Alexander Robel (Referee)

This manuscript explores the stability of Pine Island Glacier under forcing from ocean melt, using a high order model of marine ice sheet dynamics. Additionally, this is the first study to bring the concept of Early Warning Signals (EWS) to the stability transition known as the "marine ice sheet instability" (MISI). Though EWS have been explored in a limited degree in some other areas of glaciology, this is an interesting and useful application to the MISI problem, which has recently been a focus of intense study in the glaciology community.

The central concept and design of this study are sufficiently novel and important that it eventually should be published in The Cryosphere, though I think it requires some revision first. In particular, since this is the first application of EWS to the MISI problem, it needs to be clear why exactly EWS are a useful tool for studying marine ice sheet stability. Additionally, the methodological details of EWS, while established in the dynamical systems literature, are not well known in glaciology. If the authors wish other glaciologists to follow their lead in using this approach (which I think should be the case), then they need to do a better job explaining the methods they use and the assumptions inherent to these methods. I lay out these critiques in more detail below through major and minor suggestions:

Many thanks, Alex, for the positive evaluation of our study and the helpful comments. We replied to all comments and hope that our replies and changes made to the manuscript address all issues raised. Our replies below are indicated in red, changes to the manuscript summarised in blue and references can be found at the end of the document.

Major:

1. Here is the question that you should answer in this manuscript: Why/When are EWS a useful tool for understanding MISI at a particular glacier? At the moment, my interpretation (perhaps erroneous) of the purpose of EWS laid out in this manuscript is to show that there is a bifurcation (in fact several) in the model. However, you don't need EWS to show that this is the case when you have a model available, since you have the quasi-steady simulations which show the bifurcation structure of PIG. Rather, the point of EWS is to detect a bifurcation before it occurs. You do so in the context of your model, however, solely within the context of a predictive model, EWS are not strictly necessary, because the model can be run forward to determine whether a bifurcation will occur with continued forcing along some trajectory (this is the point of physical models!). However, what you could argue here is that your study is a proof-of-concept to indicate the circumstances under which we would expect to detect EWS in observations, which would be immensely useful for the community. This is what I find currently lacking in the study - any discussion of the implications of your study for observations. For example, you touch on this issue later in the paper about the fact that in the real world, ice sheets do not stay on the stable manifold because forcing is much faster than the response time, but then don't really explore whether this makes EWS useless in practice (I think no, but that isn't my take away from the current way its written). Another issue (which you don't remark on) is the fact that a 300 year averaging window for the EWS indicators is not super useful when the entirety of the observational record is 40 years long (perhaps a bit longer if we include some lower quality historical obs). This is all to say that showing EWS exist in a model is not very insightful in of itself if it doesn't provide some indication for what we should be looking for in observations (since again, we already know that there are bifurcations associated with MISI in models).

Both reviewers state that our paper needs to do a better job of explaining the background and methodology of EWS and why our results are useful in a glaciological context. We agree that both of these things could be greatly improved on and will endeavour to do so in a revision of the paper. The aim of this paper is a 'proof of concept', demonstrating that EWS can be found for the marine ice sheet instability in realistic geometries. The theoretically-proven hysteresis of MISI (Schoof 2007) is similar to a fold (aka saddle node) bifurcation. For this class of dynamic systems the existence of EWS can be theoretically proven. However, it is by no means clear that EWS can be still detected in more complex, realistic systems, as for example discussed in Dakos et al., (2015). We show for the first time for a realistic geometry (including for example ice shelf buttressing that is not considered in the theoretical hysteresis for MISI) that indeed EWS can be detected. We will add more discussion regarding the potential for observations of ice sheets to be used in the context of EWS since we agree that this is arguably the most important potential outcome of our findings. Our study is a first step in understanding how this methodology could be used. Certainly, as Alex points out, a 300 year averaging window is not necessarily useful in terms of available datasets. An important caveat that we will make much clearer in a revised paper is that the 300 year averaging window that we found was optimal does not necessarily represent a lower bound and this is something that certainly requires and warrants further investigation.

We have added an extensive new section in the paper that explain early warning theory, including a new figure. In addition, we have added much more discussion on the applications of this work in terms of ice sheet modelling and observations, as well as caveats to issues such as the window size that would be important to readers looking to apply this to their own work.

2. You haven't necessarily explained why EWS show up in certain types of systems. To me, this is key to then explaining why you are calculating these things (ACF, variance, etc). In a canonical saddle-node bifurcation, we expect the stable eigenvalue of the linearized system state to smoothly decrease towards zero as you approach the bifurcation, which causes weaker damping of noisy forcing back towards the stable manifold. So, do EWS only occur where there is a saddle-node bifurcation? How do we know that MISI at PIG has such a smooth approach to the bifurcation? i.e. if the eigenvalue associated with the stable mode is controlled at first order by bed topography (which it is in the canonical formulation of MISI, see Schoof 2012 and others), then does the topography in the vicinity of the bifurcation need to vary smoothly towards a bedrock peak to produce EWS? Maybe these are questions for another study, but there needs to be some indication that you have grappled with the question of why you expect EWS to occur for PIG.

Although EWS have largely been used to detect saddle node bifurcations these methods can be applied to other types of bifurcation (Scheffer, 2009), for example they have successfully been used in the context of Hopf bifurcations (Chisholm & Filotas, 2009). We will add this point to the revised paper. The second comment, regarding how bed roughness might affect the performance of EWS, is something we looked at during our modelling work but ended up not including in the paper submission. In fact, this was originally a major motivation of the PIG experiments because there was no guarantee that observing critical slowing in the idealised MISMIP experiment would work for a realistic topography. Before we did the PIG experiment we superimposed varying types and amounts of Perlin noise onto the smooth MISMIP bed, such that the overall retrograde bed slope became more obscured by smaller scale bed features. Repeating the same experiments on these new beds and extracting the relaxation time to predict the tipping point became successively less accurate as more noise was added. However even for a very bumpy bed there was still a trend towards longer relaxation times as the tipping point was approached and the *accuracy of the predicted tipping point was still good enough that we had confidence this approach would work for a real glacier. The resolution of both our PIG model and the bed topography are such that we resolve as much of this 'bumpiness' as possible in these simulations. Discussion around both of these points will be added into a revised version of the manuscript.*

We have added further discussion in the manuscript around why early warning signals show up in certain systems and that this is not just true for saddle-node bifurcations. We have also added a section to the flowline experiment results describing the extended experiments we did whereby bed bumpiness was increased, and discussed the implications of this in the manuscript.

3. Related to the issue above, more of the detail about how the EWS indicators are calculated would be helpful to bring into the main text, since this is a topic most TCD Interactive comment Printer-friendly version Discussion paper TC readers are not familiar with. How do you ensure statistical significance? Could we see EWS away from a bifurcation by chance? Why not?

The issue of so-called 'false alarms', whereby an EWS is detected and incorrectly anticipates a bifurcation, is something that has been written about extensively in the literature and indeed it is entirely possible using this methodology. In our paper we do various statistics on the sensitivity and the significance of the indicators that we have calculated. These are currently in an appendix and would probably warrant being moved into the main text which we will do, along with reference to the possibility of false alarms.

The explanation of how early warning indicators are calculated has been expanded and is now consolidated into one part of the manuscript.

4. You say that you force your model with variability from a "surrogate model" based on ocean variability in the Amundsen Sea region, but don't provide further details. First, more detail is needed on the surrogate model. Second, presumably this surrogate model produces ocean variability with significant power in the decadal range, as many studies (e.g. from Jenkins and others) have found that such variability is important in this region. However, the typical formulation of EWS (e.g. Lenton et al. 2008, and previous studies) assumes a martingale process for the noise forcing (i.e. white noise) which is not the case here. Can you explain why this doesn't affect your interpretations of EWS indicators? Third, it is unclear whether the steady-state and quasi-steady-state simulations used to make the bifurcation diagrams in Figure 3 include noise forcing. If not, then this is concerning, because it is well known that marine ice sheets have a different steady-state with and without noise in the forcing (e.g. Robel et al. 2018, Hoffman et al. 2019, Mikkelsen et al. 2018 (but for non-marine ice sheets)). This could be quite important in your simulations, since the location of the bifurcation is important to know for calculating EWS.

The surrogate model was an AR based surrogate and does indeed contain a lot of decadal variability and this is important to capture as Alex and other authors have shown. We will provide more details on how this surrogate was made in a revised version of the paper. The second important benefit of adding this variability to our melt rate forcing is that, by perturbing the model, it helps to extract information on the system response time to identify critical slowing. It is true that the paper by Lenton 2008 shows results from earlier work by Held and Kleinen 2004 and in this case they used white noise with the same aim. The variability we add to our forcing is not white noise but there is no reason why this should affect our ability to extract information on the system response time. Indeed, *other authors have found critical slowing in a wide variety of paleo data (e.g. Dakos et al. 2008) and the natural variability that drove these systems is presumably also not white noise. The key to be able to detect EWS is to add some perturbation with a measurable impact on the system state and our surrogate time series succeeds in that with the added benefit of replicating natural variability as closely as possible. Indeed, using natural variability to test for EWS arguable has more relevance to this system than using white noise. With regards to our steady-state simulations (the squares and circles in figure 3b) no natural variability was added to these simulations. With the addition of strong variability in the forcing it is possible that some of these steady states might be nudged far enough that they end up in a different basin of attraction. However, we disagree with the reviewer that this is concerning. The location of the bifurcation for the purposes of searching for EWS was not defined based on these points but on the 'quasi steady state' points and so have no bearing on our results and merely serve to demonstrate that several hystereses exist. Having said that, it is more consistent to add the same variability in these runs. We have re-run these steady state simulations with the same natural variability that was added to the main PIG simulations and this has no significant impact on the location of these points but we will use these results, rather than the ones shown in this version of the paper, for a revised submission.*

We have added a more detailed explanation of our surrogate time series and re-calculated all quasi-steady simulations to include the same variability as the transient simulations. This has not significantly affected any of the results and the figure now shows these steady state values instead of the previous ones that did not include variability.

Minor:

Line 23: please define what you mean by "tipping element" for the uninitiated *This is now explained*

Line 29: grounding line flux *Done*

Line 47: the 2012 Schoof JFM paper makes more sense to cite here *Changed the citation*

Line 48: what do you mean by catastrophic?

We use the word catastrophic in the sense that it is sometimes in literature on tipping points, that is a large change in the system state. We have made an effort to keep all terminology clear and consistent and since this is the only instance of that word in the paper we will rephrase it.

A wide variety of words are used in this context, another example is abrupt, but this implies a rapid response that is not necessarily true. We opted to call this a 'qualitative shift' to convey that the system is substantially different once it crosses a bifurcation point

Line 89: So is accumulation held constant in time or does it have a seasonal cycle? Be more specific here, because its important to know if there is variability in more than just ocean melt.

Accumulation is held constant in time, specifically to avoid adding variability in more than one forcing which could potentially muddy our results.

Clarified that accumulation is constant in time

Line 119: Again, to be clear, it isn't necessarily the case that EWS exist for all tipping points (if we define any bifurcation as a tipping point).

We addressed this comment above and will add a clarification on this point.

Added this clarification

Line 126: it would help here to explain exactly how you force the simulations to produce the grey dashed and black lines in Figure 3 (also in caption). How fast is the forcing? How large are the step increments to determine the steady-states? How do you determine when there is a tipping point (just a large enough jump in the grounding line?)? What if you have a tipping point that causes the grounding line to retreat only a small amount?

We will expand on this in a revised version of the manuscript.

The paragraph of line 126 is talking about the experiment in appendix A and not the one that generated the results in figure 3. We have added some of these details to the figure caption and the rest in the relevant section in the main body of text.

Line 119-129: This whole paragraph is confusing

Much of this paragraph has been rewritten

Line 136: What do you mean by short tem "weather noise"? Isn't this the thing that is detected by EWS?

This is a term that has been used in EWS literature to describe rapid but low amplitude fluctuations in the system state that do not represent quantities of interest

The wording has been changed to remove this term

Line 184: What do you mean by "equivalent to a random walk"?

A random walk describes an evolving system that consists of a succession of random steps but this is probably unnecessary jargon and will be removed.

This sentence is removed and instead we describe the DFA in more detail without this jargon in section 2

Line 197-198: is this hysteresis related to the domain extent? If you aren't simulating flow from outside the domain, then when the whole domain collapses you won't be able to regrow the glacier for any parameter (because theres no catchment).

The catchment is not completely gone and the grounding line remains within the basin but presumably you are correct that by extending the domain it might make it easier to regrow the glacier. That does not detract from our point in this case but we will include this as a possible explanation.

This point has been added

Line 203-205: what about much shorter windows? (related to point 1 above related to observations). How short of a window would actually be calculable from observations (a decade? does this start to run into the AC time scale of forcing?)

This is addressed above and will be discussed in more detail in the revised paper.

We have added substantial discussion on this point

Line 214-215: this is a confusing sentence which leads me to think that you are saying the EWS are not actually "early". To what extent does this depend on the speed of the trend in forcing? Can you test it for different trend rates?

We feel that the next sentence addresses this question to some extent but we can expand on that further. The EWS are identifying tipping in the transient simulation for which they are calculated, and this would be equivalent to the 'steady-state' simulations if the forcing was increased sufficiently slowly, but since the transient simulation evolves quicker than the glacier can adjust then these are not the same. To what extent the forcing rate affects various things is a potentially large and complex research question that we believe is beyond the scope of our study.

We have added discussion of this point in Sect. 5

Figure 3a: If the black line doesn't fall on top of the gray dashed line, then the black line simulation isn't really quasi-steady. Why not call it something else? Also, it is unclear how the grey dashed line is determined, and which parts are stable and unstable?

We will think of a word other than 'quasi steady' as this is possibly confusing. The dashed grey line is just a line through the individual model results represented by symbols, all symbols are steady states but it is possible that these are not true stable steady states.

We have clarified how the dashed grey line is determined in the figure caption. We discussed alternative names for this simulation but in the end we decided to continue with this name which is the same as has been used in similar studies on ice sheet hysteresis (e.g. the recently published paper by Garbe et al.) and instead we have added a better explanation defining what we mean by this.

Figure 3b: please explain the different between "tipping point" and "instability onset". I can guess that the latter has to do with the region in which an unstable manifold exists (i.e. there is hysteresis), but I'm not sure readers will necessarily pick up on this without you explaining it explicitly.

Instability onset is not the best choice of words here, and the legend and caption have been changed to EWI window to reflect that this is the portion of the time series that was analysed using early-warning indicators

Figure 4: Looking at this (and Fig. B2), it seems clear to me that the length of time ahead of the tipping point needed to detect EWS is directly correlated to the speed of the forcing. Yet, that isn't really made clear here or in the text.

The rate at which the forcing is changed is constant throughout the simulation (ignoring the added variability)

We have added discussion on the rate of forcing in Sect. 5

Figure B2: can you extend this to shorter windows?

Done

Appendix A: Related to some of the issues I raised above, it may be valuable to bring the flowline simulations into the main text to demonstrate, in a very simple system where the exact location of the tipping points are known, how EWS work

We have added extensive explanation of how early-warning signals work to address these concerns but have chosen to keep this section in the appendix to avoid the confusion of describing two very different model simulations with different goals in the same paper

References

Dakos, V. et al. Resilience indicators: prospects and limitations for early warnings of regime shifts. Phil. Trans. R. Soc. B 370, 20130263 (2015)

Chisholm, R. A. & Filotas, E. Critical slowing down as an indicator of transitions in two-species models. J. Theor. Biol. 257, 142–149 (2009).

Scheffer, M. et al. Early-warning signals for critical transitions. Nature reviews. 461. 53-59 (2009)

Dakos, V. et al. Slowing down as an early warning signal for abrupt climate change. PNAS. 14308-14312 (2008).

Held, H. and Kleinen, T. Detection of climate system bifurcations by defenerate fingerprinting. GRL. 31. L23207 (2004).

Brock, W. A. and Carpenter, S. R. Interacting regime shifts in ecosystems: implication for early warnings. Ecological Monographs 80, 353-367 (2010)

Garbe, J. and Albrecht, T. and Leverman, A. and Donges, J. F. and Winkelmann, R. The hysteresis of the Antarctic Ice Sheet, *Nature*, **585**, 538-544 https://doi.org/10.1038/s41586-020-2727-5 (2020)

Anonymous Referee #2

In this manuscript the authors seek to detect the onset of marine ice sheet instability (MISI) in model simulations of Pine Island Glacier, using techniques that have previously been applied to other complex systems. The novelty of this study lies in the application of critical slowing indicators to confirm MISI events. It provides an interesting framework for evaluating vulnerabilities in ice sheets that will be of interest to the TC scientific community. However, particularly due to its novelty within glaciology, some aspects of the paper need improving to aid the clarity, and it would benefit from further exploration/discussion of the usefulness of the techniques beyond the modelling example provided here. I have outlined these below, followed by line-by-line comments.

Many thanks for the positive evaluation of our study and the helpful comments. We replied to all comments and hope that our replies and changes made to the manuscript will address all issues raised. Our replies are in red and the changes to the manuscript summarised in blue.

The paper includes a nice explanation and accompanying schematics of hysteresis; however, the critical slowing description and explanation of the indicators is less intuitive. This may partly be due to the structure – Appendix A offers a useful demonstration of how critical slowing manifests in a carefully controlled simple experiment. I think it is safe to assume most TC readers will not be familiar with these concepts, and therefore I do not think this example should necessarily be tucked away in an appendix. A diagram of critical slowing in a similar vein to Figure 1 would be helpful, or some additional annotation to Figure A1. There is a disconnect between the flowline example in Appendix A, and the methods used for determining the onset of a tipping point in the main set of experiments. Could you show how critical slowing in the flowline experiment can be demonstrated with the various indicators you use in your main experiments? This would help show how these indicators are related to the increased recovery time from a stepwise perturbation as the tipping point is approached.

The other reviewer also highlighted that various elements currently in the appendix could be moved into the main body of text and we will do so. We will also give a more detailed explanation of critical slowing since it is true that many TC readers will not be familiar with this concept and will produce a new figure in the introduction to hopefully aid this explanation. Our two sets of experiments, one using the MISMIP setup and the other using the PIG geometry, currently identify critical slowing in two different ways. This is intentional as these have different goals and we do not think it would be useful to directly compare them. The benefit of the MISMIP experiment is that it is simple and the way we extract the change in response time directly is very easy to understand. It is also an experiment that many readers of TC and the wider glaciological community are very familiar with. However, since the bed is smooth and the perturbations to the model are stepwise, we can not detect EWS in the flowline setup using the same approach as was used for the PIG experiment.

We have added a new section to the paper that gives a more detailed explanation of critical slowing and early warning indicators, including a new figure that explains these concepts diagrammatically. We have added further explanation to the differences between the flowline results and main results to clarify why the indicators could not be used in the flowline case.

My other main comment is about the usefulness of this method in detecting EWS in reality. Would the 300-year optimal window size apply to other catchments? What kind of observational datasets are required to implement this analysis, in a way that would act as a useful EWS for MISI? The measurement used in this study (grounding line flux) does not exist (at least not at the quality/resolution required here) prior to the satellite era, so what is the alternative, given the 300-

year window size? In your model simulations the forcing is applied gradually in order to avoid "one tipping point cascading into the next and result in three individual tipping points being misinterpreted as only one event" (L168). What are the implications of this for detecting tipping points in observations, where the system is not necessarily able to return to a quasi-steady-state with changing forcings? Do you have a sense of whether the indicators would hold up if the forcing is more rapid? Further discussion of these issues would strengthen the paper.

Many of these points are raised by reviewer 1 and we have addressed them at length in that reply. To summarise our response: we will add more discussion around the implications of our results for finding EWS in observational records. We do not believe the 300-year window size to be a lower bound and did not try to reduce this systematically, but this will also be discussed in a revised version of the paper. Since the length of record necessary for EWS as well as the variable used to detect EWS are critical parts of what observations can or cannot be used, this is a very important research question but one that requires considerable work to address and is beyond the scope of this initial study. Regarding the last point about how cascading tipping points and a rapid forcing might affect our ability to detect critical slowing, this is known to influence the ability to detect EWS (Dakos et al., 2015). For example in ecosystems, interacting regime shifts can muffle or magnify variance near critical transitions (Brook and Carpenter, 2010). In this study, as explained in detail in the response to reviewer 1, we first of all want to make the nontrivial case, that ESW – that exist theoretically for MISI – can actually be detected in a realistic geometry that is of great interest with respect to MISI at the moment. We would argue that this issue raised by the reviewer is very important and an interesting research questions but beyond the scope of this study.

We have added extensively to the manuscript to discuss all the points raised here by the reviewer: the usefulness of the method, the window size, observational datasets required and cascading tipping points.

Other comments:

L13: "Self-amplifying retreat" this could be considered an overstatement. Selfsustaining retreat would be more accurate (and more in keeping with language further on in the manuscript).

Changed as suggested

L18: "early warning indicators robustly detect critical slowing for the marine ice sheet instability". It might be worth removing the term "critical slowing" from the abstract, and instead using a less jargon-y alternative, e.g. "robustly detect the onset of MISI"?

Changed as suggested

L31-32: "a complex range of factors can either cause or suppress the MISI" – such as? The two papers cited refer to buttressing, what about local sea level, GIA etc? Haseloff, 2018, should be Haseloff and Sergienko, 2018.

Further citations added for other factors

L68-70: "Our results reveal the existence of multiple smaller tipping points that when crossed could easily be mis-identified as simply periods of rapid retreat, with the irreversible and the self-sustained aspect of the retreat being missed": this seems to contradict your results and conclusions. The two smaller tipping points are not irreversible, as the system can return to previous state through stronger perturbations in the opposite direction – shown by the hysteresis loops in Figure 3.

Unhelpfully, the word irreversible is used by various authors to mean different things in this context and we have tried to be consistent throughout the paper and we clarify our meaning in lines 53-54. We are not aware of a word that uniquely describes one or other type of irreversibility but this is an important point to clarify.

We now differentiate between the two types of behaviour here and throughout the manuscript

L106: Basal melt rates: it is not clear from this paragraph whether basal melt occurs under grounded ice. How do you treat partially grounded elements? This has been shown to be important in modelling grounding line retreat (e.g. Seroussi and Morlighem, 2018, doi: 10.5194/tc-12-3085-2018).

Melting is only ever applied to fully floating elements.

This point is added to the revised manuscript

L153-159: Unlike the other paragraphs in this section, this paragraph does not contain an outcome of your decision-making process – which of the criteria will you use?

We find that indicators do seem to reach critical values ahead of a tipping point and but this is a result of our study rather than a decision made during the experiment design.

This paragraph has been rewritten to clarify how we present our results and why

L180-181: "Furthermore, the indicator reaches a critical value relatively close in time to when the MISI event gets underway". Clarify that the critical value is 1.

Done

L183-185: "For this early warning indicator. . .". I don't understand this sentence and it seems like it would be better suited (with added detail) to the methods. I thought both indicators have a critical value of 1 (section 2.2), so why does scaling the DFA help with comparison to ACF?

The DFA indicator does not have a critical value of 1, hence why scaling it makes sense.

This is hopefully now much clearer as a result of the more detailed explanation of the DFA indicator

L188-189: "although variance cannot be used directly to predict when that threshold will be crossed" – perhaps this is obvious, but why not? Because there is no critical value? But crossing the critical value doesn't seem like a robust way of detecting the exact onset of MISI either, considering some of the trend lines in Fig. 4 cross x=0 before they reach the critical value?

Yes, because variance has no critical value. It is true that our trend lines do not cross critical values exactly at the tipping point, that would be remarkable given the complexity of the model, however we do find our indicators reach critical values very close to a tipping point, making this a useful EWS.

We have clarified why variance cannot be used and also why it is useful later in the discussion

L275: What do you mean by "i.e. a record length of 600 years" – how does that relate to the window size? Is that the minimum record length required?

This point is addressed at length elsewhere in our response.

L258: Basin of attraction has not been defined/explained.

Reworded this sentence

Figure A1: Panel A, grounding line position in km: clarify the direction of retreat.

This is now annotated on the figure

The tipping points and early-warning indicators for Pine Island Glacier, West Antarctica

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Abstract. Mass loss from the Antarctic Ice Sheet is the main source of uncertainty in projections of future sea-level rise, with important implications for coastal regions worldwide. Central to this is the marine ice sheet instability: once a critical threshold, or tipping point, is crossed, ice-internal dynamics can drive a self-~~amplifying~~sustaining retreat committing a glacier to irreversible, rapid and substantial ice loss. This process might have already been triggered in the Amundsen Sea region, where Pine Island and Thwaites glaciers dominate the current mass loss from Antarctica, but modelling and observational techniques have not been able to establish this rigorously, leading to divergent views on the future mass loss of the WAIS. Here, we aim at closing this knowledge gap by conducting a systematic investigation of the stability regime of Pine Island Glacier. To this end we show that early warning indicators robustly detect ~~critical slowing for~~the onset of the marine ice sheet instability. We are thereby able to identify three distinct tipping points in response to increases in ocean-induced melt. The third and final event, triggered by an ocean warming of approximately 1.2 °C from the steady state model configuration, leads to a retreat of the entire glacier that could initiate a collapse of the West Antarctic Ice Sheet.

1. Introduction

The West Antarctic Ice Sheet (WAIS) is a ~~tipping element~~ major component of the earth system susceptible to tipping point behaviour, known as a tipping element (Lenton *et al.*, 2008~~) and its~~). Its collapse, potentially driven by the Marine Ice Sheet Instability (MISI~~)~~ (; Feldmann and Levermann, 2015), would result in over 3m of sea level rise (Fretwell *et al.*, 2013). Key to the MISI are the conditions at the grounding line - the transition ~~line at~~across which ~~the~~ grounded ice begins to float on the ocean forming ice shelves. In steady state, ice flux across the grounding line balances the surface accumulation upstream. If a grounding line retreats over a region of bed where ~~ice~~grounding line flux increases and is not balanced by a corresponding increase in accumulation, the net mass balance is negative and retreat will continue (Weertman, 1974; Schoof, 2007). Conversely, grounding line advance leading to an increasing accumulation greater than the change in flux will lead to a continued advance. In this regime, a small perturbation ~~in forcing~~ can result in the system crossing a tipping point, beyond which a positive feedback propels the system to a contrasting state (Fig. 1c). A complex range of factors can either cause or suppress the MISI (Haseloff, 2018; Pegler, 2018, O'Leary *et al.*, 2013; Gomez *et al.* 2010; Robel *et al.* 2016) and the difficulties in predicting this behaviour are a major source of uncertainty for future sea level rise projections (Church *et al.*, 2013; Bamber *et al.*, 2019; Oppenheimer *et al.*, in press; Robel *et al.* 2019).

One area of particular concern is the Amundsen Sea region. Pine Island (PIG) and Thwaites glaciers, the two largest glaciers in the area, are believed to be particularly vulnerable to the MISI (Favier *et al.*, 2014; Rignot *et al.*, 2014). Palaeo and observational records of PIG show a history of retreat, driven by both natural and anthropogenic variability in ocean forcing (Jenkins *et al.*, 2018; Holland *et al.*, 2019). One possible MISI driven retreat might have happened when PIG unpinned from a submarine ridge in the 1940s (Jenkins *et al.*, 2010~~,~~; Smith *et al.*, 2016). Recent modelling studies indicate that a larger scale MISI event may now be underway for both Pine Island and Thwaites glaciers that would lead to substantial and sustained mass loss throughout the coming centuries (Favier *et al.*, 2014~~,~~; Jenkins *et al.*, 2016~~,~~; Joughin *et al.*, 2010). Being able to identify a MISI driven retreat and differentiate this from an externally forced retreat where a tipping point has not been crossed is vital information for projections of future sea level rise.

~~This~~The tipping behaviour of the MISI is an example of a ~~fold (or~~ saddle-node (or fold) bifurcation in which three equilibria exist; an upper and lower stable branch and a middle unstable branch (Fig. 1c~~) (~~; Schoof, ~~2007~~2012). Starting on the upper stable branch, perturbing the system beyond a tipping point (x_1 in Fig. 1c) will induce a ~~catastrophic~~qualitative shift to the lower and contrasting stable state. Importantly (and in contrast to a system such as that shown in Fig.1a and 1b), in order to restore conditions to the state prior to a collapse it is not sufficient to simply reverse the forcing to its previous value. Instead, the forcing must be taken back further (to point x_2), which in some cases may be far beyond the parameter space that triggered the initial collapse. This type of behaviour is known as hysteresis. A large change in response to a small forcing is not necessarily indicative of a hysteresis, as shown in Fig.1b. Tipping points are crossed in both Fig. 1c and Fig. 1d and both cases are often referred to as irreversible, although the two are distinct in that only Fig. 1d is irreversible for any change in the tested range of the control parameter. Hereafter we will refer to the former as irreversible, in line with previous studies, and the latter as permanently irreversible, to differentiate the two. Diagnosing whether~~ or not~~ a tipping point has been crossed without some prior knowledge of the system is not generally possible without reversing the forcing to see if a hysteresis has occurred. An alternative approach to identify tipping points is based on a process known as *critical slowing*, which is known to precede ~~fold~~saddle-node bifurcations of this type (Wissel, 1984; van Nes and Scheffer, 2007; Davos *et al.*, 2008; Scheffer *et al.*, 2009). Critical slowing is a general feature of non-linear systems and refers to an increase in the time a system takes to recover from perturbations as a tipping point is approached (Wissel, 1984). We will explore both hysteresis and critical slowing as indicators of tipping points in our model simulations.

~~Here, we~~In Section 2, we explain critical slowing and early warning indicators in the context of the MISI. We then map out the stability regime of Pine Island Glacier using numerical model simulations. We force the model with a slowly increasing ocean melt rate and identify three periods of rapid retreat with the methodology explained in Sect. ~~2~~3.1. Using statistical tools from dynamical systems theory~~, introduced in Sect. 2.2,~~ we find critical slowing preceding each of these retreat events and go on to demonstrate that these are indeed tipping points in Sect. ~~3~~4. This is confirmed by analysing the hysteresis behaviour of the glacier, showing the existence of unstable grounding line positions. To our knowledge, this is the first time that the stability regime of Pine Island Glacier has been investigated in this detail and the first time that tipping point indicators have been applied to ice sheet model simulations. Our results reveal the existence of multiple smaller tipping points that when crossed could easily be mis-identified as simply periods of rapid retreat, with the irreversible and the self-sustained aspect of the retreat being missed.

2. Critical slowing and early warning indicators

As certain classes of complex systems approach a tipping point, they show early warning signals which can allow us to anticipate or even predict the onset of a tipping event by means of early warning indicators (EWIs; Wissel, 1984). Early warning signals have been found to precede, for example, collapse of the thermohaline circulation (Held and Kleinen, 2014; Lenton, 2011), onset of epileptic seizures (Litt, 2001; McSharry and Tarassenko, 2003), crashes in financial markets (May *et al.*, 2008; Diks *et al.*, 2018), onset of glacial terminations (Lenton, 2011) and wildlife population collapses (Scheffer *et al.*, 2001). Although most commonly used to detect the onset of saddle-node bifurcations, of which the MISI is an example, they are not strictly limited to bifurcations of this type and have, for example, also been successfully used to indicate the onset of Hopf bifurcations (Chisholm & Filotas, 2009).

2.1 Critical slowing preceding the Marine Ice Sheet Instability

Critical slowing is one example of an early warning signal that has been used in the past for both model output and observational records such as paleoclimate data, with the aim of detecting an approaching bifurcation (Held and Kleinen, 2004; Livina and Lenton, 2007; Dakos *et al.* 2008; Lenton *et al.* 2009; Lenton *et al.* 2012b). Critical slowing is so called because, as a non-linear system is gradually forced towards a bifurcation, that system will become more 'sluggish' in its response to perturbations (see middle panel of Fig. 2). This can be shown mathematically, because the dominant eigenvalue of the system tends to zero as a bifurcation point is approached (Wissel, 1984), or, equivalently, the recovery time (i.e. the time it takes for a system to return to a steady state after small perturbations) tends to infinity. While critical slowing is a general behaviour of the dynamics underlying the MISI, the question remains whether it can be reliably detected in the context of a complex glacier where many other processes are at play.

As a first step to addressing this question, we model a MISI in an idealised flowline setup of a marine ice sheet. In this setup, we determine the change in recovery time before a tipping point directly through multiple stepwise perturbations of the control parameter (Appendix A). Our setup closely resembles the MISMIP experiments (Pattyn *et al.*, 2012) and indeed hints of critical slowing can be identified in that paper (Fig. 2 in Pattyn *et al*. 2012). The results in Appendix A show that critical slowing is easily identified preceding both MISI driven advance and retreat bifurcations. This demonstrates that there is at least the potential that critical slowing could be found in a less simplified modelling framework. This is not a priori clear and, for example, adding noise to the bed topography reduces the ability to identify early warning, as detailed in the Appendix. Identifying critical slowing in this stepwise perturbation manner is appealing because it directly extracts the change in response time that we are searching for, however it is not practical for a realistic model forcing which would not normally take the form of a step function. A more general approach, which we adopt for our simulation of PIG, is to use EWIs to analyse the recovery time of the system as it is forced with natural variability.

2.2. Early warning indicators

As the field of EWIs has expanded, more methods have been developed for extracting critical slowing information from model results and observational records. These methods seek to approximate the system recovery time from some measure of the system state. The challenge is that, for most real-world applications, natural forcing does not take the form of a step function and the system is continuously perturbed and so cannot return to a true steady state. However, if the recovery time of a system is indeed increasing, the response to a continual stochastic forcing could be detected as a tendency for each measurement of the system state to be more similar to the previous measurement, sometimes referred to as an increase in "memory" of small perturbations. This is shown conceptually and with examples extracted from our PIG model in Fig. 2. One common way to measure this effect is by sampling the data at discrete time intervals and calculating the lag-1 autocorrelation, i.e. the correlation between values that are one time interval apart (examples given in Fig. 2). This measure, which we refer to hereafter as the *ACF indicator*, should increase as a tipping point is approached (Dakos *et al.*, 2008; Ives, 1995). Since recovery time tends to infinity as the bifurcation is approached, successive system states should become more and more similar and the ACF indicator should tend to one. An alternative measure that also seeks to identify changes in recovery time is to use the Detrended Fluctuation Analysis algorithm (Livina and Lenton, 2007; Lenton *et al*., 2012a; Lenton *et al*., 2012b). This first calculates the mean-centered cumulative sum of the time series, splits the result into epochs of length n which are detrended and then calculates the RMS $F(n)$ for each epoch. This is repeated for epochs of different length and finally an exponent α can be fitted in log-log space such that $F(n) \propto n^{\alpha}$. This exponent yields information on the self-correlation of the original time series, whereby a value of 0.5 corresponds to uncorrelated white noise and greater values indicate increasing "memory" up to a maximum of 1.5. To aid comparison with the ACF indicator, we rescale the exponent so that it reaches a critical value of 1 and call this the *DFA indicator* (Livina and Lenton, 2007). These indicators can be supported by analysing the variance of the system state which is expected to increase as a tipping point is approached.

3. Methods

We conduct a quasi-steady modelling experiment whereby we subject PIG to slowly increasing rates of basal melt underneath its adjacent ice shelf (Fig. ~~2). We use basal melt rate as the control parameter,~~3). Conducting a transient simulation with an evolving basal melt that exactly tracks the equilibrium curve (Fig. 1c) is not computationally feasible or necessary for our purposes. Thus, we adopt this quasi-steady modelling approach in which the forcing increases slowly enough that it approximates the steady state behaviour, but faster than the long response timescales of the glacier would require to be truly in equilibrium. Quasi-steady state experiments have previously been successfully applied to identify the tipping point of the Greenland Ice Sheet with respect to the melt-elevation feedback (Robinson *et al.*, 2012) and to identify hysteresis of the Antarctic Ice Sheet (Garbe *et al.*, 2020). In Garbe *et al.* (2020) it was shown that such transient experiments enable identification of hysteresis behaviour, while the exact shape of the curve must be mapped out with equilibrium simulations. We accompany the quasi-steady simulations with simulations that run to a true steady state for constant values of the control parameter at discrete values (these simulations continue until the change in ice volume is approximately equal to zero). We use basal melt rate as the control parameter, i.e. the parameter that we will change to drive the system towards a tipping point. We make this choice since erosion of ice shelves by the intrusion of warm ocean currents is widely accepted as the mechanism responsible for the considerable changes currently observed in this region (Shepherd *et al.*, 2004~~,~~; Rignot *et al.*, 2014; Rignot 1998; Joughin *et al.*, 2010; Park *et al.*, 2013~~,~~; Gudmundsson *et al.*, 2019). Sub-ice-shelf melt rates are increased linearly (with additional variability as explained below) from a value that generates a steady state for the present-day glacier configuration. Based on the numerical experiments we then evaluate ~~early warning indicators~~EWIs to test for critical slowing.

~~2~~

3.1 Model description

All simulations use the community Úa ice-flow model (Gudmundsson *et al.*, 2012; Gudmundsson 2013, Gudmundsson 2020), which solves the dynamical equations for ice flow in the shallow ice stream approximation (SSTREAM or SSA~~) (~~; Hutter, 1983). Bedrock geometry for the Pine Island Glacier domain is a combination of the R-Topo2 dataset (Schaffer *et al.*, 2016) and, where available, an updated bathymetry of the Amundsen Sea Embayment (Millan *et al.*, 2014). Surface ice topography is from CryoSat-2 altimetry (Slater *et al.*, 2018). Depth-averaged ice density is calculated using a meteoric ice density of 917 kg m^{-3} together with firn depths obtained from the RACMO2.1 firn densification model (Ligtenberg *et al.*, 2011). Snow accumulation is a climatological record obtained from RACMO2.1 and constant in time (Lenaerts *et al.*, 2012).

Viscous ice deformation is described by the Glen Steineman flow law $\dot{\varepsilon} = A\tau_E^n$ with exponent $n = 3$ and basal motion is modelled using a Weertman sliding law $u_b = C{\tau_b}^m$ with exponent$m = 3$. ~~Rheology~~The constituitive law and the sliding law use spatially varying parameters for the ice rate factor (A) and basal slipperiness(C), respectively, to initialise the model with present day ice velocities. These are obtained via ~~inverse~~optimization methods using satellite observations of surface ice velocity from the Landsat 8 dataset (Scambos *et al.* 2016; Fahnestock *et al.* 2016). An optimal solution is obtained by minimising a cost function that includes both the misfit between observed and modelled velocities and regularisation terms~~, to avoid overfitting.~~. An additional term in the cost function penalises initial ~~changes in~~rates of ice thickness change in order to ~~avoid large transients~~ensure that these are close to zero at the start of simulations. This approach helps to provide a steady-state configuration of PIG from which we can conduct our perturbation experiments.

The Úa model solves the system of equations with the finite element method on an unstructured mesh, generated with mesh2d (Engwirda *et al.* 2014). The mesh remains fixed throughout the simulation to avoid contaminating the time series with errors resulting from remapping fields onto a new mesh. The mesh is refined in regions of high strain rate gradients, fast ice flow and around the grounding line. The region of grounding line mesh refinement, in which the average element size is ~750m, extends upstream sufficiently far so that the grounding line always remains within this region until after the final MISI collapse.

Basal melt rates are calculated using a widely used, local quadratic dependency on thermal forcing:

$$M = f\gamma_T\left(\frac{\rho_w c_p}{\rho_i L_i}\right)^2 \left(T_0 - T_f\right)\left|T_0 - T_f\right|,$$

where γ_T is the constant heat exchange velocity, ρ_w is sea water density, c_p is the specific heat capacity of water, ρ_i is ice density, L_i is the latent heat of fusion of ice, T_0 is the thermal forcing and T_f is the freezing temperature (Favier *et al.* 2019). Melt rates are only applied beneath fully floating elements to ensure that no melting can possibly occur upstream of the grounding line (Seroussi and Morlinghem, 2018). The initial melt rate factor (f) is chosen such that the model finds a steady state with a grounding line approximately coincident with its position as given in Bedmap2 (Fretwell *et al.* 2013). This melt rate factor ~~forms~~is the aforementioned control parameter ~~for our Pine Island simulations and is increased linearly to drive~~ that drives changes in the ~~region to collapse. The~~model, some of which may be identifiable as tipping points.

To effectively extract information about the system's recovery time using the statistical methods outlined ~~below rely~~in Sect. 2, we need to perturb the model in a way that has some measurable impact on ~~extracting~~ the system ~~recovery time from small perturbations.~~state. A slow and monotonically increasing forcing would make our chosen approach impractical and is arguably as unrealistic as a stepwise perturbation. We therefore add ~~a synthetic melt rate~~ natural variability to ~~our linearly increasing forcing, generated as a surrogate time series of an~~ the linearly increasing melt rate factor (f). There is strong evidence that the inferred and observed changes of PIG over the last century can be linked to changes in thermocline depth of the Amundsen Sea ~~ocean temperature proxy~~ shelf, which in turn is influenced by a Rossby wave train originating in the Pacific Ocean (Jenkins *et al.* 2018). Following Jenkins *et al.* (2018), we use a ~130 year time series of central tropical pacific sea surface temperature anomaly as a proxy for relevant variability in our melt rate forcing. We create an autoregressive (AR) model-based surrogate from this time series using the Yule-Walker method to fit the AR model and minimum description length to determine the maximum order of the model. This new surrogate time series has the same decadal variability that would be expected for the melting beneath PIG and can be extended to any length required. As shown in more detail below, ~~this ensures~~by superimposing this signal onto the linearly increasing melt rate factor we ensure that the system response contains sufficient variability to extract information about critical slowing and thereby ~~facilitating~~enable the calculation of ~~early warning indicators~~EWIs.

~~2.2~~ 3.2 Detecting critical slowing

We have already established the control parameter for our model, but another important decision to make is what model output should be used as a measure of the system state.~~Critical slowing and early warning indicators~~

~~As complex systems approach a tipping point, they show early warning signals which can allow us to anticipate or even predict the onset of a tipping event (Wissel, 1984). Evidence of these early warnings have been found to precede, for example, collapse of the thermohaline circulation (Held and Kleinen, 20014; Lenton, 2011), onset of epileptic seizures (Litt, 2001; McSharry and Tarassenko, 2003), crashes in financial markets (May *et al.*, 2008, Diks *et al.*, 2018), onset of glacial terminations (Lenton, 2011) and wildlife population collapses (Scheffer *et al.*, 2001). Critical slowing is one example of an early warning signal that has been used extensively in various studies of this kind. Using this approach to detect a MISI event is first validated with an idealised flowline setup of a marine ice sheet, in which we determine the change in relaxation time before a tipping point directly through multiple stepwise perturbations of the control parameter (Appendix A). Identifying critical slowing in this way is straightforward but is not practical for a realistic model forcing which would not normally take the form of a step function. A more general approach, which we adopt for our simulation of PIG, is to analyse the response time of the system as it is forced with natural variability using statistical tools.~~

~~A first decision to make is what quantity to measure in order to look for critical slowing in our model simulations and there are a number of possible options.~~ One choice could be changes in ice volume, since it can be related to sea level rise and ice sheet model simulations tend to focus on this result. However, ice volume varies very smoothly over time, making it difficult to detect changes in the system recovery time. Instead, we use the integrated grounding line flux, which shows much more variability and whose change is directly related to the MISI mechanism. ~~Prior~~As with other studies of this type, the model output is processed prior to the calculation of ~~critical slowing indicators,~~EWIs. This consists of aggregating the ~~model~~ output ~~is aggregated~~(i.e. data binning) to remove ~~short term 'weather noise'~~high frequency variability not related to the system recovery time, and ~~detrended~~detrending to remove nonstationarities (detrending is included in the ~~Detrended Fluctuation Analysis (~~DFA~~)~~ algorithm and therefore not required before calculation of the DFA indicator). Detrending was done using a Gaussian kernel smoothing function that has been shown to perform better than linear detrending (Lenton *et al.* ~~2012~~2012a). A smoothing bandwidth was selected that removed long term trends without overfitting the model time series. Indicators are calculated over a moving window with a length of 300 years. The optimal window length is further discussed below.

From the processed time series, we calculate three different ~~early warning indicators~~EWIs:

1. Critical slowing is measurable as an increase in the state variable auto-correlation. We measure this here using the lag-1 auto-correlation function (Dakos *et al.*, 2008; Scheffer *et al.*, 2009; Held and Kleinen, 2004) applied to the grounding line flux over a 300 year moving window preceding each tipping point (~~hereafter referred to as the~~ ACF indicator).
2. Similarly, ~~detrended fluctuation analysis~~ DFA (Peng *et al.*, 1994~~) (hereafter referred to as the DFA indicator~~) measures increasing auto-correlation in a time series and we apply this with the same moving window approach.
3. An additional consequence of critical slowing is that variance will increase as a tipping point is approached (Scheffer *et al.*, 2009). We calculate variance of grounding line flux for each moving window and this can be used in conjunction with other indicators to increase robustness.

~~Two different criteria are frequently used to assess early warning indicators and determine whether a tipping point is being approached. The first is simply to determine whether or not an indicator increases in the run up to a tipping event.~~As described in Sect. 2, recovery time should tend to infinity as a tipping point is approached. This corresponds to the ACF and scaled DFA indicators reaching a critical value of one. In practice, for a complex model there are a wide variety of reasons why a tipping point might be crossed before the EWI reaches a critical value. For example, this can be a result of variability in the control variable pushing the system over a tipping point despite its long-term mean still being some distance from its critical value. For this reason, most studies adopt an alternative approach of looking for a consistent increase in the EWIs in the run up to a tipping event. This is often measured by calculating the nonparametric Kendall's τ coefficient, which equals one if the indicator is monotonically increasing with time (Dakos *et al.* 2008; Kendall, 1948). ~~A second criteria is whether the indicators reach a critical value (in both cases a value of one) at the onset of the tipping event and thus can be used to predict when that event will happen. The second is clearly more useful but often fails in high complexity models. This failure can be a result of variability in the control variable pushing the system over a tipping point despite its long-term mean being far from its critical value.~~ This single value enables a simple interpretation of our results, since $0 < \tau < 1$ means the EWI is tending to increase with time, suggesting an imminent tipping point. We present our results in terms of both of these criteria.

~~3~~4. Results

The quasi-equilibrium simulation shows three potential tipping points with respect to the applied melt (Fig. ~~3~~4). Upon crossing each threshold, indicated by the numbered blue dots in Fig. ~~3~~4, PIG undergoes periods of not only rapid but (as we show ~~later~~below) also self-sustained and irreversible mass loss. At this stage, relying only on a record of changes in ice volume resulting from an increasing forcing (solid black line in Fig. ~~3~~4), one can only speculate that these are indeed tipping points and more analysis is necessary to confirm this hypothesis, as we go on to later. The last of the three events causes ~~an~~a permanently irreversible collapse within the entire model domain (Fig ~~3a).~~4a). We focus our results on these three major changes in the glacier configuration and ignore any possible smaller tipping points that do not result in significant grounding line retreat or changes in ice volume. We increase basal melt rates gradually and in a quasi-steady-state manner to ensure that successive retreat events can be isolated, and their effects do not overlap during the simulation. A more rapidly increasing forcing could lead to one tipping point cascading into the next and result in three individual tipping points being misinterpreted as only one event.

Grounding line positions before each of these retreat events and after the final collapse are shown in Fig. ~~2~~3. Events 1 and 2 each contribute approximately 20mm of sea-level rise while event 3, which arises after slightly more than doubling current melt rates, contributes approximately 100mm. The actual sea level rise that would result from this third and largest event is likely to be larger since in our simulation the effects stop at the domain boundary and in reality neighbouring drainage basins would be affected.

~~3~~4.1 Early warning for the marine ice sheet instability

The three periods of MISI-driven retreat after a tipping point has been crossed can be identified clearly using ~~early warning indicators~~EWIs (Fig. ~~4~~5). The ACF ~~early warning~~ indicator increases and tends to one as the tipping points are approached (Fig. ~~4a~~5a-c), indicating a tendency to an infinitely long ~~relaxation~~recovery time as predicted by theory~~ (Wissel, 1984).~~. We calculate Kendall's τ coefficient to identify trends in the indicator, with a value of one representing a monotonic increase in the indicator with time~~ (SI).~~. The positive Kendall's τ coefficient shows that in all three cases, the lag-1 auto-correlation increases before the onset of unstable retreat. Furthermore, the ACF indicator reaches a critical value of one relatively close in time to when the MISI event gets underway.

These findings are supported by the DFA indicator~~. For this early warning indicator, a scaling exponent is calculated that reaches a critical value at 1.5, equivalent to a random walk (rescaled here to reach a critical value at unity to aid comparison with the ACF indicator) (Livina and Lenton, 2007). Also here, the~~, described in Sect. 2. The Kendall's τ coefficient indicates a significant increase of the indicator when approaching the tipping points~~.~~ and the indicator trends towards a critical value of one. We show the change in normalised variance calculated over each time window and in all cases this increases ahead of the tipping points being crossed with a positive Kendall's τ coefficient. The increase in variance gives greater confidence to the findings of the other two ~~early warning indicators that a tipping point is being crossed~~EWIs, although variance cannot be used directly to predict when that threshold will be crossed~~.~~ since it does not approach a critical value before a tipping point is crossed.

~~3~~4.2 Hysteresis of Pine Island Glacier

In order to verify that we have correctly identified tipping points using the ~~early-warning indicators~~EWIs, we run the model to steady state for a given melt rate to search for hysteresis loops that indicate the presence of unstable grounding line positions. These simulations start from either the initial model setup (advance steady state) or the configuration just prior to the final tipping point (retreat steady state~~) and the~~). The model is run for ~~a~~ range of ~~fixed~~ melt ~~rate~~rates between these two states, with the mean melt factor held constant and the same natural variability applied as in the forward simulation, until ~~it~~the modelled ice volume reaches a steady state. The first two tipping events show relatively small but clearly identifiable hysteresis loops (Fig. ~~3b~~4b), for which recovery of the grounding line position requires reversing the forcing beyond the point at which retreat was triggered (i.e. as shown in Fig. 1c). The third event marks the onset of an almost complete collapse of PIG (Fig. ~~3a~~4a). Unlike the previous two, this collapse cannot be reversed to regrow the glacier for any value of the control parameter. This is an example of ~~an~~a permanently irreversible tipping point, as shown in Fig. 1d. Note that this permanent irreversibility is only true for the glacier modelled in isolation and by expanding the domain it would presumably be possible for other catchments that may not have collapsed to enable this glacier to regrow.

~~3~~4.3 Robustness of the indicators

We carry out several tests to assess the robustness of the EWIs and their sensitivity to the processing that is done on the model output prior to calculating each indicator. Two parameters in this processing step are the bin size into which data are aggregated and the bandwidth of the smoothing kernel that removes long term trends in the time series. To check that the increasing trends in our indicators are a robust feature of our results, regardless of these choices, we conducted a sensitivity analysis. The parameters were varied by +/- 50% and the indicators were recalculated for each resulting time series. As before, we assess the utility of an indicator by whether it shows an increasing trend before each tipping point, as measured by a positive Kendall's τ coefficient. The results of this sensitivity analysis are presented for each MISI event in Fig. 6. Kendall's τ coefficient is positive for all tested combinations of parameters and all MISI events, although MISI event 2 is particularly insensitive to these parameter choices whereas the spread in Kendall's τ coefficient is greater for the other two events.

In general, critical slowing will only occur close to a tipping point. Determining how close to a tipping point a system must be in order to anticipate the approaching critical transition, i.e. the prediction radius, is an important question and also informs the selection of palaeo-records that could be used to detect an upcoming MISI event. We show results for a window size of 300 years (i.e. a record length of 600 years), which is the shortest window size for which the DFA indicator provides an accurate prediction for all tipping events. We explored the prediction radius of our model by calculating Kendall's τ for the ACF and DFA indicators and the variance for a range of window lengths, see Fig. 7. For the main tipping event, preceded by the longest stable period, the indicators gradually lose their ability to anticipate a tipping event as more data is included further from the event. The same is true for the two smaller tipping events, but the drop off is quicker such that the indicators break down for window lengths > 500 years. These results suggest that the prediction radius is relatively small and window sizes that are too large, and hence include data far from a tipping point, become less useful for the application of EWIs.

In addition to a sensitivity analysis, it is important to check that trends in the calculated indicators are statistically significant and not the result of random fluctuations. We follow the method originally proposed by Dakos *et al.* (2012) and produce surrogate datasets from the model time series that have many of the same properties but should not contain any critical slowing trends. We generate 1000 of these datasets using an autoregressive AR(1) process based surrogate. For each of these datasets we calculate the ACF and DFA indicators and variance in the same way as with the model time series and then estimate the trend with values of Kendall's τ coefficient. We calculate the probability of our results being a result of chance for each indicator and for all three combined as the proportion of cases for which the surrogate dataset was found to have a higher correlation than the model time series. We find that P<0.1 in all but one instance for the ACF and DFA indicators but variance trends were generally less significant (Table 1). However, the combined probability that all three indicators would be equally positive as a result of chance was less than 0.02 for the first MISI event and less than 0.005 for the second two events.

~~We carry out several tests to assess the robustness of the early warning indicators, see Appendix B and C. Varying parameter choices in the time series processing steps can in some cases reduce the strength of the positive Kendall's τ correlation before each tipping point, but in general the trends are consistent and not dependant on the selected parameters (Fig. B1). A window size of 300 years is the smallest for which we find that the early warning indicators can provide a useful prediction for when a tipping point will occur (Fig. B2). Larger window sizes lead to a steadily reducing positive Kendall's τ correlation as a larger proportion of the time series is far away from a tipping event. To test whether the observed Kendall's τ correlations could have been obtained by chance we generate randomised surrogates of the model time series, process them in the same way and calculate Kendall's τ correlations. In this framework, the likelihood of obtaining our trend statistics by chance is estimated by the proportion of surrogate time series with a larger Kendall τ value than detected in the original data. We find that individually the DFA and ACF indicators are significant at the 10% level in all but one case and the significance of obtaining such positive trends for all three indicators at the same time is significant at the 1% level or less for all three MISI events (Table C1).~~

5. Discussion

The indicators we have tested provide early warning of tipping points as they are approached in our transient simulation with gradually increasing melt rates. Tipping points driven by the MISI represent potential 'high impact' shifts in the earth climate system, since they may lead to considerable changes in the configuration of the Antarctic Ice Sheet that are effectively irreversible on human timescales. Computational models are frequently used to forecast future changes of the Antarctic Ice Sheet in response to various greenhouse gas emission/warming scenarios. Predictive studies of this kind sometimes label periods of rapid retreat as 'unstable' without further analysis of the type done here (e.g. Joughin *et al.* 2014; Ritz *et al.* 2015; Favier *et al.* 2014) or avoid making this diagnosis altogether (DeConto and Pollard, 2016). Here, we have demonstrated that EWIs robustly approach critical thresholds preceding tipping points driven by the MISI. Our results show that EWIs can be used as a method to identify instabilities without the need of the aforementioned modelling approach based on computationally expensive equilibrium simulations.

It is important to clearly understand what critical threshold is identified by the ~~early warning indicators.~~EWIs. In Fig. ~~3~~4 the simulated steady-states show the crossing of the tipping point earlier than identified by the indicators in the transient simulation. Since the time-scales of ice are longer than the forcing time-scale, the ice-sheet system modelled here does not evolve along the steady-state branch (as shown schematically in Fig. 1c). Relaxation to a steady-state takes centuries to millennia in the simulations. This means that while technically the critical value of the control parameter (basal melt rate) might have already been crossed, the glacier state could still be reverted in the transient simulation at that point, if the basal melt rate was reduced below the critical threshold. This is true until the system state variable crosses its critical value (point x_t in Fig. ~~1c) – and this is the point picked up by the early warning indicators. This complication in interpreting early warning indicators is inherent to ice dynamics because of its long response time scales~~1c) – and this is the point identified by the EWIs. This complication in interpreting EWIs is inherent to ice dynamics because of its long response time scales. We find that both the ACF and DFA indicators not only increase as a tipping point is approached, as shown by positive Kendall coefficients, but also generally approach the critical value of one, although with varying degrees of precision (Fig. 5). This enhances their predictive power, since by extending a positive trend line it is possible to approximate what value of the control parameter will eventually cause a tipping point to be crossed. While our experiments in Appendix A showed that critical slowing down can accurately predict onset of tipping points in an idealised setup, applying this method to a more complex case study may have failed and in this context our finding that these indicators largely retain their predictive power is very encouraging. One area of additional complexity in our model of Pine Island Glacier compared to the setup in Appendix A is the bed geometry, which is obtained from observations and so much less smooth than the synthetic retrograde bed used in the MISMIP experiments. We explored how the addition of 'bumpiness' to bed geometry affects the performance of EWIs and found that this reduces how clearly we can resolve the change in response time (Appendix A). This effect may account for the fact that EWIs do not precisely reach a value of one at the bifurcation point but confirming this would require further testing.

4

There are several important caveats to the use of EWIs as presented here. Firstly, and as explained above, the tipping point identified is that of the transient system not in steady state. Although the transient behaviour is arguably of greater societal relevance and an ice sheet is unlikely to ever truly be in steady state, this is an important distinction to make. Secondly, the predictive power of this method decreases as the distance to tipping increases and must eventually break down altogether. This effect can be clearly seen in Fig 7 as the Kendall's τ coefficient decreases with increasing window length. Thirdly, there is a risk of so-called 'false alarms' and 'missed alarms' (Lenton, 2011). False alarms, whereby a positive trend in an indicator that is incorrectly interpreted as a tipping point being imminent, can occur for a wide variety of reasons. First and foremost, interpreting EWIs requires robust statistical analysis and judicious data processing to ensure that the response time being measured is that of the critical mode (Lenton, 2011). It is possible that rising autocorrelation is a result of other processes, and using more than one indicator together with changes in variance can help mitigate this risk (Ditlevesen, 2010). It is also possible for a tipping point to be crossed with no apparent warning i.e. 'missed alarms'. This could happen if the internal variability in a system is high so that it changes state before a bifurcation point is reached, or similarly if the forcing is too sudden. This last point is particularly pertinent, since we intentionally perturb our model slowly and do not explore how a change in forcing rate affects the performance of our chosen EWIs. Increasing the forcing rate might present further problems by leading to cascading tipping points. For example, if the control parameter changed rapidly after crossing the first tipping point it might cause the system to cross the second tipping point in a way that disguised that two distinct tipping points had been crossed. This issue of cascading tipping points is that one that we intentionally avoid in our experiments to simplify the analysis and is known to influence EWIs (Dakos *et al.*, 2015, Brock and Carpenter, 2010).

In this paper we have presented an application of EWIs on model output to anticipate tipping points. This is a useful approach in and of itself, since it could be used in model studies to detect bifurcations in the system with minimal computational expense, or to check whether a model might be on a trajectory to cross a tipping point at some point in time beyond the simulation. Alternatively, it may be possible to use this method on observational data, palaeo records, or some combination thereof. This raises the question of what data might qualify as useful for the application of EWIs, which can be broken down further into (1) the type of data needed and (2) the length of record necessary. As mentioned previously, ice volume or related measures of an ice sheet's size do not show sufficient variability for information on the recovery time to be extracted. Ice speed however can change significantly over very short timescales, for example many ice streams show large variability over timescales as short as tidal periods (Anandakrishnan *et al.*, 2003, Gudmundsson, 2006, Minchew *et al.*, 2017). Ice flux was chosen in this study since it is closely related to the MISI mechanism and because flux is proportional to velocity, but it is possible that other metrics related to ice velocity might also exhibit critical slowing in a similar way. With regards to record length, we find in this study a minimum window length of 300 years must be applied to obtain reliable early warning of tipping points. However, this does not mean that is this represents the minimum window size in general and is likely sensitive to a number of the choices in our methodology. For example, this value is likely to be sensitive to the rate of forcing applied to the system. In the limiting case of a forcing rate approaching zero, the necessary window length must increase since EWIs are only expected to work relatively close to the tipping point. Both of these points require further study in order to establish suitable datasets for prediction of MISI onset.

6. Conclusions

Conducting quasi-steady numerical experiments, whereby the underside of the PIG ice shelf is forced with a slowly increasing ocean-induced melt, we have established the existence of three distinct tipping points. Crossing each tipping point initiates periods of irreversible and self-sustained retreat of the grounding line (MISI) with significant contributions to global sea level rise. The tipping points are identified through *critical slowing*, a general behavioural characteristic of non-linear systems as they approach a tipping point. ~~Early warning indicators~~EWIs have been successfully applied to detect critical slowing in other complex systems. We here show that they robustly detect the onset of the marine ice sheet instability in the simulations of the realistic PIG configuration which is promising for application of early warning to further cryospheric systems and beyond.

While the possibility of PIG undergoing unstable retreat has been raised and discussed previously, this is to our knowledge the first time the stability regime of PIG has been mapped out in this fashion. The first and second tipping events are relatively small and could be missed without careful analysis of model results but nevertheless are important in that they lead to considerable sea level rise and would require a large reversal in ocean conditions to recover from. The third and final tipping point is crossed with an increase in sub-shelf melt rates equivalent to a +1.2°C change in ocean temperatures and leads to a complete collapse of PIG. Long-term warming and shoaling trends in Circumpolar Deep Water (Holland *et al.*, 2019), in combination with changing wind patterns in the Amundsen Sea (Turner *et al.*, 2017), can expose the PIG Ice Shelf to warmer waters for longer periods of time, and make temperature changes of this magnitude increasingly likely.

Appendix A: Flowline experiments

The MISI has been a major focus of modelling efforts within the glaciological community in recent years. In an effort to assess how ice-flow models capture this behaviour, a model inter-comparison experiment was performed to calculate the hysteresis loop of advance and retreat of a marine ice sheet on a retrograde slope, known as MISMIP experiment 3 (referred to as EXP 3 hereafter, Pattyn *et al.*, 2012). As a first step to establishing whether critical slowing can be observed prior to the MISI, we undertook a slightly modified version of this experiment using the Úa ice-flow model (Gudmundsson 2012, Gudmundsson 2013, see methods). In our modified experiment, the marine ice sheet is forced towards tipping points through step perturbations in the control parameter as before, but with smaller steps and the additional constraint that the model must be in steady state after each perturbation before moving onto the next. In this experiment the chosen control parameter is the ice rate factor, a parameter linked to ice viscosity and temperature.

Following each perturbation in the ice rate factor, we analyse the e-folding ~~response~~relaxation time (T_R) of the state variable (in this case, grounding line position) to directly extract the ~~relaxation~~recovery time ~~(T_R)~~ of the model as it approaches each tipping point (both advance and retreat). Theory predicts that $T_R \rightarrow \infty$ close to a tipping point and that the point at which ${T_R}^{-2}$ (as plotted versus the control parameter) reaches 0 thus identifies the critical value of the control parameter, beyond which a tipping point is crossed (Wissel 1984). We show this plot for both the advance and retreat scenarios of EXP 3 in Fig. A1. In both cases the relaxation time decreases as predicted by theory, even far from the tipping point. A linear fit through the last six perturbations yields a good agreement with theory and accurately predicts the critical value of the control parameter when compared to the analytical solution (red arrows in Fig. A1) given by Schoof (2007). Critical slowing still occurs outside of this range (equivalent to a change in ice temperature of >5 °C) but using these more distant points to forecast the tipping point would yield a less accurate prediction. These results therefore provide some insight into how far from the ~~basin of attraction~~tipping point we can expect the predicted linear response.

Appendix B: Indicator sensitivity

Before calculating the ACF and DFA indicators, the model output grounding line flux was processed, as described in the methods. Two parameters in this processing step are the bin size into which data are aggregated and the bandwidth of the smoothing kernel that removes long term trends in the time series. In order to check that the increasing trends in our indicators are a robust feature of our results, regardless of these choices, we conducted a sensitivity analysis. The parameters were varied by +/- 50% and the indicators were recalculated for each resulting time series. As before, we assess the utility of an indicator by whether it shows an increasing trend before each tipping point, as measured by a positive Kendall's τ coefficient. The results of this sensitivity analysis are presented for each MISI event in Fig. B1. Kendall's τ coefficient is positive for all tested combinations of parameters and all MISI events, although MISI event 2 is particularly insensitive to these parameter choices whereas the spread in Kendall's τ coefficient is greater for the other two events.

In general, critical slowing will only occur close to a tipping point. Determining how close to a tipping point a system must be in order to anticipate the approaching critical transition, i.e. the prediction radius, is an important question and also informs the selection of palaeo-records that could be used to detect an upcoming MISI event. We show results for a window size of 300 years (i.e. a record length of 600 years), which is the shortest window size for which the DFA indicator provides an accurate prediction for all tipping events. We explored the prediction radius of our model by calculating Kendall's τ for the ACF and DFA indicators and the variance for a range of window lengths, see Fig. B2. For the main tipping event, preceded by the longest stable period, the indicators gradually lose their ability to anticipate a tipping event as more data is included further from the event. The same is true for the two smaller tipping events, but the drop off is quicker such that the indicators break down for window lengths > 500 years. These results suggest that the prediction radius is relatively small and window sizes that are too large, and hence include data far from a tipping point, lose their ability to serve as early-warning indicators.

Appendix C: Indicator significance

In addition to a sensitivity analysis, it is important to check that trends in the calculated indicators are statistically significant and not the result of random fluctuations. We follow the method originally proposed by Dakos *et al.* (2012) and produce surrogate datasets from the model time series that have many of the same properties but should not contain any critical slowing trends. We generate 1000 of these datasets using an autoregressive AR(1) process based surrogate. For each of these datasets we calculate the ACF and DFA indicators and variance in the same way as with the model time series and then estimate the trend with values of Kendall's τ coefficient. We calculate the probability of our results being a result of chance for each indicator and for all three combined as the proportion of cases for which the surrogate dataset was found to have a higher correlation than the model time series. We find that $P<0.1$ in all but one instance for the ACF and DFA indicators but variance trends were generally less significant (Table C1). However, the combined probability that all three indicators would be equally positive as a result of chance was less than 0.02 for the first MISI event and less than 0.005 for the second two events.

One major simplification in this idealised experiment is the bed geometry, which is synthetic and arguably unrealistically smooth. To test whether the addition of ‘bumpiness’ to the bed affects how accurately the critical value of the control parameter can be predicted, we conducted further experiments in which the bed was made successively less smooth. One simple but flexible way to generate the desired roughness is to add Perlin noise to the bed. Perlin noise is a commonly used method in terrain generation that adds noise at a number of levels with successively smaller wavelengths and amplitudes. The number of levels is denoted by the octave, the rate at which each octave changes frequency is the lacunarity and the rate at which each octave changes amplitude is the persistence. We made the common choice of a lacunarity greater than one and a persistence less than one, meaning that each octave adds noise of higher frequency and lower amplitude. For a starting octave amplitude of 25m the difference between the analytical solution and linear fit is less than 1%, but this grows to ~5% with an amplitude of 50m (note the change in height from peak to trough in the retrograde region of the smooth bed is ~120m). This suggests that more realistic bed geometries with increased roughness might make the task of predicting tipping points more challenging than it is in this simplified case.

Code availability

The source code of the Úa ice-flow model is available from https://github.com/ghilmarg/UaSource (last access: 30 June 2020) and raw model output is available from the authors upon request.

Author ~~Contribution~~contribution

SHRR and RR conceived the study, SHRR conducted the modelling experiments, JFD contributed to the statistical analysis and surrogate time series, JDR provided an initial model setup. SHRR and RR wrote the manuscript with contributions from all authors.

Competing interests

The authors declare that they have no conflict of interest.

References

Anandakrishnan, S. and Alley, R.: Tidal forcing of basal seismicity of ice stream C, West Antarctica, observed far inland, *J. Geophys.Res.*, **102**, 15813–15196, (1997) Bamber, J. L., Oppenheimer, M., Kopp, R. E., Aspinall, W. P., Cooke, R. M. Ice sheet contributions to future sea-level rise from structured expert judgment. *Proceedings of the National Academy of Sciences*, **116 (23),** 11195-11200 https://doi.org/10.1073/pnas.1817205116, (2019)

Brock, W. A. and Carpenter, S. R. Interacting regime shifts in ecosystems: implication for early warnings. *Ecological Monographs* **80**, 353-367 (2010)

Chisholm, R. A. & Filotas, E. Critical slowing down as an indicator of transitions in two-species models. *J. Theor. Biol.* **257**, 142–149 (2009)

Church, J. A. *et al.* in *Climate Change 2013: The Physical Science Basis. Contribution of Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change* (eds Stocker, T. F. *et al.*) Ch. 13, 1137–1216 (Cambridge Univ. Press, 2013)

Dakos *et al.* Slowing down as an early warning signal for abrupt climate change, *Proceedings of the National Academy of Sciences*, **105 (38),** 14308-14312, https://doi.org/10.1073/pnas.0802430105, (2008)

Dakos, V. et al. Resilience indicators: prospects and limitations for early warnings of regime shifts. *Phil. Trans. R. Soc. B*, **370**, 20130263 (2015)

DeConto, R., Pollard, D.: Contribution of Antarctica to past and future sea-level rise. *Nature* **531,** 591–597 https://doi.org/10.1038/nature17145 (2016)

Diks, C., Hommes, C., and Wang, J.: Critical slowing down as an early warning signal for financial crises?, *Empirical Economics*, https://doi.org/10.1007/s00181-018-1527-3, (2018)

Ditlevsen, P. D. and Johnsen, S. J. Tipping points: Early warning and wishful thinking. *Geophys. Res. Let.* **37**, L19703 (2010)

Dutrieux, P. *et al.*: Strong sensitivity of Pine Island ice-shelf melting to climatic variability. *Science*, **343,** 174-178. https://doi.org/10.1126/science.1244341, (2014)

Engwirda, D.: Locally-optimal Delaunay-refinement and optimisation-based mesh generation, Ph.D. Thesis, *School of Mathematics and Statistics, The University of Sydney*, September, (2014) Fahnestock, M., Scambos, T., Moon, T., Gardner, A., Haran, T., and Klinger, M.: Rapid large-area mapping of ice flow using Landsat 8, *Remote Sens. Environ.*, **185**, 84–94, https://doi.org/10.1016/j.rse.2015.11.023, (2016)

Favier *et al.* Retreat of Pine Island Glacier controlled by marine ice-sheet instability. *Nature Climate Change*, **4,** 117-121. https://doi.org/10.1038/nclimate2094, (2014)

Favier, L., Jourdain, N. C., Jenkins, A., Merino, N., Durand, G., Gagliardini, O., Gillet-Chaulet, F., and Mathiot, P.: Assessment of sub-shelf melting parameterisations using the ocean–ice-sheet coupled model NEMO(v3.6)–Elmer/Ice(v8.3) , *Geosci. Model Dev.*, **12**, 2255–2283, https://doi.org/10.5194/gmd-12-2255-2019, (2019)

Feldmann, J. and Levermann, A. Collapse of the West Antarctic Ice Sheet after local destabilization of the Amundsen Basin, *Proceedings of the National Academy of Sciences*, 112 (46), 14191-14196, https://doi.org/10.1073/pnas.1512482112, (2015)

Fretwell, P. et al.: Bedmap2: improved ice bed, surface and thickness datasets for Antarctica, *The Cryosphere*, **7,** 375–393, https://doi.org/10.5194/tc-7-375-2013, (2013)

Garbe, J. and Albrecht, T. and Leverman, A. and Donges, J. F. and Winkelmann, R. The hysteresis of the Antarctic Ice Sheet, *Nature*, **585**, 538-544 https://doi.org/10.1038/s41586-020-2727-5 (2020)

Gomez, N., Mitrovica, J., Huybers, P. *et al.* Sea level as a stabilizing factor for marine-ice-sheet grounding lines. *Nature Geosci* **3,** 850–853 (2010).

Gudmundsson, G. H.: Fortnightly variations in the flow velocity of Rutford Ice Stream, West Antarctica., *Nature*, **444**, 1063–1064 (2006)

Gudmundsson, G. H., Krug, J., Durand, G., Favier, L., and Gagliardini, O.: The stability of grounding lines on retrograde slopes, *The Cryosphere*, **6**, 1497–1505, https://doi.org/10.5194/tc-6-1497-2012, (2012)

Gudmundsson, G. H.: Ice-shelf buttressing and the stability of marine ice sheets, *The Cryosphere*, **7**, 647–655, https://doi.org/10.5194/tc-7-647-2013, (2013)

Gudmundsson, G. H., Paolo, F. S., Adusumilli, S., and Fricker, H. A. Instantaneous Antarctic ice- sheet mass loss driven by thinning ice shelves. *Geophysical Research Letters*, 46, 13903–13909. https://doi.org/10.1029/2019GL085027, (2019) Gudmundsson, G. H. GHilmarG/UaSource: Ua2019b (Version v2019b). https://doi~~:~~.org/10.5281/zenodo.3706624 (2020)

Hansen, J. E.: A slippery slope: How much global warming constitutes 'dangerous anthropogenic interference'? *Climatic Change* **68,** 269–279 (2005).

Haseloff, M., and Sergienko, O.: The effect of buttressing on grounding line dynamics. *Journal of Glaciology*, **64**(245), 417-431. https://doi~~:~~.org/10.1017/jog.2018.30 (2018)

Held, H. and Kleinen, T.: Detection of climate system bifurcations by degenerate fingerprinting, *Geophysical Research Letters*, **31**, 1–4, https://doi.org/10.1029/2004GL020972, (2004)

Holland, P. R., Bracegirdle, T. J., Dutrieux, P., Jenkins, A. and Steig, E. J.: West Antarctic ice loss influenced by internal climate variability and anthropogenic forcing. *Nature Geoscience*, **12**. 718-724. https://doi.org/10.1038/s41561-019-0420-9, (2019)

~~Jenkins, A. *et al*. Decadal Ocean Forcing and Antarctic Ice Sheet Response: Lessons from the Amundsen Sea. *Oceanography*, **29,** 106-117. https://doi.org/10.5670/oceanog.2016.103, (2016)~~

Hutter, K.: Theoretical Glaciology, D. Reidel, Dordrecht, Netherlands. (1983)

Ives, A. R.: Measuring Resilience in Stochastic Systems. *Ecological Monographs* **65**, 217-233. (1995)

Jenkins, A. *et al.*: West Antarctic Ice Sheet retreat in the Amundsen Sea driven by decadal oceanic variability, *Nature Geoscience*, **11**, 733–738, https://doi.org/10.1038/s41561-018-0207, (2018)

Jenkins, A., Dutrieux, P., Jacobs, S. *et al.* Observations beneath Pine Island Glacier in West Antarctica and implications for its retreat. *Nature Geosci* **3,** 468–472, https://doi.org/10.1038/ngeo890, (2010)

Jenkins, A. *et al*. Decadal Ocean Forcing and Antarctic Ice Sheet Response: Lessons from the Amundsen Sea. *Oceanography*, **29,** 106-117. https://doi.org/10.5670/oceanog.2016.103, (2016)

Joughin, I., Smith, B. E., & Holland, D. M.: Sensitivity of 21st century sea level to ocean-induced thinning of Pine Island Glacier, Antarctica, *Geophysical Research Letters*, **37**, https://doi.org/10.1029/2010GL044819 (2010)

Joughin, I., Smith, B. E. and Medley, B.: Marine ice sheet collapse potentially under way for the Thwaites Glacier basin, West Antarctica, *Science*, **344**(6185), https://doi.org/10.1126/science.1249055 (2014)

Kendall, M. G.: Rank correlation methods, Oxford, Griffen, (1948)

Lenaerts, J. T. M., van den Broeke, M. R., van de berg, W. J., van Mejigaard, E., Kuipers Munneke, P.: A new, high-resolution surface mass balance map of Antarctica (1979-2010) based on regional atmospheric climate modeling. *Geophys. Res. Lett.*, **39**, L04501, https://doi~~:~~.org/10.1029/2011GL050713, (2012)

Lenton, T. M. *et al.*: Using GENIE to study a tipping point in the climate system. *Phil. Trans. R. Soc. A*, **367**, 871-884 (2009)

Lenton, T. M. *et al.*: Tipping elements in the Earth's climate system, *Proceedings of the National Academy of Sciences*, **105 (6),** 1786-1793, https://doi.org/10.1073/pnas.0705414105, (2008)

Lenton, T. M.: Early warning of climate tipping points, *Nature Climate Change*, **1**, 201–209, https://doi.org/10.1038/nclimate1143, (2011)

Lenton, T. M., Livina, V. N., Dakos, V., and Scheffer, M.: Climate bifurcation during the last deglaciation?, *Climate of the Past*, **8**, 1127–1139, https://doi.org/10.5194/cp-8-1127-2012, (~~2012~~2012a)

Lenton, T. M., Livina, V. N., Dakos, V., van Nes, E. H., & Scheffer, M.: Early warning of climate tipping points from critical slowing down: comparing methods to improve robustness. *Philosophical transactions. Series A, Mathematical, physical, and engineering sciences*, **370**, 1185–1204. https://doi.org/10.1098/rsta.2011.0304 (2012b),

Ligtenberg, S. R. M., Helsen, M. M., van den Broeke, M. R.:An improved semi-empirical model for the densification of Antarctic firn. The Cryosphere, 5, 809-819, https://doi~~: doi:~~.org/10.5194/tc-5-809-2011, (2011)

Litt, B., Esteller, R., Echauz, J., D'Alessandro, M., Shor, R., Henry, T., Pennell, P., Epstein, C., Bakay, R., Dichter, M., and Vachtsevanos, G.: Epileptic Seizures May Begin Hours in Advance of Clinical Onset: A Report of Five Patients, *Neuron*, **30**, 51–64, https://doi.org/~~https://doi.org/~~10.1016/S08966273(01)00262-8, (2001) Livina, V. N. and Lenton, T. M.: A modified method for detecting incipient bifurcations in a dynamical system, *Geophysical Research Letters*, **34**, 1–5, https://doi.org/10.1029/2006GL028672, (2007)

May, R., Levin, S. A., and Sugihara, G.: Ecology for bankers, *Nature*, **451**, 893–895, https://doi.org/10.1038/451893a, (2008)

McSharry, P. E., A., S. L., and Tarassenko, L.: Prediction of epileptic seizures: are nonlinear methods relevant?, *Nature Medicine*, **9**, 241–242, https://doi.org/10.1038/nm0303-241, (2003)

Millan, R., Rignot, E., Bernier, V., Morlighem, M., and Dutrieux, P.: Bathymetry of the Amundsen Sea Embayment sector of West Antarctica from Operation IceBridge gravity and other data, Geophys. Res. Lett., 44, 1360– 1368, https://doi~~:~~.org/10.1002/2016GL072071, (2017)

Minchew, B. M., M. Simons, B. Riel, and P. Milillo: Tidally induced variations in vertical and horizontal motion on Rutford Ice Stream, West Antarctica, inferred from remotely sensed observations, *J. Geophys. Res. Earth Surf.*, 122, 167–190, https://doi.org/10.1002/2016JF003971 (2017)

O'Leary, M., Hearty, P., Thompson, W., Raymo, M. E., Mitrovica, J. X. and Webster, J. M.: Ice sheet collapse following a prolonged period of stable sea level during the last interglacial. *Nature Geosci* **6,** 796–800, https://doi.org/10.1038/ngeo1890 (2013)

Oppenheimer, M., B.C. Glavovic , J. Hinkel, R. van de Wal, A.K. Magnan, A. Abd-Elgawad, R. Cai, M. Cifuentes-Jara, R.M. DeConto, T. Ghosh, J. Hay, F. Isla, B. Marzeion, B. Meyssignac, and Z. Sebesvari, 2019: Sea Level Rise and Implications for Low-Lying Islands, Coasts and Communities. In: *IPCC Special Report on the Ocean and Cryosphere in a Changing Climate* [H.-O. Pörtner, D.C. Roberts, V. Masson-Delmotte, P. Zhai, M. Tignor, E. Poloczanska, K. Mintenbeck, A. Alegría, M. Nicolai, A. Okem, J. Petzold, B. Rama, N.M. Weyer (eds.)]. In press.

Park, J. W., Gourmelen, N., Shepherd, A., Kim, S. W., Vaughan, D. G., and Wingham, D. J.: Sustained retreat of the Pine Island Glacier, *Geophysical Research Letters*, **40**, 2137–2142, https://doi.org/10.1002/grl.50379, (2013)

Pattyn, F. *et al.*: Results of the Marine Ice Sheet Model Intercomparison Project, MISMIP, *The Cryosphere*, **6**, 573–588, https://doi.org/10.5194/tc-6-573-2012, (2012) Pegler, S.: Marine ice sheet dynamics: The impacts of ice-shelf buttressing. *Journal of Fluid Mechanics*, **857**, 605-647. https://doi~~:~~.org/10.1017/jfm.2018.741 (2018)

Peng, C.K., Buldyrev, S. V., Havlin, S., Simons, M., Stanley, H. E., and Goldberger, A. L.: Mosaic organization of DNA nucleotides, *Phys. Rev. E*, **49**, 1685–1689, https://doi.org/10.1103/PhysRevE.49.1685, (1994)

Rignot, E. J.: Fast Recession of a West Antarctic Glacier, *Science*, **281**, 549-551, https://doi.org/10.1126/science.281.5376.549, (1998)

Rignot, E., Mouginot, J., Morlighem, M., Seroussi, H., and Scheuchl, B. Widespread, rapid grounding line retreat of Pine Island, Thwaites, Smith, and Kohler glaciers, West Antarctica, from 1992 to 2011, *Geophys. Res. Lett.*, **41,** 3502– 3509, https://doi~~:~~.org/10.1002/2014GL060140, (2014)

Ritz, C., Edwards, T., Durand, G., Payne, A. J., Peyaud, V. and Hindmarsh, R. C. A.: Potential sea-level rise from Antarctic ice-sheet instability constrained by observations. *Nature* **528,** 115–118 https://doi.org/10.1038/nature16147 (2015)

Robel, A. A., Schoof, C., and Tziperman, E.: Persistence and variability of ice-stream grounding lines on retrograde bed slopes, *The Cryosphere*, **10**, 1883–1896, https://doi.org/10.5194/tc-10-1883-2016 (2016)

Robel, A. A., Seroussi, H. and Roe, G. H. Marine ice sheet instability amplifies and skews uncertainty in projections of future sea-level rise, *Proc. Natl. Acad. Sci.*, **116**, 14887-14892, https://doi~~:~~.org/10.1073/pnas/1904822116, (2019)

Robinson, A., Calov, R. & Ganopolski, A. Multistability and critical thresholds of the Greenland ice sheet. *Nature Clim Change* **2,** 429–432, https://doi.org/10.1038/nclimate1449, (2012)

Scambos, T., Fahnestock, M., Moon, T., Gardner, A., and Klinger, M.: Global Land Ice Velocity Extraction from Landsat 8 (GoLIVE), Version 1, NSIDC: National Snow and Ice Data Center, Boulder, Colorado USA, https://doi.org/10.7265/N5ZP442B, http://nsidc.org/data/golive, (2016)

Schaffer J, Timmermann R, Arndt JE, Kristensen SS, Mayer C, Morlighem M, et al.: A global, high-resolution data set of ice sheet topography, cavity geometry, and ocean bathymetry, *Earth System Science Data* **8**(2), 543–557, https://doi.org/10.5194/essd8-543-2016, (2016) Scheffer, M., Carpenter, S., Foley, J. A., Folke, C. and Walker, B. Catastrophic shifts in ecosystems, *Nature*, **413,** 591-596, https://doi.org/10.1038/35098000, (2001)

Scheffer *et al.* Early-warning signals for critical transitions, *Nature*, **461,** 53-59, https://doi.org/10.1038/nature08227, (2009)

Schoof, C. Ice sheet grounding line dynamics: Steady states, stability, and hysteresis, *J. Geophys. Res.*, **112,** F03S28, https://doi.org/10.1029/2006JF000664, (2007)

Schoof, C. Marine ice sheet stability. *J. Fluid. Mech.* **698**, 62-72. https://doi.org/10.1017/jfm.2012.43, (2012)

Seroussi, H. and Morlighem, M.: Representation of basal melting at the grounding line in ice flow models, *The Cryosphere*, **12**, 3085–3096, https://doi.org/10.5194/tc-12-3085-2018, (2018)

Shepherd, A., Wingham, D., and Rignot, E. Warm ocean is eroding West Antarctic Ice Sheet, *Geophys. Res. Lett.*, 31, L23402, https://doi~~:~~.org/10.1029/2004GL021106, (2004)

~~Schoof, C. Ice sheet grounding line dynamics: Steady states, stability, and hysteresis, *J. Geophys. Res.*, **112,** F03S28, doi:10.1029/2006JF000664, (2007)~~

Slater, T. *et al.*: A new digital elevation model of Antarctica derived from CryoSat-2 altimetry, *The Cryosphere*, **12**, 1551–1562, https://doi.org/10.5194/tc-12-1551-2018, (2018)

Smith, J. A. *et al.* Sub-ice-shelf sediments record history of twentieth-century retreat of Pine Island Glacier, Nature, **541**, 77-80, https://doi.org/10.1038/nature20136, (2016)

Turner, J., Orr, A., Gudmundsson, G. H., Jenkins, A., Bingham, R. G., Hillenbrand, C.-D., and Bracegirdle, T. J., Atmosphere-ocean-ice interactions in the Amundsen Sea Embayment, West Antarctica, *Rev. Geophys.*, **55**, 235- 276, https://doi:10.1002/2016RG000532, (2017)

van Nes, E. H. and Scheffer, M. Slow Recovery from Perturbations as a Generic Indicator of a Nearby Catastrophic Shift, *The American Naturalist*, **169 (6),** 738-747, https://doi.org/10.1086/516845, (2007)

Weertman, J. Stability of the Junction of an Ice Sheet and an Ice Shelf. *Journal of Glaciology*, **13 (67),** 3-11. https://doi~~:~~.org/10.3189/S0022143000023327, (1974) Wissel, C.: A universal law of The characteristic return time near thresholds, *Oecologia*, **65**, 101–107, https://doi.org/10.1007/BF00384470, (1984)

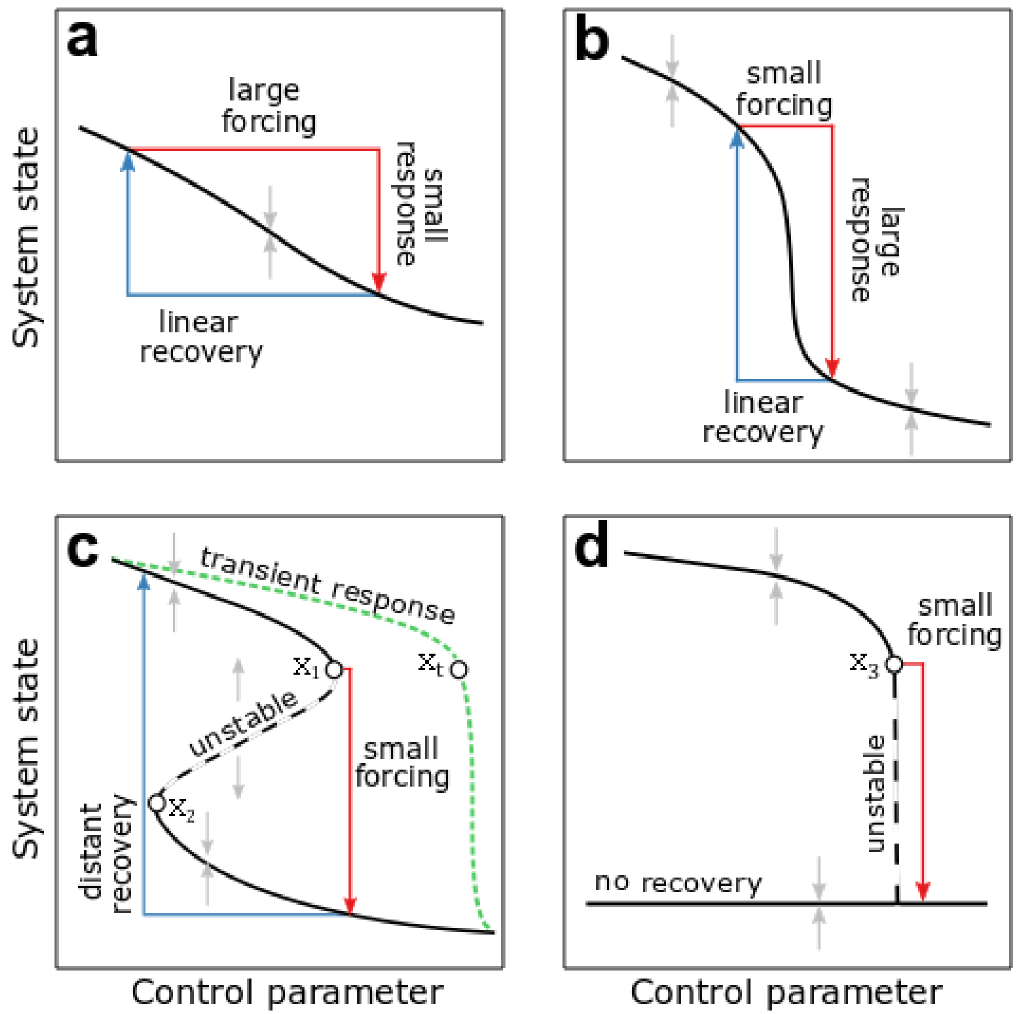
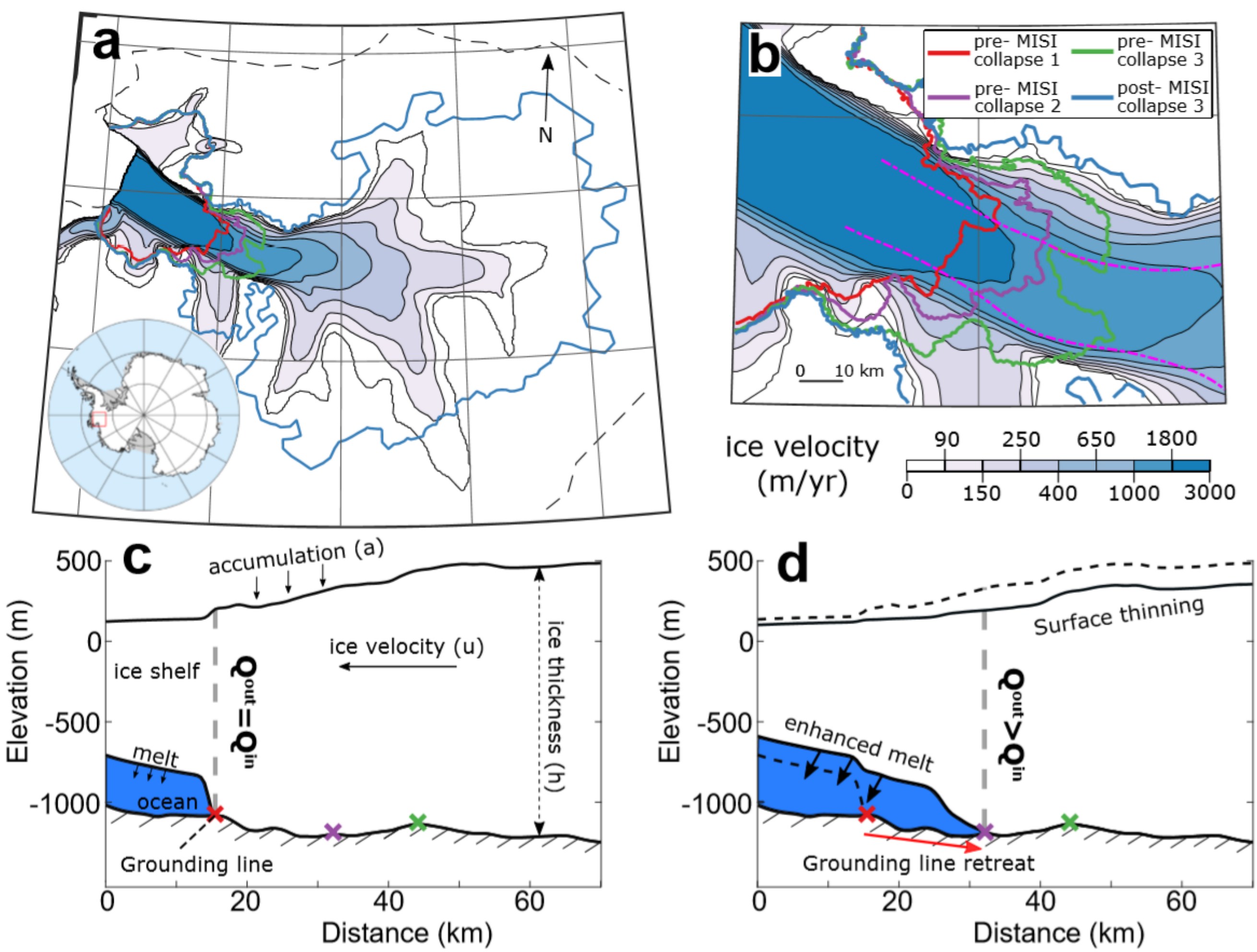
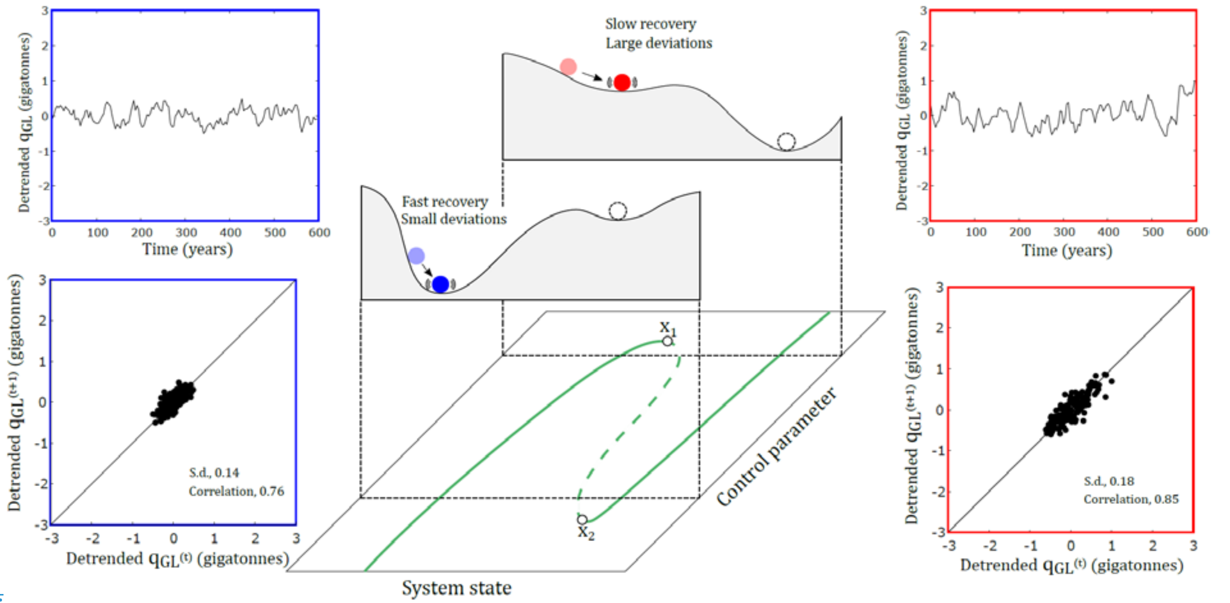


Figure 1. Possible range of behaviours for a system state (e.g. ice flux) in response to perturbations in a control parameter (e.g. ocean temperature). A system can respond to a perturbation (a) in a linear way that is directly recoverable with a reversal of the forcing, (b) with a large response to a small perturbation but that is still directly recoverable, (c) with a large response to a small perturbation that is irreversible (hysteresis behaviour), and (d) with a large response that is irreversible for any change in the ~~control parameter.~~tested range of the control parameter, a behaviour we refer to as permanently irreversible (no recovery possible even if forcing is reversed well below the initial level). Tipping points are crossed only in panels (c) and (d) and are indicated by x_1, x_2 and x_3. Panel (c) also shows a transient response in which the system state lags behind changes in the control parameter as is the case for ice sheets and thus crosses the irreversible system state at a later point, x_t.

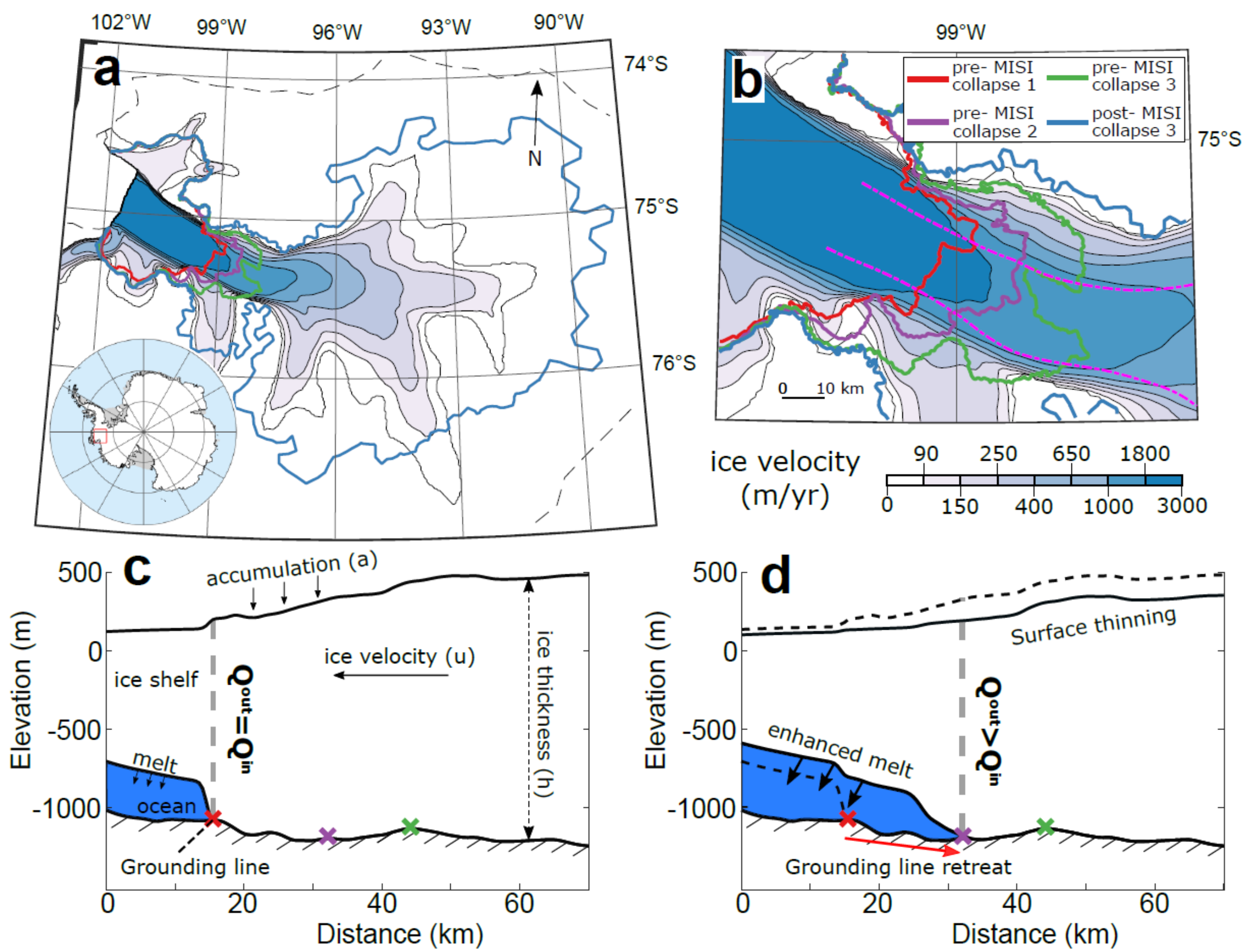


a
N
b
pre- MISI collapse 1
pre- MISI collapse 2
pre- MISI collapse 3
post- MISI collapse 3
0 10 km
ice velocity (m/yr)
90 250 650 1800
0 150 400 1000 3000
c
accumulation (a)
ice shelf
ice velocity (u)
ice thickness (h)
$Q_{out}=Q_{in}$
melt
ocean
Grounding line
Elevation (m)
Distance (km)
d
Surface thinning
enhanced melt
$Q_{out}>Q_{in}$
Grounding line retreat

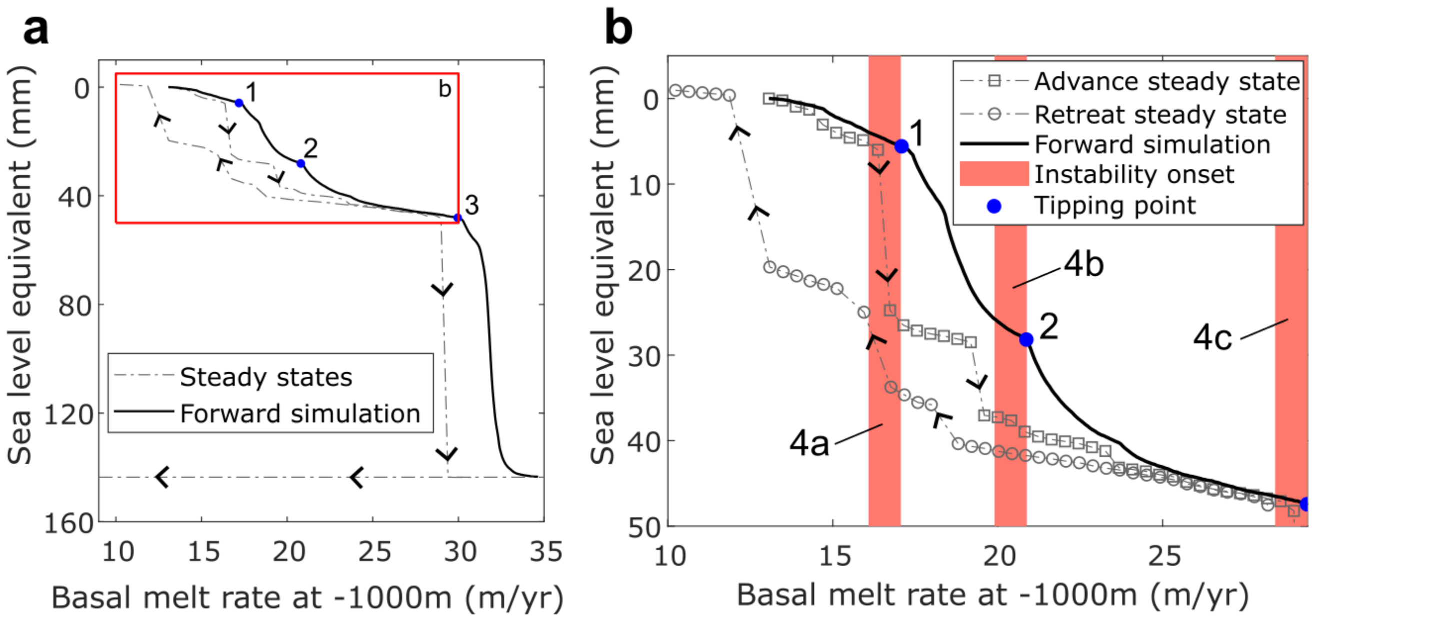


~~Figure 2.~~

Figure 2. Critical slowing can serve as an indicator that the system is approaching a tipping point. This can be understood conceptually using the common 'ball on a slope' analogy (middle panel), where the ball represents the system state and minima are stable equilibrium states. Two example cases are superimposed onto their corresponding positions in the hysteresis plot of MISI shown by the green line and equivalent to Fig. 1c. The processed model results demonstrate how critical slowing manifests itself, as shown in the blue and red panels at the sides. If the system is far from a tipping point (blue case), the system state (which is in this study the grounding line flux q_{GL}, upper left panel) recovers quickly from perturbations in the control parameter (which is here the basal melt variability). This means that from one measurement (at time t) to the next (at time t+1) the grounding line flux changes rapidly and has a low lag-1 auto-correlation (lower left panel). Conversely, close to a tipping point (red case), critical slowing manifests and the system state responds more slowly to perturbations in the control parameter (upper right panel). Since the state variable is changing more slowly, successive measurement are more similar, resulting in a higher lag-1 auto-correlation (lower right panel).



[Figure 3.](#) **Marine Ice Sheet Instability events for Pine Island Glacier. Shown are (a) grounding line positions before and after the three MISI driven glacier collapses with (b) a zoom to the initial events (coloured lines). The colormap indicates initially modelled ice velocity and the model domain boundary is indicated by a dashed black contour in panel a . Panels (c) and (d) show a transect through the main trunk of PIG, calculated as an average of properties between the two dashed magenta lines in (b). The vertical section along the transect is shown (c) at the initial steady state where fluxes (Q_{in} and Q_{out}) are in balance and (d) during a MISI event where retreat causes an increase in Q_{out}, pushing the glacier to be out of balance and leading to further retreat.**



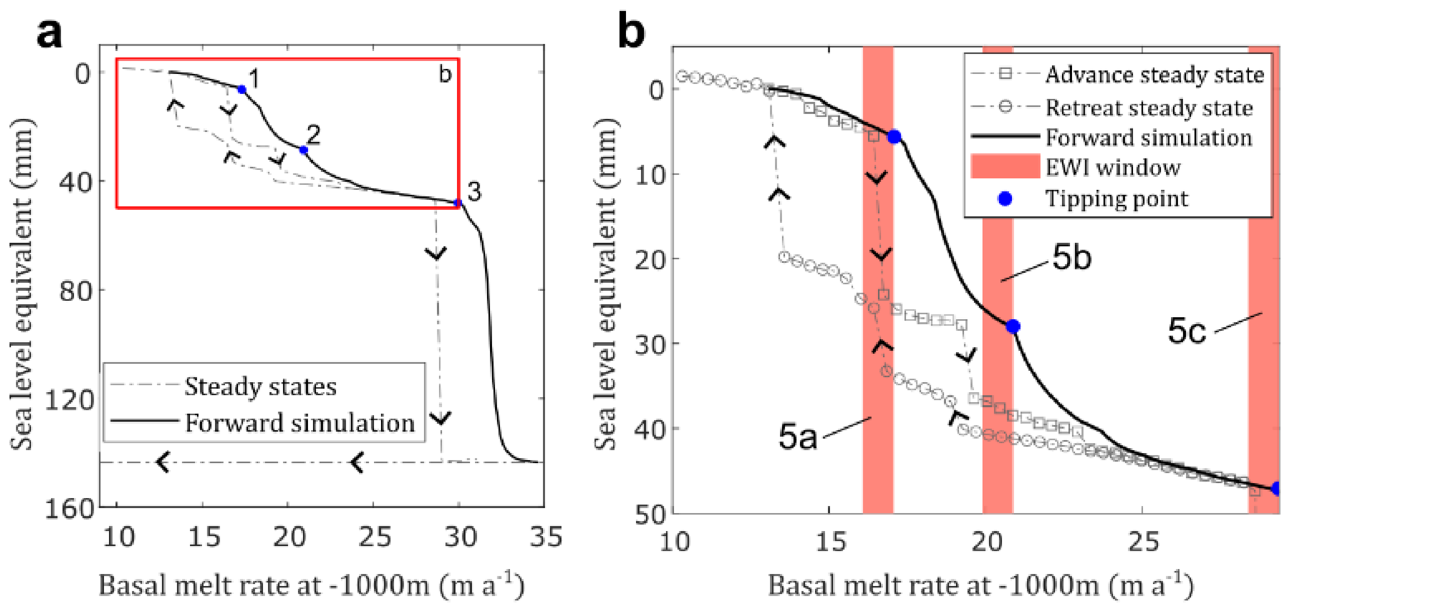


Figure ~~3~~4. Change in system state in terms of sea level equivalent ice volume as a function of the control parameter, which is the melt rate at the ice-ocean interface. (a) The model is run forward with a slowly increasing basal melt rate (solid black line) and shows three distinct tipping points (blue ~~dots).~~dot). From the start of the transient simulation to the third tipping point is approximately 10kyrs. The steady states for a given melt rate in both an advance and retreat configuration are plotted as dashed grey lines, arrows indicate the direction of the hysteresis. Panel (b) focuses on the model response before the larger tipping point (event 3) and shows the three windows that we analyze for early warning indicators as shaded red boxes (Fig. ~~4).~~5). Circle and square symbols represent steady state configurations for a given forcing and the dashed grey line is a linear interpolation between these points. Each step in melt rate for the steady state runs is approximately equivalent to 0.4m/yr of basal melting, or 250 years in the transient simulation.

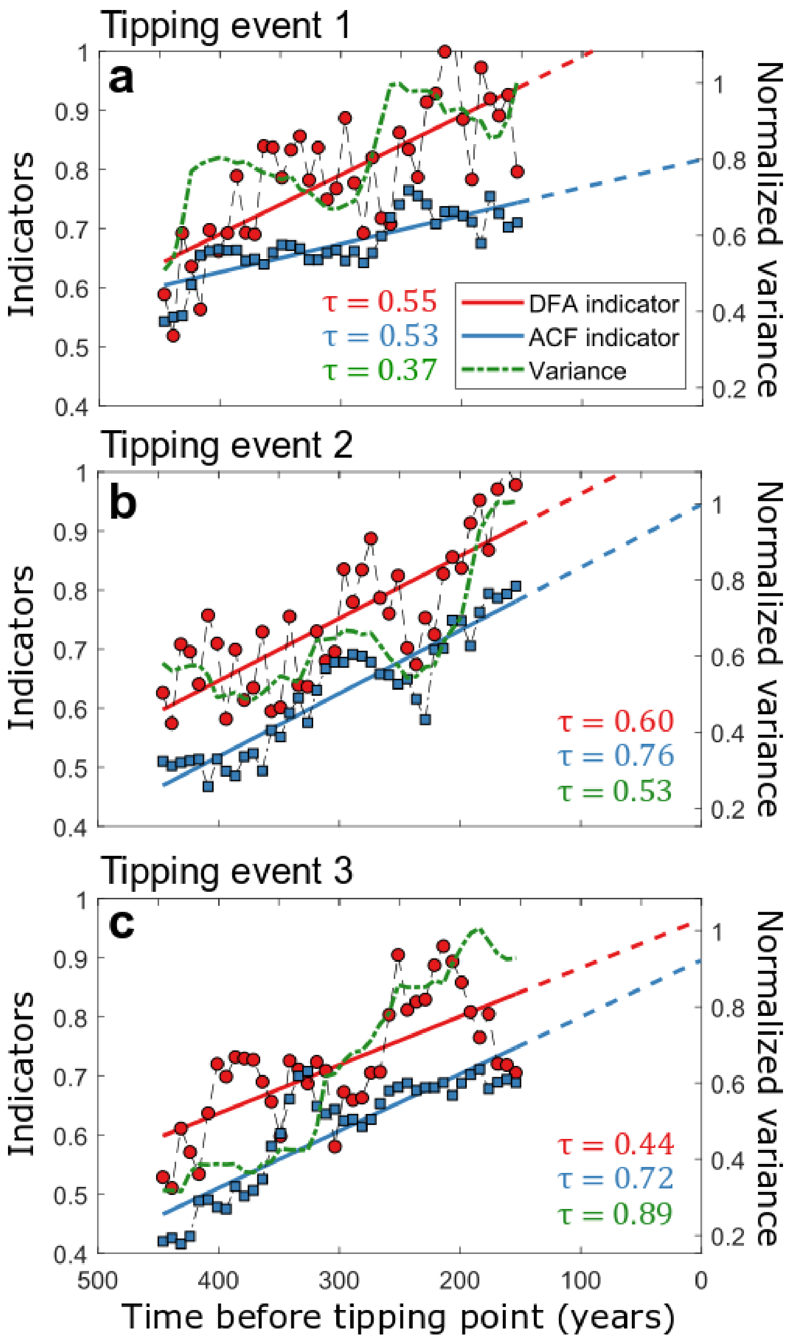


Figure ~~4. Early warning indicators~~5. EWIs for the marine ice sheet instability in Pine Island Glacier. Each panel shows the ~~early warning indicators~~EWIs preceding each of the three MISI tipping event marked in Fig 3b, along with the linear trend extrapolated to the point in the simulation when the respective tipping event occurs. Increasing trends in all indicators are shown by a positive Kendall's τ coefficient which measures the correlation between each indicator and time between -1 and 1.

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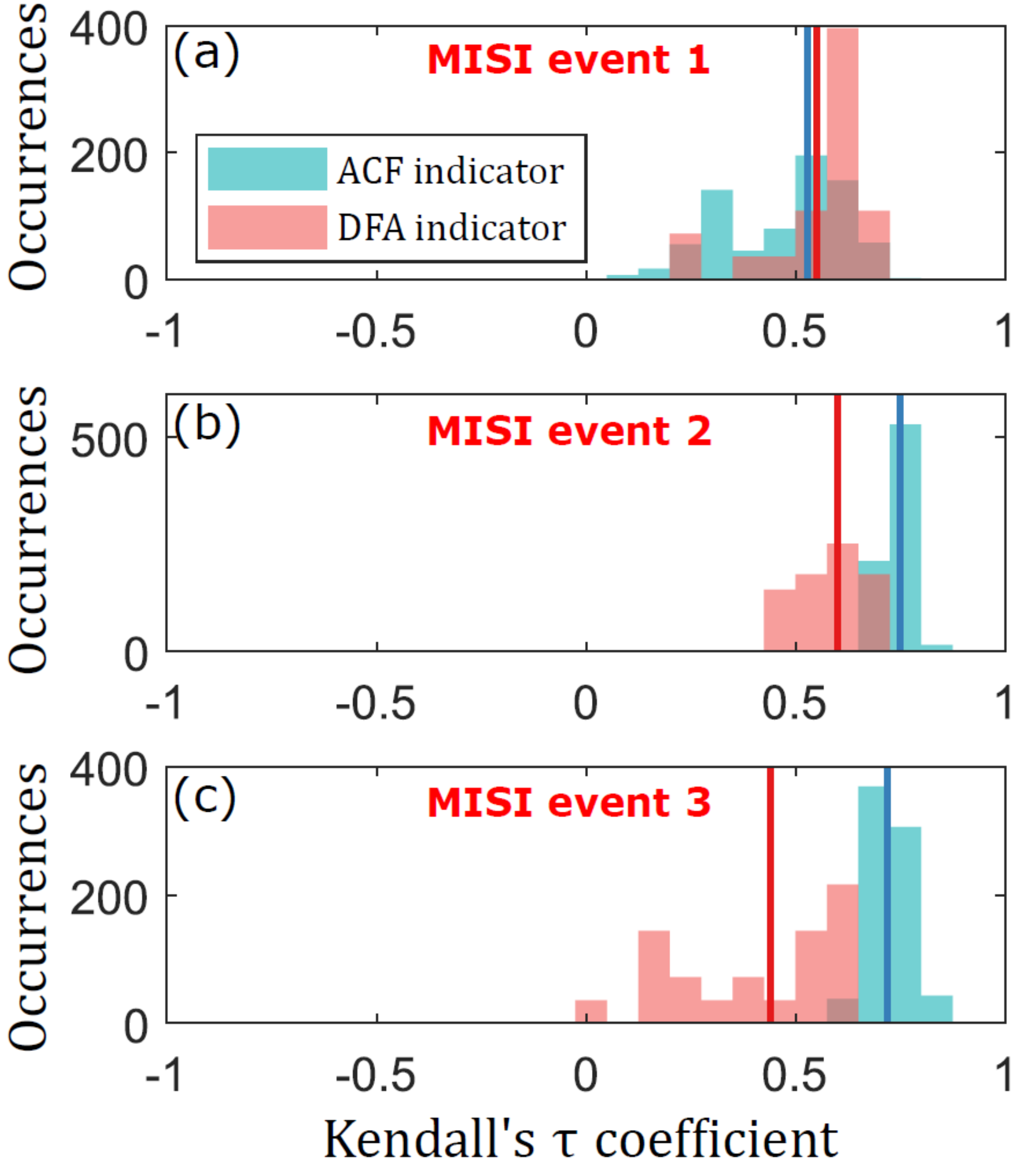


Occurrences
400
200
0
(a)
MISI event 1
ACF indicator
DFA indicator
-1
-0.5
0
0.5
1
Occurrences
500
0
(b)
MISI event 2
-1
-0.5
0
0.5
1
Occurrences
400
200
0
(c)
MISI event 3
-1
-0.5
0
0.5
1
Kendall's τ coefficient

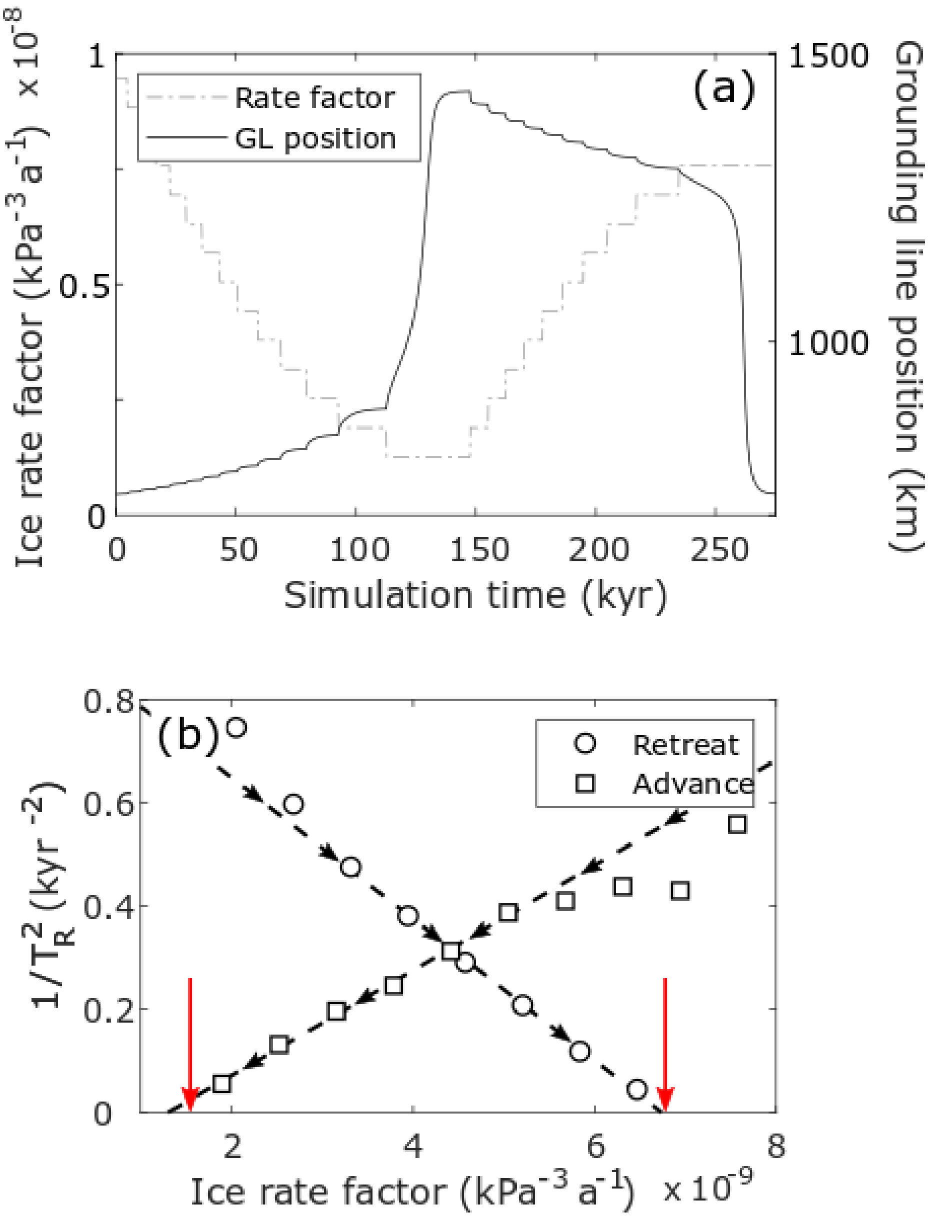


Figure 6. Sensitivity analysis for the ACF and DFA indicators. Each occurrence is the Kendall's τ coefficient for a different choice of filtering bandwidth and data aggregation. The solid red and blue lines show the Kendall's τ coefficient for the DFA and ACF indicators respectively, as calculated for the choice of parameters used in Fig. 5.

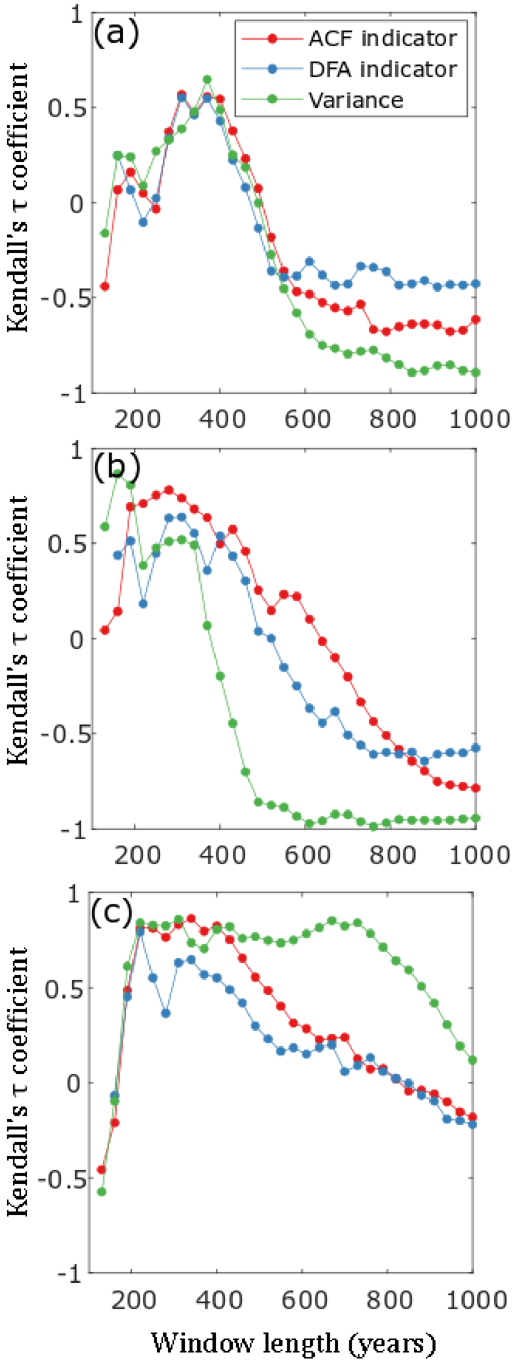


Figure 7. The effect of window length on the predictive power of EWIs for the MISI. The three panels show the change in Kendall's τ coefficient as calculated for each indicator versus window length for MISI events 1, 2 and 3 (panels a, b and c respectively).

Event Number	Indicator name	Indicator value	Probability	Total Probability
MISI event 1	DFA	0.55	0.041	0.0198
	ACF	0.53	0.122	
	Variance	0.37	0.315	
MISI event 2	DFA	0.60	0.022	0.0030
	ACF	0.76	0.012	
	Variance	0.53	0.207	
MISI event 3	DFA	0.44	0.099	0.0044
	ACF	0.72	0.026	
	Variance	0.89	0.018	

Table 1. Probability of the Kendall's τ correlation for each indicator being a result of chance. One thousand surrogate time series of the state variable are generated and the indicators and Kendall's τ correlations calculated for each one. The probability of a Kendall's τ value is then the fraction of these surrogate time series with a higher correlation coefficient. The total probability is the fraction of surrogates for which all three indicators have a higher correlation coefficient than is observed in the original model time series.

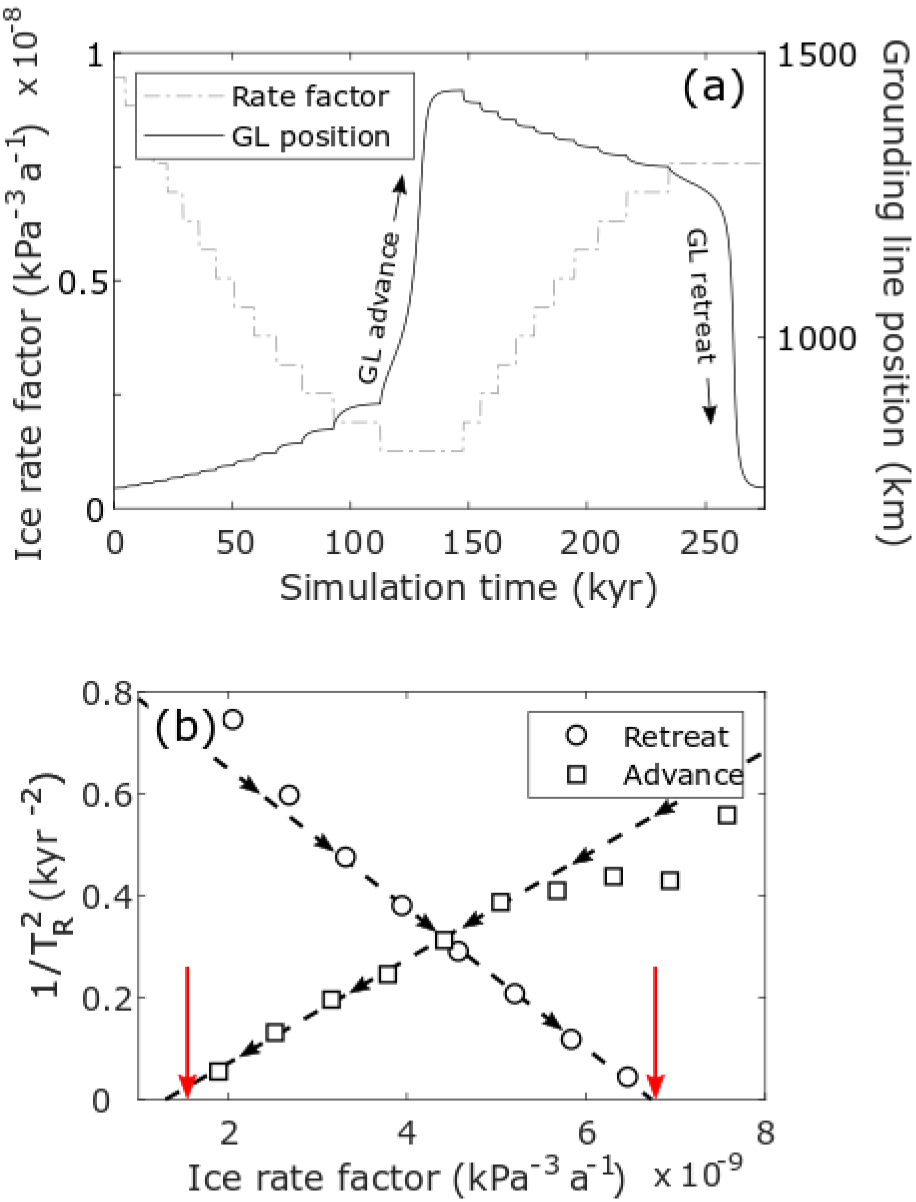
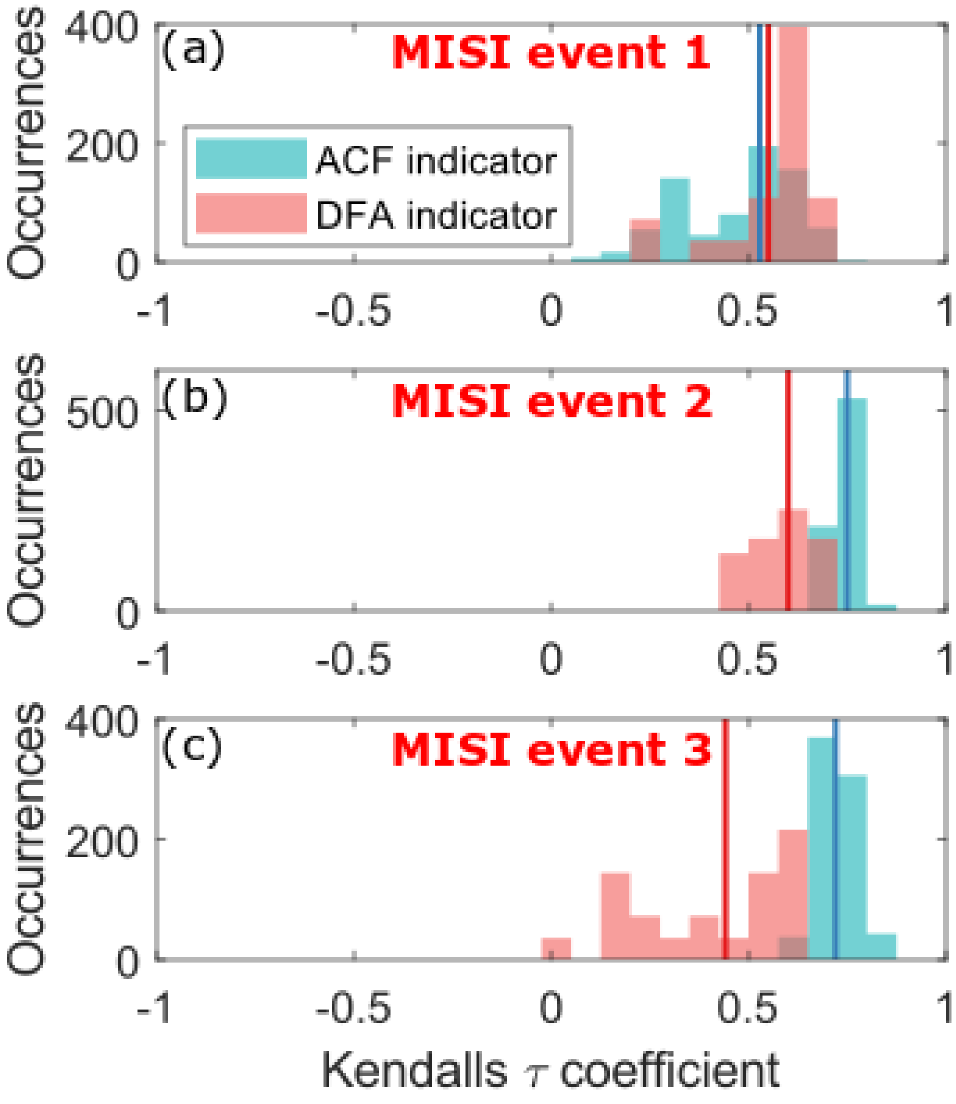
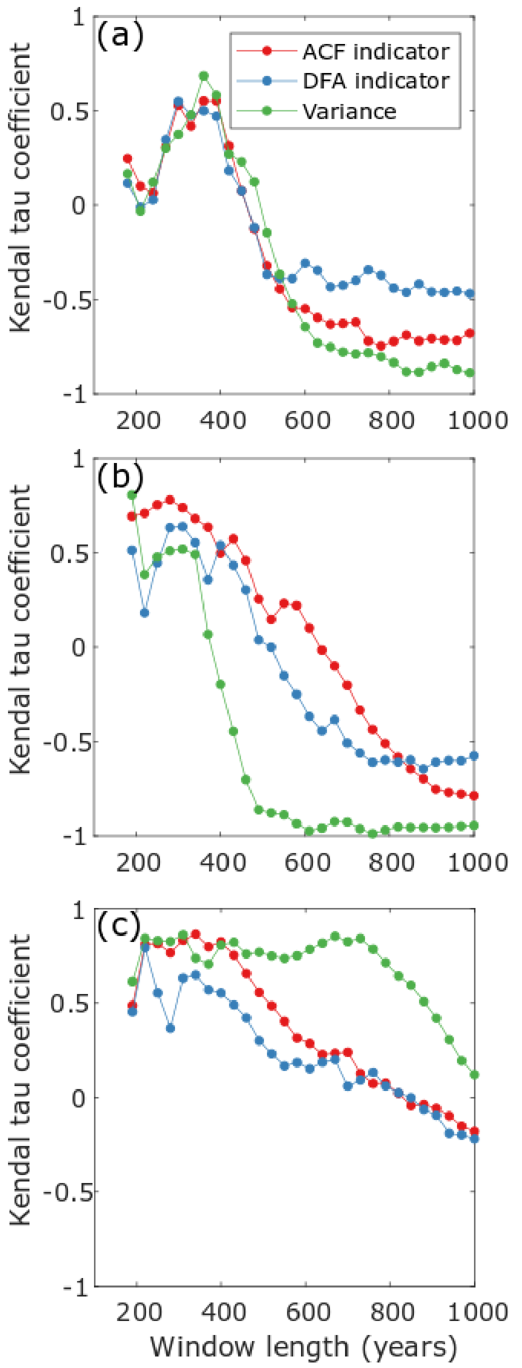


Figure A1. Results of EXP 3, showing change in GL position with time resulting from step perturbations in the ice rate factor (panel a). The calculated inverse relaxation time for each corresponding step change in rate factor in both the advance (square symbols) and retreat (circular symbols) phase is shown in panel b. The dashed line in panel b is a line of best fit, calculated for the five steps in rate factor that preceded the advance or retreat MISI phase. Red arrows indicate the rate factors for which the analytical solution predicts a MISI event and black arrows show the direction of the forcing towards each tipping point.



835 Figure B1. ~~Sensitivity analysis for the ACF and DFA indicators. Each occurrence is the Kendall's τ coefficient for a different choice of filtering bandwidth and data aggregation. The solid red and blue lines show the Kendall's τ coefficient for the DFA and ACF indicators respectively, as calculated for the choice of parameters used in Fig.~~ 3.



~~Figure B2. The effect of window length on the predictive power of early warning indicators for the MISI. The three panels show the change in Kendall's~~ $\cancel{\tau}$ ~~coefficient as calculated for each indicator versus window length for MISI events 1, 2 and 3 (panels a, b and c respectively).~~

~~**Event Number**~~	~~**Indicator name**~~	~~**Indicator value**~~	~~**Probability**~~	~~**Total Probability**~~
~~**MISI event 1**~~	~~DFA~~	~~0.55~~	~~0.041~~	~~0.0198~~
	~~ACF~~	~~0.53~~	~~0.122~~	
	~~Variance~~	~~0.37~~	~~0.315~~	
~~**MISI event 2**~~	~~DFA~~	~~0.60~~	~~0.022~~	~~0.0030~~
	~~ACF~~	~~0.76~~	~~0.012~~	
	~~Variance~~	~~0.53~~	~~0.207~~	
~~**MISI event 3**~~	~~DFA~~	~~0.44~~	~~0.099~~	~~0.0044~~
	~~ACF~~	~~0.72~~	~~0.026~~	
	~~Variance~~	~~0.89~~	~~0.018~~	

~~Table C1. Probability of the Kendall's τ correlation for each indicator being a result of chance. One thousand surrogate time series of the state variable are generated and the indicators and Kendall's τ correlations calculated for each one. The probability of a Kendall's τ value is then the fraction of these surrogate time series with a higher correlation coefficient. The total probability is the fraction of surrogates for which all three indicators have a higher correlation coefficient than is observed in the original model time series.~~