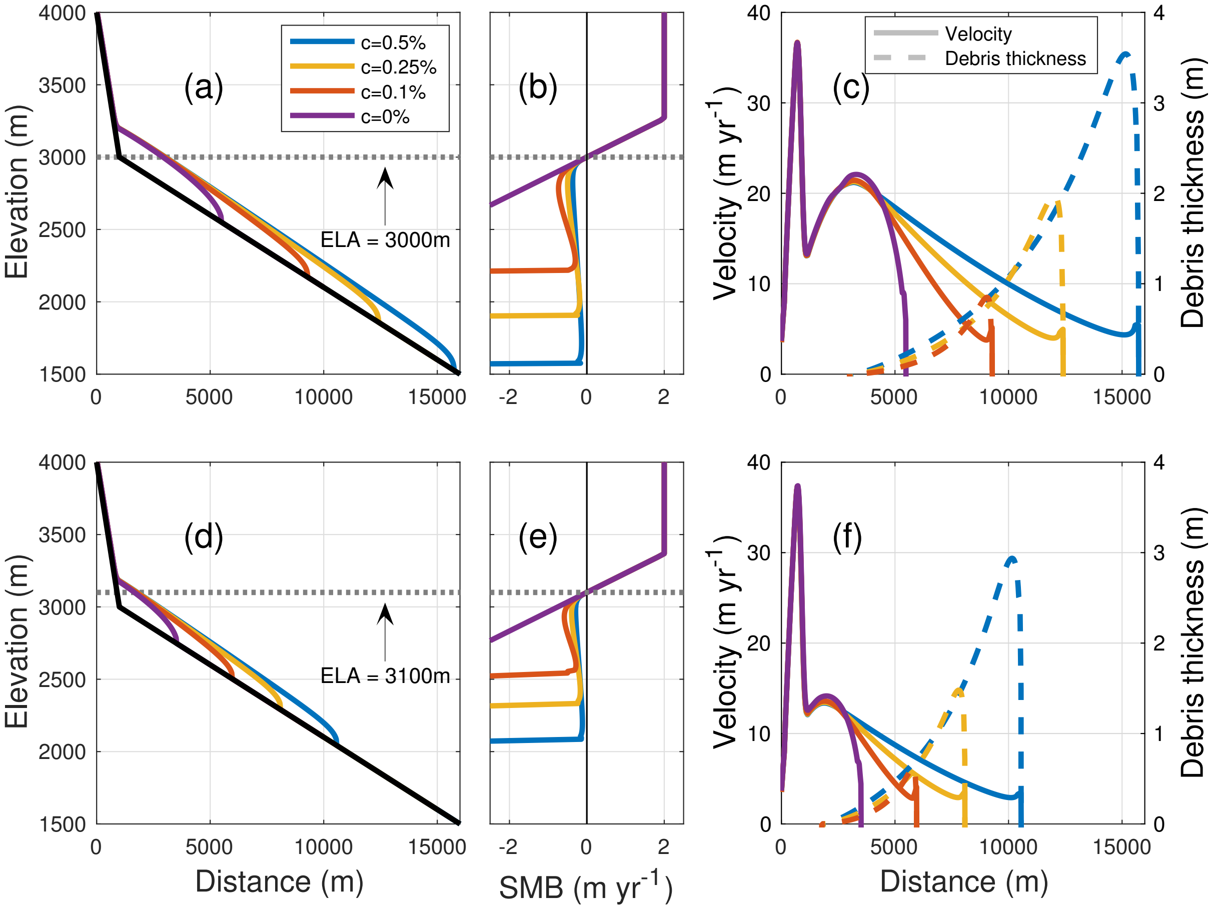

Figure 2. Steady state glacier geometry profiles (a and c), and profiles of surface velocity (solid lines) and debris thickness (dashed lines) (b and d), corresponding to ELA = 3000 m and 3100 m for four different debris concentrations.

length are plotted together for each glacier, where we have set the cold (ELA = 3000 m) steady state volume $V = 1$ (length $L = 1$) and warm (ELA = 3100 m) steady state volume $V = 0$ (warm length $L = 0$) for ease of comparison. For the debris-free case, shown in Fig. ~~3a~~4a, the volume and length curves follow each other closely but there is a small but noticeable time lag between the volume response and the length response, which is more evident during retreat. The debris-covered response, in Fig. ~~3b~~4b, shows a substantial lag time between the volume response and the length response. This lag in the length response is, at roughly 250 years, much larger during the retreat but is also observable during the advance, where a 50 year lag is observed at the onset of advance. An additional difference between the glaciers is found near the end of the retreat phase. The transient debris-covered glacier volume overshoots the final steady state volume, observable starting just before $T = 500$ yr in Fig. ~~3b~~4b, before recovering to its final volume. During this overshoot and recovery in volume, the transient glacier length monotonically decreases and never goes below its final steady state length. In contrast, the transient debris-free glacier volume has no overshoots: it monotonically decreases during retreat.

In Figure ~~4~~5, we again compare the debris-free case with the $c = 0.25\%$ debris-covered case but this time we look at the respective glacier thickness profiles during retreat. To facilitate comparison across spatial and temporal scales, we plot the normalized glacier thickness profiles for equivalent relative transient evolution times during retreat for both glaciers. In Fig. ~~4a~~5a, we see that immediately as the debris-free glacier thins, it also retreats in a roughly uniform way with thinning approximately matched by reduction in glacier extent. This can be thought of as a manifestation of a volume-area scaling law $V = cA^{\gamma}$ (e.g. Bahr et al., 1997), which essentially says that a debris-free glacier volume is linked to its area by a power law. Note that for our flowline model, the equivalent scaling law is a relationship between area and length. The debris-covered glacier profile shown in Fig. ~~4b~~ 5b does not follow the same pattern. As the glacier thins during the period of relative time $\Delta t = 0$ to $\Delta t = 0.6$, there is no discernible change in the glacier extent. Initially, most of the thinning occurs in the upper half of the ablation zone but ceases there rapidly after $\Delta t = 0.1$ to 0.2 relative time. By $\Delta t = 0.6$, the entire region from relative length $x = 0$ to $x = 0.6$ is at or slightly below its final steady state ice thickness even though the original glacier extent has not changed yet. Only by $\Delta t = 0.8$ do we finally see the glacier terminus start to retreat, with the last roughly 35% of the glacier appearing as a thin, ~~stagnating~~not very dynamic, and soon to be disconnected terminus (see Fig. ~~S2 for corresponding stagnating~~ S1 in the supplementary material for corresponding velocity profiles). Although the velocity is nonzero, this type of terminus is often described as stagnating because dynamic replacement of ice is close to zero. In the final 20% of the total retreat time, the stagnant terminus has completely disappeared and the central region that had previously over-thinned has now recovered to its steady state thickness and has become the new terminus. We revisit this interesting terminus behaviour in ~~section 3.6~~ the section 4.1 below.

Table 3. Comparison of numerical e-folding volume response times (in years) due to step changes between steady states at ELA = 3000 m and ELA = 3100 m for different debris concentrations. The columns marked T_{num} represent the numerical volume response time and the other columns represent the theoretical estimate of Jóhannesson et al. (1989) developed for clean ice using different methods of calculating the surface mass balance at the terminus (see section ~~4.3~~ 4.4 for details).

	τ_v – retreat					τ_v – advance				
c	T_{num}	T_1	T_2	T_3	T_4	T_{num}	T_1	T_2	T_3	T_4
0	77	52	–	–	–	133	108	–	–	–
0.1	154	29	456	54	222	265	48	694	87	340
0.25	256	22	646	42	207	396	34	955	66	321
0.5	385	17	800	34	179	529	25	1237	50	273

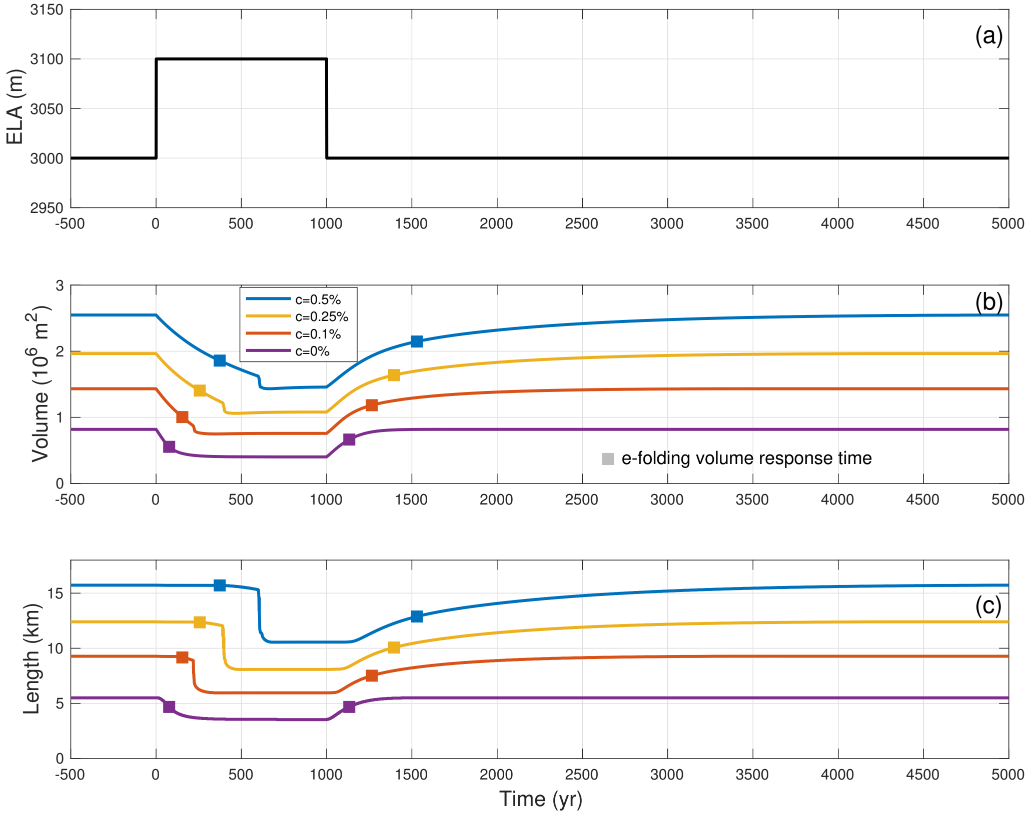

Figure 3. Step change in ELA between steady states (a) leading to transient volume response (b) and transient length response (c) for glaciers with different debris concentrations (coloured lines). The filled-in squares in both (b) and (c) represent the e-folding volume response time.

260 3.3 Response to ~~white noise~~ random climate forcing

We have investigated debris-covered glacier response to step changes in the climate and it is natural to query whether these results will have any bearing on a more realistic fluctuating climate input. To investigate this issue in a somewhat less idealized setting, we initialize the model to a steady state corresponding to an ELA of 3050 m. Then we force the model using a varying climate signal consisting of a 5000 year long time series made up of random fluctuations between ELA = 3000 m and ELA =
265 3100 m which corresponds in the Alps to a change in air temperature of about 0.8°C (Linsbauer et al., 2013). The fluctuations occur at fixed intervals of 100 years, which is close to but a bit larger than the clean ice response time during retreat, and they have a mean of ELA = 3050 m. This ~~white noise ELA~~ random climate forcing is shown in Fig. ~~5a~~ 6a and the respective transient volume and length time series are shown in Fig. ~~5b and 5c~~ 6b and 6c for a debris-free glacier and three debris-covered

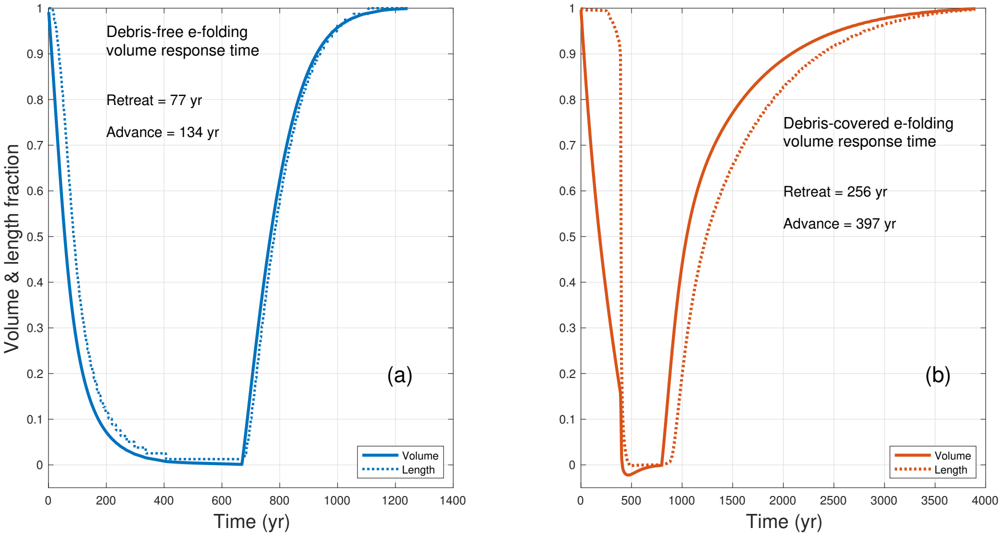

Figure 4. Normalized transient volume and length response for a step change in ELA between two steady states for (a) a debris-free glacier and (b) a debris-covered glacier with $c = 0.25\%$.

glaciers with different debris concentrations.

The behaviour of the debris-free glacier (purple line at the bottom of Fig. ~~5b and 5c~~6b and 6c) exhibits a relatively rapid volume and length response, which can be seen by how quickly the solid curve, representing the transient, moves back towards the dashed line, which represents the steady state value for the mean climate of ELA = 3050 m. For the debris-covered glaciers with lower concentration (red and yellow lines in Fig. ~~5b~~6b), the volume responds only marginally more slowly. The difference is more pronounced for the glacier with the greatest debris concentration (blue line) where the transient volume never goes below the mean climate steady state volume beginning from $T = 2800$ yr. Note that the light blue shading corresponds to colder than average time periods and white shading corresponds to warmer than average time periods.

The asymmetric response is much more pronounced in the transient length time series. While the glacier with the lowest debris concentration (red line in Fig. ~~5c~~6c), exhibits a marginally slower response compared to the debris-free case (solid purple line), the glaciers more heavily laden with debris, shown in yellow and blue, have transients lengths that are almost throughout more extended than the mean climate steady state length and tend to advance more than retreat. This is especially true for the $c = 0.5\%$ case, which after 5000 yr is more than 1 km longer than one would expect for the mean value climate. Hence, due to the lag in the length response to a warming climate, debris-covered glaciers preferentially show the effects of the colder climate. Put another way, debris-covered glaciers remember periods of cold climate more than warm ones. This also

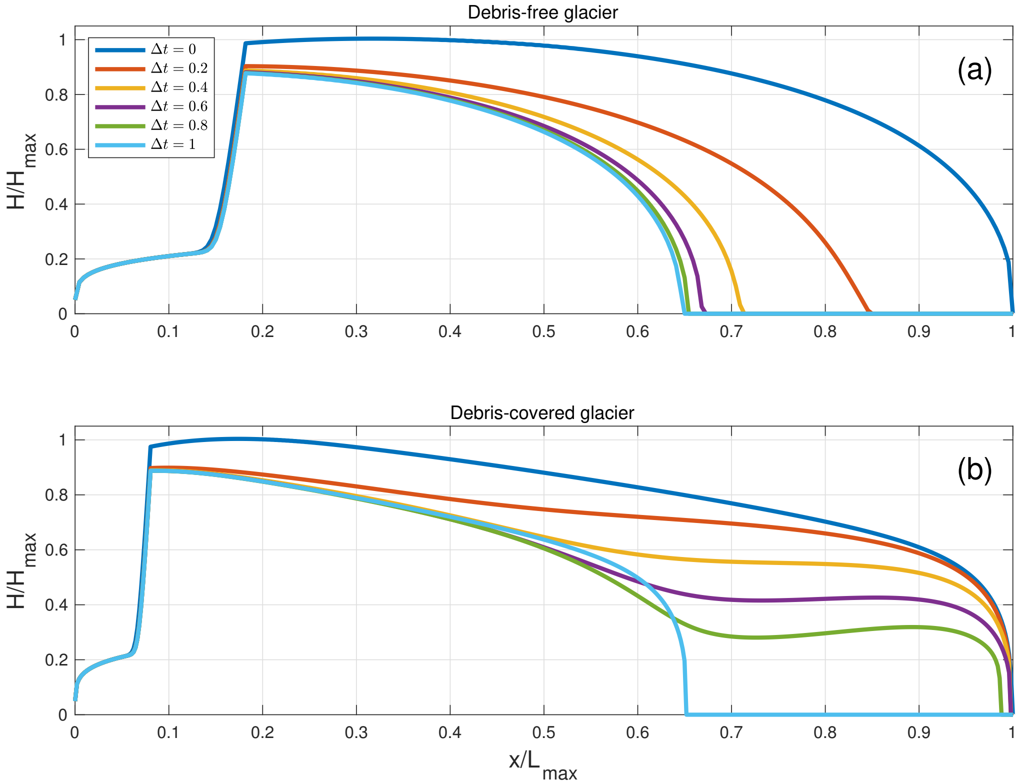

Figure 5. Glacier thickness profiles relative to the maximal initial thickness for (a) a debris-free glacier and (b) a debris-covered glacier with $c = 0.25\%$ at different times during a transient retreat. The different coloured lines refer to the time relative to the time it takes to retreat to steady state, where a time of $t = 0.1$ corresponds in the debris-free case to about 38 years and in the debris-covered case to about 40 years.

suggests that time-averaged length of a debris-covered glacier under random climate forcing will be longer than the steady state length for the equivalent constant climate forcing.

3.4 Effect of cryokarst on response

Most of the debris-covered glaciers observed in the present day have varying amounts of ice cliffs and supraglacial ponds present on their tongues which are known to enhance surface ablation (Benn et al., 2012). However, the long-term effect of such cryokarst features on thinning and glacier dynamics is poorly understood. With this in mind, we repeat the above experiments for four debris-covered glaciers, all with a medium debris concentration $c = 0.25\%$ by including the dynamic cryokarst model introduced in Section 2.1.5 and perform runs for different maximum local cryokarst area fraction of $\lambda_m = 0, 5, 10$, and 20% using driving stress thresholds $\tau_d^+ = 110$ kPa and ~~$\tau_d^+ = 60$~~ $\tau_d^- = 60$ kPa. Since we dynamically couple the onset and

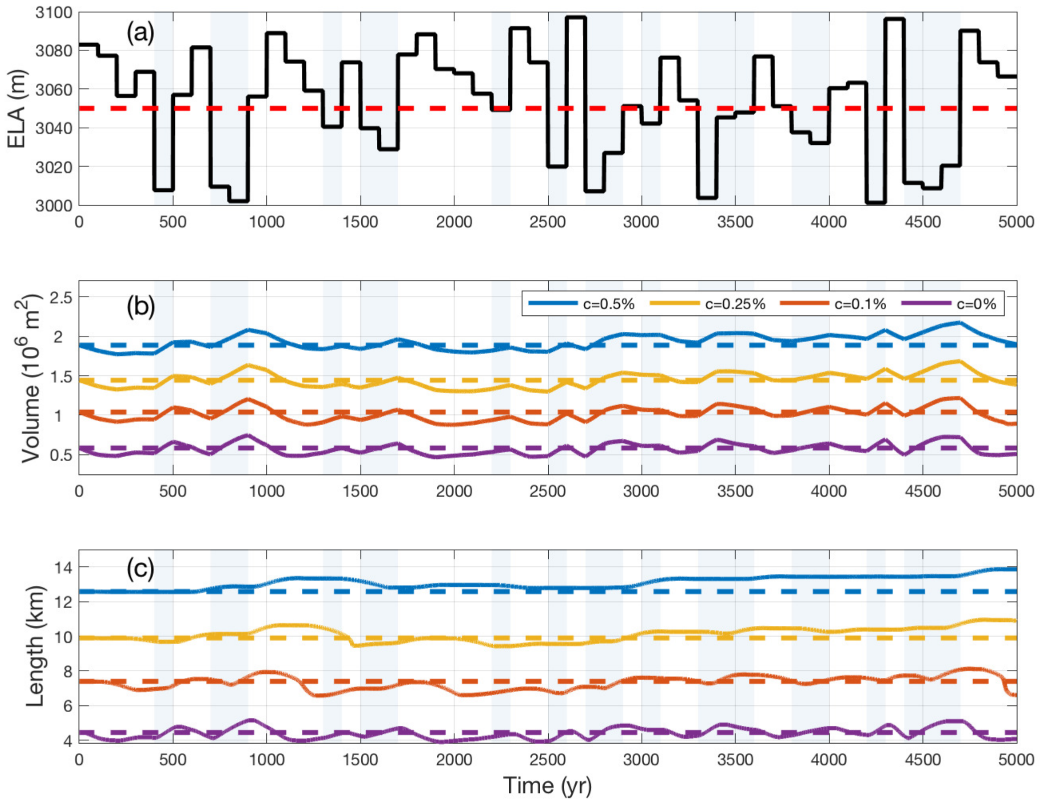

Figure 6. ~~White noise~~ Random climate forcing (a) and the corresponding transient volume response (b) and transient length response (c) for glaciers of different debris concentrations. The dashed lines represent (a) the mean value climate of ELA = 3050 m and the corresponding steady state (b) volume and (c) length. The light blue background shading represents temporal periods during which the climate forcing is colder than the mean climate.

intensity of melt enhancement from cryokarst to the driving stress using equations (9) and (10), the effect of cryokarst is only felt during periods of mass loss and we focus exclusively on this in Figure ~~6.~~ 7. The purple lines in Fig. ~~6a and 6b~~ 7a and 7b correspond to the case with no cryokarst features and therefore they show the same retreat as the yellow lines in Fig. ~~2b and 2c~~3b and 3c.

The addition of cryokarst has a noticeable effect on both the volume and length response of a debris-covered glacier. In Fig. ~~6a~~7a, there is a clear reduction in e-folding volume response time of a couple of decades visible (see Table 4 for the exact values), which is more pronounced with the presence of enhanced cryokarst. The effect on glacier length response is even more striking, with a difference of more than a century between the timings of the onset of retreat. The actual amount of equivalent bare ice for each glacier is shown as a percentage of the entire ablation zone area in Fig. ~~6c~~7c. Even for the smallest amount of cryokarst modelled, which accounts for only 2% of equivalent bare ice in the ablation area, there is a shortening of roughly 70 yr in the timing of the main phase of retreat. Despite this evident effect the presence of cryokarst has on length response, there is still a significant lag observed compared to the clean ice case, as in all modelled cases the glaciers are still at their maximum pre-step change extents even by the respective e-folding volume response times. Note that the choice of driving stress thresholds effects the strength of the cryokarst effect on the response (see Fig. ~~S3 and S4~~ S2 and S3 in the supplementary material).

To aid in the visualization of the effect of cryokarst on the transient glacier dynamics, we plot driving stress, cryokarst area fraction, and melt rate in Fig. 8 at five different time steps, spaced fifty years apart, corresponding to the case of $\lambda_m = 10\%$. As the driving stress drops below the upper threshold τ_d^+, shown in Fig 8a, the cryokarst area fraction is observed to increase, shown in Fig 8b. This leads to an increase in melt rate, depicted in Fig 8c. The maximum melt rates occur at $t = 250$ and 300 yr, shown in yellow and purple, when the driving stress is at or below the lower threshold τ_d^- and the corresponding cryokarst area fraction λ at the terminus achieves the maximum value λ_m. This increased melt rate contributes to a rapid retreat, corresponding to the green line for $t = 350$.

Table 4. Comparison of numerical e-folding volume response times (in years) due to step changes between ELA = 3000 m and ELA = 3100 m for a debris concentration of $c = 0.25\%$ and with the presence of several different values of maximum terminal cryokarst fraction λ_m.

Debris conc. c	Max. cryo. fraction λ_m	Vol. resp. time τ_v
0.25	0	256
0.25	0.05	234
0.25	0.1	219
0.25	0.2	200

3.5 Response to ~~white noise~~ random climate forcing with presence of cryokarst

Figure ~~7~~ 9 shows the same ~~white noise~~ random climate forcing experiment with $c = 0.25\%$ as in Fig. ~~5 but~~ 6 but cyrokarst with driving stress thresholds of $\tau_d^+ = 110$ kPa and $\tau_d^- = 60$ kPa, and four different maximum local cryokarst area fractions $\lambda_m = 0, 5, 10,$ and 20%. As before, the $\lambda_m = 0$ glacier, corresponding to the purple curves in Fig. ~~7~~9, is identical to the results already plotted in Fig. ~~5.~~ 6. in yellow. Since there is no dynamical effect during periods of advance, we do not see much difference in either the volume or length change rate in colder climate regimes; see in particular between 3000 and 5000 yr model time. However, during retreat the difference is visible especially during the warm climate dominated period of $T = 1300$ to 3000 yr. In Fig. ~~7b~~ 9b and even more clearly in Fig. ~~7c~~9c, the transient volume and length of the cryokarst covered glaciers exhibit a shorter memory and are therefore able to retreat much more quickly than the corresponding cryokarst-free glacier.

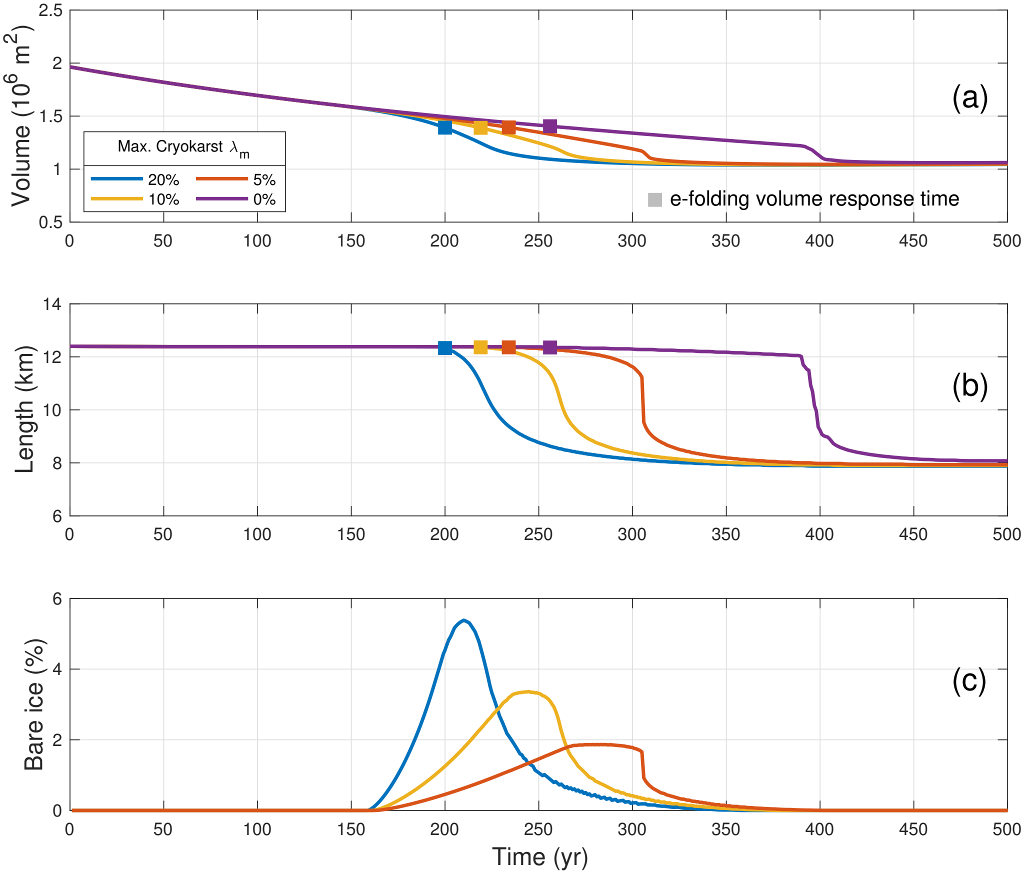

Figure 7. Transient volume response (a) and length response (b) for debris-covered glaciers with terminal cryokarst features retreating from steady state after a 100 m step change in ELA. Each colour represents a different value of the maximum cryokarst percentage λ_m. In all cases, the debris concentration is $c = 0.25\%$ and the driving stress thresholds are $\tau_d^+ = 110$ kPa and $\tau_d^- = 60$ kPa. The filled in squares in (a) and (b) represent the e-folding volume response time. The percentage of total debris-covered length that has a bare ice equivalent surface mass balance due to the presence of cryokarst is shown in (c).

Despite this faster response, all of the modelled debris-covered glaciers still respond with much more delay than a debris-free glacier with the same climate forcing, as shown in the black dotted line in Fig. ~~7b and 7c~~9b and 9c, which has been rescaled in the magnitude for both length and volume for ease of comparison. As noted above, the timing in onset of retreat is rather sensitive to the choice of the upper and lower driving stress thresholds. To compare with results using different threshold values, please refer to supplementary figures ~~S5 and S6~~S4 and S5.

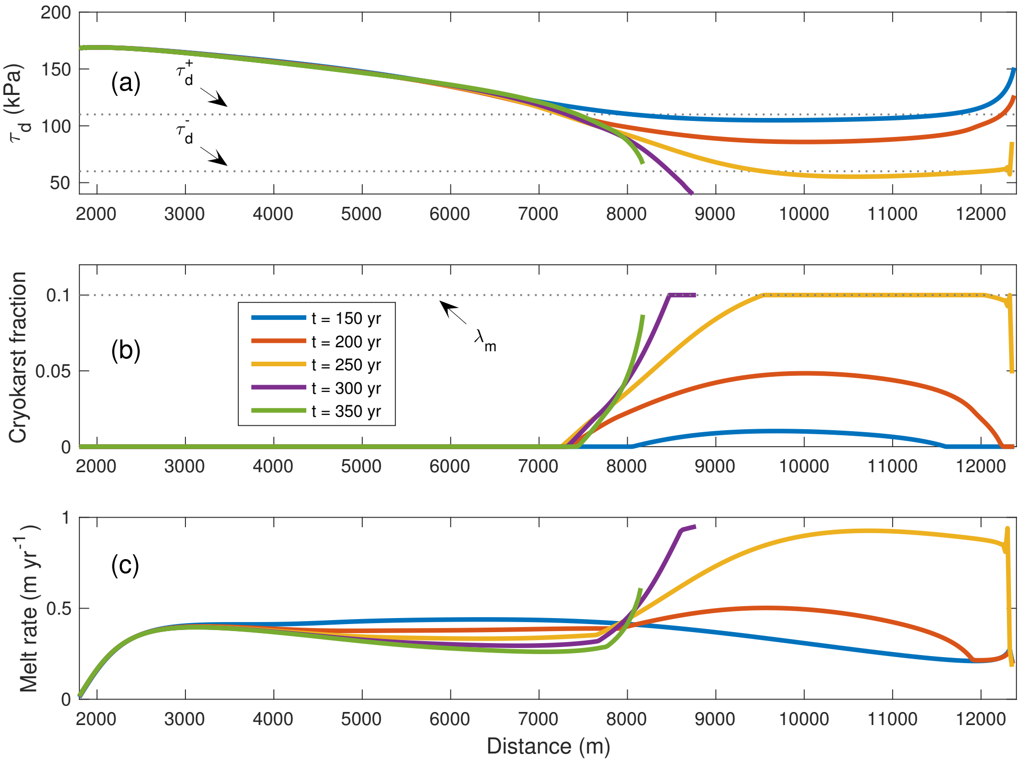

Figure 8. Profiles of driving stress (a), cryokarst fraction (b), and melt rate (c) at five different time steps for the case of maximum cryokarst area fraction $\lambda_m = 10\%$. The dashed lines in (a) denote the driving stress thresholds used in the cryokarst parameterization.

~~Thinning rate, flux divergence, and surface mass balance averaged over the final 200 m of the glacier terminus before the terminus during advance (a), retreat (b), and retreat with cryokarst (c), for $c = 0.25\%$, $\lambda_m = 0.05$, $\tau_d^+ = 110$ kPa, and $\tau_d^- = 60$ kPa. In all cases, the initial condition is a steady state at ELA = 3050 m followed by a 50 m step change in ELA at time $t = 0$.~~

4 Discussion

We explored the transient response of a debris-covered glacier to changes in climate forcing using a flowline model that couples ice flow with debris melt out and advection and also includes an ad-hoc representation of the effects of dynamically coupled cryokarst features at the glacier terminus. Several interesting results related to dynamics were obtained, which we discuss separately in light of observational data, previous studies, and model limitations.

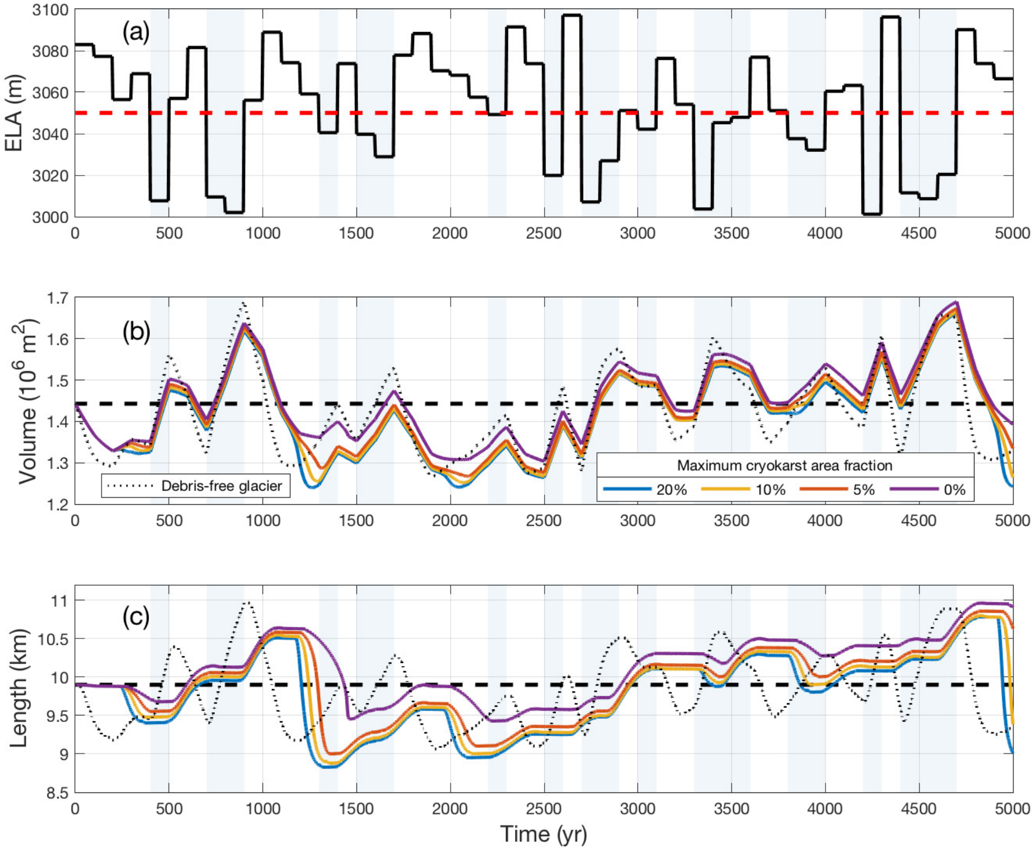

Figure 9. ~~White noise~~ Random climate climate forcing (a) and the corresponding transient volume response (b) and transient length response (c) for glaciers of different maximum cryokarst fraction λ_m. The dashed lines represent (a) the mean value climate of ELA = 3050 m and the corresponding steady state (b) volume and (c) length. The light blue background shading represents temporal periods during which the climate forcing is colder than the mean climate. In all cases, the debris concentration is $c = 0.25\%$ and the cryokarst driving stress thresholds are $\tau_d^+ = 110$ kPa and $\tau_d^- = 60$ kPa.

4.1 Terminus behaviour during transient response

The results of our numerical experiments indicate that debris-covered glaciers have an asymmetric response to climate forcing, with a visible lag in response during a retreat, and that the magnitude of the lag is reduced in the presence of terminal cryokarst. To better understand this behaviour, we further examine the debris-covered terminus region during advance and retreat and consider the relative magnitudes of surface mass balance a and flux divergence $\partial Q / \partial x$ on the rate of thickness

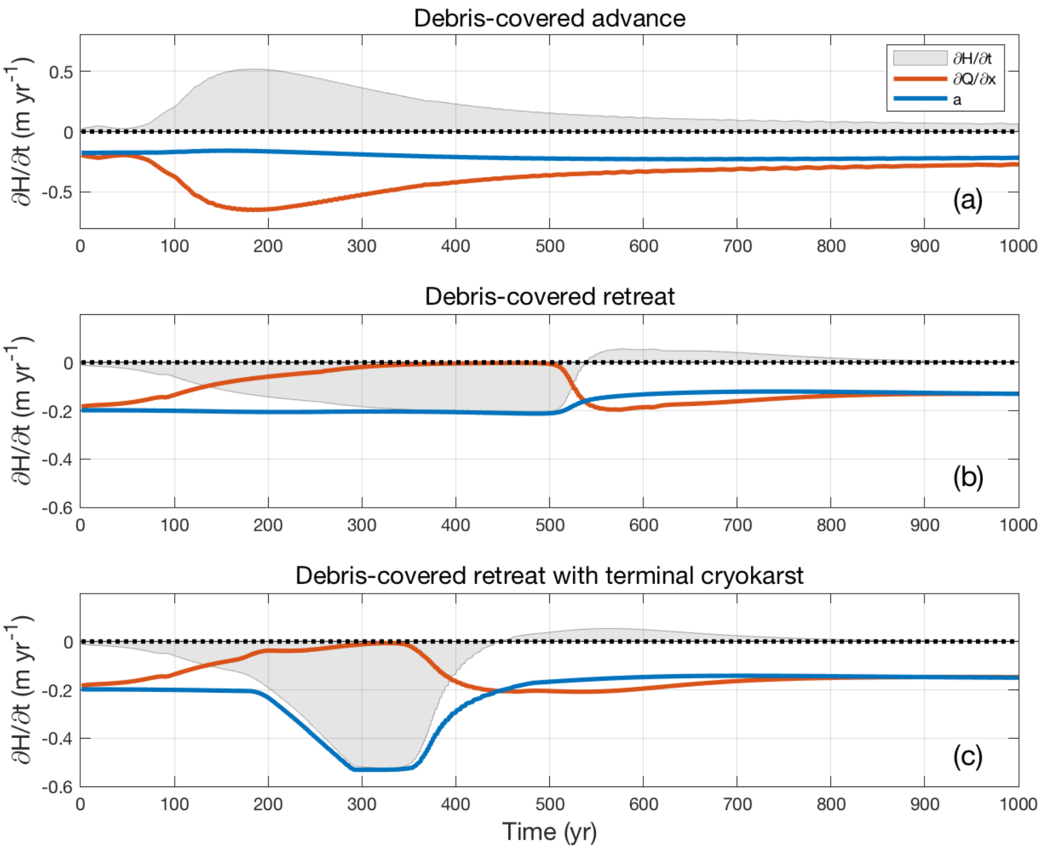

Figure 10. Thinning rate, flux divergence, and surface mass balance averaged over the final 200 m of the glacier terminus before the terminus during advance (a), retreat (b), and retreat with cryokarst (c), for $c = 0.25\%$, $\lambda_m = 0.05$, $\tau_d^+ = 110$ kPa, and $\tau_d^- = 60$ kPa. In all cases, the initial condition is a steady state at ELA = 3050 m followed by a 50 m step change in ELA at time $t = 0$.

change $\partial H/\partial t$. ~~Anderson et al. (2019a)~~ Anderson et al. (2021) used a similar approach to study thinning at the terminus but here we are primarily interested in the retreat rate. Fig. ~~8~~ 10 shows the components of the mass conservation equation (1) for the moving region consisting of the last 200 m of debris-covered area for the cases of advance, retreat, and retreat with maximum local cryokarst area fraction $\lambda_m = 5\%$ and driving stress thresholds $\tau_d^+ = 110$ kPa and $\tau_d^- = 60$ kPa. The initial condition for all three panels is steady state for an ELA = 3050 m, with the advance and retreat due to ELA step changes of ± 50 m at $T = 0$.

In each plot, the grey region represents the thickening or thinning rate, with the area under the curve representing the total thickness change of the last 200 m during the entire 1000 yr of the advance or retreat. In all three cases, during the first 200 yr the surface mass balance a, plotted in blue, does not change appreciably from the pre-step change value of $a = -0.2$ m yr^{-1}. This is because the debris at the terminus is thick enough to make the glacier here relatively insensitive to small changes in climate forcing. For the advancing glacier, depicted in Fig. ~~8a~~10a, the change in surface mass balance is minimal and the thickening rate is driven by an increase in the magnitude of the flux divergence (red line), that peaks at around $T = 150$ yr, the time taken for the increased ice flux to propagate down the glacier to the terminus.

For the retreating glacier, shown in Fig. ~~8b~~10b, the thinning rate is clearly driven by a decrease in flux divergence, which eventually drops to zero at roughly $T = 300$ yr, at which point the glacier terminus stagnated. It remains so until roughly $T = 500$ yr, when the total amount of thinning is large enough that the stagnant terminus finally disappears. After this, there is a small amount of thickening at the terminus as the glacier readjusts to the overshoot caused by the collapse of the stagnant terminus.

When a small amount of cryokarst features is added to the terminus during retreat, representing at most roughly 2% of the total debris-covered area (Fig ~~8c~~10c), the glacier behaves identically to the cryokarst–free case up until roughly $T = 190$ yr. From then on the terminus dynamics become stagnant enough that the cryokarst features begin to develop and within several decades cause an increase in the melt rate by a factor of more than two. This significantly speeds up the thinning on the tongue and hence the retreat rate, with the bulk of the retreat completed about 100 yr earlier than in the cryokarst–free case.

5 ~~Discussion~~

~~We explored the transient response of a debris-covered glacier to changes in climate forcing using a flowline model that couples ice flow with debris melt out and advection and also includes an ad-hoc representation of the effects of dynamically coupled cryokarst features at the glacier terminus. Several interesting results related to dynamics were obtained, which we discuss separately in light of observational data, previous studies, and model limitations.~~

4.1 ~~Debris-covered glacier memory~~

~~The~~

4.1 Debris-covered glacier memory

We showed that the memory of a debris-covered glacier ~~was shown to be~~ is selective, exhibiting an effective hysteresis, with periods of relatively cold climate having a sustained effect on the volume and in particular on the length. Strictly speaking this is not a true hysteresis since if the glacier is allowed a lengthy relaxation period of several centuries, the resulting equilibrium is independent of history. Previous numerical simulations of the transient response of debris-covered glaciers focused only on the effects of sudden debris input in the form of an avalanche (Vacco et al., 2010; Menounos et al., 2013). Such a one-time debris input leads to an advance in glacier extent and foreshadows the results of our study, where a constant debris source and changing climate forcing gives rise to a more complex response.

The glacier termini seem to struggle to retreat in warmer periods even if they are sustained over a century, and hence debris covered glaciers have the tendency to either advance or stagnate in a fluctuating climate (Fig. ~~5~~6). This also means that for debris covered glaciers, no unique glacier length exists for a given climate, but rather that the length of debris covered glaciers is determined by the history of repeated cold phases. ~~This is a novel result which has~~ Furthermore, debris-covered glaciers under random climate forcing are expected to have a longer average length than the steady state length corresponding to the equivalent constant climate forcing. These are novel modelling results which have important implications not only for the observed present day extended extents of debris-covered glaciers but also on historical reconstructions. For example, inferences of past climate from historical glacier extents that do not take into account the asymmetric memory of debris-covered glaciers risk misrepresenting the climate as being colder than it actually was (Clark et al., 1994).

Observational data (Quincey et al., 2009; Scherler et al., 2011; Ragettli et al., 2016) show that many debris-covered glaciers have strongly extended and stagnating tongues which is consistent with our modelling and our interpretation that debris-covered glaciers remember rather the colder climates of the past and are therefore quite far out of balance with the present climate. ~~Note, that this asymmetric memory is much more pronounced for the adjustment in glacier length than in volume.~~ However, since the observational record is not long enough to provide data on meaningful timescales and is heavily biased towards retreating glaciers, it is currently only possible to study this phenomenon fully using numerical experiments.

Note, that this asymmetric response to climate forcing is much more pronounced for the adjustment in glacier length than in volume. In the ~~white noise~~ random climate experiments, volume change and hence average thinning behaviour are surprisingly similar for all debris concentrations and the clean ice case (Fig. ~~5~~6), which agrees with the general observations of relatively high mass loss despite the occurrence of substantial debris cover (Pellicciotti et al., 2015; Brun et al., 2018). Such rapid mass loss is governed by two processes. Initially, the warming has a strong impact on the upper accumulation area where debris is still thin. Then the lower tongue with thick debris cover stagnates and dynamic ice replacement diminishes ($\partial Q/\partial x$ goes to zero, Fig. ~~8b~~10b) and hence the ice simply ~~melt~~ melts away. As this stagnant area is extensive the related total volume loss is therefore substantial.

The results presented used a ~~white noise~~ random climate forcing with a particular ELA range and time interval between random step changes. Different climate forcing signals are possible, as are many different shapes and sizes of glaciers with varying debris thickness profiles. However, the general qualitative results are expected to be the same, though for example a longer time interval between random steps (approaching the debris-covered response time) will reduce the debris-covered memory effect. An obvious extension of this work is to undertake a detailed sensitivity analysis, using a variety of climate signals, glacier geometries, and debris thickness profiles, in order to better understand the conditions under which this selective memory effect becomes significant.

4.2 Cryokarst effect in modulating response

Our results suggest that cryokarst features which dynamically develop during a retreat on the stagnating terminus substantially speed up the length response and also noticeably reduce the volume response time. This is important for any long-term modelling studies involving debris-covered glaciers, as neglecting the effects of cryokarst results in an overestimation of transient response times during a warming phase. Furthermore, the resulting earlier and more enhanced mass loss rates agree better with the current observations of rapid thinning (Pellicciotti et al., 2015; Brun et al., 2018; Mölg et al., 2019) but the terminus response is still strongly delayed and requires warm periods of substantial durations (several centuries) to cause substantial retreat (Fig. ~~7~~9). This suggests that todays thinning may still be related to the warming after the Little Ice Age, or alternatively, it may be a consequence of our rather ad-hoc approach and threshold for the onset of cryokarst.

Numerous previous studies ~~(Sakai et al., 2000, 2002; Benn et al., 2012; Buri et al., 2016; Miles et al., 2016; Ragettli et al., 2016; Rounce e~~ (Sakai et al., 2000, 2002; Benn et al., 2012; Buri et al., 2016; Kraaijenbrink et al., 2016; Watson et al., 2017; Miles et al., 2016; Ragettli et have investigated the role of ice cliffs and supraglacial ponds on the enhancement of melt on debris-covered glaciers and indicate some link between stagnation in dynamics and the development of such cryokarst features. Our model is, however, the first attempt to couple the effects of these features to glacier dynamics ~~and explore its~~ in a numerical model in order to explore the impact on glacier thinning. Although the ad-hoc approach used here is admittedly simplistic, it does allow for the general effect of cryokarst to be incorporated dynamically without requiring knowledge of the details of the physical processes, which are not yet completely worked out and would greatly complicate the numerics since they are sub-grid scale. The parameters chosen resulted in behaviour consistent with fractional area observations ~~(Mölg et al., 2019; Anderson et al., 2019b; Steiner et al., 2019, give a local~~ (Mölg et al., 2019; Anderson et al., 2021; Steiner et al., 2019, give a local area fraction up to 10%) and the approximate timing of the cryokarst evolution also matches the observation that stagnating glaciers tend to have more cryokarst (Benn et al., 2012).

The main limitation to this component of the model is that the choice of driving stress thresholds for the onset of cryokarst features is not well constrained by observations or directly linked to a sub-grid process based model. Hence it is clear that a better understanding of the link between glacier dynamics and the formation of cryokarst is needed and a more sophisticated model that faithfully represents the large scale, long-term effect of ice cliff and supraglacial pond evolution on the local surface mass balance would be useful for future studies.

4.3 Transient response time

The thinning and hence the volume response during retreat occurs in two distinct phases: a first relatively rapid response in the debris-free zone directly caused by enhanced melting, followed by a slower response in the debris-covered zone punctuated by the collapse of the stagnant terminus, and caused by the stagnation of the debris covered tongue. Although this has been indicated conceptually by general observations, Banerjee and Shankar (2013) gave the first dynamical explanation for this behaviour using a simplified representation of the effects of debris cover. Here we use a more physically realistic model which includes debris evolution coupled to the ice flow and our results confirm their dynamical explanation. An important implication of this result, also pointed out by Banerjee and Shankar (2013), is that a simple volume response time scale to characterize the transient response of a glacier to climate forcing, developed for debris-free glaciers in Jóhannesson et al. (1989) and Harrison et al. (2001), does not seem possible for debris-covered glaciers because of its more complicated retreat behaviour.

To illustrate this, we calculate for our step change experiments (Sec. 3.2) the theoretical volume response time of Jóhannesson et al. (1989), which is given by

$$\tau_v = \frac{H_m}{-a_t}, \tag{11}$$

where H_m is the maximum ice thickness and a_t is the surface mass balance at the glacier terminus. It is, however, not that clear how to define the terminal surface mass balance, as the glacier has both a debris-covered and a debris-free zone (frontal cliff) near the terminus. This is even more problematic when there is a zone of cryokarst at the terminus, so we neglect that case here. We choose four different terminus locations to extract the surface mass balance at the terminus from the modelling results and which depend on the location of extraction: for the response time T_1, the terminal surface mass balance is taken on the debris-free terminal ice cliff; for T_2, it is taken on the debris-covered zone just up-glacier from the ice cliff; for T_3, it is taken as an average of the surface mass balances from T_1 and T_2; and for T_4, it is taken from the average over the last 300 m (or roughly ten ice thicknesses) including the debris-free ice cliff. The results of these response time calculations for both retreat and advance are found in Table 3. As is evident by comparison with the corresponding numerical volume response times, none of these approaches gives reasonable theoretical predictions (Table 3) and results in either strongly over or underestimated response times, depending on whether the debris-free ice cliff is excluded. Note that using the theoretical volume response time of Harrison et al. (2001) does not make sense here, as this calculation takes into account the gradient of the surface mass balance near the terminus, which is close to zero wherever there is debris cover.

The presence and variability of debris cover brings into play additional dynamics that affect not only the volume response but also the geometry. The transient glacier thickness profile during a retreat showed two distinct shapes, depending on whether the stagnant and unsustainable tongue was still present. This time dependent glacier shape suggests that the volume–area power law scaling relationship that exists for debris-free glaciers (e.g. Bahr et al., 1997) is unlikely to exist in such a simple form for debris-covered glaciers. Volume–area scaling for debris-free glaciers, which rests on both theoretical arguments and observational data, shows that debris-free glaciers keep essentially the same shape even if they are not in steady state. This is clearly not true for the debris-covered glaciers modelled in our study.

Future work in establishing a way to understand and predict volume response times would be very beneficial here, as it would allow the approximate assessment of the large scale volume and length response to climate forcing without the need to run detailed, computationally expensive models for each glacier.

4.4 Steady state velocity–debris thickness relationship

The steady state profiles resulting from our model show an inverse relationship between debris thickness and ice flow velocity, consistent with both observations ~~(Mölg et al., 2019) and other~~ (Anderson and Anderson, 2018; Mölg et al., 2019) and numerical studies (Anderson and Anderson, 2016, 2018). It is natural to ask to what extent the debris thickness profile depends on the ice flow model and the debris transport model used. That question can be answered for the steady state case without assuming anything about the ice flow and considering only conservation of mass. In steady state and for the debris-covered domain, eqs. (1) and (3) can be written as one equation:

$$\kappa \frac{\partial(\bar{u}H)}{\partial x} + \frac{\partial(\bar{u}D)}{\partial x} = 0, \tag{12}$$

where $\kappa = cu_s/\bar{u}$. Integrating from the location of initial debris emergence (~~in our case the position of the ELA~~not necessarily at the ELA, as in our numerical model) to an arbitrary point x further down glacier and rearranging, we obtain an expression for steady state debris thickness D given by

$$D(x) = \frac{\kappa \bar{u}_e H_e}{\bar{u}} - \kappa H, \tag{13}$$

where $\bar{u}_e$ and H_e are depth averaged ice velocity and ice thickness at the point of initial debris emergence, respectively. Near the terminus, the ice thickness $H(x)$ approaches zero and hence, the terminal debris thickness D_{tr} can be expressed as

$$D_{tr} \simeq \frac{\kappa Q_e}{\bar{u}_{tr}}, \tag{14}$$

where $Q_e = \bar{u}_e H_e$ is the ice flux at the initial debris emergence point and $\bar{u}_{tr}$ is the ice velocity at the terminus. Note that we have not assumed SIA or any other ice flow model here, so there is no issue with vanishing velocity at the terminus, although even in that case one can require nonzero velocity at the terminus as in our debris transport model. Equation (14) is similar to an equation derived by Anderson and Anderson (2018) but with the important difference that we have allowed variable velocity in this derivation, rather than assuming a constant velocity along the entire glacier.

Consistent with Anderson and Anderson (2018), Eq. (14) suggests that debris thickens towards the terminus as the velocity decreases. An interesting consequence ~~is that for a~~ of our formulation makes use of the fact that we have allowed for a variable velocity. In the case of a debris-covered glacier near steady state with an approximately uniform debris concentration, one can infer this concentration by measuring the representative velocity and debris thickness at the terminus and the velocity and ice thickness at the emergence location.

An additional feature of these model results is that for a fixed climate, the debris thickness profile in steady state appears to be approximately independent of concentration. Although this agreement is not perfect, one can see that the dashed lines indicating debris thickness in panels Fig. ~~1b and 1d~~ 2c and 2f are within 10% of each other for most of the ablation zone. For example, the two glaciers present at $x = 10$ km both have a debris layer of about 1 m thick even though their respective debris concentrations differ by a factor of two. The exception is the upper ablation zone just below the position of debris emergence (e.g. ELA), as there debris thicknesses are very low and relative differences therefore large. The differences in this zone seems to have a profound impact on the downstream velocity and ice flux gradient and hence seem to govern the final steady state glacier length. The very similar debris thicknesses for all concentrations in the lower part of the tongue directly imply almost identical surface mass balance at the same locations (for surface mass balance profiles see Fig S1).

4.5 Model limitations

We have used a well-tested but relatively simple ice flow model, the shallow ice approximation, ~~for our simulations rather than a full-Stokes model and it is important to understand how much this choice matters for the qualitative analyses performed in our study. For the case of debris-free glaciers, this question was examined in Leysinger Vieli and Gudmundsson (2004) and the finding was that there is little difference in response times between these two models and when interested in glacier evolution. We expect this result would also be true in our case, as the representation of debris transport in our model does not depend on the~~ which seemed to adequately capture the ice physics. ~~Our~~ Our debris transport model did not resolve the englacial transport of debris and assumed that all debris immediately starts melting out at the ELA. This approximation is ~~reasonable~~ a reasonable first order approach, especially for debris that is deposited onto the glacier surface near the effective ELA ~~But this~~ or during transient simulations. This assumption works less well for glaciers whose debris deposition zone is far above the ELA, since the resulting emergence location will be located much further down glacier. ~~However, this~~ It will likely not change any of our qualitative results but just shift the debris emergence location downstream from the ELA, resulting in the same general pattern of thickening of debris along the glacier and therefore essentially the same transient behaviour. It is also not an accurate method for determining the steady-state glacier extent for different climate scenarios, since due to the assumption of constant debris concentration, the total debris input into the system necessarily scales with the size of the accumulation area and accumulation rate. This is a significant shortcoming of the model, since it results in larger glaciers associated with colder climates possessing stronger debris source terms, which is not supported by observations (Banerjee and Wani, 2018). However, the general conclusions with regard to transient behaviour and delay in response of debris-covered glaciers still holds. Further, since the changes in debris input stay in general relatively small and since the glacier does not exhibit a response until the additional debris has worked its way from the accumulation zone to the surface of the ablation zone, which can take decades to centuries, this becomes more important for final steady-state length and volume changes and is not expected to appreciably change the transient response of a debris-covered glacier far from equilibrium. To address this issue a model with the capability to track englacial debris transport would be required, which is beyond the scope of this study and is computationally much more expensive.

~~A more~~ Another important issue is what happens at the terminus. In addition to the limitations of the cryokarst model discussed above, a further concern is the choice of boundary condition at the terminus. Our boundary condition is qualitatively similar to that used in Anderson and Anderson (2016), in that for both models there is a sub-grid scale rule for defining the interface between the debris-covered surface and an exposed ice terminus. The rare observations of advancing glaciers support the use of a terminal ice cliff (see Appendix A) but during retreat this is less commonly observed. Even so, the boundary condition we use captures the effects of a stagnating tongue and therefore still seems largely consistent with observations of terminus dynamics. More detailed observations of the termini or debris-covered glaciers and their effect on the glacier dynamics would be of benefit for future modelling studies.

5 Conclusions

We have presented a model that captures the essential processes governing debris-covered glacier dynamics while in a second step also integrating the effect of evolving cryokarst features on glacier evolution. Using this model, we have investigated the transient response of debris-covered glaciers to changes in climate. The results show that for a retreat the length response is strongly delayed compared to the volume response and that in general volume response times are much longer than for clean ice glaciers. This implies that periods of cold climate have a longer lasting effect on the transient volume and particularly on the length of debris-covered glaciers than do periods of warm climate. Such glaciers therefore tend to advance or stagnate in length in a fluctuating climate and hence glacier length is not representative of climate but rather depends on the history of cold phases. The modelled extended but generally stagnant glacier tongues in a warming climate are in agreement with observations. With regard to volume loss, the model is however much more responsive and can produce similar to observed substantial thinning and hence mass loss on the extended tongues due to stagnation or more specifically the cessation in local dynamic replacement of ice.

When cryokarst features are dynamically included in the model, it enhances both terminus thinning and the retreat rate and produces similar mass loss rates to those observed today. However, our cryokarst-model was rather simple and the related parameters not well constrained, underscoring the need for a better understanding of the evolution of ice cliffs and supraglacial ponds so that they may be more accurately represented in long-term modelling studies.

Currently existing theoretical volume response times do not appear to be relevant for debris-covered glaciers because there is not a consistent way to define the surface mass balance at the terminus that gives theoretical values that match the numerical response times. Taking into account only the debris-covered area greatly overestimates the response time and this is likely due to the more complicated dynamics caused by the presence of the debris layer.

Code availability. Code used to generate the analyses used in this study is available at https://doi.org/10.5281/zenodo.4546678.

Appendix A: Boundary condition at the terminus

The choice of debris boundary condition near the terminus is not trivial, as many simple approaches lead to numerical simulations that exhibit unacceptable behaviours. Debris must leave the system before reaching the glacier terminus as otherwise the glacier can effectively grow without bound since debris may continue to thicken down-glacier indefinitely, thereby almost entirely insulating the glacier from melt. Hence, the debris boundary condition must be applied at a point up-glacier from the glacier terminus. However, boundary conditions that are defined at a location which depends on grid size, such as a fixed number of grid points from the glacier terminus, run the risk of exhibiting grid-size dependency in the numerical simulations. For example, we found that applying a debris flux condition one grid point from the glacier terminus results in a steady state glacier length that is heavily grid-size dependent, with the length varying by many times the mesh size. This is an undesirable outcome since the glacier physics should be independent of the numerical discretization used.

Observations of debris-covered glaciers that terminate in an exposed dynamically active ice cliff are numerous, occurring in relatively recent aerial images (e.g. Tsijiore Nouve, shown in Fig. A1a), early glaciological literature (Ogilvie, 1904), and even historical paintings (Escher von der Linth, 1794, as shown in Fig. A1b). While the terminus shown in Fig A1a is from a period of positive mass balance in the Alps (e.g. Mölg et al., 2019) and may represent an advancing glacier tongue, ice cliff termini have also been observed on several retreating debris-covered glaciers in the Himalaya (Evan Miles, personal communication, 2020). Motivated by these observations, we define the point at which the debris leaves the glacier to coincide with the location x^* of a terminal ice cliff of critical thickness H^*, as shown in Fig. A2. All the surface debris transported past the ice cliff location x^* slides down the cliff and out of the system. Since the critical ice thickness will generally fall between two grid points, a sub-grid interpolation is performed to determine the exact location of the ice cliff and the surface mass balance of the cell containing the ice cliff is then defined as a weighted average of debris-covered and bare ice melt rates.

For x^* between grid points x_i and x_{i+1}, the surface mass balance at x_{i+1} is given by:

$$a_{i+1} = a\frac{x^* - x_i}{\Delta x} + \tilde{a}\frac{x_{i+1} - x^*}{\Delta x}, \tag{A1}$$

where a is the debris-covered surface mass balance, $\tilde{a}$ is the debris-free surface mass balance, and Δx is the grid spacing. This boundary condition is found to be grid size independent in the sense that it results in steady state glacier extents that converge to a constant value ~~, within an error of less than the grid spacing.~~ as the grid size is decreased.

A similar boundary condition was used in Anderson and Anderson (2016), where a debris-free region was also employed at the terminus. In their approach, the size of the terminal ice cliff depends directly on the grid spacing but the debris flux can be varied. In contrast, our terminal ice cliff is roughly independent of grid size, since it depends on the critical ice thickness H^*, but the debris flux is fixed by the local ice velocity.

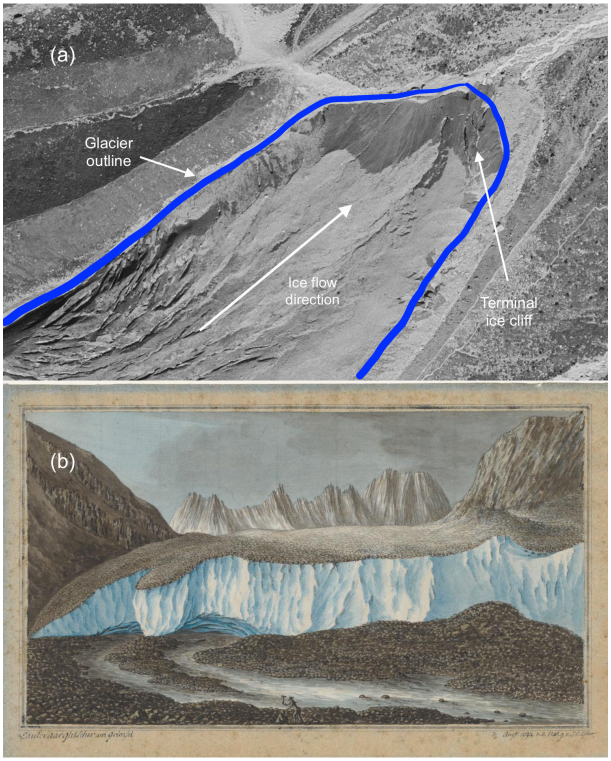

Figure A1. Examples of debris-covered glaciers with an ice cliff terminus: (a) satellite image of Tsijiore Nouve Glacier in 1988, reproduction with permission from Swisstopo (BA20059) and (b) a painting of the terminus of Unteraargletscher from 1794 (Escher von der Linth, 1794).

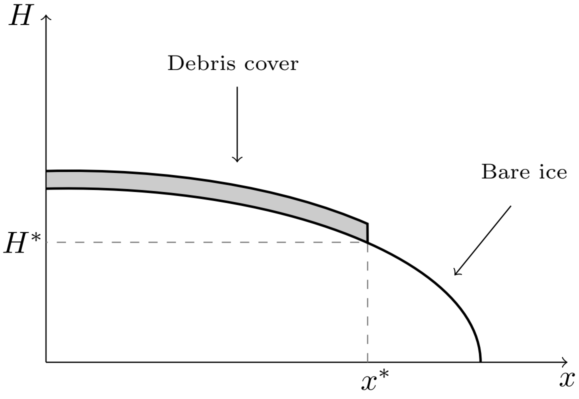

Figure A2. Schematic representation of the model terminus boundary condition. Debris covers the glacier only until a point x^*, corresponding to a critical glacier thickness H^*, and past this point the glacier is debris-free.

Appendix B: Cryokarst model

The model coupling cryokarst features to glacier dynamics described in Section 2.1.5 requires threshold values of driving stress, τ_d^+ and τ_d^-, that define the presence of cryokarst near the terminus, which in turn reduces the insulating effect of the debris locally. We choose the values of the thresholds by examining the modelled driving stress of a debris-covered glacier during retreat and choosing threshold values that seem consistent with the onset of stagnation. The upper threshold between 100 and 125 kPa is consistent with driving stresses observed at the upper limit of the cryokarst zone at Zmuttgletscher in the Alps during its retreat from the Little Ice Age to today (100 to 130 kPa; Mölg et al., 2020). In Fig. B1a, we plot τ_d at 50 yr intervals during retreat following a step change between two steady states at ELA = 3000 m and ELA = 3100 m. In Fig. B1b and B1c, we show the corresponding glacier thickness H and velocity u during the retreat. Note that for driving stresses below the upper threshold (100 to 125 kPa) velocities drop to virtually zero (Fig B1c).

To illustrate the sensitivity of the model to variations in the thresholds, in Fig. B2 we rerun the experiments shown above in Fig. 6 but for different values of lower threshold τ_d^- while keeping constant $\tau_d^+ = 125$ kPa and $\lambda_m = 10\%$. For larger values of the lower threshold τ_d^-, the full effect of cryokarst is felt sooner and therefore the volume and length response occur sooner (Fig. B2). However, the percentage of bare ice equivalent, shown in Fig. B2c, is less for larger τ_d^- values as the zone of stagnation driven cryokarst has less time to develop.

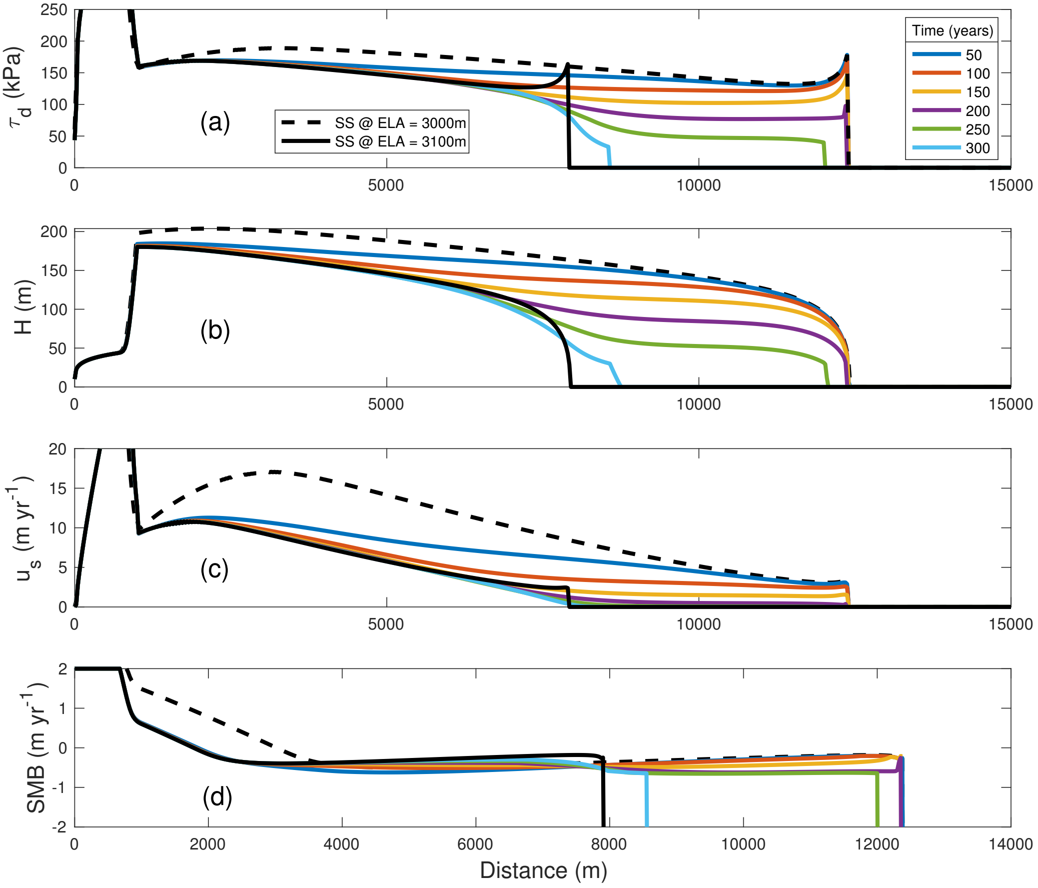

Figure B1. Effect of glacier retreat in response to step change in ELA from 3000 m to 3100 m on profiles of (a) driving stress τ_d, (b) ice thickness H, ~~and~~ surface velocity u_s, and surface mass balance at time intervals of 50 yr. The dashed black and solid black lines, respectively, refer to the initial and final steady states (SS) of the retreat experiment.

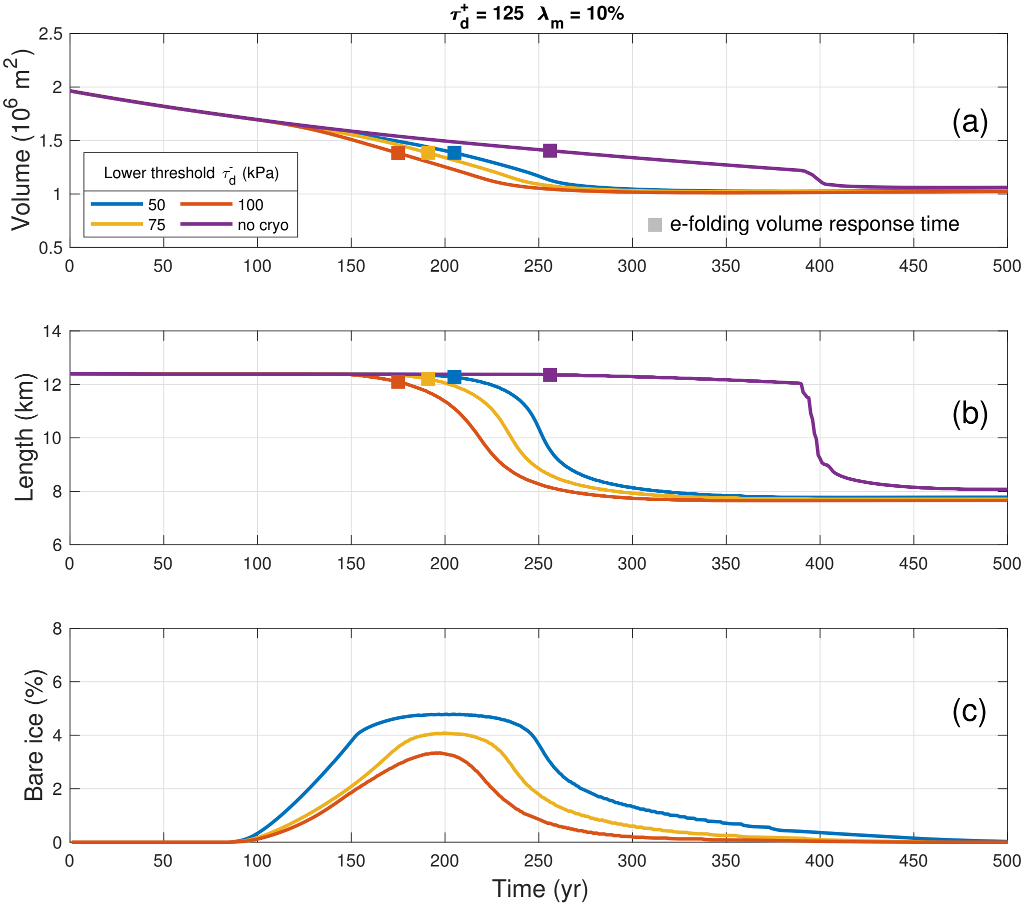

Figure B2. Sensitivity of model to different lower threshold values τ_d^- shown by coloured lines for (a) transient volume response, (b) transient length response and (c) bare ice equivalent debris-covered area during a retreat after a 100 m step change in ELA from 3000 m to 3100 m. In all cases, the debris concentration is $c = 0.25\%$, the upper driving stress threshold is $\tau_d^+ = 125$ kPa, and the maximum cryokarst fraction is $\lambda_m = 10\%$.

Author contributions. JF and AV designed the study. JF wrote the code and performed the numerical experiments. JF interpreted the data and wrote the paper with input from AV.

Competing interests. None.

Acknowledgements. We thank Tobias Bolch and Nico Mölg for helpful discussions. We also thank the scientific editor, Harry Zekollari, and two reviewers, Leif Anderson and Fabien Maussion, for numerous useful suggestions that greatly improved the quality of the final manuscript. This research was financially supported by the Swiss National Science Foundation grant number 169775, entitled "Understanding and quantifying the transient dynamics and evolution of debris-covered glaciers".

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