Supplementary material for

Temperature and strain controls on ice deformation mechanisms: insights from the microstructures of samples deformed to progressively higher strains at -10, -20 and -30°C

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Contents of this file

Introduction

Supplementary material provides the sensitivity test of the co-latitude range (Text S1, Figure S1) in the plot of MUD as a function of co-latitude for the calculation of opening-angle of *c*-axis cone; the mathematical background (Text S2) for the conversion between engineering strain/strain rate, true axial strain/strain rate and octahedral shear strain/strain rate.

Text S1 Sensitivity test of the co-latitude range chosen for the plot of MUD as a function co-latitude

In order to quantify cone opening-angles, we counted the number of *c*-axes that lie at a given angle ("co-latitude") from the compression axis. In practice we counted the *c*-axes between two co-latitudes separated by a co-latitude range and calculated the MUD for this co-latitude range to plot on a graph of MUD as a function of co-latitude. We tested the sensitivity of co-latitude range for the plot of MUD as a function of co-latitude.

Fig. S1 shows the distribution of MUD as a function of co-latitude with the co-latitude ranges of 2°, 4°, 6° and 8° for three different examples, including PIL165 (with random CPO), PIL177 (with cone-shaped c-axis CPO) and PIL166 (with narrow-cone-shaped c-axis CPO). The MUD distribution as a function of co-latitude is generally consistent for different co-latitude ranges. However, the MUD value for *c*-axes that lie within ~2° to the centre of the stereonet is sensitive to the value of co-latitude range. For sample PIL177, which is characterised by a cone-shaped *c*-axis CPO, there is a peak at ~2° in the MUD distribution with the co-latitude range of 2°. This peak cannot be recognized from the contoured *c*-axis CPO.

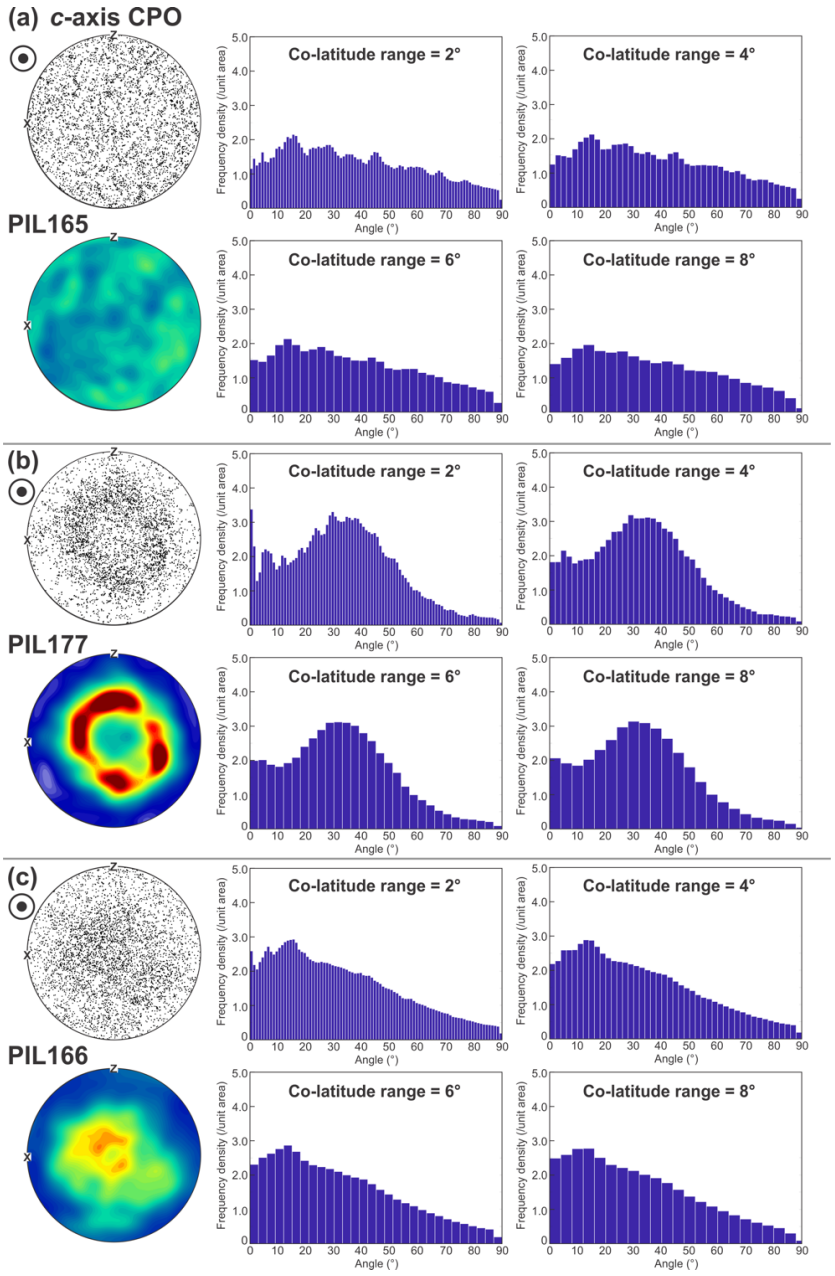

Figure S1. Sensitivity test of co-latitude range on samples of **(a)** PIL 165, characterised by a random CPO, **(b)** PIL177, characterised by a cone-shaped c-axis CPO and **(c)** PIL166, characterised by a narrow-cone-shaped c-axis CPO. For each sample, the upper left box is the point pole figure with 5000 randomly selected points and the lower left box is the corresponding contoured *c*-axis CPO. The centre of the stereonet is parallel to compression. The right part contains MUD distribution as a function of co-latitude with the co-latitude ranges of 2°, 4°, 6° and 8°.

Text S2 Mechanical data processing

The raw data of the axial displacement readings as a function of time were processed to obtain strain and strain rate. Engineering axial strain e, true axial strain ε and octahedral shear strain γ are most frequently used parameters to quantify the sample deformation in published literatures. In this study, ε and γ were converted to e for a direct comparison of the data from this study and literatures.

In an uniaxial compression experiment, we define the axial stretch λ (Eq. (S1)) as the ratio of the sample length ($L(t)$) at time t and the initial sample length (L_0).

$$\lambda = \frac{L(t)}{L_0} \quad \text{(Equation S1)}$$

The true axial strain ε, at time t, is given by:

$$\begin{aligned} \varepsilon &= -\ln\left(\frac{L(t)}{L_0}\right) \\ &= -\ln(\lambda) \end{aligned} \quad \text{(Equation S2)}$$

The engineering axial strain e at time t is given by:

$$\begin{aligned} e &= \frac{L_0 - L(t)}{L_0} \\ &= 1 - \lambda \end{aligned} \quad \text{(Equation S3)}$$

Therefore, the conversion between true axial strain ε and engineering axial strain e can be expressed as:

$$\varepsilon = -\ln(1-e) \quad \text{(Equation S4)}$$

The octahedral shear strain γ at time t is given by:

$$\gamma = \frac{1}{3}\sqrt{(\epsilon_1 - \epsilon_2)^2 - (\epsilon_2 - \epsilon_3)^2 - (\epsilon_3 - \epsilon_1)^2}, \quad \text{(EquationS5)}$$

where ϵ_1, ϵ_2 and ϵ_3 represent principle engineering strains. For uniaxial compression, we suppose the horizontal shortening is radially uniform ($\epsilon_2 = \epsilon_3$). Therefore, Eq. (S5) can be simplified as:

$$\gamma = \frac{\sqrt{2}}{3}(\epsilon_1 - \epsilon_3), \quad \text{(Equation S6)}$$

where the principle axial engineering strain ϵ_1 equals to e. We suppose the sample volume remains unchanged during uniaxial compression. The sample volume V is given by:

$$\begin{aligned} V &= \pi {R_0}^2 L_0 \\ &= \pi R(t)^2 L(t), \end{aligned} \quad \text{(Equation S7)}$$

where R_0 is the initial sample radius, $R(t)$ is the sample radius at time t. Therefore, $R(t)$ is given by:

$$R(t) = R_0\sqrt{\frac{L_0}{L(t)}}$$

$$= R_0 \frac{1}{\sqrt{\lambda}} \quad \text{(Equation S8)}$$

The principle radius engineering strain ϵ_3 is given by:

$$\epsilon_3 = \frac{R_0 - R(t)}{R_0}$$
$$= 1 - \frac{R(t)}{R_0}$$
$$= 1 - \frac{1}{\sqrt{\lambda}}$$
$$= 1 - \frac{1}{\sqrt{1-e}} \quad \text{(Equation S9)}$$

Therefore, Eq. (S6) can be converted to:

$$\gamma = \frac{\sqrt{2}}{3}\left(e + \frac{1}{\sqrt{1-e}} - 1\right) \quad \text{(Equation S10)}$$

In this study, the axial displacement remains constant, but the sample length keeps decreasing with time. Consequently, the axial strain rate increases with time. The engineering axial strain rate ($\dot{e}$) is given by:

$$\dot{e} = \frac{d(e)}{dt} \quad \text{(Equation S11)}$$

The relationship between true axial strain rate ($\dot{\varepsilon}$) and engineering axial strain rate ($\dot{e}$) is given by:

$$\dot{\varepsilon} = \frac{d(\varepsilon)}{dt}$$
$$= -\frac{d}{dt}(\ln(1-e))$$
$$= \frac{1}{1-e}\frac{d(e)}{dt}$$
$$= \frac{\dot{e}}{1-e} \quad \text{(Equation S12)}$$

The relationship between octahedral shear strain rate ($\dot{\gamma}$) and engineering axial strain rate ($\dot{e}$) is given by:

$$\dot{\gamma} = \frac{d(\gamma)}{dt}$$
$$= \frac{\sqrt{2}}{3}\frac{d}{dt}\left(e + \frac{1}{\sqrt{1-e}} - 1\right)$$
$$= \frac{\sqrt{2}}{3}\left(\frac{d(e)}{dt} + \frac{d}{dt}\left(\frac{1}{\sqrt{1-e}}\right)\right)$$
$$= \frac{\sqrt{2}}{3}\left(1 + \frac{1}{2(1-e)^{\frac{3}{2}}}\right)\frac{d(e)}{dt}$$

$$= \frac{\sqrt{2}}{3}\left(1 + \frac{1}{2(1-e)^{\frac{3}{2}}}\right)\dot{e} \qquad \text{(Equation S13)}$$