Supplemental Material

The following calibration procedure is used to convert the measured voltages at the receiver output into calibrated scattering matrix values. This calibration is applied in the IDL code in post-processing.

Assumptions

The following assumptions are made:

1. The cross-polarization isolation of the antennas is high so that the cross-coupling between polarization states is assumed to be zero.
2. The average phase difference between the VV and HH scattering coefficient is zero when viewing an isotropic media, such as snow viewed at zero-degrees incidence angle.
3. The average phase difference between the VH and HV scattering coefficient is zero when viewing an isotropic media, such as snow viewed at zero-degrees incidence angle.
4. The expected value of the magnitude of the VV and HH scattering coefficients are equal when viewing isotropic media.
5. The expected value of the magnitude of the VH and HV scattering coefficients are equal for all reciprocal media.

Methodology

The measured voltage at the output of the receiver is given in terms of the scattering matrix and receiver and transmitter distortion matrices:

$$\mathbf{V} = \mathbf{ASFT} \tag{1}$$

where

$$\mathbf{V} = \begin{bmatrix} V_{vv} & V_{vh} \\ V_{hv} & V_{hh} \end{bmatrix}$$

is a 2x2 matrix of complex voltages measured at the receiver output with matrix elements, $V_{i,j}$ where i, j represent either vertical (v) or horizontal (h) receiver channels;

$$\mathbf{S} = \begin{bmatrix} S_{vv} & S_{vh} \\ S_{hv} & S_{hh} \end{bmatrix}$$

is the complex scattering matrix of the scene under observation, with scattering matrix elements, $S_{i,j}$ where i, j represent either vertical (v) or horizontal (h) polarization.

The receiver distortion matrix $\mathbf{A}$ and transmit distortion matrix, $\mathbf{F}$ are given by:

$$\mathbf{A} = \begin{bmatrix} a_{00} & 0 \\ 0 & a_{11} \end{bmatrix}; \ \mathbf{F} = \begin{bmatrix} f_{00} & 0 \\ 0 & f_{11} \end{bmatrix}$$

where the main diagonal terms are complex-valued and the off-diagonal terms are set to zero since the antennas are assumed to have high cross-polarization isolation (> 25 dB). Finally, the matrix describing the transmit polarization for successive pulses is given by

$$\mathbf{T} = \begin{bmatrix} T_V & 0 \\ 0 & T_H \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

where the transmit complex amplitudes are assumed to be equal and set to 1. Equation (1) can be solved for the scattering matrix by simple matrix manipulation (Geldsetzer et al., 2007):

$$\mathbf{S} = \mathbf{A}^{-1}\mathbf{V}\mathbf{F}^{-1}$$

where the inverse of the receiver distortion matrix is

$$\mathbf{A}^{-1} = \frac{1}{a_{00}a_{11}} \begin{bmatrix} a_{11} & 0 \\ 0 & a_{00} \end{bmatrix}$$

and similarly for the transmit distortion matrix, such that the scattering matrix elements can be computed from the measured receiver voltages as

$$\begin{bmatrix} S_{vv} & S_{vh} \\ S_{hv} & S_{hh} \end{bmatrix} = \frac{1}{a_{00}a_{11}f_{00}f_{11}} \begin{bmatrix} a_{11} & 0 \\ 0 & a_{00} \end{bmatrix} \begin{bmatrix} V_{vv} & V_{vh} \\ V_{hv} & V_{hh} \end{bmatrix} \begin{bmatrix} f_{11} & 0 \\ 0 & f_{00} \end{bmatrix} \qquad (2)$$

Let $\alpha = \frac{a_{00}}{a_{11}}$ and $\beta = \frac{f_{00}}{f_{11}}$, then (2) may be written as

$$\begin{bmatrix} S_{vv} & S_{vh} \\ S_{hv} & S_{hh} \end{bmatrix} = \frac{1}{a_{00}f_{00}} \begin{bmatrix} V_{vv} & \beta V_{vh} \\ \alpha V_{hv} & \alpha\beta V_{hh} \end{bmatrix} \qquad (3)$$

The absolute calibration of the normalized radar cross-section is handled separately, thus the factor $(a_{00}f_{00})^{-1}$ which simply scales the scattering matrix by a complex value, may be ignored.

Determining α and β from measured data

When viewing a surface target at normal incidence angle (elevation =0°), the expected value of the phase difference between S_{vv} and S_{hh} is zero, i.e., $\left\langle S_{vv} S_{hh}^{*} \right\rangle = 0$ where the angle brackets indicate expected value taken over the range of azimuth angles when scanning at zero degrees elevation. The expected value of the phase difference between the measured voltages at the output of the receiver, V_{vv} and V_{hh} is not in general zero for such isotropic media, due to the effects of the receiver and transmitter distortion matrices. For such an isotropic target, the system will measure a phase angle, θ between the co-polarized signals:

$$\theta = \quad \left\langle V_{vv} V_{hh}^{*} \right\rangle = \tan^{-1} \left\langle V_{vv} V_{hh}^{*} \right\rangle .$$

Similarly, for the cross-polarized measured voltages the system will measure a phase angle, ϕ:

$$\phi = \quad \left\langle V_{vh} V_{hv}^{*} \right\rangle = \tan^{-1} \left\langle V_{vh} V_{hv}^{*} \right\rangle$$

With some algebra, it can be shown that

$$\alpha = \varphi_{\alpha} = \tan^{-1}\left(\frac{a_{00}}{a_{11}} \right) = \frac{\theta + \phi}{2} \tag{4}$$

$$\beta = \varphi_{\beta} = \tan^{-1}\left(\frac{f_{00}}{f_{11}} \right) = \frac{\theta - \phi}{2} \tag{5}$$

Assuming $\left\langle \left| S_{vv}^{2} \right| \right\rangle = \left\langle \left| S_{hh}^{2} \right| \right\rangle$ for isotropic media, (3) gives the following product:

$$\left| \alpha\beta \right| = \sqrt{\frac{\left\langle \left| V_{vv}^{2} \right| \right\rangle}{\left\langle \left| V_{hh}^{2} \right| \right\rangle}} \ . \tag{6}$$

Assuming $\left\langle \left| S_{vh}^{2} \right| \right\rangle = \left\langle \left| S_{hv}^{2} \right| \right\rangle$ for any reciprocal media, (3) gives the following ratio:

$$\left| \frac{\alpha}{\beta} \right| = \sqrt{\frac{\left\langle \left| V_{vh}^{2} \right| \right\rangle}{\left\langle \left| V_{hv}^{2} \right| \right\rangle}} \ . \tag{7}$$

Combining (6) and (7) gives the following expressions for $|\alpha|$ and $|\beta|$:

$$|\alpha| = \left(\frac{\langle |V_{vv}^2| \rangle \langle |V_{vh}^2| \rangle}{\langle |V_{hh}^2| \rangle \langle |V_{hv}^2| \rangle} \right)^{.25} \tag{8}$$

$$|\beta| = \left(\frac{\langle |V_{vv}^2| \rangle \langle |V_{hv}^2| \rangle}{\langle |V_{hh}^2| \rangle \langle |V_{vh}^2| \rangle} \right)^{.25} \tag{9}$$

Expressions (4), (5), (8), and (9) fully determine the magnitude and phase of the receiver and transmitter distortion matrices and can be used in (3) to compute the scattering matrix from the measured voltage matrix.