Reviewer 1

General comments:

The authors have proposed a method to characterize the magnitude and timing of seasonal glacier ice velocity signals by fitting the best possible sinusoid to the velocity data obtained from optical remote sensing. It is well known that the optical data has obvious data gaps in polar regions during winters and is affected by cloud cover. This method is proposed to resolve seasonal velocity variations from such a dataset, but needs a large number of (>1000) multi-year velocity observations. The manuscript is well written, but I have a couple of points that may be useful to further improve this work. My major concern is about the applicability of this method to regions other than polar areas.

Major comments:

P5. I did not understand how observations over finite integration times make it difficult to resolve seasonal variability in case of repeat SAR imagery...

The sentence in question is from the abstract, and it previously read:

The task of generating continuous ice velocity time series that resolve seasonal variability is made difficult by the finite integration time over which ice velocities are measured...

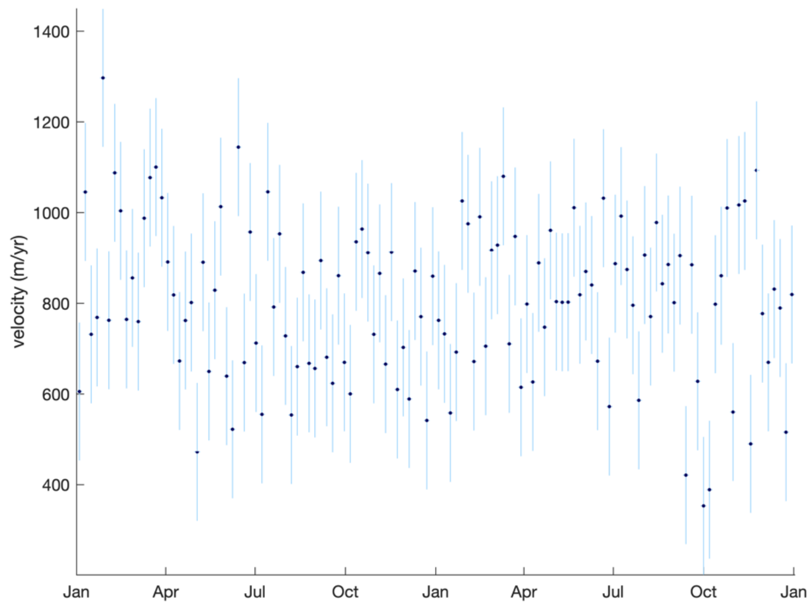
We have clarified the wording to more effectively convey that measurements of the total displacement that occurs over several months to a year will offer no direct insight into velocity variability that occurs within those months. The section now reads:

The task of generating continuous ice velocity time series that resolve seasonal variability is made difficult by a spotty satellite record that contains no optical observations during dark, polar winters. Furthermore, velocities obtained by feature tracking are marked by high noise when image pairs are separated by short time intervals, and contain no direct insights into variability that occurs between images separated by long time intervals.

...For instance, Sentinel-1SAR data is available throughout the year with a 6-day temporal cycle and a number of studies have resolved seasonality using SAR data (e.g. Sentinel-1, TanDEM-X)....

The 6-day Sentinel 1 repeat pattern does not reliably translate to continuous global coverage. Bandwidth limitations have resulted in minimal Sentinel coverage over Antarctica. Elsewhere, such as in southwest Greenland, SAR feature tracking struggles to correlate features between repeat images.

Even in the presence of perfect data coverage, short, 6-day integration times present their own challenges for feature tracking. Namely, the 1-10 m displacement uncertainty achieved with Sentinel 1 SAR image pairs is independent of image temporal separation. This translates to a velocity uncertainty on the order of 100 m/yr or more for an image pair separated by 6 days, meaning that any seasonal signals smaller than that would likely be difficult to distinguish from the scatter of the measurement noise. For example, here is a synthetic case of two years of perfect data coverage, without any missing 6-day image pairs. Here we synthetically measure a sinusoidal variation with an amplitude of 50 m/yr, but with gaussian displacement error (standard deviation 2.5 m) added to each synthetic measurement. Despite perfect coverage with 6-day image pairs, the 50 m/yr velocity variation is not clearly evident.



Matlab code to create the plot above is included at the bottom of this response letter.

...I think the focus of this paper should be optical data, its limitations during polar winters and cloud cover and how your method can still resolve seasonality using optical data.

The paper is focused almost entirely on optical data, its limitations, and how our method can still resolve seasonality using optical data. However, the method is fully agnostic to whether feature-tracked velocities were obtained from optical or SAR data, so we have been careful to write the paper in a way that does not preclude its application to SAR data.

P30-35. The authors should highlight these significant gaps which still limit our understanding of ice dynamics change on different time scales. I agree that a number of studies on the seasonal ice dynamics of glaciers in Arctic, Antarctic and other glaciated regions are available in bits and pieces, but they do provide a great degree of evidence that help us understand the physical processes which govern the ice dynamics on different time scales. I can't imagine how a consistent global mapping of seasonal ice dynamics looks like, which the authors have pointed to and how, if accomplished, they better our understanding...

The case for consistent, large-scale mapping can be made by drawing a parallel to Rignot et al.'s first comprehensive mapping of Antarctic secular ice velocity, which was published in 2011. Dozens of regional studies of ice velocity had already been published at that time, yet the application of a consistent measurement technique and synthesis into a single map has already informed nearly a thousand peer reviewed studies. We allude to the potential insights that could be gained from comprehensive mapping with this new sentence, which we have added to the end of the paragraph:

A comprehensive mapping of the world's seasonal ice dynamics would permit direct inter-comparison of seasonal evolution in regions with different driving processes; provide a basis for analysis of long-term changes in seasonal behavior; and supply models with a zeroth-order understanding of global ice climatology.

...It is also not clear why such an approach relies on optical imagery, even though we have a year-around consistent and global SAR imagery by missions like Sentinel-1. These points should be addressed in the Introduction to better form a basis or need for this study.

The method we present is not limited to optical imagery, and can just as easily be applied to SAR image pairs. Following this suggestion, we have now clarified that point in the abstract by stating,

In this paper, we describe a method of analyzing optical- or SAR-derived feature-tracked velocities...

We reiterate in the final paragraph of the Discussion section,

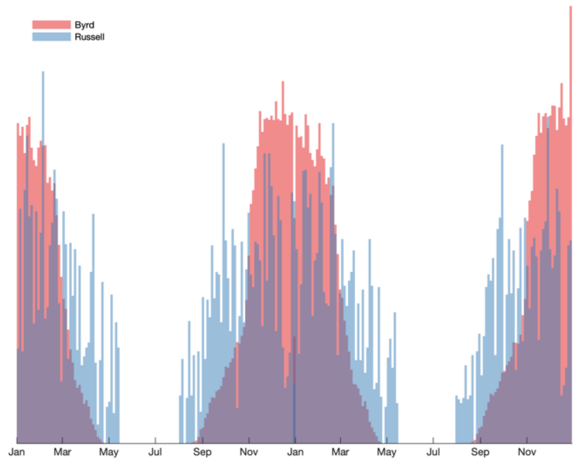
The methods presented in this paper have focused primarily on optical satellite data because no other type of sensor provides such a long record of ice velocity. As more radar data become available, particularly since the launch of Sentinel 1a/b, the problem of missing winter data will be eliminated, but the methods presented in this paper will still hold.

Figure 7. Nice figure. But when I compared this with Figure 4, I drew a couple of points that need to be addressed. ITS_LIVE velocity data for Russel Glacier in Greenland is much more dense and appears to be well distributed around the year as compared to Byrd Glacier, Antarctica...

The apparent difference in data density is likely just a matter of figure size. Figure 4 fills the entire page width, and as stated in the caption it shows 14,208 image pairs of Byrd Glacier. In contrast, the panels of Figure 7 are much smaller and show 5189 image pairs of Russell Glacier. We used the same linewidths and marker sizes in both figures, which likely makes the Russell Glacier data appear more dense, given the overall difference in size of the two figures. We include the number of image pairs in the caption to clarify this point.

...I wonder how such a large number of wintertime velocities are available in Greenland using optical data. Are they averaged for the entire polar wintertime?...

To be clear, no images have been acquired during any winter at Russell Glacier. This is also true at Byrd Glacier. Here is a normalized histogram of image collection times for both. We've offset the dates of the Russell Glacier images by 183 days to allow direct comparison:



The lower latitude of Russell Glacier (67N) compared to Byrd Glacier (80S) allows a longer image collection season at Russell, but still, months go by each winter without any image acquisitions. If we wish to inspect feature-tracked velocities directly, all we can do is infer average velocities between images captured in the fall and spring, and we display them following the standard convention of using a horizontal line to connect the acquisition times of the two images. We find horizontal lines alone create a confusing mess to inspect visually, so we place a dark dot at the center point of each line. The caption of Figure 4 states,

Light gray horizontal bars connect the acquisition times of each image...Center dates t_m are shown as dark gray dots for visual clarity.

In Figures 4 and 7, the horizontal bars that span winters and dark dots placed in their centers may create the impression that data are available during the winter, but that is simply a convention of displaying this type of data because, as far as we know, there isn't a clearer way to visualize it.

...I expect that this dense and well distributed velocity data is the main reason why you have a great sinusoidal fit here, isn't it? Because ideally the method should resolve the missing velocities in winters using the data points for the rest of the year. By the way, it appears to me that ITS_LIVE observations in case of Russel Glacier are already resolving seasonal variations without applying your proposed method. An example, where velocity data is sparse like for Byrd Glacier, would be much more convincing how well your proposed method can resolve seasonality...

There are only 5189 ITS_LIVE observations at Russell compared to 14,208 at Byrd. We do not have a years-long record of GPS at Byrd for comparison, but the robustness of our results there makes it clear that the signal is physical, it is persistent, and it can be resolved by our method using any random selection of 1000 or more image pairs.

The central problem our method solves is that velocity variability from feature tracking cannot be interpreted by eye when image pairs are separated by months or more. Figure 1 illustrates this point—the patterns that appear visually in the image pair data in Figures 4 and 7 do not necessarily reflect the underlying patterns of velocity variability, either in amplitude or in phase.

...Russel Glacier is the best case scenario (P270). An example from any mountain glaciers from Alaska, European Alps and high Asia would enhance the applicability of the proposed method. The optical data is available around the year in these regions and is only affected by cloud cover. At present the method is proposed to work in these regions, but the potential challenges are not highlighted.

Russell Glacier is not necessarily a best-case scenario within the global ITS_LIVE dataset. As far as we can tell, it suffers from rather typical image issues, including summer surface meltwater and cloudiness. It was selected for analysis only because of its known seasonality and the availability of a decade-long GPS record that we could use for validation. The least-squares method we describe does not suffer from winter data gaps, but it does not benefit from the gaps either. As we state in the abstract and in the main text, our method is *agnostic* to data gaps in winter.

Minor comments:

P5,25 or elsewhere. Instead of using “accelerations”, I would recommend to use “velocity variations” because both acceleration and deceleration are governed by physical processes.

We have replaced the word *accelerations* with the term *velocity variations*.

P25. Can your approach resolve velocity changes during such short time-scales? If yes, you should highlight in the paper what additional information is required in your method in order to retrieve such signals. If not, there is no need to include it in the Introduction.

In this introductory paragraph we provide context for the timescales and physical processes that we aim to resolve. We indicate that some previous studies have reported on interannual variability driven by ocean forcing, while other studies have considered the effects of tides on glacier movement. The timescales we wish to resolve are in between fortnightly and interannual, but currently the potential influence of ocean forcing and tides on seasonal timescales are not well understood. We feel it is appropriate to provide this brief background to help readers place this our paper in the context of previous work.

Figure 1. Example-1 shows a hypothetical scenario, especially the velocity time series shown in blue. We have plenty of SAR data and derived ice velocities for the polar glaciers. What about showing a real case here?

Real cases are shown in Figures 4 and 7, but visually deciphering what’s happening in them is nearly impossible. Rather than overwhelm viewers with thousands of image pairs, we created this simple (but mathematically precise) cartoon to illustrate the fundamental elements of the problem that we solve in this paper.

P45: satellite image pairs » optical remote sensing image pairs

The sentence in question reads,

The method we present applies to ice velocity datasets...which have been derived by feature tracking techniques applied to satellite image pairs.

As it is written, the sentence is correct because our method also applies to SAR image pairs.

P85: It is not clear how the weights are assigned here. Are these based on residuals?

Line 81-83 states,

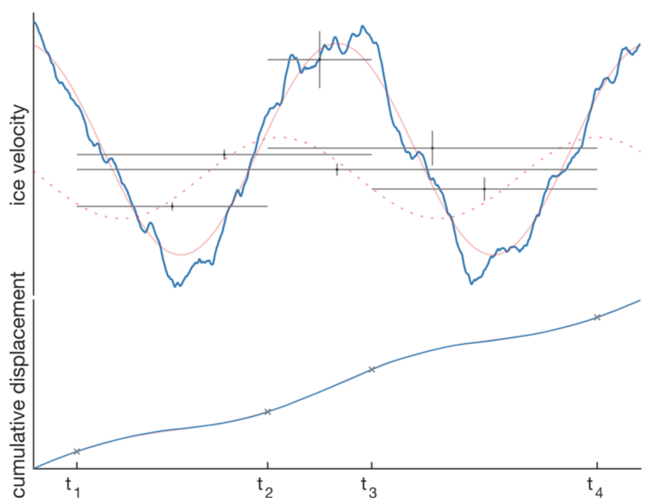
We assign the velocity weights w_v in the polynomial fit using the formal error estimates σ_v from the ITS_LIVE data such that $w_v = \sigma_v^{-2}$.

Figure 4: The colors (blue and green) in the legend and fits are inconsistent.

Fixed.

P105. "Instead, we operate on the displacements associated with each image pair, taken as the integrals of velocities" should be shown as a different figure as this is an important step of the paper. It would be better to see how various displacement estimates at different epochs ranging from days to years are prepared for any sinusoidal fit.

We have taken this suggestion, as we now plot the displacement time series along with the velocity time series to make it a bit more clear. Here is the updated Figure 1 and caption:



The upper panel shows an ice velocity time series in blue, which integrates to form the cumulative displacement time series shown in the lower panel. We use satellite images to measure ice velocity as the cumulative displacement of crevasses and other glacier features that occurs between acquisition times of any two satellite images. Here, four images taken at times t_1 through t_4 provide six unique combinations of image pairs that yield the measurements depicted as horizontal gray lines connecting the acquisition times of each image pair. Vertical gray lines *show measurement uncertainty and a black dot is placed at the center times of each image pair for visual clarity. A dashed sinusoid is fit to velocity measurements at the center times of each image pair to highlight the inadequacy of fitting directly to velocity data for determination of seasonal amplitude or phase. This paper describes an alternate, exact approach, wherein sinusoids are fit to accumulated displacements, which are then converted to velocity to produce the light red velocity sinusoid shown in the upper panel.*

P130. I recommend the authors to make a relationship between Va and Vs here. In other words, the authors should derive equation 4 from above equations or make a relationship between them. What is the goal here? Minimizing the Va?

We see that we failed to explain how we got to Equation 4. It is just the integral of Equation 1, but that was not made clear. We have modified the text, which now says

If our first estimates of A and φ are correct, then seasonal variability must have aliased each initial estimate of interannual variability by a certain amount v_a *The amount of velocity aliasing equates the seasonal displacement over time, which we can obtain by dividing the definite integral of Eq. 1 by dt, or...*

Matlab code

```
% Dates of first image for years of coverage every six days:
t = (1:6:365*2)';

% Dates of second image in each pair:
t = [t t+6];

% Displacement error in each image pair (m):
sigma_d = 2.5;

% Corresponding velocity error in each image pair (m/yr):
sigma_v = sigma_d/(6/365);

% Measurement noise:
v_noise = sigma_v.*randn(size(t(:,1)));

% Formal estimates of error (equates to 2.5 m in each image pair):
v_err = sigma_v.*ones(size(t(:,1)));

% "Measured" velocity time series with sinusoidal variation of 50 m/yr
% amplitude with a max value on day 91 of each year:
v = 800+sineval([50 91],t(:,1))+v_noise;

% Plot the results:
itslive_tsplot(t,v,v_err)
axis tight
datetick('x','mmm','keeplimits')
box off
ylabel 'velocity (m/yr)'
```

Reviewer 2

General comments

The authors present a workflow to fit a sinusoidal function to a data set of clustered velocity estimates on ice sheets and outlet glaciers. The work is well written, and the authors clearly identify the need to extract more concise information of this vast collection of Eath observation data. The steps taken by the authors are explained, but in general there is a tendency to highlight the strong points of the methodology in their argumentation. Being a methodology paper, there might be a reason to keep this presentation brief, but it might be more than worthwhile to emphasis points of improvement and why certain decisions are taken.

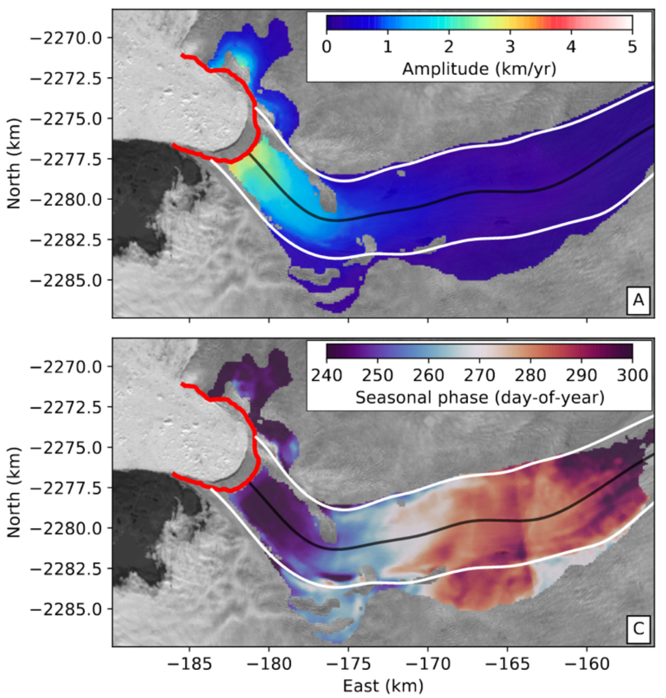
My main concern with this work stems from the property that the authors define seasonal variation as a cycle. In this way the reader is pushed into a certain narrative, which limits how to approach this issue. The authors are correct about the sinusoidal variation of the forcing (the sun and the seasons), but this does not mean the ice velocity has the same reaction. Personally, I see the seasonal variation more as a perturbation, to which there is a reaction time/response, a peak and fade out/reorganization. Thus a perturbation (including a sinus function, but also a lot of other responses) occurs every year, due to surface melt run-off, but the time span does not need to extend towards a whole year, as is assumed here. If we look at other studies short spikes are clearly visible (Kjeldsen et al. 2017, 10.1002/2017GL074081; Derkacheva et al.2020; 10.3390/rs12121935), or in the dynamics of a surging icecap (Dunse et al. 2015,10.5194/tc-9-197-2015) where a step function is seen, that is initiated by meltwater perturbation. So I miss a discussion on how good a sinus-function is as a model. There is only testing of how good the observations meets the model description, and not how good the model fits the observation. Putting everything on "background interannual variability" is a bit easy

The primary concern here is that without any regard for the shape, timing, or source of annual forcing mechanisms, we have gone ahead with an assumption that a sinusoid is a reasonable approximation of any seasonal behavior. To be clear, we have not assumed anything about the shape of forcing functions, nor do we directly discuss any potential driving mechanisms in this manuscript. Though we do have understanding of how some types of seasonal velocity signals evolve, currently, our state of knowledge is such that we do not fully understand where seasonal forcing mechanisms exist, what they are, or what their shapes may be.

We do, however, provide a method for gaining insights into a glacier’s response to seasonal variability in forcing, and for this we use a sinusoid. The sinusoid does not assume anything about the shape of the forcing mechanism or even the shape of glacier’s response. Rather, the sinusoid *describes* the cyclic behavior in the simplest way possible.

We contend that we must understand the most basic level of behavior before we can begin to discuss aberrations from it. This means that before we can begin fitting higher order functions or investigating how seasonal cycles change from year to year, we must first identify *where* seasonal variability exists, *how significant* it is in terms of the overall displacement cycle of the glacier, and in *what season* of the year ice velocity tends to reach its maximum.

The value of the sinusoidal approach can be seen in a new preprint by Riel et al. (https://doi.org/10.5194/tc-2020-193), which was submitted to *The Cryosphere* after the reviews for our manuscript were posted. Using the exact approach we describe, they map the seasonal amplitude and phase of variability to observe traveling waves in Sermeq Kujalleq. The map here provides evidence for kinematic behavior that begins at the glacier terminus and travels upstream each year.



Sermeq Kujalleq does not exhibit perfectly sinusoidal behavior, but the simple two-term description of seasonal variability reduces complexity and makes interpretation straightforward. If more complex terms are desired, we point out in the paper that additional sinusoids can be added, and it eventually it will be possible to build a Fourier series with this approach.

Another question arising is the wording of climatological velocity, I am not able to figure out what the authors mean with this term. This directly also brings me to a second point on the sinus fitting, as it is treated as a cyclic function similar to (Menchew etal., 2017, 10.1002/2016JF003971). They look at a tidal time span, where the forces are highly repetitive in magnitude and phase. However, if this is the case for seasonal glacier velocities is not so clear, as the amplitude of glacier velocity seem to correlate with surface mass balance. This has been observed with GPS in Greenland (van deWal et al. 2015, 10.5194/tc-9-603-2015) or on Nordenskiöldbreen, Svalbard (van Peltet al. 2018 10.1029/2018GL077252). But the sinus function of the authors does not take the change in amplitude, from year to year into account. However, this (to me) would be a climatological velocity (if I had to guess what the authors mean).

We have clarified the definition of climatology with the addition of this sentence:

In this paper, we describe a robust method of measuring the climatology*—or average seasonal cycle—of ice flow dynamics, with the ultimate goal that our method may be used to map the typical magnitude and timing of the seasonal glacier dynamics worldwide.*

Other influencing phenomena, like the ocean/front position have similar seasonal amplitude change (Kehrl et al. 2016, 10.1002/2016JF004133). By putting all these into a cyclic function, the signals of phase and amplitude might smooth out. In connection to this, at high latitudes, the coverage is concentrated towards the summer season. Hence, how do short term perturbation propagate into the velocity estimation? From the synthetic test the methodology can be "considered agnostic", but this is true for reconstruction purposes of a sinusoidal function. It is also not clear where the authors are after, the onset of speed-up, or "identify the seasonal maximum velocity"? Other studies/data/methods are able to find the timing of such speed-up events (Altena et al.2017 10.3389/feart.2017.00053, Vijay et al. 2019, 10.1029/2018GL081503), though not as precise or automatic as presented here, but are less constrained. So, there are some issues on the amplitude, but also on the phase. The argument of the authors for using a sinus, as it is "elegant" is a bit weak in my opinion. It would very much strengthen the manuscript, if these influencing effects/considerations are highlighted, as it gives handholds on the way forward.

We clearly hear the reviewers concern that a sinusoid is not a perfect representation of glacier variability and that in some cases it may even be a poor representation. We completely agree with this assessment but disagree that a sinusoidal approximation is a bad first guess in the absence of a universal model of variability. We could assume a sawtooth or step function, or even a pricewise model but we're unconvinced that any of these models would not suffer from the same shortcomings. The problem is how can we use heterogeneous data to compare across vastly different climate conditions, glacier characteristics, ocean forcing, etc., to identify where glaciers are fluctuating seasonally. A sinusoid is the most basic assumption we can make and provides a starting point for the contextualization of global glacier variability that will increase in nuance and complexity with time. We justify the simplicity of our assumption as it is highly generic and provides a logical first step toward global characterization.

The authors have formulated their estimation procedure by decoupling the x- and y-velocities. Is there a certain reason for this? I can imagine it can be beneficial, to include co-variance functions, so outliers in one dimension are also excluded in the other. In addition, given these phase angles are estimated independently, do the authors see a difference between both axis. If so, this would imply a change inflow direction, if not what would that mean? Also, why did the authors do filtering (using the MAD), and not do robust least squares, or at least use such procedure in the estimation? Neither is it clear to me why several iterations are applied, see (https://ccrma.stanford.edu/~jos/filters/Sum_Sinusoids_Same_Frequency.html), or is the estimation not restricted to a yearly cycle? Or is the iteration not done on the residuals?

Some of the most compelling insights that we expect to gain from this method will involve transverse motion that could not be detected if we were to assume that flow variations only occur in the direction of mean flow. For example, we have applied our method to Drygalski Ice Tongue and found the same side-to-side motion that has previously been found using in-situ measurements (https://ui.adsabs.harvard.edu/abs/2013AGUFM.C21A0624L/abstract).

In addition to floating ice, we may find transverse flow anomalies in grounded areas with strong seasonality of basal water pressure. For example, if the basal pressure on one side of a glacier rises while the other side remains unchanged, the lopsided acceleration in flow would cause a divergence of and change in flow direction, even if only small. The method we describe is able to resolve displacements of just a few meters left or right of a mean flowline, meaning the maps can be created by separating the x and y components of variability could hint at underlying drivers of change.

The authors run tests on synthetic data, by imposing corruption to individual velocity estimates. This noise is done on an individual basis, which is partly due to measurement noise. But there is also dependent noise, as displacement estimates are derived for pairs of images. Hence, when one image is corrupted for some reason, there is a high probability it propagates to all displacements it is part of. However, this issue is not included into the analysis, though of importance (and due to the synthetic nature, is possible to generate). This would give more insights then the 32 velocity estimates, stated now.

The reviewer makes a very good point but there is a subtlety to the data that makes this not the case. The largest source of error when tracking features between two images acquired with the same viewing geometry (repeat image) is the geolocation error in each image that typically manifests itself as a scalar displacement in x and y. These correlated errors have been corrected by the autoRIFT algorithm that produces the ITS_LIVE velocity data. The correction process works by examining the initial measured velocities over all stable surfaces in an image such as rock. The average measured velocity over rock is equates to an offset error across the entire scene. After the offset errors are removed in x and y, the remaining errors in each image pair can be considered to be uncorrelated.

Minor comments:

In general the manuscript is well written, the authors write in their mother tongue, so concerning this issue I am not able to do any better. But for a global audience the wording is sometimes a bit hard; I have learned quite a lot of new words. For sake of easy reading, and not having to go back and forth to a dictionary, please consider changing words a bit. Think of, "unwieldy" or "egregious”.

We appreciate this feedback, as we wish to make our work accessible to all interested readers, regardless of personal background. We have looked through the manuscript to ensure that (aside from a few necessary technical words) there is no language in this revision that wouldn’t be found in in standard English-language news outlets.

I have tried to understand from the text what is done, and also looked in the code to be able to zoom into the plots/data. But the provided code and plotting does not work, as some functions ("itslive_tsplot" or "itslive_seasonal_deets") are absent.

We appreciate the reviewer's effort in digging into the code that we included as a supplement to the manuscript. It appears this comment regards the make_fig04.m script, which contains the code that can be used to recreate Figure 4 of the manuscript. We've double-checked and cannot identify any missing functions, but it seems likely that the confusion stems from our inclusion of the itslive_seasonal_deets function at the end of the make_fig04.m script. It's a relatively new feature that Matlab can call functions that are included at the end of a standard non-function .m script, and we've taken advantage of the feature to keep our bundle of supplemental code as tidy as possible.

Regarding the missing itslive_tsplot function, we note that it is included among the ITS_LIVE functions we've posted to GitHub (www.github.com/chadagreene/ITS_LIVE). The README.txt file that describes what's included in the supplemental material states at the top that some functions necessary to run the scripts are part of the toolboxes on my GitHub page, including ITS_LIVE tools, Antarctic Mapping Tools for Matlab (Greene et al., 2017), and the Climate Data Toolbox for Matlab (Greene et al., 2019).

title: be a bit more specific, maybe change to "Reconstructing seasonal oscillations" also include "glacier ice".

We note that the application of this method is not limited to glaciers. For example, we have applied our method to the Ross Ice Shelf and found the same patterns of seasonal variability reported this week by Klein et al. (https://doi.org/10.1017/jog.2020.6).

6: "dark polar winters" > "at high latitudes"

The sentence in question describes the problem of observing seasonal variability using optical data, when no optical data are available for several months each year due to lack of sunlight. Thus, we describe the situation that there are "...no optical observations throughout *dark, polar winters*." In our view, the suggested wording, "*at high latitudes*" does not directly address seasonality nor make mention of the solar illumination that's necessary for optical data acquisition. We prefer to keep the wording as it is.

8: "climatological average winter velocities" what is meant here?

We have clarified the definition of climatology with the addition of this sentence:

In this paper, we describe a robust method of measuring the climatology*—or average seasonal cycle—of ice flow dynamics, with the ultimate goal that our method may be used to map the typical magnitude and timing of the seasonal glacier dynamics worldwide.*

15: "sufficient quantity of data" is this due to quantity of data, the consistency of campaigns/ monitoring programs or simple availability of large computing power. Or is it opening up of the archives, making historical flow estimation possible (Cheng et al.2019 10.5194/isprs-archives-XLII-2-W13-1735-2019).

The conference paper by Cheng et al is intriguing, and if they are able to successfully employ the method of processing outlined in that paper, it will be interesting to see what the velocity fields from the ARGON era look like. But as far as we can tell, that dataset has not been produced or has not been made widely available.

18: "all of the world's ice" large bodies of glacial ice

We have clarified that annual velocity mosaics are now available for "*nearly* all of the world's *land* ice".

20: "upended glaciology" Remote sensing is able to get geometric information about the (sub)surface, and is of great aid. To some extent this is a game changer, but it might be fair to also give credit to automatic weather stations, or put it into persective. This have been other great advancement in glaciology, think for example of (Ohmuraet al., 1992; Oerlemans et al., 1998).

The introductory paragraph briefly mentions some of the recent advancements in remote sensing that have gotten us where we are today, and then the paragraph identifies the types of insights we want to gain from this new abundance of satellite data. We feel that a discussion of automatic weather stations would be a distraction from the main points we wish to communicate in this manuscript.

31: There is also an large collection of studies at the intermediate timescale, which isleft out here, dealing with surges. For sake of completeness, this might be included

We do reference a paper on surge dynamics by Yasuda and Furuya, but we have tried to keep the focus of this manuscript primarily on cyclic behavior.

37: "the logistical challenge of" what is meant here?

We have modified the sentence to now read,

...due in part to the ***technical*** *challenge of working with optical data in polar regions, where the surface is not touched by sunlight for months-long periods each winter.*

The technical challenge of working with optical data in polar regions is that several months go by each winter when optical data are not collected, because the sun does not illuminate the surface during those months. The full text of this manuscript provides a detailed description of the technical challenges of working with this data.

39: "robust extraction" using a robust pre-processing technique, is different from a robust methodology. Given its stiffness (non adaptive) towards one model (a sinusoidal) this might not be a correct formulation. It might be "precise"...?

Following this suggestion, we have changed the word *robust* to *precise*.

40: "primarily focused on Antarctica" maybe better: on the ice sheets and their outlets....?

We designed the study with the primary goal of understanding Antarctic seasonal ice dynamics, because that's where data is most limited, and it's where the fewest studies have been published about the subject. The examples we provide and the statistics in all of the synthetic tests are based on Antarctic data. Accordingly, we stand by the statement that

Our study is primarily focused on Antarctica, where seasonal variability is poorly understood, and where data limitations currently present the greatest challenges to making such measurements.

53: "by feature tracking" add: over longer time spans

We have made the suggested change.

55: "the true magnitude" change to "a" instead of "true" or "a well fit"

We have removed the phrase true magnitude, as suggested. The sentence now reads,

...by fitting a cyclic function to the time series of displacements rather than average velocities, we show that it is possible to accurately recover the magnitude and phase of seasonal velocity variability.

56: I miss another possibility here, which is common practice in inSAR displacement estimation, being inversion (e.g.: Bontemps et al. 2018, 10.1016/j.rse.2018.02.023, Li et al. 2020, 10.1016/j.rse.2020.111695). This does not make it necessary to work with average time stamps or a model, and can resolves to very small time steps.

It is unclear how the inversion techniques described by Bontemps et al. or Li et al. should be included in this study.

64: " first or second image" first and second? maybe be more clear or use "master-slave" "chip-search space" etc.

We note that the community is moving away from the terms *master* and *slave* (https://comet.nerc.ac.uk/about-comet/insar-terminology/).

The sentence in question states,

Each satellite image may serve as the first or second image in multiple image pairs...

The words "first" and "second" refer to the sequence in time. The suggested wording does not adequately convey temporal sequence, and the meaning of "chip-search space" may not be widely understood.

100: "most robust means" why is this robust, where do you get the reliability from?

We have removed the word “robust”.

fig4: maybe it is good to note, why there are two groups of points, as one is +- a year and the other at short time intervals in summer. btw: the purple line nicely follows the annual velocity clusters!

Following this suggestion, we have edited the caption of Figure 4 to describe the two groups of points as follows:

The clustering of these 14,208 measurements taken near the grounding line of Byrd Glacier typifies ITS_LIVE image pair data, with short Δt measurements providing direct, but noisy observations of velocity variability throughout each summer, while much lower-noise winter estimates can only give insight into the total displacement that occurs during the dark, winter months.

151: I don’t think this is sensitivity, but more an analysis to get an idea how good the recovery is. As the model is corrupted with noise and then an attempt is made to reconstruct the model. If I understand correctly.

The Sensitivity Analysis section is where we test the sensitivity of the method to several different parameters. We determine the sensitivity of the technique to the number of image pairs used, the level of background interannual variability, the amplitude of the underlying signal, and the phase of the underlying signal. The parameters of these tests are tabulated in Table 1: Sensitivity test parameters.

181: "recover", change to we "are able to describe the variation by seasonal cycles" or alike.

The passage in question reads,

We conducted several tests to determine the accuracy with which we can recover the amplitudes and phases of seasonal cycles in synthetic velocity time series.

We feel that the present wording will be more easily understood than the suggested wording.

258: " is remarkable" subjective wording, please change

Following this suggestion, we have changed the word *remarkable* to *notable*.

258: "robust", precise/accurate might fit better

Following this suggestion, we have changed the word *robust* to *precise*.

267: "minuscule variations" subjective wording

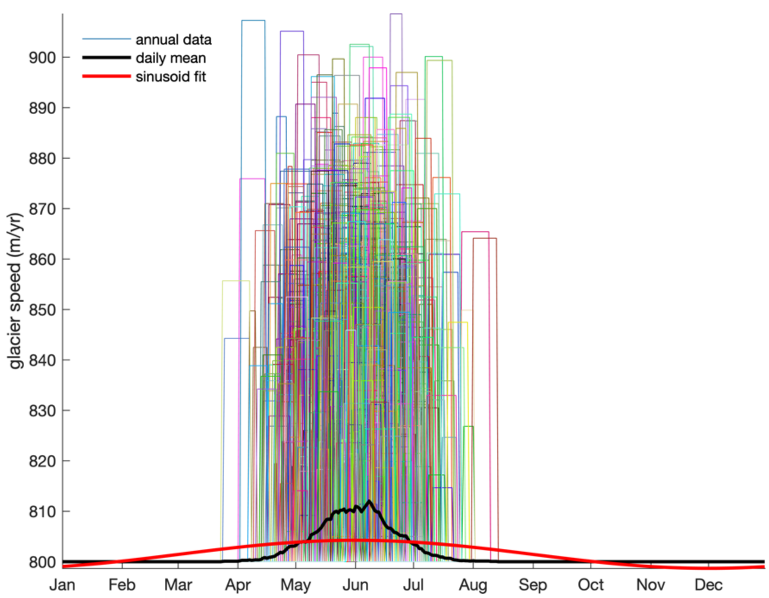
We have replaced the word *minuscule* with *tiny*, to describe the displacements on the order of a meter or so that can be detected with this method.

283: " in the climatological sense, nature does not consistently time such events as calving or increases in basal water pressure with any greater precision than the method we have presented to detect them". What is meant here?

We have added a definition of climatology to clarify that the climatology refers to the average seasonal cycle taken over many years.

Transient events such as calving or impulses of water into a subglacial hydrological system often occur on a yearly cycle, but the corresponding glacier speedup may only last for a few days. It may seem that a spike in velocity only lasting a few days of the year would be poorly represented by a sinusoid that continuously varies throughout the year, but this sentence points out that mother nature does not time glacier calving to occur on the very same day each year. As a result, when we take the average annual cycle from many years of data, we find there is generally a *season* of glacier acceleration rather than just a few days of acceleration.

To illustrate this point, here we consider a glacier whose velocity is exactly 800 m/yr most days of the year, but each summer it accelerates by 50±10 m/yr for a duration of 10±3 days, centered on June 1±20 days. We generate 1000 years of this pattern, and then consider the mean cycle of daily speeds and compare it to the sinsuoid fit. Using the Matlab code provided at the bottom of this document, we get the following plot:



What we see in the daily mean velocity curve is that in the climatological sense, the period of high summer velocity lasts for months, even though the average high-velocity event only lasts for 10 days. Fitting a sinusoid to the daily means finds a peak velocity on June 1, which is exactly the prescribed center date of the high-velocity period. Of course, the amplitude is severely underestimated by the sinusoid in this scenario, but the passage in question regards timing, not amplitude. It states, *While it is true that a glacier can accelerate in response to a transient event and return to an equilibrium velocity within just a few days (Stevens et al., 2015; Andrews et al., 2014), in the climatological sense, nature does not consistently time such events as calving or increases in basal water pressure with any greater precision than the method we have presented to detect them.*

285: "In most cases, a sinusoid will likely capture the majority of velocity variance throughout the year, and represent the fundamental mode of subannual variability in ice velocity." Please justify this claim, as this is the corner stone which the whole study is build upon.

We agree that this claim was not well supported, and on reflection we see that it may have been untrue. We have removed the claim from the text.

291+: It seems the authors put all the misfits of the sinus model on the inaccuracies of the GPS measurements, while this sensor measures all kinds of physics decadal, annual, daily,...

We do not claim that the small misfit between GPS and our method is solely the fault of the GPS data. Rather, we simply point out that the GPS record contained several long gaps, and it is possible that any disagreement between the two methods could reflect the different times during which data were collected. Here in the Discussion section it is appropriate to acknowledge the possible causes of any mismatch between the GPS and ITS_LIVE data, and consider what that might mean for applying this method elsewhere. The passage states,

...the [GPS] receiver's harsh polar environment has led to several long gaps during which no GPS position data were acquired. This suggests that as a means of measuring ice dynamic climatology, our method might not only meet, but exceed the performance of the in situ GPS receiver while providing insights into dynamic behavior as far back as the mid 1980s.

We feel it is important to point out that GPS data—although absolutely vital to this type of work—is not perfect. Most GPS receivers do not collect data over polar winter, yet the method of image analysis we describe is able to approximate winter ice velocities. Similarly, very few locations on Earth offer GPS measurements even today, yet the ITS_LIVE dataset offers global coverage, and in some locations that coverage extends back to the 1980s.

305: "our method can extract" please add "..by describing ice flow as an oscillation ..."in some way

Following the suggestion, the passage now reads,

...our method can extract the amplitude of seasonal variability with a precision on the order of about 1 m/yr ***by describing ice flow as an oscillation****, provided the level of background interannual variability does not overwhelm the overall signal.*

309: "independent of the amplitude and phase of the seasonal velocity variability", this is not convincingly given.

The sentence in question reads,

Ability to detect seasonal amplitudes is independent of the amplitude and phase of the seasonal velocity variability, but phase accuracy benefits with increasing amplitude of seasonal variability.

Please see Figures 5g, 5i, and 6a.

313: "fully three dimensional understanding" what is meant here?

We have fixed this sentence. It now reads,

...by providing a method that can be employed independently in the two dimensions of Cartesian coordinates, we hope to gain a ***more complete*** *understanding of how dynamic signals propagate through the world's ice.*

322: "egregious outliers" egregious=outliers

We have replaced *egregious* with *extreme* to make the text more accessible to non-native English speakers. To be clear, we are not discussing outliers that are just three or four standard deviations away from the mean. Rather, we are discussing the extreme outliers that can land hundreds of standard deviations away from the rest of the bunch. We now describe them as *extreme* outliers.

Matlab code

```
% Create synthetic time series:
N = 1000; % number of years of the time series
dur = 10 + round(3*randn(N,1)); % (days) duration of fast flow each summer
dur(dur<1) = 1; % just in case randn resulted in any negative durations.
st = 153 + round(20*randn(N,1)-dur/2); % DOY of fast flow period.
vs = 50+20*randn(N,1); % Magnitude of summer speedup
vs(vs<5) = 5;  % just in case randn made vs negative
va = 800; % background/winter ice velocity

% Define the rectangular function for each year:
v = va*ones(N,365);
for k = 1:N
   ind = st(k):(st(k)+dur(k)); % indices corresponding to fast-flow event
   v(k,ind) = va+vs(k);
end

%% Plot the time series

cm = rand(N,3); % colormap
figure
hold on
for k = 1:N
   hi(k) = plot(1:365,v(k,:),'color',cm(k,:));
end
axis tight
box off
hm = plot(1:365,mean(v),'k','linewidth',2);

% Fit a sinusoid:
ft = sinefit(1:365,mean(v),'terms',3) % Climate Data Tools (Greene et al.,
2019)
hf = plot(1:365,sineval(ft,1:365),'r','linewidth',2);

datetick('x','mmm','keeplimits')
ylabel 'glacier speed (m/yr)'

legend([hi(1),hm,hf],'annual data','daily mean','sinusoid
fit','location','northwest')
legend boxoff
```

Detecting seasonal ice dynamics in satellite images

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Abstract. Fully understanding how glaciers respond to environmental change will require new methods to help us identify the onset of ice acceleration events and observe how dynamic signals propagate within glaciers. In particular, observations of ice dynamics on seasonal timescales may offer insights into how a glacier interacts with various forcing mechanisms throughout the year. The task of generating continuous ice velocity time series that resolve seasonal variability is made difficult by ~~the finite integration time over which ice velocities are measured from optical and repeat SAR imagery, and by~~ a spotty satellite record that contains no optical observations ~~throughout~~ during dark, polar winters. Furthermore, velocities obtained by feature tracking are marked by high noise when image pairs are separated by short time intervals, and contain no direct insights into variability that occurs between images separated by long time intervals. In this paper, we describe a method of analyzing optical- or SAR-derived feature-tracked velocities to characterize the magnitude and timing of seasonal ice dynamic variability. Our method is agnostic to data gaps and is able to recover climatological average winter velocities regardless of the availability of direct observations during winter. Using characteristic image acquisition times and error distributions from Antarctic image pairs in the ITS_LIVE dataset, we generate synthetic ice velocity time series, then apply our method to recover imposed magnitudes of seasonal variability within ± 1.4 m yr^{-1}. We then validate the techniques by comparing our results to GPS data collected on Russell Glacier in Greenland. The methods presented here may be applied to better understand how ice dynamic signals propagate on seasonal timescales, and what mechanisms control the flow of the world's ice.

1 Introduction

Earth-observing satellites have been in orbit for over half a century, but it was only in 2011 that a sufficient quantity of data had been collected to complete the first pan-Antarctic map of ice velocity (Rignot et al., 2011). Since then, new satellites have led to follow-on mappings that identified regions of changing ice flow (Gardner et al., 2018), and today data are being collected at such a rate that velocity mosaics can be generated every year for nearly all of the world's land ice (Joughin, 2017; Mouginot et al., 2017; Gardner et al., 2019). So for a field of study that was born in an age of *in situ* stake networks and dead reckoning, the data revolution of the past decade has completely upended glaciology. Where once our challenge was to squeeze as much information as possible from a few sparse field measurements, our biggest challenge now lies in processing massive, often unwieldy datasets, and finding all the meaningful signals that lie hidden in this new abundance of data.

One of the most direct and insightful ways to understand how ice moves, what controls its flow, and how it responds to changes in its environment, is to observe dynamic variability under a wide range of periodic forcings. Long-term trends and interannual variability (e.g., Moon et al., 2012; Christianson et al., 2016; Greene et al., 2017a; Dehecq et al., 2019) can provide a sense of how glaciers respond to sustained forcing, but on much shorter timescales we are able to see how ~~accelerations~~ velocity variations initiate and what physical processes control a glacier's movement. Several targeted studies have shown that glaciers can exhibit observable dynamic responses to ocean tides, and that tidal signals can propagate well inland of the grounding line on daily to fortnightly timescales (e.g., Anandakrishnan et al., 2003; Walter et al., 2012; Rosier and Gudmundsson, 2016; Minchew et al., 2017; Robel et al., 2017), but in many glaciers around the world, a significant gap exists in our understanding of how ice dynamic changes develop between tidal and interannual timescales.

It is certainly understood that many mountain glaciers speed up and slow down throughout the year (Burgess et al., 2013; Armstrong et al., 2017; Yasuda and Furuya, 2015; Kraaijenbrink et al., 2016), and that some of Greenland's glaciers respond to seasonal cycles of subglacial hydrology or calving dynamics (Joughin et al., 2008; Howat et al., 2010; Bartholomew et al., 2010; Sole et al., 2013; Moon et al., 2015; King et al., 2018). Seasonal variability has even been reported in a few studies of Antarctic glaciers (Nakamura et al., 2010; Zhou et al., 2014; Greene et al., 2018); but to date, no global-scale mapping of seasonal dynamics of the world's ice has been completed, due in part to the ~~logistical~~ technical challenge of working with optical data in polar regions, where the surface is not touched by sunlight for months-long periods each winter. A comprehensive mapping of the world's seasonal ice dynamics would permit direct inter-comparison of seasonal evolution in regions with different driving processes; provide a basis for analysis of long-term changes in seasonal behavior; and supply models with a zeroth-order understanding of global ice climatology.

In this paper, we describe a ~~method that allows for the robust extraction of seasonal changes in ice flow , and~~ precise method of measuring the *climatology*—or average seasonal cycle—of ice flow dynamics, with the ultimate goal that our method may be used to map the typical magnitude and timing of the seasonal ~~dynamics of all the world's ice~~glacier dynamics worldwide. Our study is primarily focused on Antarctica, where seasonal variability is poorly understood, and where data limitations currently present the greatest challenges to making such measurements. We test the sensitivity of our method on several thousand synthetic ice velocity time series, then validate it by applying the method to satellite data covering Russell Glacier in Greenland, where we compare our results to GPS observations that show persistent seasonality.

2 Feature-tracked velocity data

The method we present applies to ice velocity datasets such as GoLIVE (Scambos et al., 2016) or ITS_LIVE (Gardner et al., 2019), which have been derived by feature tracking techniques applied to satellite image pairs. For a detailed review of the principles of feature tracking we refer readers to Scambos et al. (1992) or Fahnestock et al. (2016), but for the work presented here it is essential to know only that feature-tracked velocities are measured as the integrated surface displacement that occurs between the acquisition times of two satellite images of a given location. That is, each measurement represents an *average* velocity between image acquisition dates, and the required passage of time between images precludes direct measurement of instantaneous velocity at any given time. As a result, any high-frequency variability that occurs between images is not represented, and seasonality may appear missing or deprecated in velocity measurements obtained by feature tracking over long

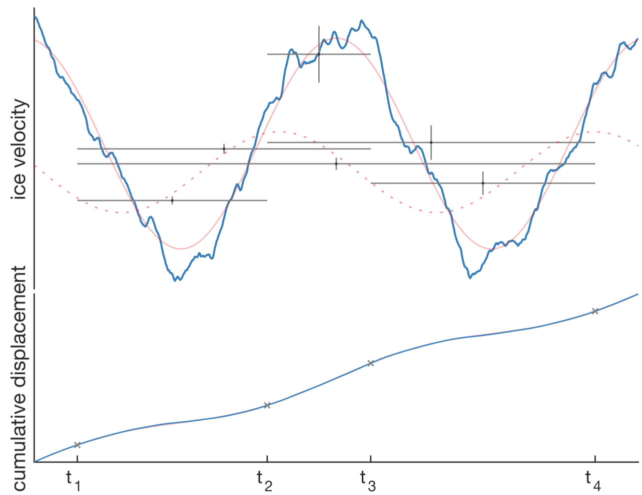

Figure 1. Example scenario: ~~For a true continuous~~ The upper panel shows an ice velocity time series ~~like the one shown~~ in blue, ~~feature tracking measures~~ which integrates to form the cumulative displacement time series shown in the lower panel. We use satellite images to measure ice velocity as the ~~total~~ cumulative displacement of crevasses and other glacier features that occurs between acquisition times of any two satellite images~~of the flowing ice~~. Here, four images taken at times t_1 through t_4 provide six unique combinations of image pairs that yield ~~six velocity~~ the measurements ~~, which are~~ depicted as horizontal gray lines connecting the acquisition times of each image pair. Vertical gray lines show measurement uncertainty and a black dot is placed at the center times of each image pair for visual clarity. ~~The pink~~ A dashed sinusoid is fit to velocity measurements at the center times of each image pair to highlight the inadequacy of fitting directly to velocity data ~~to determine the~~ for determination of seasonal amplitude or phase~~of seasonal variability~~. This paper describes an alternate, exact approach, wherein sinusoids are fit to accumulated displacements~~rather than~~ , which are then converted to velocity ~~time series~~to produce the light red velocity sinusoid shown in the upper panel.

timespans (Fig. 1). Nonetheless, by fitting a cyclic function to the time series of displacements rather than average velocities, we show that it is possible to ~~recover the true~~ accurately recover the magnitude and phase of seasonal velocity variability.

This paper focuses primarily on the Landsat image pairs that populate the ITS_LIVE dataset, in part because their record extends back as far as 1985 in some locations. The long Landsat record may help ascertain a climatological seasonal cycle of ice dynamics at any given location; however, we face the limitation that optical satellites like Landsat do not collect data throughout the dark winters in polar regions, where land ice is most prevalent (see Fig. 2). Despite the lack of direct observation in winter, we will show that the magnitude and timing of seasonal variability can be accurately retrieved from Landsat data, regardless of when in the year the maximum velocity occurs.

For any given 240×240 m pixel in Antarctica, the ITS_LIVE dataset may contain from a few dozen to more than 10,000 velocity measurements (see Fig. 3), which are taken as the ice displacement observed between two satellite images collected on different days. Each satellite image may serve as the first or second image in multiple image pairs, resulting in many overlapping measurements of ice velocity, as shown in Fig. 4. Georegistration error of each image leads to some visible disagreement

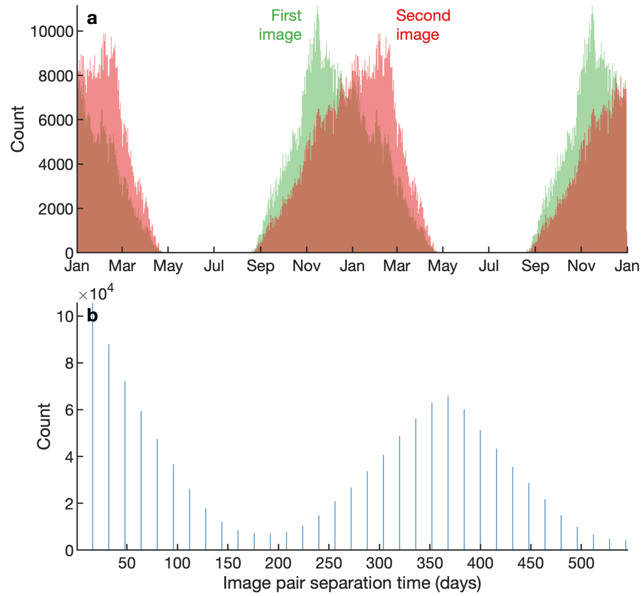

Figure 2. Antarctic image pair acquisition times. Over 1.8 million Landsat image pairs provide Antarctic ice velocities in the ITS_LIVE dataset. **a:** The seasonal cycle of Landsat image collection shows the effects of the solar cycle on Antarctic sampling. The cycle is repeated for visual continuity over the summer. **b:** ~~Images~~ Displacement fields in the ITS_LIVE dataset are processed for all image pairs separated by 16 to 544 days, with half of all image pairs representing 80 days of displacement or less, but a distinct secondary peak corresponds to a Δt value of 1 year.

between the overlapping velocity measurements, but despite the noise, a coherent pattern of interannual variability is apparent as the clusters of velocity measurements move up or down from year to year.

3 Method of analyzing image pair velocity data

3.1 Assess and remove interannual variability

The first step toward quantifying seasonal variability for any pixel is to remove any interannual variability from the time series. Interannual variability can be determined by smoothing the velocity data using any of several common methods. A polynomial fit is robust and computationally efficient, but requires choosing a polynomial order, which is subjective and can lead to overfitting or underfitting the data. Alternatively, a moving average makes no prior assumption about the shape of the velocity curve, and can adapt to any arbitrary interannual variability. After exploring several approaches, we find the best results by first detrending the time series with a low-order polynomial, then using a hybrid of a moving average and a spline fit, which we describe below.

When assessing interannual variability, we temporarily ignore the duration over which each image pair measured ice displacement, and simply assign the average velocity to the center date t_{m} of each image pair. Using the range of times t_{m} in the

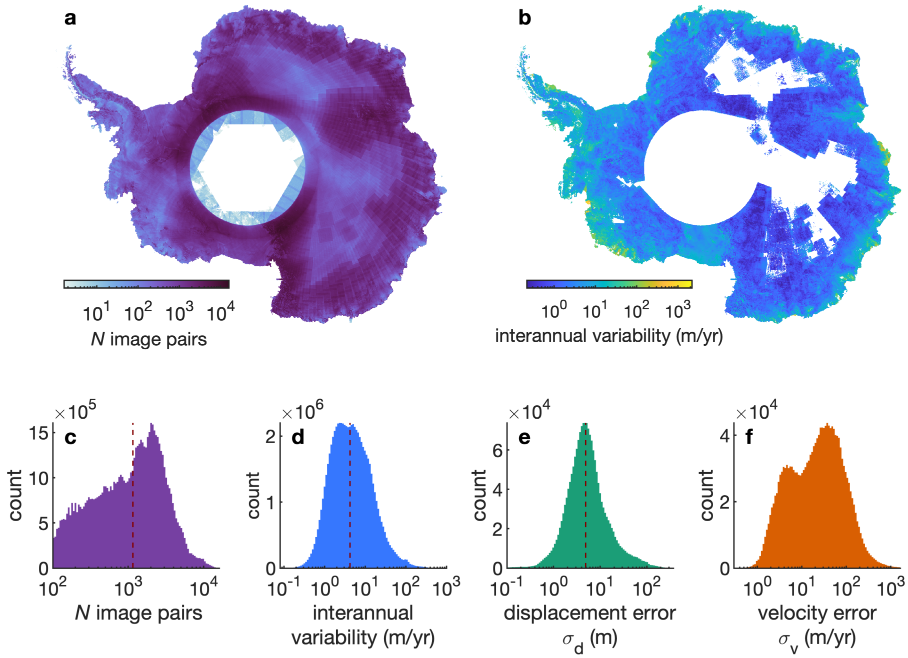

Figure 3. ITS_LIVE velocity data statistics. Maps of **a:** the number of image pairs that contribute to each 240×240 m pixel in the ITS_LIVE summary velocity mosaic and **b:** the standard deviation of annual velocity mosaics from 2013 to 2018. Histograms in panels **c** and **d** show only the values from **a** and **b** that correspond to grid cells with at least 100 contributing image pairs and whose mean velocity is at least 15 $\mathrm{m\,yr^{-1}}$. Displacement errors σ_d shown in panel **e** result from satellite image georegistration error, and when divided by values of Δt corresponding to each image pair results in the velocity errors σ_v shown in panel **f**. Because displacement errors are distributed symmetrically, the bimodal distribution of velocity errors roughly correspond to the two peaks of Δt values over which velocity is measured (shown in Fig. 2). Vertical lines in panels **c–e** indicate median values of 1153 image pairs per grid cell, 4.2 $\mathrm{m\,yr^{-1}}$ interannual variability, and 4.9 m displacement error.

time series, we require at least two years of velocity data and detrend using a polynomial whose order is chosen as one quarter of the range of t_m in years, rounded up to the nearest integer. The result is that for up to four years of data, the time series is linearly detrended; from four to eight years of data, the time series is quadratically detrended, and so on. We assign the velocity weights w_v in the polynomial fit using the formal error estimates σ_v from the ITS_LIVE data such that $w_\mathrm{v} = \sigma_\mathrm{v}^{-2}$. An example of a fourth-order polynomial fit to the velocity data is shown in Fig. 4.

After detrending the time series with a weighted polynomial fit, we characterize any residual interannual variability with a spline fit to the mean velocities of each year. We take the weighted mean velocity of all measurements whose t_m lies within 183 days of winter solstice of that year. We assign the weighted mean velocity of each year to the weighted mean date of those velocities, then interpolate with a shape-preserving piecewise cubic hermite polynomial to obtain a measure of interannual variability corresponding to each image pair's center date t_m.

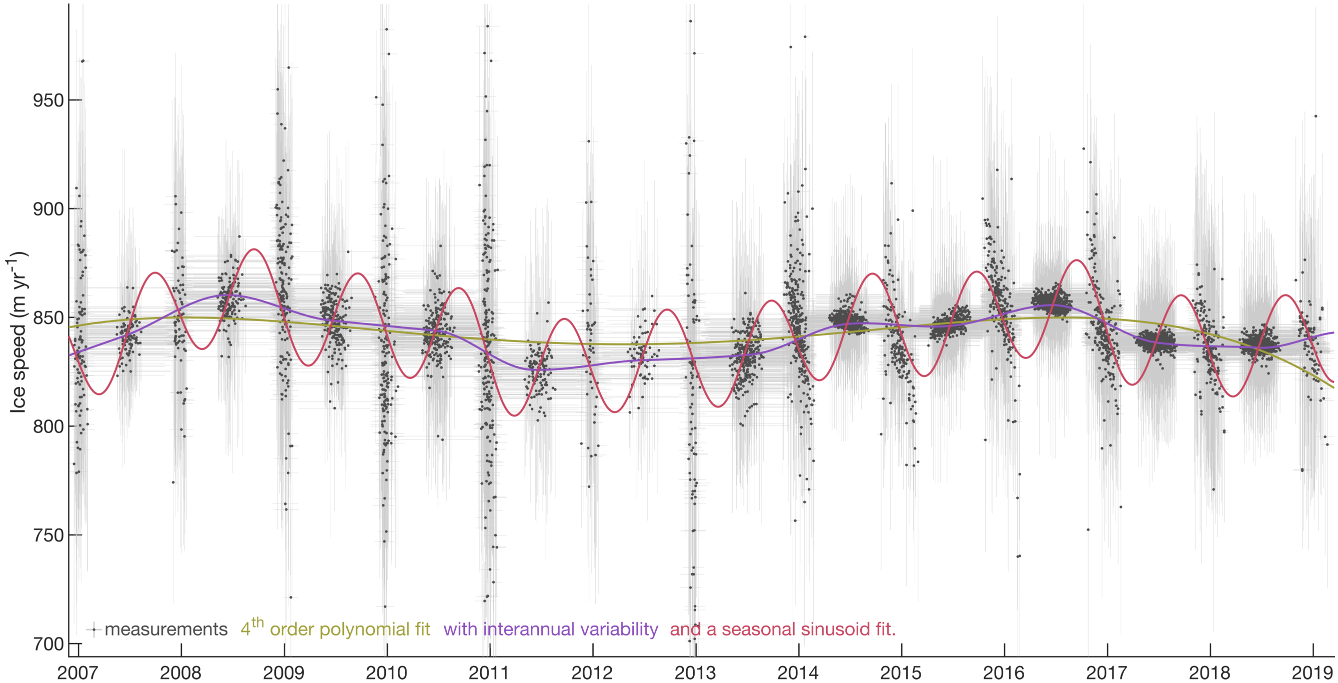

Figure 4. Velocity time series for an ITS_LIVE pixel. ~~These~~ The clustering of these 14,208 measurements taken near the grounding line of Byrd Glacier ~~typify~~ typifies ITS_LIVE image pair data, with short-Δt measurements providing direct, but noisy observations of velocity variability throughout each summer, while much lower-noise winter estimates can only give insight into the total displacement that occurs during the dark, winter months. Light gray horizontal bars connect the acquisition times of each image pair and vertical bars show $\pm 1\sigma_\mathrm{v}$ uncertainty. Center dates t_m are shown as dark gray dots for visual clarity.

In the method described thus far, most summer image pairs whose Δt is small contribute much less to the weighted mean annual velocities than do long Δt image pairs that span winter. This is because velocity error by any feature-tracking algorithm stems strictly from displacement error σ_d, while timing error is essentially zero. So although summer offers more image pair measurements, their shorter Δt values are associated with greater velocity error σ_v and therefore they are significantly downweighted in the calculation of annual mean velocities. In other words, it is possible that either due to the higher weights of winter velocities or the higher quantity of measurements available during summer, the weighted mean annual velocities could be biased toward one season or the other. We account for this possibility by iteratively solving for interannual and seasonal variability, as we describe later in Section 3.3.

3.2 Assess seasonal variability

We characterize the magnitude and timing of seasonal variability as a simple sinusoid that can be applied in the x and y directions separately to build a two dimensional understanding of how ice moves throughout the year. Limitations of the sinusoidal approximation are discussed later in Sect. 6, but we justify our approach as it is the simplest ~~, most robust~~ means of describing cyclic behavior, and by nature it is constrained to capture the sum total of ice displacement that occurs throughout the year.

After characterizing interannual variability, we subtract it from the velocity time series at the center date of each image pair. The residuals v_r after removing interannual variability can then be assumed to contain only seasonal variability and noise. To address blunders, we remove outliers whose absolute value exceeds 2.5 robust standard deviations of v_r (see Appendix A). The remaining task is to fit a sinusoid to the v_r time series, but given that each velocity measurement is a single value that represents several weeks to more than a year, we cannot fit a sinusoid directly to the velocity time series or assume that values of v_r correspond to the image pair center dates t_m. Instead, we operate on the displacements associated with each image pair, taken as the integrals of velocities v_r.

We seek to define seasonal velocity variability v_s in the form

$$v_\mathrm{s}(t) = A\sin\left(2\pi(\phi+t)\frac{t+\phi}{T}\right) + C_0, \tag{1}$$

where A and ϕ are the amplitude and phase of the seasonal velocity cycle, respectively, ~~and~~ t represents time in decimal years, and $T = 1$ year is the period of oscillation. By our method, the constant C_0 is ideally zero after detrending and removing interannual variability, but we include it here in case some residual offset is present in the v_r time series.

For a more robust least-squares solution, we employ a trigonometric identity to rewrite Eq. 1 as

$$v_\mathrm{s}(t) = C_1\sin(\left(2\pi t)\frac{t}{T}\right) + C_2\cos(\left(2\pi t)\frac{t}{T}\right) + C_0, \tag{2}$$

which is related to Equation 1 by $A = \sqrt{C_1^2 + C_2^2}$ and $\phi = \mathrm{atan2}(C_2, C_1)$.

Again, we cannot solve Eq. 1 or Eq. 2 directly with image-pair velocities, because they do not represent instantaneous velocity at any known times t. Instead, image pair data track displacements, which equate to the integral of velocities between times t_1 and t_2 when the two images were acquired. Accordingly, we can take the definite integral of Eq. 2 to solve for the seasonal displacement cycle d_s as,

$$d_\mathrm{s} = \frac{C_1}{2\pi}C_1\frac{T}{2\pi}\left[\cos(\left(2\pi t_1)\frac{t_1}{T}\right) - \cos(\left(2\pi t_2)\frac{t_2}{T}\right)\right] + \frac{C_2}{2\pi}C_2\frac{T}{2\pi}\left[\sin(\left(2\pi t_2)\frac{t_2}{T}\right) - \sin(\left(2\pi t_1)\frac{t_1}{T}\right)\right] + C_0(t_2 - t_1). \tag{3}$$

We solve for the coefficients of Eq. 3 using a least-squares approach with weights given by $w_\mathrm{d} = (\sigma_\mathrm{v}\cdot\Delta t)^{-2}$. This type of approach has previously been applied to study Earth deformation (e.g., Hetland et al., 2012), and is similar to an approach that has been used to analyze ice dynamic responses to tidal forcing (Milillo et al., 2017; Minchew et al., 2017).

By employing Eq. 3 to solve for A and ϕ, we obtain a first approximation of the seasonal variability of ice velocity. However, the possibility remains that the initial estimate of interannual variability may have been partly aliased by seasonal variability due to uneven temporal sampling. Thus, we refine our estimates of interannual and seasonal variability by iterative means.

3.3 Iteratively refine interannual and seasonal variability estimates

If our first estimates of A and ϕ are correct, then ~~we know that the amount by which seasonal variability~~ seasonal variability must have aliased each initial estimate of interannual variability ~~is given by~~ by a certain amount v_a. The amount of velocity aliasing equates the seasonal displacement over time, which we can obtain by dividing the definite integral of Eq. 1 by Δt, or

$$v_\mathrm{a} = \cancel{\frac{A}{2\pi\Delta t}}\cancel{\{\cos} A\frac{T}{2\pi(t_2 - t_1)}\left[\cos\left(2\pi\cancel{(t_2+\phi)}\frac{t_2+\phi}{T}\right) - \cos\left(2\pi\cancel{(t_1+\phi)}\frac{t_1+\phi}{T}\right)\right]\cancel{\}}. \tag{4}$$

After obtaining initial estimates of A and ϕ, we subtract v_a from the original detrended velocity measurements and repeat the process of calculating interannual and seasonal variability. In most cases, we find that initial estimates of A and ϕ are reasonably close to their true values, and that just a few iterations are sufficient to yield accurate final estimates of seasonal amplitude and phase. We explore the number of requisite iterations in Sect. 4.3 and show the effects of iterating in Fig. 5.

Figure 4 shows an example of our method applied to a time series of 14,208 ITS_LIVE image pairs acquired in a single pixel near the grounding line of Byrd Glacier, Antarctica. Because no strong long-term velocity trends are present, the green fourth-order polynomial fit is relatively flat, albeit with a slight downturn at the unconstrained end of the time series. The blue curve of interannual variability follows the multi-year rises and falls of velocity much more closely, and is uncontaminated by seasonal variability on this tenth iteration of the solution. The red curve adds seasonal variability with an amplitude of 23 m yr^{-1} to the interannual variability, and given that it is a least-squares solution, it represents the only solution that minimizes the misfit between the curve and all available observations. Indeed, while at first glance it may appear visually that the timing of image acquisitions themselves are the only seasonal pattern in the ITS_LIVE data plotted in Fig. 4, close inspection shows a persistent pattern of summer slowdown in the image pair data that becomes especially clear after the 2013 launch of Landsat 8.

4 Sensitivity analysis

To assess uncertainty of our method, we generate synthetic random continuous velocity time series, then artificially sample them using random subsets of image acquisition times from Landsat image pairs that cover Antarctica. We then apply the methods described in Section 3 to determine how accurately seasonal ice dynamics can be measured, and under what conditions.

4.1 Synthetic time series generation

Any synthetic velocity time series used for testing should resemble the true nature of ice dynamics in its variability on all timescales. Accordingly, we create realistic interannual variability by matching the distribution of the standard deviations of velocities in each grid cell in the ITS_LIVE annual velocity mosaics from 2013 to 2018. Figure 3 shows a map of interannual variability for the grid cells that contain all six years of data, and a histogram of those values for grid cells that contain at least 100 total image pairs and a minimum mean velocity of 15 m yr^{-1}. Within this subset, the median interannual variability is characterized by a standard deviation of 4.2 m yr^{-1}.

To create each synthetic time series of interannual variability, we generate uniformly distributed random values centered about zero, at daily temporal resolution. We apply a first-order low-pass Butterworth filter with a cutoff period of 548 days to each random time series to ensure that no annual cycles are present, and then we discard the first and last 548 values of each time series to eliminate edge effects of the filter. We then scale the remaining time series such that its standard deviation matches a prescribed level of interannual variability.

Our handling of interannual variability is an attempt to mimic the observable ways in which Antarctic glaciers speed up and slow down from year to year. Ideally, we would also carry forth in a similar manner for seasonal variability, imposing cyclic behavior that matches the true character and distribution of the types of seasonal variability that exist in nature. However, as the intent of this paper is to develop the methods that will be necessary to understand where and how ice velocities vary on seasonal timescales, we cannot at present create synthetic seasonal variability distributions to match what truly exists in nature. Instead, to each synthetic time series we add a sinusoid with a period of 365.25 days, a random phase, and a random amplitude between 0 and 100 $\mathrm{m\,yr^{-1}}$.

4.2 Synthetic time series sampling

Each synthetic time series is artificially sampled using characteristic acquisition times and error distributions from Antarctic Landsat-derived ITS_LIVE image pairs. In Fig. 3 we see that among grid cells containing at least 100 image pairs, and whose mean velocity is at least 15 $\mathrm{m\,yr^{-1}}$, we can expect a median of 1153 image pairs per grid cell. Accordingly, we use the acquisition times (Fig. 2) and corresponding error estimates (Fig. 3) from random subsets of 1153 image pairs (sampled from all 1.8 million Antarctic image pairs) to artificially sample each synthetic time series. Each synthetic velocity measurement is calculated from the cumulative sum of daily velocities that occurred between the first and second image in a given pair, but before dividing the total displacement by the time Δt between images, a random gaussian value of displacement is added according to the formal error estimates associated with that image pair in the ITS_LIVE dataset. The result is a time series of synthetic measurements that resemble the acquisition times and error characteristics of ITS_LIVE image pairs, but whose underlying continuous velocity signal it represents is known.

4.3 Seasonal amplitude and phase recovery

We conducted several tests to determine the accuracy with which we can recover the amplitudes and phases of seasonal cycles in synthetic velocity time series. The parameters of each test are detailed in Table 1.

In the first test, we sought to understand how many iterations are necessary to achieve a stable solution. In this test, we generated 10,000 synthetic velocity time series, each having interannual variability with a standard deviation of 4.2 $\mathrm{m\,yr^{-1}}$. Sinusoidal seasonal variability was added to each time series, characterized by a random phase and a uniform distribution of amplitudes between 0 and 100 $\mathrm{m\,yr^{-1}}$. Each time series was then synthetically sampled using the acquisition times and error characteristics of 1153 random image pairs in the ITS_LIVE dataset. Following the method described in Sect. 3, we analyzed each time series by assessing interannual variability in the synthetic measurements, removing it, removing outliers, and then obtaining the amplitude and phase of each seasonal cycle. After accounting for any aliasing that would have been caused by the

Table 1. Sensitivity test parameters. To understand how various factors influence measurement sensitivity, we isolate and vary a number of key characteristics of the synthetic time series and the data analysis method. In the first test, we vary only the number of iterations described in Section 3.3, to determine how many iterations are necessary to achieve a stable solution. We then test the effects of sampling velocity time series with 32 to 10,000 synthetic image pairs. In the remaining tests, we prescribe characteristics of the synthetic velocity time series to understand the influence of interannual variability, the amplitude of the seasonal cycle, and the timing of the seasonal cycle on measurement accuracy.

test	iterations	N image pairs	interannual rms (m yr^{-1})	seasonal amplitude A (m yr^{-1})	seasonal phase	image times & velocity errors	Figure
iterations	1–15	1153	4.2	0–100	0–2π	RFD	5a,b
N image pairs	10	32–10,000	4.2	0–100	0–2π	RFD	5c,d
interannual rms	10	1153	0–3500	0–100	0–2π	RFD	5e,f
seasonal amplitude	10	1153	4.2	0–500	0–2π	RFD	5g,h
seasonal phase	10	1153	4.2	0–100	0–2π	RFD	5i,j
overall Antarctic	10	RFD	RFD	0–100	0–2π	RFD	6

RFD indicates *randomized from distribution* of all 1.8 million Antarctic ITS_LIVE image pairs.

seasonal amplitude and phase (Sect. 3.3) that were measured in the first iteration, we conducted a second iteration of assessing interannual and seasonal variability. We then followed with a third iteration, and so on, up to 15 iterations. Results in Fig. 5a,b show that solutions tend to converge after the first few iterations, but it is possible that in some situations more iterations could be necessary, depending on sampling and the characteristics of the velocity time series. Accordingly, we use 10 iterations in all of the tests that follow.

We conducted a second test to determine how many image pairs are necessary to detect seasonal cycles in a velocity time series. We found that with just 32 image pairs, we were able to recover phase with sufficient accuracy to correctly identify the season of maximum velocity (Fig. 5c,d). We also found that performance improves dramatically with increasing image pairs, until errors in phase and amplitude approach their asymptotes between 500 and 1000 image pairs. Beyond that, the number of image pairs used in the analysis has a negligible impact on the accuracy of seasonal signal detection.

In a third test, we found that one factor more than any other threatens the accuracy of seasonal variability detection. Given a velocity time series sampled by 1153 image pairs, seasonal amplitude error increases approximately linearly with the level of background interannual variability (Fig. 5e,f). This is because despite our attempts to account for interannual variability, it is inevitable in a time series of finite length that some residual variability will influence the least-squares fit of the seasonal cycle. Nonetheless, if we consider $\pm$45 days of phase uncertainty to be a threshold that indicates accurate detection of the season of maximum velocity, our method performs adequately in the presence of interannual variability with a standard deviation exceeding 100 m yr^{-1}. Further analysis suggests that for any level of interannual variability, the amplitude of the seasonal cycle must be at least one third of the standard deviation of interannual variability to reliably be detected (see Sect. A1). We note, however, that because we have used the simple metric of the standard deviation of velocity to describe all forms of

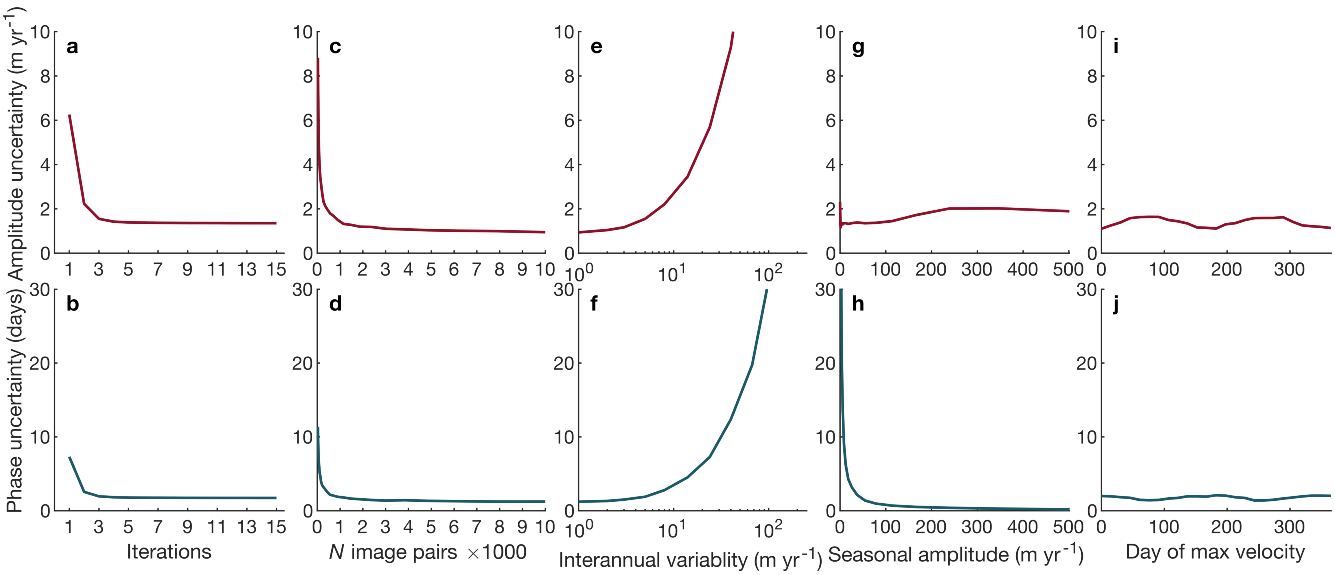

Figure 5. Sensitivity test results. The tests outlined in Table 1 indicate that **a–b:** amplitude and phase errors level off after the first few iterations of removing interannual variability and solving for seasonal variability; **c–d:** at least 500 to 1000 image pairs are necessary for the most accurate, stable solutions; **e–f:** strong interannual variability can drastically affect seasonal amplitude and phase detection; **g:** seasonal amplitude measurement uncertainty is independent of the seasonal ice velocity amplitude itself, but **h:** seasonal phase is measured most accurately when the seasonal ice velocity amplitude is strong; and **i–j:** although some faint residual effects of the seasonal bias in sampling persist after 10 iterations, amplitude and phase accuracy are effectively independent of the timing of the seasonal ice velocity variability.

interannual variability, it is likely that our method will perform better than these error estimates suggest wherever interannual variability is dominated by a long-term trend.

The final four panels of Fig. 5 provide valuable insights into the capabilities and sensitivities of our seasonal detection method. Most notably, that for a constant level of background interannual variability, seasonal amplitude errors are independent of the amplitude of the seasonal cycle. However, phase detection benefits with increasing seasonal amplitude. This should not be surprising, as a signal must not only exist, but also be sufficiently strong for its phase to be accurately measured. The last two panels of Fig. 5 show that for all practical purposes our seasonal amplitude and phase estimates can be considered insensitive to the timing of the underlying seasonal signal, despite having no images during winter.

Thus far we have determined that 10 iterations are more than sufficient for the model of seasonal variability to converge. We have also found that our measurements can be considered agnostic to the timing of ice velocity variability and to the timing of satellite image acquisitions, when applied to at least 500 to 1000 image pairs. With this understanding, we now apply our method to 100,000 synthetic time series that typify the interannual ice velocity variability and satellite image acquisitions that have been measured across the Antarctic continent as a whole. Interannual variability in the synthetic time series was randomly sampled from the distribution shown in Fig. 3d and a uniform distribution of seasonal amplitudes from 0 to 100 m yr^{-1} were added to the interannual variability. Each synthetic ice velocity time series was observed with a number of synthetic image

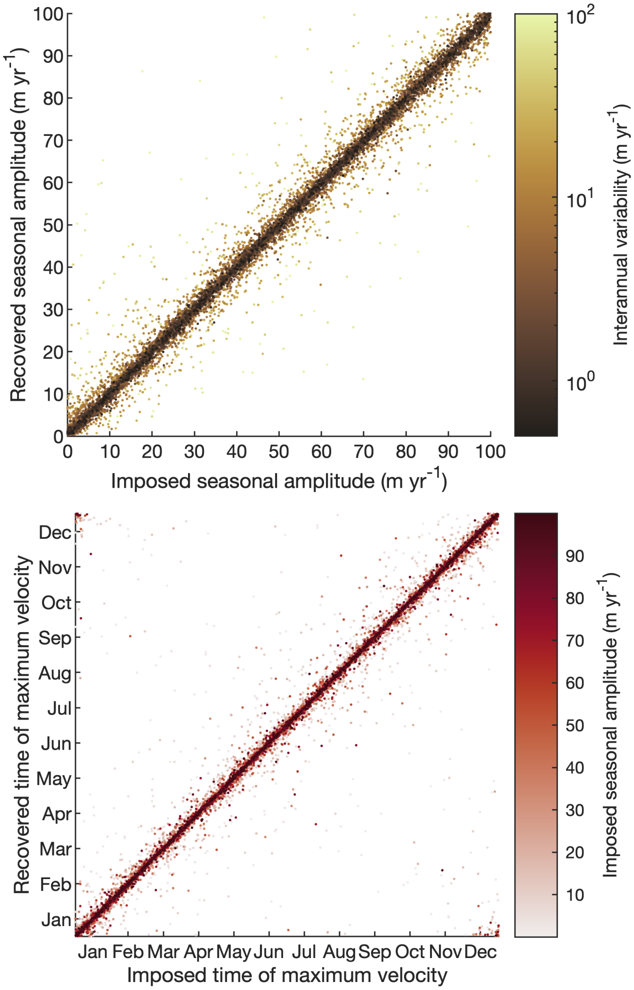

Figure 6. Pan-Antarctic seasonal signal recovery. Following the parameters of the final test in Table 1, we examine the performance of our method using the complete distributions of Antarctic Landsat image acquisition times, ITS_LIVE velocity errors, and measured Antarctic interannual ice velocity variability. Among the results where 500 or more image pairs were used to analyze the time series, amplitude uncertainty was found to be 1.4 $\mathrm{m\,yr^{-1}}$ and phase uncertainty is 2 days. For visual clarity, only 10,000 randomly selected results are shown of the 100,000 tests we performed for overall Antarctic parameter distributions.

pairs taken randomly from the distribution of values in Fig. 3c that exceed 500 image pairs, and displacement errors randomly sampled from the distribution in Fig. 3e were added to each synthetic measurement before dividing by the times Δt that separate each image pair. The results of this test, shown in Fig. 6, indicate that with the Landsat images that have been acquired to date, we should be able to measure seasonal amplitudes within $\pm 1.4\ \mathrm{m\,yr^{-1}}$ and phase within about ± 2 days.

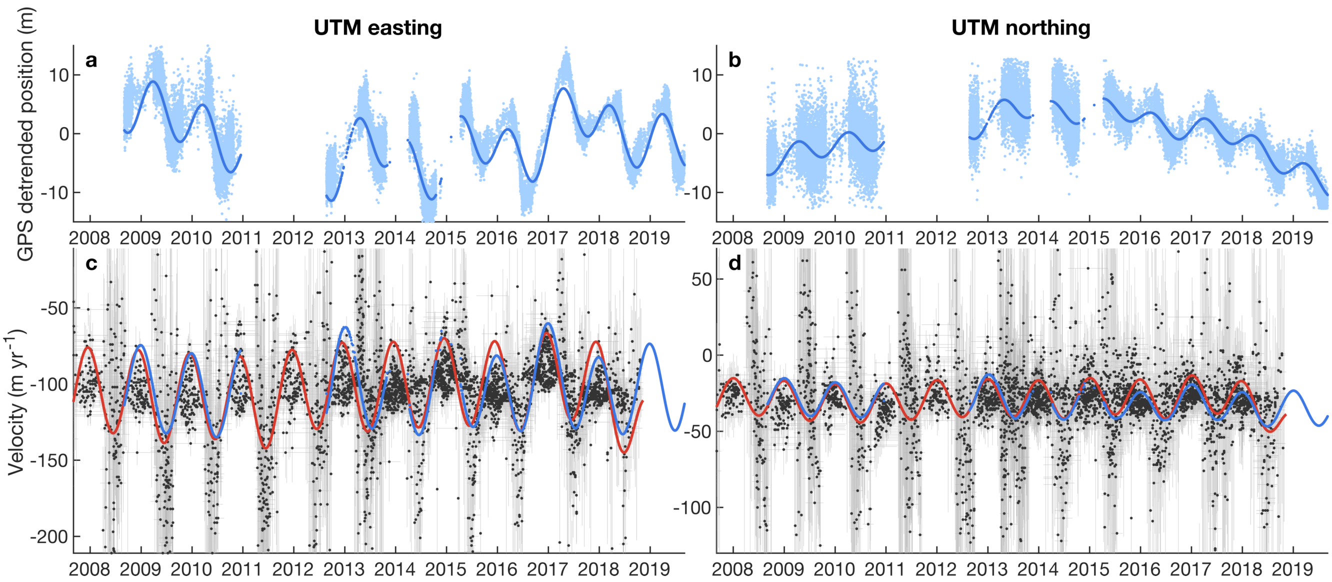

Figure 7. GPS validation at Russell Glacier, Greenland. Hourly time series of **a–b:** detrended positions of a GPS receiver are shown as raw data in light blue and the sum of interannual variability and a sinusoid fit in dark blue. The amplitudes of positional seasonal variability are only 4.5 m in the easting (x) direction and 1.8 m in the northing (y) direction, yet these minor seasonal displacements are easily detected by applying the methods described in Sect. 3 to ITS_LIVE Landsat image pair velocities. **c–d:** Velocity measurements from 5189 ITS_LIVE image pairs are plotted in gray with our fit to the measurements in red. The blue velocity curves are simply the time derivatives of the corresponding GPS position curves plotted in panels a and b. The level of agreement between the two independent measurements is detailed in Table 2.

5 Validation with in situ GPS observations

To confirm that we can extract seasonal ice velocities from feature-tracking data, we compare an *in situ* GPS position time series against results obtained by applying our method to ITS_LIVE velocities that were generated from 15 m resolution optical Landsat 7 and 8 imagery. We focus on Russell Glacier in Greenland, where GPS data provide more than a decade of observations and where seasonal velocity variability has previously been reported Van de Wal et al. (2015). We use the GPS-derived positions from the PROMICE (Van As, 2011) KAN_L station (64.4822°N, 49.5358°W, 530 masl; see Appendix B). Although we ultimately aim to characterize Antarctic seasonality, we use this record from Greenland because we are not aware of any such decade-long GPS records in Antarctica that offer winter position data and capture seasonal cycles of glacier movement that can be used for validation. The ITS_LIVE data covering Greenland are identical to the Antarctic image pair data, so we directly apply the methods discussed above without any adjustments. Likewise, our methods could just as easily be applied to ITS_LIVE data covering Alaska, the Canadian Arctic, Svalbard, the Russian Arctic, Patagonia, or High Mountain Asia.

Table 2. ITS_LIVE analysis compared to GPS. The secular mean velocities and x and y components of the seasonal characteristics of the glacier velocity time series shown in Fig. 7. ~~Despite an imperfect overlap in the~~ The acquisition ~~times~~ dates of the two independent measurements differ slightly, yet agreement is within a few percent by every measure. Uncertainty estimates are described in Appendix A2.

	GPS	ITS_LIVE	difference
secular mean $\bar{v}_x$	-102.4	-104.0	-1.6 $\mathrm{m\,yr^{-1}}$
$v_{s,x}$ amplitude	28.5	28.8±0.9	0.3 $\mathrm{m\,yr^{-1}}$
day of max $v_{s,x}$	Dec. 27	Dec. 13±5	-14 days
secular mean $\bar{v}_y$	-30.6	-29.5	1.2 $\mathrm{m\,yr^{-1}}$
$v_{s,y}$ amplitude	11.5	12.4±0.9	0.9 $\mathrm{m\,yr^{-1}}$
day of max $v_{s,y}$	Jan. 3	Jan. 1±11	-2 days

Figure 7a,b shows the linearly detrended GPS position data, along with a model fit that represents the sum of interannual variability and a seasonal sinusoid fit to the position data (see Appendix B). From the GPS data, we obtain velocity time series by taking the time derivative of the position fits, which are shown in Fig. 7c,d. As we expect when taking the derivative of a sinusoid, the peaks in velocity occur 91 days before peaks in position, and velocity amplitudes are $2\pi\,\mathrm{yr^{-1}}$ times the amplitudes of the seasonal position fits.

We compare the GPS velocity time series to velocities obtained from 5189 ITS_LIVE image pairs in the pixel closest to the fixed median location of the GPS receiver throughout the course of its data collection. Key findings are listed in Table 2. Secular mean velocities in the x and y directions measured by the two independent datasets agree on the order of $1\,\mathrm{m\,yr^{-1}}$. Seasonal velocity amplitudes also show agreement on the order of $1\,\mathrm{m\,yr^{-1}}$, as we expect on this glacier where interannual variability has a standard deviation of $3.6\,\mathrm{m\,yr^{-1}}$ in the x direction and $3.4\,\mathrm{m\,yr^{-1}}$ in the y direction (see Appendix A2).

The largest disagreement in Table 2 lies in the phase of the x direction, which at 14 days is only about 4% of a year. We note further that disagreements listed in Table 2 do not necessarily reflect errors in the ITS_LIVE data or the methods we have employed to process it. Rather, disagreements may simply reflect that the GPS receiver did not record data from late 2010 to mid 2012, and no GPS data were recorded during two of the winters that followed. Meanwhile, we use ITS_LIVE data from images that were collected more than a year before the start of the GPS record, but image pair data have not yet been processed corresponding to the final months of the GPS record. In addition to these differences in timing of data collection, we also note that the GPS record represents a Lagrangian measurement of ice velocity, whereas the ITS_LIVE data record an Eulerian measurement at a pixel that the GPS receiver passed through only once in its decade-long life. Nonetheless, agreement between the GPS solutions and our method of image pair data analysis is ~~remarkable~~notable, and lends credence to this method as a ~~robust~~ precise means of measuring seasonal variability in ice velocity.

6 Discussion

The method we present in this paper requires a multi-year record with at least several hundred image pairs to confidently identify the amplitude and phase of seasonal variability. Some key regions of interest, such as Totten Glacier in Antarctica, do not yet offer enough cloud-free images to meet this threshold, so a few more years of data collection may be required before our methods can be successfully implemented there. In addition, it may be difficult or impossible by our methods to detect seasonality in places of interest such as Pine Island and Thwaites glaciers, which are currently undergoing dramatic interannual change that could confound our measurements of seasonal variability.

Nonetheless, Fig. 7 illustrates the sensitivity of our method to ~~minuscule~~ tiny variations in glacier flow when conditions are favorable. The vertical scale of the upper panels spans just ± 15 m; yet, by our method of analyzing 15 m resolution Landsat Band 8 images, we are able to detect minor nuances in position that occur entirely within this range. We know of no other sensor or dataset that can offer such insights into these kinds of subtle variabilities of ice flow that have occurred over the past few decades.

The most significant limitation of the method we present may lie in our approximation of seasonal variability as a sinusoid. Where seasonal dynamic variability has previously been documented, it has been found that in some cases a sawtooth function or other higher-order fits might match seasonal variability more closely than a sinusoid (Moon et al., 2014; Van de Wal et al., 2015; Vijay et al., 2019). In particular, although events such as springtime calving or summer melt may occur on an annual cycle, a glacier's complete response to them may only last for only a few days (Schoof, 2010). To approximate such events as smoothly varying sinusoids will underestimate the magnitude of any brief glacier acceleration, while potentially giving a false sense that the ice responds to an impulse event continually throughout the entire year.

Despite the tendency of a sinusoid to oversimplify complex time series, we contend that no other description of seasonal variability is as elegant or robust over decadal timescales, and that understanding seasonal ice dynamics must begin with a zeroth order description of the amplitude and phase of ice velocity variability. While it is true that a glacier can accelerate in response to a transient event and return to an equilibrium velocity within just a few days (Stevens et al., 2015; Andrews et al., 2014), in the climatological sense, nature does not consistently time such events as calving or increases in basal water pressure with any greater precision than the method we have presented to detect them.

~~In most cases, a sinusoid will likely capture the majority of velocity variance throughout the year, and represent the~~ The approach we present captures the fundamental mode of subannual variability in ice velocity ~~. The approach we present~~ and conserves all ice displacement that occurs throughout the year~~, and the~~ . The simple two-term explanation of amplitude and phase provides a robust description of seasonality that is less prone to error than higher-order fits such as three-term sawtooth functions. By providing a simple measure of amplitude and phase, we offer a straightforward method to compare how neighboring glaciers respond to a common seasonal forcing or investigate how dynamic signals propagate upstream or downstream in a given glacier.

In Sect. 5 we applied our method to ITS_LIVE velocity measurements and compared against GPS position data, with the intention that the *in situ* GPS station would provide a reliable ground-truth reference. However, although the two independent measurements show close agreement whenever GPS data were logged, the receiver's harsh polar environment has led to several long gaps during which no GPS position data were acquired. This suggests that as a means of measuring ice dynamic climatology, our method might not only meet, but exceed the performance of the *in situ* GPS receiver while providing insights into dynamic behavior as far back as the mid 1980s. The particular ITS_LIVE grid cell ~~we have~~ in Greenland that we analyzed in this paper is hardly unique in its potential to provide historical context, as decades-long velocity records exist in most of the 240×240 m pixels which cover nearly all of the world's ice.

The methods presented in this paper have focused primarily on optical satellite data because no other type of sensor provides such a long record of ice velocity. As more radar data become available, particularly since the launch of Sentinel 1a/b, the problem of missing winter data will be eliminated, but the methods presented in this paper will still hold. When an abundance of feature-tracked velocities from radar become available, Eq. 3 may then be easily modified to include additional terms to simultaneously solve for cyclic velocity variability on tidal timescales as well as seasonal variability.

7 Conclusions

Given a relatively continuous time series of at least 500 to 1000 image pairs, our method can extract the amplitude of seasonal variability with a precision on the order of about 1 m yr^{-1} by describing ice flow as an oscillation, provided the level of background interannual variability does not overwhelm the overall signal. We find that if the amplitude of seasonal variability is at least a third of the standard deviation of interannual variability, the method we describe can reliably detect the season of maximum ice velocity. Ability to detect seasonal amplitudes is independent of the amplitude and phase of the seasonal velocity variability, but phase accuracy benefits with increasing amplitude of seasonal variability.

With the method we describe, we may begin to map seasonal ice dynamic variability on a global scale, in a consistent and meaningful manner; and by providing a method that can be employed independently in the two dimensions of Cartesian coordinates, we hope to gain a ~~fully three dimensional~~ more complete understanding of how dynamic signals propagate through the world's ice.

Code and data availability. Data analysis in this paper relied upon Antarctic Mapping Tools for MATLAB (Greene et al., 2017b) and the Climate Data Toolbox for MATLAB (Greene et al., 2019). All data analyzed in this paper and code necessary to generate the figures are included as a supplement to the paper, or are available online at http://www.github.com/chadagreene. PROMICE data is freely available at http://www.promice.dk. ITS_LIVE global image-pair velocity data is freely available at its-live.jpl.nasa.gov.

Appendix A: Additional uncertainty analysis

Phase and amplitude uncertainty in Sect. 4.3 were calculated from the differences between imposed and recovered values. Differences in phase were wrapped such that their absolute values did not exceed 182.62 days. During this work, we found that in some cases, simple standard deviations of differences could be strongly influenced by a few ~~egregious~~ extreme outliers, and as a result, standard deviations of differences (and similarly, root-mean-square errors) did not reflect the true gaussian distributions of errors. So for a more robust and meaningful measure of error distributions, we define σ as 1.4826 times the median of the absolute value of differences.

A1 Seasonal uncertainty from interannual variability

In addition to the tests described in Sect. 4.3, we analyzed 120,000 synthetic time series to better understand the scenarios in which interannual variability may confound our ability to detect seasonal cycles in an ice velocity time series. For every unique combination of 31 values of seasonal amplitudes from 0 to 1000 $\mathrm{m\,yr^{-1}}$ and 26 values of interannual variability from 0 to 3500 $\mathrm{m\,yr^{-1}}$, we generated 150 synthetic time series, sampled them with 1153 synthetic image pairs, and attempted to recover the seasonal cycles we imposed. A striking relationship emerged, shown in Fig. A1, in which we see a strong demarcation between a zone of accurate phase detection and a zone where phase cannot be determined with any confidence.

Letting phase uncertainty of 45 days be the threshold indicating whether the season of maximum ice velocity can be accurately determined, in Fig. A1 we see that above the noise floor of about 1 $\mathrm{m\,yr^{-1}}$ seasonal amplitude, the 45 day phase uncertainty contour marks an approximately linear relationship between interannual variability and seasonal amplitude. Indeed, the dashed blue line that nearly coincides with the 45 day phase uncertainty contour shows the simple slope whereby seasonal amplitude is one third of the standard deviation of interannual variability, with zero offset from the origin. This tells us that there is a quite simple relationship between the amplitude of a seasonal cycle, the level of background interannual variability, and our ability to detect phase—As long as the seasonal amplitude is above the noise floor of about 1 $\mathrm{m\,yr^{-1}}$ and interannual variability does not exceed three times the amplitude of the seasonal cycle, we can accurately detect the seasonal cycle.

A2 Uncertainty estimates for ITS_LIVE/GPS comparison

The values of seasonal amplitude and phase uncertainty listed in Table 2 were each calculated as the robust standard deviations of amplitude and phase errors from 5000 synthetic time series sampled by 5189 image pairs. In the x direction, ITS_LIVE image pairs indicate an interannual variability of 3.85 $\mathrm{m\,yr^{-1}}$ in our pixel of interest, with a seasonal amplitude of 28.8 $\mathrm{m\,yr^{-1}}$ and a maximum velocity occurring around December 13th of each year. We generated 5000 synthetic time series matching these criteria, and were able to recover the 28.8 $\mathrm{m\,yr^{-1}}$ seasonal amplitude within σ=0.87 $\mathrm{m\,yr^{-1}}$ and phase within σ=4.7 days. Similarly for the y direction, from 5000 synthetic time series with an interannual variability of 3.38 $\mathrm{m\,yr^{-1}}$, seasonal amplitude of 14.4 $\mathrm{m\,yr^{-1}}$, and maximum velocity occurring on January 1 of each year, we recover seasonal amplitudes within σ=0.91 $\mathrm{m\,yr^{-1}}$ and phase within σ=10.7 days.

Appendix B: GPS processing

We use hourly position data from the PROMICE KAN_L GPS station on Russell Glacier (Van As, 2011). The KAN stations utilize the Trimble SAF270-G antenna with a single L1 frequency to minimize power usage. L1 signals have previously been

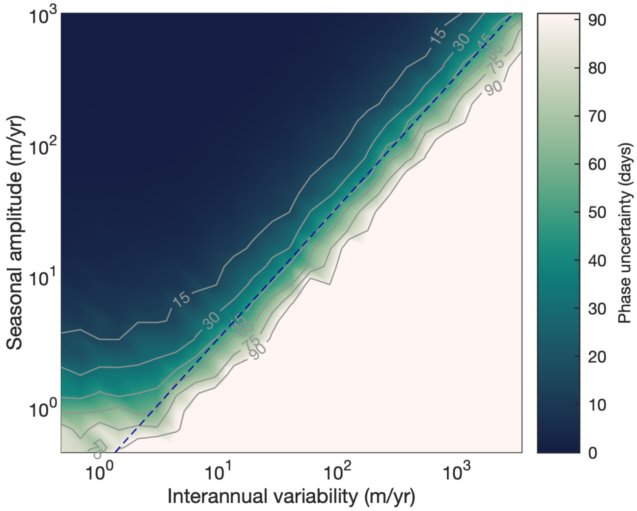

Figure A1. Interannual variability and seasonal signal detection. The dark green region of this plot indicates scenarios in which phase can be accurately detected with 1153 image pairs. When seasonal amplitudes are above the noise floor of about $1\,\mathrm{m\,yr^{-1}}$, the amplitude of seasonal variability must be at least a third of the standard deviation of interannual variability to be detected, and this relationship is marked by a dashed blue line. The linear relationship coincides with the 45 day phase uncertainty contour that determines whether the season of maximum ice velocity can be accurately measured.

used in studies of short-term ice motion with success in the Russell Glacier region (e.g., Van de Wal et al., 2015). Positions are recorded hourly during the spring/summer observation period and daily over winter. In addition to position data, PROMICE reports information about horizontal dilution of precision (HDOP) and station tilt data; both of which are used in addition to position data to perform post-processing quality control. HDOP provides information on uncertainties related to satellite geometry, and we discard positions for which the HDOP exceeds a value of 5. We manually remove offsets of a few meters that occurred during four site visits on 28 April 2015, 16 July 2016, 1 Sept 2017, and 28 Aug 2018. We then remove any outliers, which we define as all points whose detrended x or y positions lie more than 2.5 robust standard deviations (see Appendix A1) away from zero. After detrending the position data we remove interannual variability following the spline-fitting method described in Sect. 3.1, then fit sinusoids to the residual x and y position time series using the `sinefit` function in MATLAB (Greene et al., 2019). The detrended raw position data are shown in Fig. 7 along with a model fit, which is taken as the sum of interannual and seasonal variability.

Author contributions. CAG conceived of and carried out the study with guidance and technical assistance from ASG. LCA post-processed and interpreted the GPS data. CAG wrote the manuscript with input from ASG and LCA.

Competing interests. The authors declare that they have no competing interests.

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