Reviewer 2

General comments

The authors present a workflow to fit a sinusoidal function to a data set of clustered velocity estimates on ice sheets and outlet glaciers. The work is well written, and the authors clearly identify the need to extract more concise information of this vast collection of Eath observation data. The steps taken by the authors are explained, but in general there is a tendency to highlight the strong points of the methodology in their argumentation. Being a methodology paper, there might be a reason to keep this presentation brief, but it might be more than worthwhile to emphasis points of improvement and why certain decisions are taken.

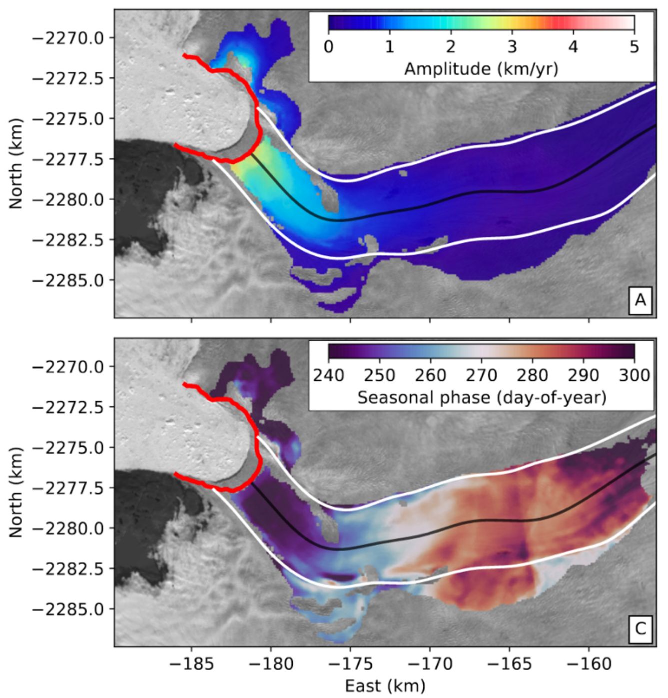
My main concern with this work stems from the property that the authors define seasonal variation as a cycle. In this way the reader is pushed into a certain narrative, which limits how to approach this issue. The authors are correct about the sinusoidal variation of the forcing (the sun and the seasons), but this does not mean the ice velocity has the same reaction. Personally, I see the seasonal variation more as a perturbation, to which there is a reaction time/response, a peak and fade out/reorganization. Thus a perturbation (including a sinus function, but also a lot of other responses) occurs every year, due to surface melt run-off, but the time span does not need to extend towards a whole year, as is assumed here. If we look at other studies short spikes are clearly visible (Kjeldsen et al. 2017, 10.1002/2017GL074081; Derkacheva et al.2020; 10.3390/rs12121935), or in the dynamics of a surging icecap (Dunse et al. 2015,10.5194/tc-9-197-2015) where a step function is seen, that is initiated by meltwater perturbation. So I miss a discussion on how good a sinus-function is as a model. There is only testing of how good the observations meets the model description, and not how good the model fits the observation. Putting everything on "background interannual variability" is a bit easy

The primary concern here is that without any regard for the shape, timing, or source of annual forcing mechanisms, we have gone ahead with an assumption that a sinusoid is a reasonable approximation of any seasonal behavior. To be clear, we have not assumed anything about the shape of forcing functions, nor do we directly discuss any potential driving mechanisms in this manuscript. Though we do have understanding of how some types of seasonal velocity signals evolve, currently, our state of knowledge is such that we do not fully understand where seasonal forcing mechanisms exist, what they are, or what their shapes may be.

We do, however, provide a method for gaining insights into a glacier's response to seasonal variability in forcing, and for this we use a sinusoid. The sinusoid does not assume anything about the shape of the forcing mechanism or even the shape of glacier's response. Rather, the sinusoid *describes* the cyclic behavior in the simplest way possible.

We contend that we must understand the most basic level of behavior before we can begin to discuss aberrations from it. This means that before we can begin fitting higher order functions or investigating how seasonal cycles change from year to year, we must first identify *where* seasonal variability exists, *how significant* it is in terms of the overall displacement cycle of the glacier, and in *what season* of the year ice velocity tends to reach its maximum.

The value of the sinusoidal approach can be seen in a new preprint by Riel et al. (https://doi.org/10.5194/tc-2020-193), which was submitted to *The Cryosphere* after the reviews for our manuscript were posted. Using the exact approach we describe, they map the seasonal amplitude and phase of variability to observe traveling waves in Sermeq Kujalleq. The map here provides evidence for kinematic behavior that begins at the glacier terminus and travels upstream each year.



Sermeq Kujalleq does not exhibit perfectly sinusoidal behavior, but the simple two-term description of seasonal variability reduces complexity and makes interpretation straightforward. If more complex terms are desired, we point out in the paper that additional sinusoids can be added, and it eventually it will be possible to build a Fourier series with this approach.

Another question arising is the wording of climatological velocity, I am not able to figure out what the authors mean with this term. This directly also brings me to a second point on the sinus fitting, as it is treated as a cyclic function similar to (Menchew etal., 2017, 10.1002/2016JF003971). They look at a tidal time span, where the forces are highly repetitive in magnitude and phase. However, if this is the case for seasonal glacier velocities is not so clear, as the amplitude of glacier velocity seem to correlate with surface mass balance. This has been observed with GPS in Greenland (van deWal et al. 2015, 10.5194/tc-9-603-2015) or on Nordenskiöldbreen, Svalbard (van Peltet al. 2018 10.1029/2018GL077252). But the sinus function of the authors does not take the change in amplitude, from year to year into account. However, this (to me) would be a climatological velocity (if I had to guess what the authors mean).

We have clarified the definition of climatology with the addition of this sentence:

In this paper, we describe a robust method of measuring the climatology*—or average seasonal cycle—of ice flow dynamics, with the ultimate goal that our method may be used to map the typical magnitude and timing of the seasonal glacier dynamics worldwide.*

Other influencing phenomena, like the ocean/front position have similar seasonal amplitude change (Kehrl et al. 2016, 10.1002/2016JF004133). By putting all these into a cyclic function, the signals of phase and amplitude might smooth out. In connection to this, at high latitudes, the coverage is concentrated towards the summer season. Hence, how do short term perturbation propagate into the velocity estimation? From the synthetic test the methodology can be "considered agnostic", but this is true for reconstruction purposes of a sinusoidal function. It is also not clear where the authors are after, the onset of speed-up, or "identify the seasonal maximum velocity"? Other studies/data/methods are able to find the timing of such speed-up events (Altena et al.2017 10.3389/feart.2017.00053, Vijay et al. 2019, 10.1029/2018GL081503), though not as precise or automatic as presented here, but are less constrained. So, there are some issues on the amplitude, but also on the phase. The argument of the authors for using a sinus, as it is "elegant" is a bit weak in my opinion. It would very much strengthen the manuscript, if these influencing effects/considerations are highlighted, as it gives handholds on the way forward.

We clearly hear the reviewers concern that a sinusoid is not a perfect representation of glacier variability and that in some cases it may even be a poor representation. We completely agree with this assessment but disagree that a sinusoidal approximation is a bad first guess in the absence of a universal model of variability. We could assume a sawtooth or step function, or even a pricewise model but we're unconvinced that any of these models would not suffer from the same shortcomings. The problem is how can we use heterogeneous data to compare across vastly different climate conditions, glacier characteristics, ocean forcing, etc., to identify where glaciers are fluctuating seasonally. A sinusoid is the most basic assumption we can make and provides a starting point for the contextualization of global glacier variability that will increase in nuance and complexity with time. We justify the simplicity of our assumption as it is highly generic and provides a logical first step toward global characterization.

The authors have formulated their estimation procedure by decoupling the x- and y-velocities. Is there a certain reason for this? I can imagine it can be beneficial, to include co-variance functions, so outliers in one dimension are also excluded in the other. In addition, given these phase angles are estimated independently, do the authors see a difference between both axis. If so, this would imply a change inflow direction, if not what would that mean? Also, why did the authors do filtering (using the MAD), and not do robust least squares, or at least use such procedure in the estimation? Neither is it clear to me why several iterations are applied, see (https://ccrma.stanford.edu/~jos/filters/Sum_Sinusoids_Same_Frequency.html), or is the estimation not restricted to a yearly cycle? Or is the iteration not done on the residuals?

Some of the most compelling insights that we expect to gain from this method will involve transverse motion that could not be detected if we were to assume that flow variations only occur in the direction of mean flow. For example, we have applied our method to Drygalski Ice Tongue and found the same side-to-side motion that has previously been found using in-situ measurements (https://ui.adsabs.harvard.edu/abs/2013AGUFM.C21A0624L/abstract).

In addition to floating ice, we may find transverse flow anomalies in grounded areas with strong seasonality of basal water pressure. For example, if the basal pressure on one side of a glacier rises while the other side remains unchanged, the lopsided acceleration in flow would cause a divergence of and change in flow direction, even if only small. The method we describe is able to resolve displacements of just a few meters left or right of a mean flowline, meaning the maps can be created by separating the *x* and *y* components of variability could hint at underlying drivers of change.

The authors run tests on synthetic data, by imposing corruption to individual velocity estimates. This noise is done on an individual basis, which is partly due to measurement noise. But there is also dependent noise, as displacement estimates are derived for pairs of images. Hence, when one image is corrupted for some reason, there is a high probability it propagates to all displacements it is part of. However, this issue is not included into the analysis, though of importance (and due to the synthetic nature, is possible to generate). This would give more insights then the 32 velocity estimates, stated now.

The reviewer makes a very good point but there is a subtlety to the data that makes this not the case. The largest source of error when tracking features between two images acquired with the same viewing geometry (repeat image) is the geolocation error in each image that typically manifests itself as a scalar displacement in *x* and *y*. These correlated errors have been corrected by the autoRIFT algorithm that produces the ITS_LIVE velocity data. The correction process works by examining the initial measured velocities over all stable surfaces in an image such as rock. The average measured velocity over rock is equates to an offset error across the entire scene. After the offset errors are removed in *x* and *y*, the remaining errors in each image pair can be considered to be uncorrelated.

Minor comments:

In general the manuscript is well written, the authors write in their mother tongue, so concerning this issue I am not able to do any better. But for a global audience the wording is sometimes a bit hard; I have learned quite a lot of new words. For sake of easy reading, and not having to go back and forth to a dictionary, please consider changing words a bit. Think of, "unwieldy" or "egregious”.

We appreciate this feedback, as we wish to make our work accessible to all interested readers, regardless of personal background. We have looked through the manuscript to ensure that (aside from a few necessary technical words) there is no language in this revision that wouldn’t be found in in standard English-language news outlets.

I have tried to understand from the text what is done, and also looked in the code to be able to zoom into the plots/data. But the provided code and plotting does not work, as some functions ("itslive_tsplot" or "itslive_seasonal_deets") are absent.

We appreciate the reviewer's effort in digging into the code that we included as a supplement to the manuscript. It appears this comment regards the make_fig04.m script, which contains the code that can be used to recreate Figure 4 of the manuscript. We've double-checked and cannot identify any missing functions, but it seems likely that the confusion stems from our inclusion of the itslive_seasonal_deets function at the end of the make_fig04.m script. It's a relatively new feature that Matlab can call functions that are included at the end of a standard non-function .m script, and we've taken advantage of the feature to keep our bundle of supplemental code as tidy as possible.

Regarding the missing itslive_tsplot function, we note that it is included among the ITS_LIVE functions we've posted to GitHub (www.github.com/chadagreene/ITS_LIVE). The README.txt file that describes what's included in the supplemental material states at the top that some functions necessary to run the scripts are part of the toolboxes on my GitHub page, including ITS_LIVE tools, Antarctic Mapping Tools for Matlab (Greene et al., 2017), and the Climate Data Toolbox for Matlab (Greene et al., 2019).

title: be a bit more specific, maybe change to "Reconstructing seasonal oscillations" also include "glacier ice".

We note that the application of this method is not limited to glaciers. For example, we have applied our method to the Ross Ice Shelf and found the same patterns of seasonal variability reported this week by Klein et al. (https://doi.org/10.1017/jog.2020.6).

6: "dark polar winters" > "at high latitudes"

The sentence in question describes the problem of observing seasonal variability using optical data, when no optical data are available for several months each year due to lack of sunlight. Thus, we describe the situation that there are "...no optical observations throughout *dark, polar winters*." In our view, the suggested wording, "*at high latitudes*" does not directly address seasonality nor make mention of the solar illumination that's necessary for optical data acquisition. We prefer to keep the wording as it is.

8: "climatological average winter velocities" what is meant here?

We have clarified the definition of climatology with the addition of this sentence:

In this paper, we describe a robust method of measuring the climatology*—or average seasonal cycle—of ice flow dynamics, with the ultimate goal that our method may be used to map the typical magnitude and timing of the seasonal glacier dynamics worldwide.*

15: "sufficient quantity of data" is this due to quantity of data, the consistency of campaigns/ monitoring programs or simple availability of large computing power. Or is it opening up of the archives, making historical flow estimation possible (Cheng et al.2019 10.5194/isprs-archives-XLII-2-W13-1735-2019).

The conference paper by Cheng et al is intriguing, and if they are able to successfully employ the method of processing outlined in that paper, it will be interesting to see what the velocity fields from the ARGON era look like. But as far as we can tell, that dataset has not been produced or has not been made widely available.

18: "all of the world's ice" large bodies of glacial ice

We have clarified that annual velocity mosaics are now available for "*nearly* all of the world's *land* ice".

20: "upended glaciology" Remote sensing is able to get geometric information about the (sub)surface, and is of great aid. To some extent this is a game changer, but it might be fair to also give credit to automatic weather stations, or put it into persective. This have been other great advancement in glaciology, think for example of (Ohmuraet al., 1992; Oerlemans et al., 1998).

The introductory paragraph briefly mentions some of the recent advancements in remote sensing that have gotten us where we are today, and then the paragraph identifies the types of insights we want to gain from this new abundance of satellite data. We feel that a discussion of automatic weather stations would be a distraction from the main points we wish to communicate in this manuscript.

31: There is also an large collection of studies at the intermediate timescale, which isleft out here, dealing with surges. For sake of completeness, this might be included

We do reference a paper on surge dynamics by Yasuda and Furuya, but we have tried to keep the focus of this manuscript primarily on cyclic behavior.

37: "the logistical challenge of" what is meant here?

We have modified the sentence to now read,

...due in part to the ***technical*** *challenge of working with optical data in polar regions, where the surface is not touched by sunlight for months-long periods each winter.*

The technical challenge of working with optical data in polar regions is that several months go by each winter when optical data are not collected, because the sun does not illuminate the surface during those months. The full text of this manuscript provides a detailed description of the technical challenges of working with this data.

39: "robust extraction" using a robust pre-processing technique, is different from a robust methodology. Given its stiffness (non adaptive) towards one model (a sinusoidal) this might not be a correct formulation. It might be "precise"...?

Following this suggestion, we have changed the word *robust* to *precise*.

40: "primarily focused on Antarctica" maybe better: on the ice sheets and their outlets....?

We designed the study with the primary goal of understanding Antarctic seasonal ice dynamics, because that's where data is most limited, and it's where the fewest studies have been published about the subject. The examples we provide and the statistics in all of the synthetic tests are based on Antarctic data. Accordingly, we stand by the statement that

Our study is primarily focused on Antarctica, where seasonal variability is poorly understood, and where data limitations currently present the greatest challenges to making such measurements.

53: "by feature tracking" add: over longer time spans

We have made the suggested change.

55: "the true magnitude" change to "a" instead of "true" or "a well fit"

We have removed the phrase true magnitude, as suggested. The sentence now reads,

...by fitting a cyclic function to the time series of displacements rather than average velocities, we show that it is possible to accurately recover the magnitude and phase of seasonal velocity variability.

56: I miss another possibility here, which is common practice in inSAR displacement estimation, being inversion (e.g.: Bontemps et al. 2018, 10.1016/j.rse.2018.02.023, Li et al. 2020, 10.1016/j.rse.2020.111695). This does not make it necessary to work with average time stamps or a model, and can resolves to very small time steps.

It is unclear how the inversion techniques described by Bontemps et al. or Li et al. should be included in this study.

64: " first or second image" first and second? maybe be more clear or use "master-slave" "chip-search space" etc.

We note that the community is moving away from the terms *master* and *slave* (https://comet.nerc.ac.uk/about-comet/insar-terminology/).

The sentence in question states,

Each satellite image may serve as the first or second image in multiple image pairs...

The words "first" and "second" refer to the sequence in time. The suggested wording does not adequately convey temporal sequence, and the meaning of "chip-search space" may not be widely understood.

100: "most robust means" why is this robust, where do you get the reliability from?

We have removed the word “robust”.

fig4: maybe it is good to note, why there are two groups of points, as one is +- a year and the other at short time intervals in summer. btw: the purple line nicely follows the annual velocity clusters!

Following this suggestion, we have edited the caption of Figure 4 to describe the two groups of points as follows:

The clustering of these 14,208 measurements taken near the grounding line of Byrd Glacier typifies ITS_LIVE image pair data, with short Δt measurements providing direct, but noisy observations of velocity variability throughout each summer, while much lower-noise winter estimates can only give insight into the total displacement that occurs during the dark, winter months.

151: I don’t think this is sensitivity, but more an analysis to get an idea how good the recovery is. As the model is corrupted with noise and then an attempt is made to reconstruct the model. If I understand correctly.

The Sensitivity Analysis section is where we test the sensitivity of the method to several different parameters. We determine the sensitivity of the technique to the number of image pairs used, the level of background interannual variability, the amplitude of the underlying signal, and the phase of the underlying signal. The parameters of these tests are tabulated in Table 1: Sensitivity test parameters.

181: "recover", change to we "are able to describe the variation by seasonal cycles" or alike.

The passage in question reads,

We conducted several tests to determine the accuracy with which we can recover the amplitudes and phases of seasonal cycles in synthetic velocity time series.

We feel that the present wording will be more easily understood than the suggested wording.

258: " is remarkable" subjective wording, please change

Following this suggestion, we have changed the word *remarkable* to *notable*.

258: "robust", precise/accurate might fit better

Following this suggestion, we have changed the word *robust* to *precise*.

267: "minuscule variations" subjective wording

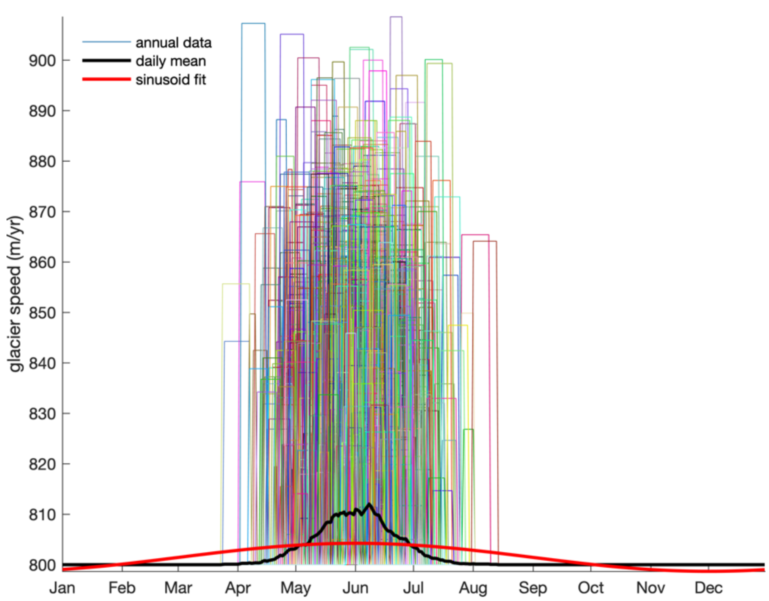
We have replaced the word *minuscule* with *tiny*, to describe the displacements on the order of a meter or so that can be detected with this method.

283: " in the climatological sense, nature does not consistently time such events as calving or increases in basal water pressure with any greater precision than the method we have presented to detect them". What is meant here?

We have added a definition of climatology to clarify that the climatology refers to the average seasonal cycle taken over many years.

Transient events such as calving or impulses of water into a subglacial hydrological system often occur on a yearly cycle, but the corresponding glacier speedup may only last for a few days. It may seem that a spike in velocity only lasting a few days of the year would be poorly represented by a sinusoid that continuously varies throughout the year, but this sentence points out that mother nature does not time glacier calving to occur on the very same day each year. As a result, when we take the average annual cycle from many years of data, we find there is generally a *season* of glacier acceleration rather than just a few days of acceleration.

To illustrate this point, here we consider a glacier whose velocity is exactly 800 m/yr most days of the year, but each summer it accelerates by 50±10 m/yr for a duration of 10±3 days, centered on June 1±20 days. We generate 1000 years of this pattern, and then consider the mean cycle of daily speeds and compare it to the sinsuoid fit. Using the Matlab code provided at the bottom of this document, we get the following plot:



What we see in the daily mean velocity curve is that in the climatological sense, the period of high summer velocity lasts for months, even though the average high-velocity event only lasts for 10 days. Fitting a sinusoid to the daily means finds a peak velocity on June 1, which is exactly the prescribed center date of the high-velocity period. Of course, the amplitude is severely underestimated by the sinusoid in this scenario, but the passage in question regards timing, not amplitude. It states, *While it is true that a glacier can accelerate in response to a transient event and return to an equilibrium velocity within just a few days (Stevens et al., 2015; Andrews et al., 2014), in the climatological sense, nature does not consistently time such events as calving or increases in basal water pressure with any greater precision than the method we have presented to detect them.*

285: "In most cases, a sinusoid will likely capture the majority of velocity variance throughout the year, and represent the fundamental mode of subannual variability in ice velocity." Please justify this claim, as this is the corner stone which the whole study is build upon.

We agree that this claim was not well supported, and on reflection we see that it may have been untrue. We have removed the claim from the text.

291+: It seems the authors put all the misfits of the sinus model on the inaccuracies of the GPS measurements, while this sensor measures all kinds of physics decadal, annual, daily,...

We do not claim that the small misfit between GPS and our method is solely the fault of the GPS data. Rather, we simply point out that the GPS record contained several long gaps, and it is possible that any disagreement between the two methods could reflect the different times during which data were collected. Here in the Discussion section it is appropriate to acknowledge the possible causes of any mismatch between the GPS and ITS_LIVE data, and consider what that might mean for applying this method elsewhere. The passage states,

...the [GPS] receiver's harsh polar environment has led to several long gaps during which no GPS position data were acquired. This suggests that as a means of measuring ice dynamic climatology, our method might not only meet, but exceed the performance of the in situ GPS receiver while providing insights into dynamic behavior as far back as the mid 1980s.

We feel it is important to point out that GPS data—although absolutely vital to this type of work—is not perfect. Most GPS receivers do not collect data over polar winter, yet the method of image analysis we describe is able to approximate winter ice velocities. Similarly, very few locations on Earth offer GPS measurements even today, yet the ITS_LIVE dataset offers global coverage, and in some locations that coverage extends back to the 1980s.

305: "our method can extract" please add "..by describing ice flow as an oscillation ..."in some way

Following the suggestion, the passage now reads,

...our method can extract the amplitude of seasonal variability with a precision on the order of about 1 m/yr ***by describing ice flow as an oscillation****, provided the level of background interannual variability does not overwhelm the overall signal.*

309: "independent of the amplitude and phase of the seasonal velocity variability", this is not convincingly given.

The sentence in question reads,

Ability to detect seasonal amplitudes is independent of the amplitude and phase of the seasonal velocity variability, but phase accuracy benefits with increasing amplitude of seasonal variability.

Please see Figures 5g, 5i, and 6a.

313: "fully three dimensional understanding" what is meant here?

We have fixed this sentence. It now reads,

*...by providing a method that can be employed independently in the two dimensions of Cartesian coordinates, we hope to gain a **more complete** understanding of how dynamic signals propagate through the world's ice.*

322: "egregious outliers" egregious=outliers

We have replaced *egregious* with *extreme* to make the text more accessible to non-native English speakers. To be clear, we are not discussing outliers that are just three or four standard deviations away from the mean. Rather, we are discussing the extreme outliers that can land hundreds of standard deviations away from the rest of the bunch. We now describe them as *extreme* outliers.

Matlab code

```
% Create synthetic time series:
N = 1000; % number of years of the time series
dur = 10 + round(3*randn(N,1)); % (days) duration of fast flow each summer
dur(dur<1) = 1; % just in case randn resulted in any negative durations.
st = 153 + round(20*randn(N,1)-dur/2); % DOY of fast flow period.
vs = 50+20*randn(N,1); % Magnitude of summer speedup
vs(vs<5) = 5;  % just in case randn made vs negative
va = 800; % background/winter ice velocity

% Define the rectangular function for each year:
v = va*ones(N,365);
for k = 1:N
   ind = st(k):(st(k)+dur(k)); % indices corresponding to fast-flow event
   v(k,ind) = va+vs(k);
end

%% Plot the time series

cm = rand(N,3); % colormap
figure
hold on
for k = 1:N
   hi(k) = plot(1:365,v(k,:),'color',cm(k,:));
end
axis tight
box off
hm = plot(1:365,mean(v),'k','linewidth',2);

% Fit a sinusoid:
ft = sinefit(1:365,mean(v),'terms',3) % Climate Data Tools (Greene et al.,
2019)
hf = plot(1:365,sineval(ft,1:365),'r','linewidth',2);

datetick('x','mmm','keeplimits')
ylabel 'glacier speed (m/yr)'

legend([hi(1),hm,hf],'annual data','daily mean','sinusoid
fit','location','northwest')
legend boxoff
```